AdS radius corrected soft photon factor from a CFT$_3$ Ward identity

Karan Fernandes

National Taiwan Normal University

6th International Conference on Holography, String Theory and Spacetime

Duy Tan University, Danang

collab w/ Nabamita Banerjee (IISER Bhopal, India) and Arpita Mitra (Pohang University of Science and Technology, South Korea)

February 22, 2023

Outline

Background and Overview : Soft photon theorem

- Soft theorems : Factorization of scattering amplitudes when an external massless particle goes soft $\omega \to 0$

$$\mathcal{M}_{n+1}(\varepsilon, k; \{p_a\}) = \text{Soft factor} \times \mathcal{M}_n(\{p_a\})$$

ε: polarization; k: momentum of soft particle;
$\{p_a\}$: hard momenta
$\mathcal{M}_i$: Amplitude with 'i' external particles

- The soft factor admits a frequency expansion

$$S_{\text{em}} = \sum_i S_{\text{em}}^{(i)}$$

$$S_{\mathrm{em}}^{(0)} = \sum_{a=1}^{n} Q_{(a)} \eta_{(a)} \frac{\epsilon(\lambda) \cdot p_{(a)}}{p_{(a)} . k} \qquad \text{leading}$$

$\eta_{(a)} = +1(-1)$ for outgoing (incoming) and $Q_{(a)}$ charges

Gauge invariance of leading soft factor

$$S_{\text{em}}^{(0)} = \sum_{a=1}^{n} Q_{(a)}\eta_{(a)} \frac{\varepsilon_{(\lambda)}.p_{(a)}}{p_{(a)}.k}$$

- Gauge transformations under $\varepsilon^{\mu}_{(\lambda)} \to \varepsilon^{\mu}_{(\lambda)} + k^{\mu}$
- $S_{\text{em}}^{(0)} \to \sum_{a=1}^{n} Q_{(a)}\eta_{(a)} = 0$ (charge conservation)
- Leading soft factor is gauge invariant and universal for all $U(1)$ invariant theories
- In general

$$\text{Soft factor} = \frac{1}{\omega}A_0 + \ln\omega^{-1}A_1 + \omega^0 A_2 + \cdots$$

- Subleading terms are theory dependent; receive loop corrections

A parametrized form of soft factor

- Consider massless hard and soft particles parametrized as

$$\varepsilon_+^\mu = \frac{1}{\sqrt{2}}(\bar{z}\,, 1\,, -i\,, -\bar{z})\,, \quad \varepsilon_-^\mu = \frac{1}{\sqrt{2}}(z\,, 1\,, i\,, -z)$$

$$k^\mu = \frac{\omega}{1+z\bar{z}}(1+z\bar{z}\,, z+\bar{z}\,, -i(z-\bar{z})\,, 1-z\bar{z})\,,$$

$$p_k^\mu = \frac{E_{\vec{p}}}{1+z_k\bar{z}_k}(1+z_k\bar{z}_k\,, z_k+\bar{z}_k\,, -i(z_k-\bar{z}_k)\,, 1-z_k\bar{z}_k)$$

- Then

$$S_{\text{em}}^{(0)} = \sum_{k=1}^{n} Q_{(k)}\eta_{(k)}\frac{1}{\sqrt{2}\omega}\frac{(1+z\bar{z})}{z-z_k} \quad (+\text{helicity})$$

$$= \sum_{k=1}^{n} Q_{(k)}\eta_{(k)}\frac{1}{\sqrt{2}\omega}\frac{(1+z\bar{z})}{\bar{z}-\bar{z}_k} \quad (-\text{helicity})$$

Soft theorem = Large gauge Ward identity

- The soft factor may also be expressed as

$$\lim_{\omega\to 0}\langle \text{out}|a_+^{\text{out}}(\omega\hat{x})\mathcal{S}|\text{in}\rangle = S_{\text{em}}^{(0)}\langle \text{out}|\mathcal{S}|\text{in}\rangle = -\lim_{\omega\to 0}\langle \text{out}|\mathcal{S}a_-^{\text{in}\dagger}(\omega\hat{x})|\text{in}\rangle$$

  with $\mathcal{S}$ the S-matrix for the hard process, $\hat{x}$ and ω the soft photon direction and frequency

- Equivalent to *large gauge Ward identity* satisfied by soft modes, with gauge parameter $\epsilon(z_k\,,\bar{z}_k)$

$$\lim_{\omega\to 0}\frac{\sqrt{2}\omega}{1+z\bar{z}}\langle \text{out}|a_+^{\text{out}}(\omega\hat{x})\mathcal{S}|\text{in}\rangle = \sum_{k=1}^{n} Q_{(k)}\eta_{(k)}\epsilon(z_k\,,\bar{z}_k)\langle \text{out}|\mathcal{S}|\text{in}\rangle$$

$$\epsilon(z_k\,,\bar{z}_k) = \frac{1}{z-z_k}(+\text{ve helicity})\,,\quad \epsilon(z_k\,,\bar{z}_k) = \frac{1}{\bar{z}-\bar{z}_k}(-\text{ve helicity})$$

  [A. Strominger, arXiv:1703.05448 [hep-th] and references therein]

Motivation

- Broad Goal: Generalizations of soft factorization in scattering on non-asymptotically flat spacetimes?
- While gauge and diff. invariant observables exist on AdS, we generally lack a definition of scattering amplitudes.

- Progress so far: "Classical soft theorems" provide $1/L^2$ corrected soft factors on AdS spacetimes [N. Banerjee, KF, A. Mitra JHEP **08**, 105 (2021); N. Banerjee, A. Bhatacharjee, A. Mitra JHEP **01**, 038(2020)]
- Weinberg's soft photon theorem derivable from $L \to \infty$ limit of a CFT$_3$ Ward identity [E. Hijano, D. Neuenfeld JHEP **11** 009 (2020)]
- Using this approach, we find that the $1/L^2$ corrected soft photon factor is that of a subleading in L^{-1} soft theorem from a CFT$_3$ Ward identity. [N. Banerjee, KF, A. Mitra, arXiv: (2022)]

Outline

Classical soft theorems (CST)

- Classical soft theorems [A. Laddha and A. Sen JHEP **10** 056 (2018); A. P. Saha, B. Sahoo and A. Sen JHEP **06** 153 (2020)] :
  Classical limits of soft photon and soft graviton factors derivable from $\omega \to 0$ limit of radiative solutions from classical scattering.

- Applicabilty: $\Delta E_{\text{scatterer}} << 1$; $\lambda_{\text{radiation}} >> b$ (imp. param.)

- Classical limits of soft factors result from formula for energy flux in $\omega \to 0$ limit

$$\lim_{\omega \to 0} \epsilon^{\mu} \tilde{a}_{\mu} (\omega \, , \vec{x}) = -i \frac{e^{i\omega R}}{4\pi R} S_{\text{em}}$$

$$\lim_{\omega \to 0} \epsilon^{\mu\nu} \tilde{e}_{\mu\nu}(\omega, \vec{x}) = -i \frac{e^{i\omega R}}{4\pi R} S_{gr}$$

  where $\tilde{a}_{\mu}$ and $\tilde{e}_{\mu\nu}(\omega, \vec{x})$ are electromagnetic and gravitational perturbations in frequency space, ϵ^{μ} and $\epsilon^{\mu\nu}$ are photon and graviton polarizations; R : distance to scatterer.

Summary of CST derivation on AdS$_4$ with small c.c.

- Result derived using probe scattering on an AdS black hole spacetime in four dimensions, with $GM << r << L$. This provides $1/L^2$ corrections to asymp. flat spacetime results.

- Soft factors from a *double scaling limit* : $\omega \to 0$ as $L \to \infty$. This limit formally provides $1/\gamma^2$ corrections; $\gamma = \omega L >> 1$.

- We derived the leading (ω^{-1}) and subleading $(\ln \omega^{-1})$ terms in the soft photon and soft graviton factors, each with their respective $1/\gamma^2$ corrections. [Existence of frequency *and* γ^{-2} expansions]

- While $1/L^n$ corrections to probe trajectory exist, they contribute to $\mathcal{O}(\gamma^{-4})$ corrected soft factor

$1/\gamma^2$ corrected soft photon theorem on AdS_4

- The leading soft photon factor result is

$$S^{(0)}_{\text{em}} = S^{\text{f}(0)}_{\text{em}} + S^{\text{L}(0)}_{\text{em}}$$

$$S^{\text{f}(0)}_{\text{em}} = \sum_{a=1}^{n} Q_{(a)}\eta_{(a)} \frac{\epsilon_\mu p^\mu_{(a)}}{p_{(a)}.k}\,,$$

$$S^{L(0)}_{\text{em}} = \frac{\omega^2}{4\gamma^2} \sum_{a=1}^{n} Q_{(a)}\eta_{(a)} \frac{\epsilon_\mu p^\mu_{(a)}}{p_{(a)}.k} \frac{\vec{p}^{\,2}_{(a)}}{\left(p_{(a)}.k\right)^2}\,,$$

- From leading (universal) term, we infer the single soft theorem

$$\lim_{\omega\to 0} \omega \left(\langle \text{out}|a_+(\omega\hat{x})\mathcal{S}|\text{in}\rangle + \langle \text{out}|\tilde{a}_+(\omega\hat{x})\mathcal{S}|\text{in}\rangle\right)$$
$$= \left(S^{\text{f}\,(0)}_{\text{em}} + S^{L\,(0)}_{\text{em}}\right)\langle \text{out}|\mathcal{S}|\text{in}\rangle$$

with $\tilde{a}_+(\omega\hat{x})$ a perturbed soft mode.

Evaluation of $\langle \text{out}|\tilde{a}_+(\omega\hat{x})\mathcal{S}|\text{in}\rangle = S_{\text{em}}^{L\,(0)}\langle \text{out}|\mathcal{S}|\text{in}\rangle$

- Parametrizing for hard massless scattering, as before

$$\epsilon^{+\mu} = \frac{1}{\sqrt{2}}\,(\bar{z}\,,1\,,-i\,,-\bar{z})\ ,$$

$$q^{\mu} = \frac{\omega}{1+z\bar{z}}\,(1+z\bar{z}\,,z+\bar{z}\,,-i(z-\bar{z})\,,1-z\bar{z})\ ,$$

$$p_k^{\mu} = \frac{E_{\vec{p}}}{1+z_k\bar{z}_k}\,(1+z_k\bar{z}_k\,,z_k+\bar{z}_k\,,-i(z_k-\bar{z}_k)\,,1-z_k\bar{z}_k)$$

we find

$$\lim_{\omega\to 0}\frac{\sqrt{2}\omega}{(1+z\bar{z})}\langle \text{out}|\tilde{a}_+\mathcal{S}|\text{in}\rangle = \frac{1}{16\gamma^2}\Bigg[\sum_{k=\text{out}}\frac{(1+z_k\bar{z}_k)^2\,(1+z\bar{z})^2}{(\bar{z}-\bar{z}_k)^2(z-z_k)^3}Q_k - \sum_{k=\text{in}}\frac{(1+z_k\bar{z}_k)^2\,(1+z\bar{z})^2}{(\bar{z}-\bar{z}_k)^2(z-z_k)^3}Q_k\Bigg]\langle \text{out}|\mathcal{S}|\text{in}\rangle$$

- Subtlety : Overall numerical constant of $\tilde{a}_+$ is *not fixed* by CST

Outline

Overview

- Flat spacetime region from $L \to \infty$ limit of AdS spacetimes.
- Flat spacetime modes from scaling AdS frequency with L
  $\omega_\kappa \sim \omega_{\text{flat}} L$
- Flat spacetime amplitudes can be derived from AdS correlation functions [A. L. Fitzpatrick and J. Kaplan,arXiv:1104.2597 [hep-th]]; A. L. Fitzpatrick, J. Kaplan, J. Penedones, S. Raju and B. C. van Rees, JHEP 11, 095 (2011)]
- $U(1)$ CFT$_3$ Ward identity $\to$ Weinberg soft photon theorem [E. Hijano and D. Neuenfeld JHEP **11** 009 (2020)]

Metric and flat limit

- Analysis in global AdS$_{d+1}$ with metric

$$ds^2 = \frac{L^2}{\cos^2\rho}\left(-d\tau^2 + d\rho^2 + \sin^2\rho\, d\Omega^2_{d-1}\right)$$

AdS boundary at $\rho = \frac{\pi}{2}$ with coordinates $x = \{\tau, z\,, \bar{z}\}$

- Define

$$\frac{r}{L} = \tan(\rho) \approx \rho \text{ (when small)}\,, \qquad \frac{t}{L} = \tau$$

- AdS metric in the $L \to \infty$ limit goes to the flat spacetime metric

$$\lim_{L\to\infty} ds^2 = -dt^2 + dr^2 + r^2 d\Omega_2^2$$

- This metric is realized as a asymptotically flat spacetime patch centered in global AdS around $\tau = 0$

CFT states and flat spacetime scattering states

- Flat spacetime scattering within $\delta \sim \mathcal{O}(L)^{-1}$ around $\tau = 0$
- Beyond this region, states taken to be asymptotically free
- $\Sigma_{\pm}^{\text{AdS}}$: Cauchy surfaces to the future $(+)$ and past $(-)$ of $\tau = 0$
- $\Sigma_{\pm}^{\text{AdS}} \to \Sigma_{\pm}^{\text{Mink}}$ as $L \to \infty$.
- Fock states on asymptotic boundaries of the flat spacetime patch from boundary large N CFT states on $\Sigma_{\pm}^{\text{AdS}}$
- Will consider this on bulk solutions of free Maxwell

Maxwell field in AdS: Boundary condition and sources

- General expression for a bulk gauge field in AdS$_4$

$$\mathcal{A}_\mu(\rho, x) \xrightarrow[\rho\to\frac{\pi}{2}]{} (\cos\rho)^1 \alpha_\mu(x) + (\cos\rho)^0 \beta_\mu(x)$$

- Fix $\beta_\mu = \tilde{A}_\mu$ as a non-dynamical source at boundary. We further assume no Coulombic fields

- Then $\alpha_\mu = j_\mu$: global $U(1)$ current (for radiative fields)

- Bulk gauge field with "magnetic boundary conditions"

$$\mathcal{A}_\mu(\rho, x) \xrightarrow[\rho\to\frac{\pi}{2}]{} (\cos\rho) j_\mu(x)$$

Maxwell field in AdS: HKLL reconstruction

- Using the boundary condition, HKLL reconstruction gives

$$\hat{\mathcal{A}}_\mu(Y) = \int d^3x' \Big[K^V_\mu(Y,;x')\epsilon^{a'b'}_{\tau'}\nabla_{a'}j^+_{b'} + K^S_\mu(Y;x')\gamma^{a'b'}\nabla_{a'}j^+_{b'} + \left(K^V_\mu(Y;x')\right)^* \epsilon^{a'b'}_{\tau'}\nabla_{a'}j^-_{b'} + \left(K^S_\mu(Y;x')\right)^* \gamma^{a'b'}\nabla_{a'}j^-_{b'}\Big] ,$$

  with $\epsilon^{a'b'c'}$ and $\nabla_{a'}$ the Levi-Civita tensor and covariant derivative on the boundary; $j^\pm$: positive and negative freq.

- $K^V_\mu(Y;x')$ and $K^S_\mu(Y;x')$: HKLL kernels from "vector" $(\Delta = 2)$ and "scalar" $(\Delta = 1)$ type solutions of Maxwell's equations.

- The boundary integral takes the form

$$\int d^3x' = \int_{\mathcal{T}} d\tau' \int d\Omega'$$

  $\mathcal{T}$ is $\{-\pi\,,0\}$ for ingoing states and $\{0\,,\pi\}$ for outgoing states

Maxwell field in AdS: Kernels in HKLL

- When $L \to \infty$, $\hat{\mathcal{A}}_\mu(Y) \to \hat{\mathcal{A}}^{\text{out}}_\mu(x)$
- For the z component we have

$$\hat{\mathcal{A}}^{\text{out}}_z(x) = \frac{1}{4\pi}\int d^3x' \int d\omega\, r\, j_l(r\omega)[[\cdots]]$$

with

$$[[\cdots]] = \Bigg[\sum_{l,m} \frac{Y^*_{lm}(\Omega')}{-l(l+1)} \partial_z Y_{lm}(\Omega)\,(i)^{-l} e^{i\omega t} e^{-i\omega L\left(\tau' - \frac{\pi}{2}\right)} D^{\bar{z}'} j^+_{\bar{z}'} + \sum_{l,m} \frac{Y_{lm}(\Omega')}{-l(l+1)} \partial_z Y^*_{lm}(\Omega)\,(-i)^{-l} e^{-i\omega t} e^{i\omega L\left(\tau' - \frac{\pi}{2}\right)} D^{\bar{z}'} j^-_{\bar{z}'}\Bigg]$$

Maxwell field in asymptotically flat spacetime

- Flat patch operator that creates an outgoing photon

$$\hat{a}_{\vec{q}}^{\text{out}(\lambda)} = \lim_{t\to\infty} i \int d^3\vec{x}\, \varepsilon^{(\lambda)\mu} e^{-iq\cdot x} \overleftrightarrow{\partial_0}\, \hat{\mathcal{A}}_\mu^{\text{out}}(x)$$

- Substituting $L \to \infty$ bulk solution, we find

$$\begin{aligned}\hat{a}_-^{\text{out}}(\omega\hat{x}) =& \frac{1}{4}\frac{1+z\bar{z}}{\sqrt{2\omega}} \int d\tau'\, e^{i\omega L\left(\frac{\pi}{2}-\tau'\right)} \int d^2z' \frac{1}{(\bar{z}-\bar{z}')} D^{z'} j^-_{z'}(x')\,,\\ \hat{a}_+^{\text{out}}(\omega\hat{x}) =& \frac{1}{4\omega}\frac{1+z\bar{z}}{\sqrt{2\omega}} \int d\tau'\, e^{i\omega L\left(\frac{\pi}{2}-\tau'\right)} \int d^2z' \frac{1}{(z-z')} D^{\bar{z}'} j^-_{\bar{z}'}(x')\end{aligned}$$

- Note: positive (negative) helicity flat modes are related with $D^{\bar{z}'} j^-_{\bar{z}'}$ $(D^{z'} j^-_{z'})$

Flat modes in terms of current

- If we define

$$\epsilon(z') = \frac{1}{z - z'}\ (+\text{ve helicity})\ ,\ \epsilon(z') = \frac{1}{\bar{z} - \bar{z}'}\ (-\text{ve helicity})$$

then we have

$$a^{\text{out}}_{-}(\omega\hat{x}) = \frac{1}{4}\frac{1 + z\bar{z}}{\sqrt{2\omega}} \int d\tau'\, e^{i\omega_{\vec{q}} L\left(\frac{\pi}{2} - \tau'\right)} \int d^2 z' \epsilon(z') D^{z'} j^{-}_{z'}\ ,$$

$$a^{\text{out}}_{+}(\omega\hat{x}) = \frac{1}{4}\frac{1 + z\bar{z}}{\sqrt{2\omega}} \int d\tau'\, e^{i\omega L\left(\frac{\pi}{2} - \tau'\right)} \int d^2 z' \epsilon(z') D^{\bar{z}'} j^{-}_{\bar{z}'}$$

- $\tau = \pm\frac{\pi}{2}$ dominant contribution for massless particles. For massive particles – contribution from Euclidean caps
- Key observation – AdS bulk gauge fields in terms of the current (for large L) can be used to find flat spacetime modes. The large gauge parameter and soft factor are contained in the expression.

Asymptotic mapping

$$\mathcal{I}^{\pm} :: \tilde{\mathcal{I}}^{\pm} \; ; \qquad i^{\pm} :: \partial\mathcal{M}_{\pm} \; ; \qquad i^{0} :: \tau \in \left\{ -\frac{\pi}{2}, \frac{\pi}{2} \right\}$$

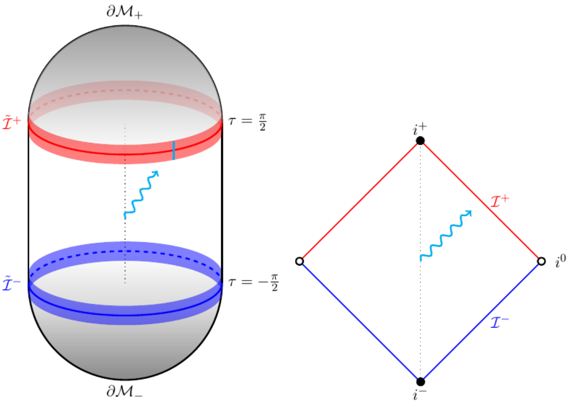


Figure: E. Hijano and D. Neuenfeld JHEP **11** 009 (2020)

Ward identity : Details

- The integrated $U(1)$ Ward identity on the boundary

$$\int d^3x'\, \alpha(x')\partial'_\mu \langle 0|T\{j^\mu(x')X\}|0\rangle$$
$$= \left(\sum_{i=1}^{n} Q_i \alpha(x'_i) - \sum_{j=1}^{m} Q_j \alpha(x'_j)\right) \langle 0|T\{X\}|0\rangle$$

$T\{X\}$: n $(\tau > 0)$ and m $(\tau < 0)$ time ordered operators with charges Q_i $(i = 1\,, \cdots\,, n)$ and Q_j $(j = 1\,, \cdots\,, m)$

- We assume $\alpha(x')$ satisfies

$$\alpha(x')|_{\tilde{\mathcal{I}}^{\pm}} = \epsilon(x')$$

- This leads to the expression for $\alpha(x')$

$$\alpha(x') = \lim_{\rho \to \frac{\pi}{2}} \int d^2\hat{x}''\, \frac{1}{4\pi} \frac{\cos^2\rho - \cos^2\tau}{(\sin\tau - \sin\rho\, \hat{x}' \cdot \hat{x}'')^2} \varepsilon(\hat{x}'')$$

Ward identity : Recovering Weinberg's soft theorem

- In the soft limit, we get the outgoing mode relations

$$\frac{1}{4}\int_{\tilde{\mathcal{I}}^+} d^3x' \, \epsilon(x') D^{\bar{z}} j^-_{\bar{z}} = \lim_{\omega\to 0} \frac{\sqrt{2}\omega}{1+z\bar{z}} \, a^{\text{out}}_+(\omega\hat{x}) \, ,$$

$$\frac{1}{4}\int_{\tilde{\mathcal{I}}^+} d^3x' \, \epsilon(x') D^{z} j^-_{z} = \lim_{\omega\to 0} \frac{\sqrt{2}\omega}{1+z\bar{z}} \, a^{\text{out}}_-(\omega\hat{x}) \, .$$

  Summing over incoming and outgoing contributions realizes the LHS as a soft photon insertion and fixes $\epsilon(x')$.

- This choice of $\epsilon(x')$ makes the RHS that of the soft theorem.

- $\langle 0|T\{X\}|0\rangle$ goes to the S-matrix on replacing the operators with the corresponding flat spacetime modes.

Outline

Procedure

- We do not correct the asymptotically flat patch or large L limits of the X operators
  Reason: We are interested in soft factor corrections of an S-matrix on the asymptotically flat spacetime patch in AdS.

- The soft factor correction comes from expanding the HKLL kernels to subleading order in $1/L^2$

$$\hat{\mathcal{A}}_z^{\text{out}}(x) = \hat{\mathcal{A}}_z^{\text{out; flat}}(x) + \hat{\mathcal{A}}_z^{\text{out};L}(x) + \text{subleading in}\,\omega$$

where

$$\hat{\mathcal{A}}_z^{\text{out; flat}}(x) = \frac{1}{4\pi}\int d^3x' \int d\omega\, r\, j_l(r\omega)[[\cdots]]$$
$$\hat{\mathcal{A}}_z^{\text{out};L}(x) = \frac{1}{16\pi}\int d^3x' \int d\omega\, \frac{r}{\gamma^2}\, j_l(r\omega) l(l+1)[[\cdots]]$$

$1/L^2$ corrected bulk modes

- $\hat{\mathcal{A}}_z^{\text{out; flat}}(x)$ recovers the flat spacetime modes as before.
- The perturbed flat mode follows from $\hat{\mathcal{A}}_z^{\text{out};L}(x)$. We find

$$\lim_{\omega\to 0}\frac{\sqrt{2}\omega}{1+z\bar{z}}\hat{a}_+^{\text{out};L}=\frac{1}{4}\int_{\tilde{\mathcal{I}}^+}d^3x'\,\epsilon_L(x')\mathcal{D}^{\bar{z}'}j^-_{\bar{z}'}$$

and likewise for the outgoing negative helicity mode.

- Recovers the LHS of the perturbed soft theorem
- However, we now have

$$\epsilon_L(x')=\frac{1}{8\pi\gamma^2}\int d\Omega_w\left[\frac{(1+z'\bar{z}')^2\,(1+z_w\bar{z}_w)^2}{(\bar{z}'-\bar{z}_w)^2\,(z-z_w)^3}\right]$$

Gauge parameter

- Going by classical soft theorem result, we should have

$$\epsilon_L^{\text{cst}}(x') = \frac{1}{16\gamma^2} \frac{(1+z'\bar{z}')^2 (1+z\bar{z})^2}{(\bar{z}-\bar{z}')^2 (z-z')^3}$$

- In contrast

$$\epsilon_L(x') = \frac{1}{8\pi\gamma^2} \int d\Omega_w \left[\frac{(1+z'\bar{z}')^2 (1+z_w\bar{z}_w)^2}{(\bar{z}'-\bar{z}_w)^2 (z-z_w)^3} \right]$$

involves an additional integration over soft mode contributions.

- Requirements on the leading soft photon factor:
  - Leading divergence that goes like ω^{-1}
  - *Collinear divergence* as $z \to z'$

Recovering classical soft theorem result

- To satisfy the collinear divergence property, we consider

$$|z_w - z'| \approx |z - z'| + \delta \approx |z_w - z| \qquad \text{with}\, \delta << 1$$

  This assumes $\{z_w\,, \bar{z}_w\}$ can be expanded about $\{z\,, \bar{z}\}$ or $\{z'\,, \bar{z}'\}$, with $|z - z'|$ as a minimal distance.

- Another way of explaining this assumption:
  $L \to \infty$ leads to $\tau \to t$ and $\rho \to r$.

- However angular separations can remain widely separated.

- Resolved by $|z - z'|$ as the smallest angular separation.

Recovering classical soft theorem result

- The integrand of $\epsilon_L(x')$ can then be expressed as

$$\frac{(1+z'\bar{z}')^2\,(1+z_w\bar{z}_w)^2}{(\bar{z}'-\bar{z}_w)^2\,(z_q-z_w)^3} = \frac{(1+z'\bar{z}')^2\,(1+z_q\bar{z}_q)^2}{(\bar{z}_q-\bar{z}')^2(z_q-z')^3} + \mathcal{O}(\delta)$$

- We can now formally integrate $\epsilon_L(x')$ to find

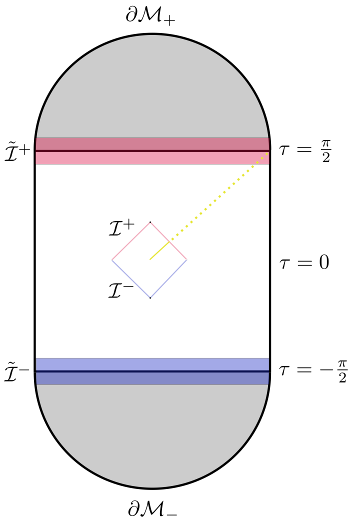
$$\epsilon_L(x') = 8\epsilon_L^{\text{cst}}(x') + \text{`` corrections''}$$

- This satisfies the $1/\gamma^2$ corrected classical soft photon theorem

- The factor of 8 identifies an overall constant of the perturbed soft photon mode.

Interpretation of the correction

Soft photons can exit the flat patch and propagate in AdS. This manifests as an asymptotic interaction at $\mathcal{I}^+$ of the patch.



Outline

Conclusions and future directions

- We investigated the soft photon theorem on AdS spacetimes.
- The results indicate an expansion about the flat spacetime patch occuring in $L \to \infty$ limit of AdS. The factor admits an expansion in frequency and $\gamma^{-2} = \frac{1}{\omega^2 L^2}$; the latter being characteristic to spacetimes with a cosmological constant.
- We established that the corrected soft photon theorem can be derived from a CFT Ward identity.

Conclusions and future directions

- The integration over angles in the $1/L^2$ gauge parameter appears to be a generic feature to higher orders

- This raises the issue of understanding the "corrections" to local gauge parameter expressions and soft factors.

- The status of soft factors on AdS beyond the flat spacetime limit remains an open issue. This could be derived in principle from $1/\gamma^n\,, n > 2$ corrections to soft factors and the hard states in the amplitude.

- Unclear if resumming $1/\gamma^n$ and frequency terms leads to an IR finite result.