# How to sit Maxwell and Higgs on the boundary of AdS


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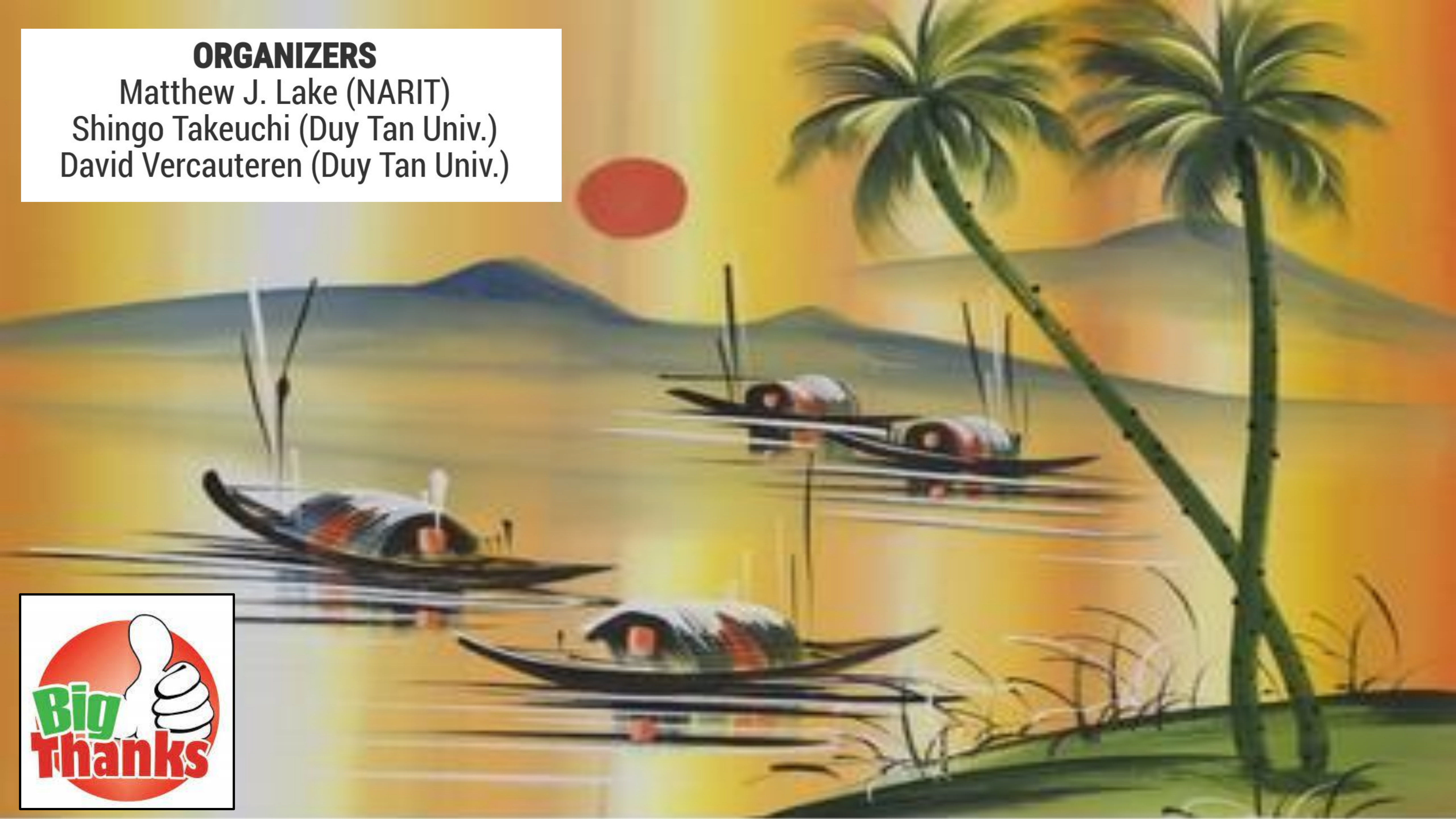


# Based on

arXiv:2211.01760 [pdf, other] hep-th doi 10.1007/JHEP02(2023)012

## Holography and magnetohydrodynamics with dynamical gauge fields

**Authors:** Yongjun Ahn, Matteo Baggioli, Kyoung-Bum Huh, Hyun-Sik Jeong, Keun-Young Kim, Ya-Wen Sun

**Abstract**: Within the framework of holography, the Einstein-Maxwell action with Dirichlet boundary conditions corresponds to a dual conformal field theory in presence of an external gauge field. Nevertheless, in many real-world applications, e.g., magnetohydrodynamics, plasma physics, superconductors, etc. dynamical gauge fields and Coulomb interactions are fundamental. In this work, we consider bottom-up ho... ▽ More

**Submitted** 3 November, 2022; **originally announced** November 2022.

**Comments:** 54 pages, 22 figures

**Journal ref:** JHEP02(2023)012

arXiv:2302.02364 [pdf, other] hep-th

## Collective dynamics and the Anderson-Higgs mechanism in a bona fide holographic superconductor

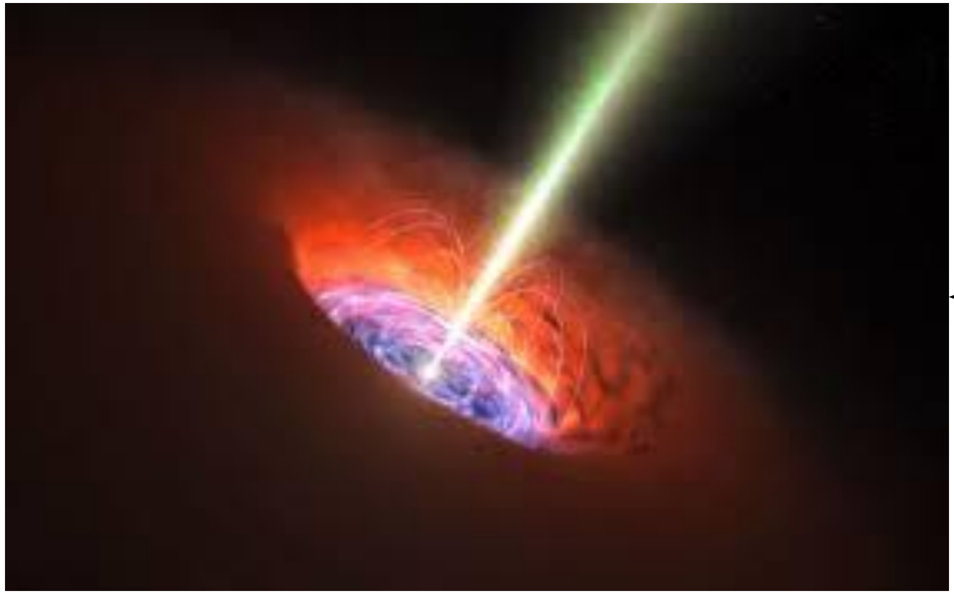
**Authors:** Hyun-Sik Jeong, Matteo Baggioli, Keun-Young Kim, Ya-Wen Sun

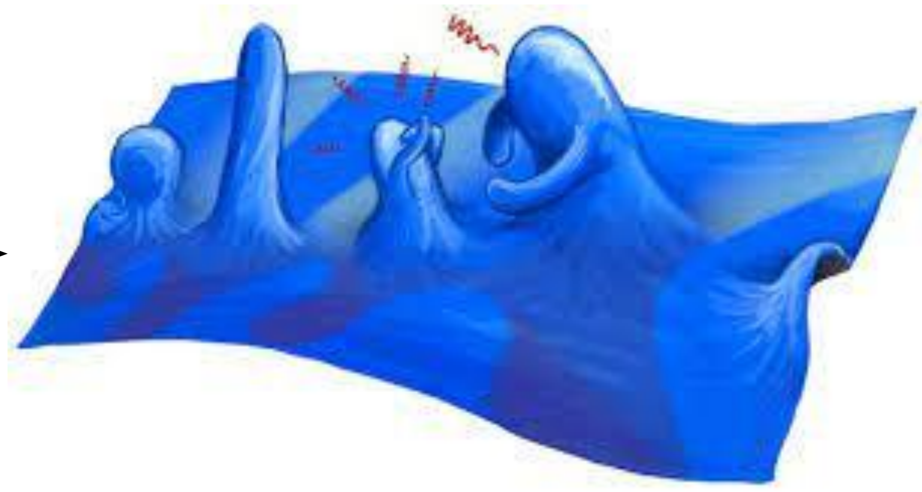
**Abstract**: The holographic superconductor is one of the most popular models in the context of applied holography. Despite what its name suggests, it does not describe a superconductor. On the contrary, the low temperature phase of its dual field theory is a superfluid with a spontaneously broken U(1) global symmetry. As already observed in the previous literature, a bona fide holographic superconductor can b... ▽ More

**Submitted** 5 February, 2023; **originally announced** February 2023.

**Comments:** 43 pages, 18 figures

# Holography






Gravitational bulk

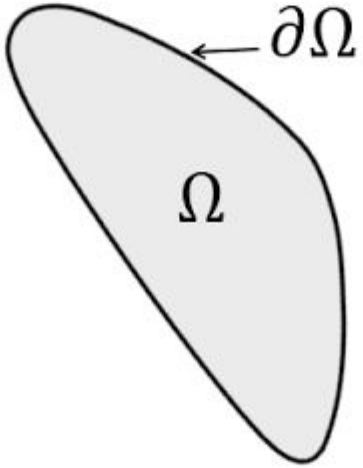
Boundary QFT

# Holography

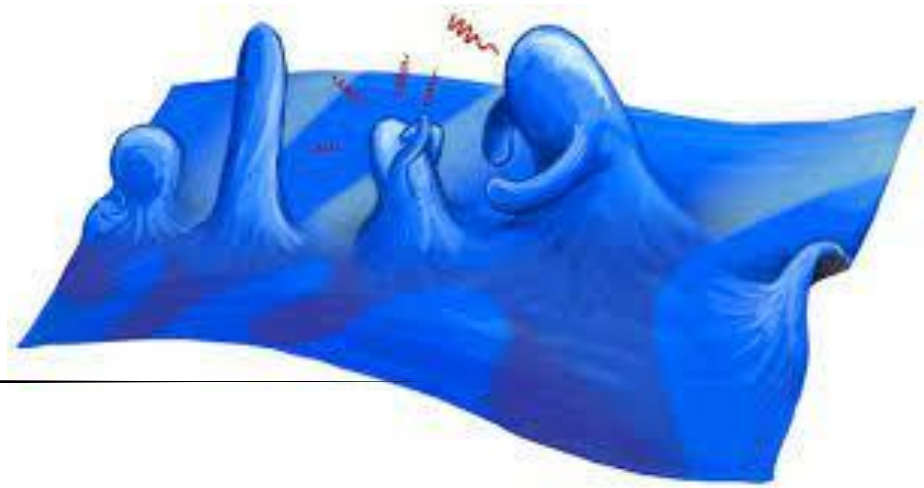
## Gravitational bulk action

$$S = \int d^{d+1}x\sqrt{-g}\,\left[R - 2\Lambda + \mathcal{L}\left(F_{\mu\nu}\right) + \ldots\right]$$

## Boundary conditions




$$A_\mu\left(r,t,\vec{x}\right) \underset{r\to\infty}{\sim} A_\mu^{(0)}\left(t,\vec{x}\right) + A_\mu^{(1)}\left(t,\vec{x}\right)\, r^{1-D}$$



Boundary QFT

# A typical example

- Temperature
- Charge
- Magnetic field

**BULK :**

$$S = \int \mathrm{d}^4x \sqrt{-g} \left( R + 6 - \frac{1}{4} F^2 \right) , \qquad F = \mathrm{d}A ,$$

$$\mathrm{d}s^2 = -f(r)\,\mathrm{d}t^2 + \frac{1}{f(r)}\,\mathrm{d}r^2 + r^2(\mathrm{d}x^2 + \mathrm{d}y^2) , \qquad A = A_t(r)\,\mathrm{d}t - \frac{B}{2} y\,\mathrm{d}x + \frac{B}{2} x\,\mathrm{d}y ,$$

$$A_\mu(r, t, \vec{x}) \underset{r\to\infty}{\sim} A_\mu^{(0)}(t, \vec{x}) + A_\mu^{(1)}(t, \vec{x})\, r^{1-D}$$

**BOUNDARY**

*Assuming standard quantization*

*Dirichlet boundary conditions*

$$Z[g_{\mu\nu}, A_\mu] = \int \mathcal{D}\Phi \exp \left[ iS_0(\Phi) + i \int d^3x \left( A_\mu J^\mu(\Phi) + \frac{1}{2} g_{\mu\nu} T^{\mu\nu}(\Phi) \right) \right]$$

# What do we get on the other side?

A QFT with a conserved U(1) current in presence of an external gauge field

$$\partial_\mu T^{\mu\nu} = F^{\nu\mu} J_\mu$$
$$\partial_\mu J^\mu = 0\,.$$

Hydrodynamics of the gauge sector

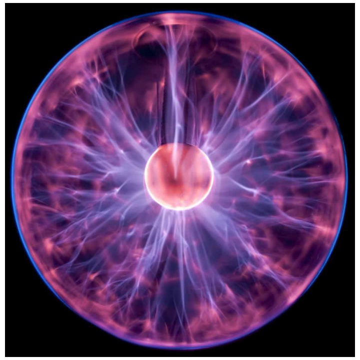
$$\omega = -iDk^2$$

Diffusion of conserved charge

# Is this everything we can do?

Unsatisfactory in many physical situations → **DYNAMICAL GAUGE FIELDS COULOMB INTERACTIONS**

electromagnetism

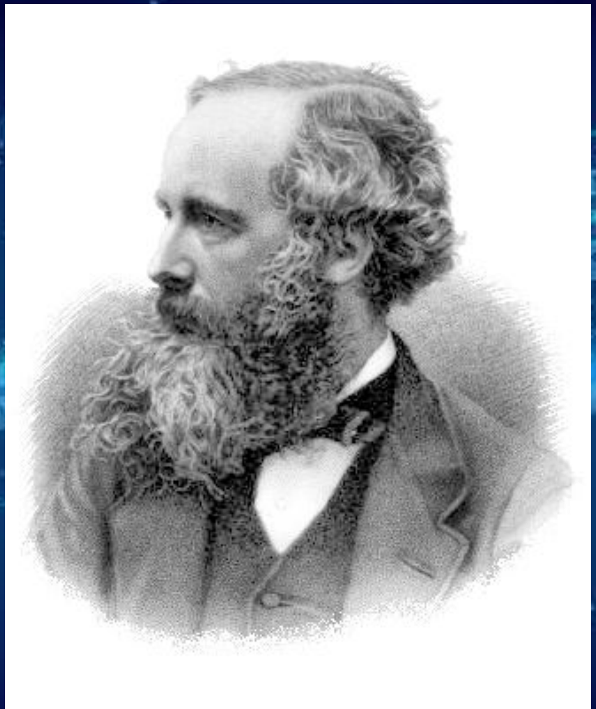


plasmas

plasmons

superconductors

# PART I



# Fancy Maxwell's equations

**Generalized global symmetries and dissipative magnetohydrodynamics**

Sašo Grozdanov, Diego M. Hofman, Nabil Iqbal

## Electromagnetism without gauge symmetries

The true global symmetry of $U(1)$ electrodynamics is actually something different. Consider the following antisymmetric tensor

$$J^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}. \tag{1.2}$$

It is immediately clear from the Bianchi identity (i.e. the absence of magnetic monopoles) that $\nabla_\mu J^{\mu\nu} = 0$. This is not related to the conservation of electric charge, but rather states that magnetic field lines cannot end.

$$\nabla_\mu J^{\mu\nu} = 0.$$

$$\nabla_\mu T^{\mu\nu} = H^{\nu}{}_{\rho\sigma} J^{\rho\sigma},$$

*Only global symmetries*
*Better for hydrodynamics, etc…*

# Magnetohydrodynamics from holography

**Generalised global symmetries in holography: magnetohydrodynamic waves in a strongly interacting plasma**

Sašo Grozdanov, Napat Poovuttikul

$$S = \frac{N_c^2}{8\pi^2}\left[\int d^5x\sqrt{-G}\left(R + \frac{12}{L^2} - \frac{1}{3e_H^2}H_{abc}H^{abc}\right)\right.$$

$$H = dB, \quad B \to B + d\lambda$$

$$B_{\mu\nu} \longrightarrow J^{\mu\nu}$$

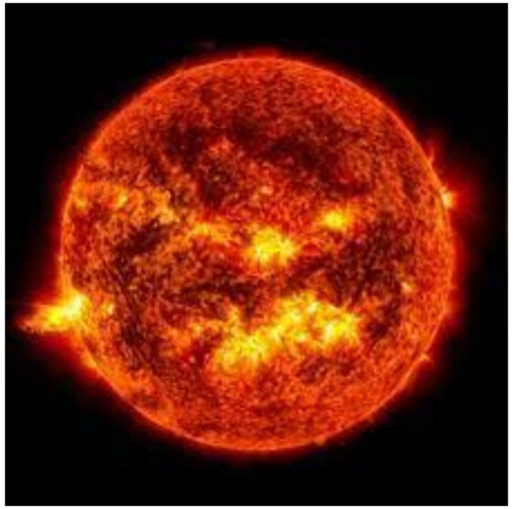
Short summary:
the standard thing but with
a higher-form ☺️

# What do they get?

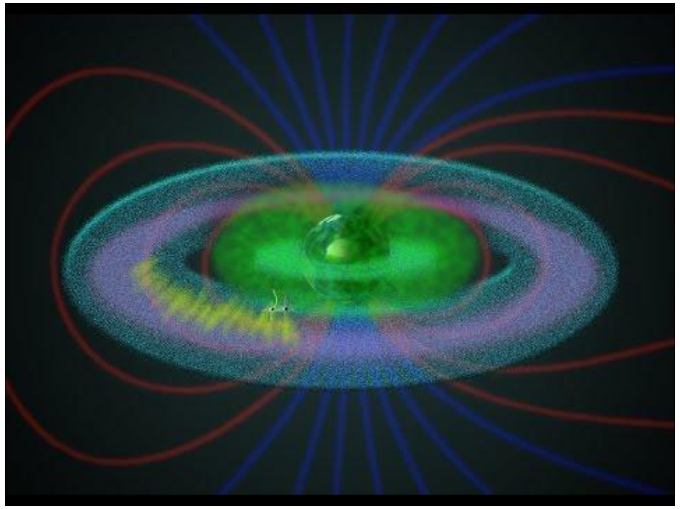
**Relativistic magnetohydrodynamics**

Juan Hernandez, Pavel Kovtun

**A QFT with a dynamical gauge field (dynamical E, B)**



Alfven waves



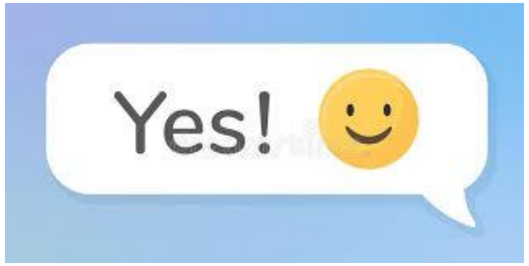
Magnetosonic waves

# Our questions/results

1. Where is the trick ?


2. Do we really need this fanciness ?

3. Is there another solution ?




4. Magnetohydrodynamics from holography (no dress code)

*The story of today …*

# Field theory break I

Let us start by considering the generating functional $Z[g_{\mu\nu}, A_\mu]$:

$$Z[g_{\mu\nu}, A_\mu] = \int \mathcal{D}\Phi \exp\left[iS_0(\Phi) + i\int d^3x \left(A_\mu J^\mu(\Phi) + \frac{1}{2}g_{\mu\nu}T^{\mu\nu}(\Phi)\right)\right] ,$$

$$S[g_{\mu\nu}, A_\mu] := -i \ln Z[g_{\mu\nu}, A_\mu]$$

$$\langle J^\mu \rangle , \quad \langle J^\mu J^\nu \rangle$$

$$\langle T^{\mu\nu} \rangle , \quad \langle T^{\mu\nu} T^{\sigma\lambda} \rangle$$

$$\langle J^\mu T^{\sigma\lambda} \rangle , \quad \langle \ldots\ldots \rangle$$

# Field theory break II

$$S_{\rm tot} = S_{\rm m}[g_{\mu\nu}, A_\mu] + \int {\rm d}^3x\sqrt{-g}\left[-\frac{1}{4\lambda}F^2 + A_\mu J^\mu_{\rm ext}\right] .$$

Maxwell kinetic term + Legendre transform (coupling to external current)

$$\delta_{A_\mu} S_{\rm tot} = \int {\rm d}^3x\,\sqrt{-g}\left[J^\mu_{\rm m} - \frac{1}{\lambda}\nabla_\nu F^{\mu\nu} + J^\mu_{\rm ext}\right]\delta A_\mu .$$

MAXWELL EQUATIONS

# Field theory break III

$$\nabla_\mu \left(T^{\mu\nu}_{\rm m} + T^{\mu\nu}_{\rm EM}\right) = F^{\lambda\nu} J_{{\rm ext}\lambda} \,, \qquad \nabla_\mu J^\mu_{\rm m} = 0 \,,$$

$$J^\mu_{\rm m} - \frac{1}{\lambda}\nabla_\nu F^{\mu\nu} + J^\mu_{\rm ext} = 0 \,, \qquad \epsilon^{\alpha\beta\gamma}\,\nabla_\alpha F_{\beta\gamma} = 0 \,,$$

Standard electromagnetism

$$T^{\mu\nu}_{\bf EM} = \frac{1}{\lambda} F^{\mu\sigma} F^{\nu}{}_{\sigma} - \frac{1}{4\lambda} F^2 g^{\mu\nu}$$

*[more details later]*

# Old but gold

*[Witten, Marolf-Ross, …]*

Boundary asymptotics : $A_\mu(r,t,\vec{x}) \underset{r\to\infty}{\sim} A^{(0)}_\mu(t,\vec{x}) + A^{(1)}_\mu(t,\vec{x})\, r^{1-D}$,

$$\alpha\, A^{(0)}_\mu(t,\vec{x}) + \beta\, A^{(1)}_\mu(t,\vec{x}) = \text{fixed}\,,$$

MIXED BOUNDARY CONDITIONS

$\beta = 0$ $\longrightarrow \mathcal{L}_{CFT} + \int d^d x\, A^{(0)}_\mu J^\mu$

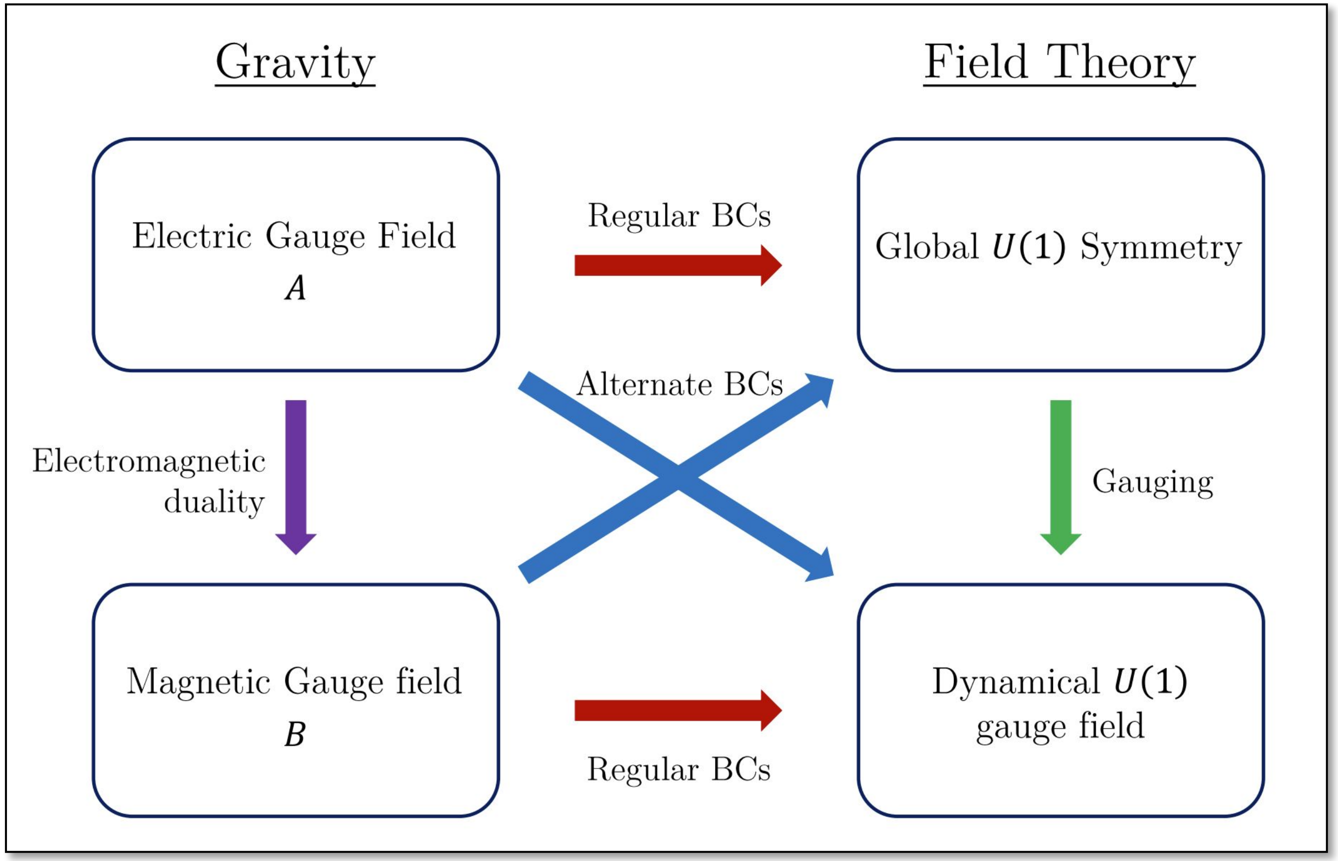
standard

# Back to the higher-form trick

arXiv:2010.06594 [pdf, other] hep-th doi 10.1103/PhysRevD.103.026011

Generalized symmetries and 2-groups via electromagnetic duality in AdS/CFT

Authors: Oliver DeWolfe, Kenneth Higginbotham




Hodge dual in the bulk does not preserve the boundary conditions !!

It changes them from Dirichlet to mixed

**Higher-forms are just a fancy way to implement mixed boundary conditions**

# On the shoulders of giants

$$\Pi^{\mu} - \frac{1}{\lambda}\partial_{\nu}F^{\mu\nu} + J^{\mu}_{\text{ext}} = 0\,, \qquad \Pi^{\mu} = \frac{\delta S_{\text{on-shell}}}{\delta A_{\mu}} = -\sqrt{-g}\,F^{r\mu}\,,$$

Our mixed boundary conditions

$$\delta J^{x\,(L)}_{\text{ext}} = Z^{(L)}_{A_1} + \frac{\lambda}{\omega^2 - k^2}\,Z^{(S)}_{A_1}\,,$$

$$\delta J^{y\,(L)}_{\text{ext}} = (\omega^2 - k^2)\,Z^{(L)}_{A_2} + \lambda\,Z^{(S)}_{A_2}\,.$$

- Lambda parametrizes the EM coupling
- Notice the factors of (w^2-k^2)

  *[back to this later]*

# Re-discovering the known

$$\epsilon = 2r_h^3 + \frac{\mu^2 r_h}{2} + \frac{B^2}{2r_h} + \frac{B^2}{2\lambda},$$

$$p = r_h^3 + \frac{\mu^2 r_h}{4} - \frac{3B^2}{4r_h} - \frac{B^2}{2\lambda}$$

$$T_{xx} = \left(p + \frac{B^2}{\mu_m}\right) \neq p$$

Thermodynamic and mechanical pressure are not equal

$$T^{\mu}{}_{\mu} = \frac{1}{4\lambda} F^2$$

Maxwell theory in 2+1 is scale invariant but not conformal invariant !!

**What Maxwell Theory in D<>4 teaches us about scale and conformal invariance**

Sheer El-Showk, Yu Nakayama, Slava Rychkov

# Magnetohydrodynamics

Step 1: EOMs

$$\nabla_\mu \left(T^{\mu\nu}_{\rm m} + T^{\mu\nu}_{\rm EM}\right) = F^{\lambda\nu} J_{{\rm ext}\lambda}\,, \qquad \nabla_\mu J^\mu_{\rm m} = 0\,,$$

$$J^\mu_{\rm m} - \frac{1}{\lambda}\nabla_\nu F^{\mu\nu} + J^\mu_{\rm ext} = 0\,, \qquad \epsilon^{\alpha\beta\gamma}\nabla_\alpha F_{\beta\gamma} = 0\,,$$

Step 2: constitutive relations

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + p\,\Delta^{\mu\nu} + \mathcal{H}^{\mu\gamma} F^\nu{}_\gamma + \Pi^{\mu\nu}\,,$$

$$J^\mu = \rho\, u^\mu - \nabla_\nu \mathcal{H}^{\mu\nu} + \nu^\mu\,, \qquad \mathcal{H}^{\mu\nu} := \frac{1}{\lambda}F^{\mu\nu} - M^{\mu\nu}_{\mathbf{m}}\,,$$

Step 3: dynamical matrix and QNMs

$$\mathcal{M}(\omega,k)\cdot s_A = 0\,, \quad \det \mathcal{M}(\omega,k) = 0\,.$$

$$s_A = \{\delta T, \delta u^{i=x,y}, \delta E^{i=x,y}, \delta B\}$$

# Zero density and zero B

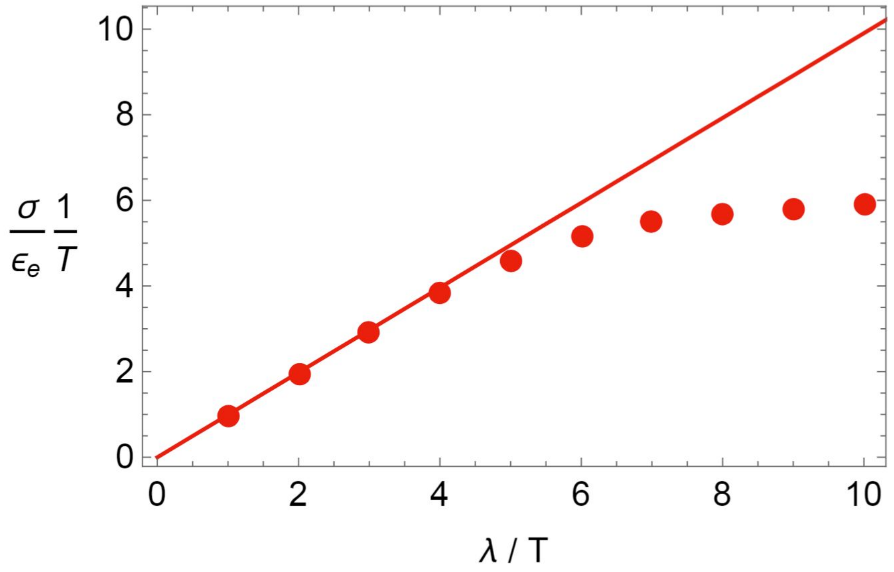
$$\omega\left(\omega + i\,\frac{\sigma}{\epsilon_{\rm e}}\right) = \frac{k^2}{\epsilon_{\rm e}\,\mu_{\rm m}}$$

"EM waves"

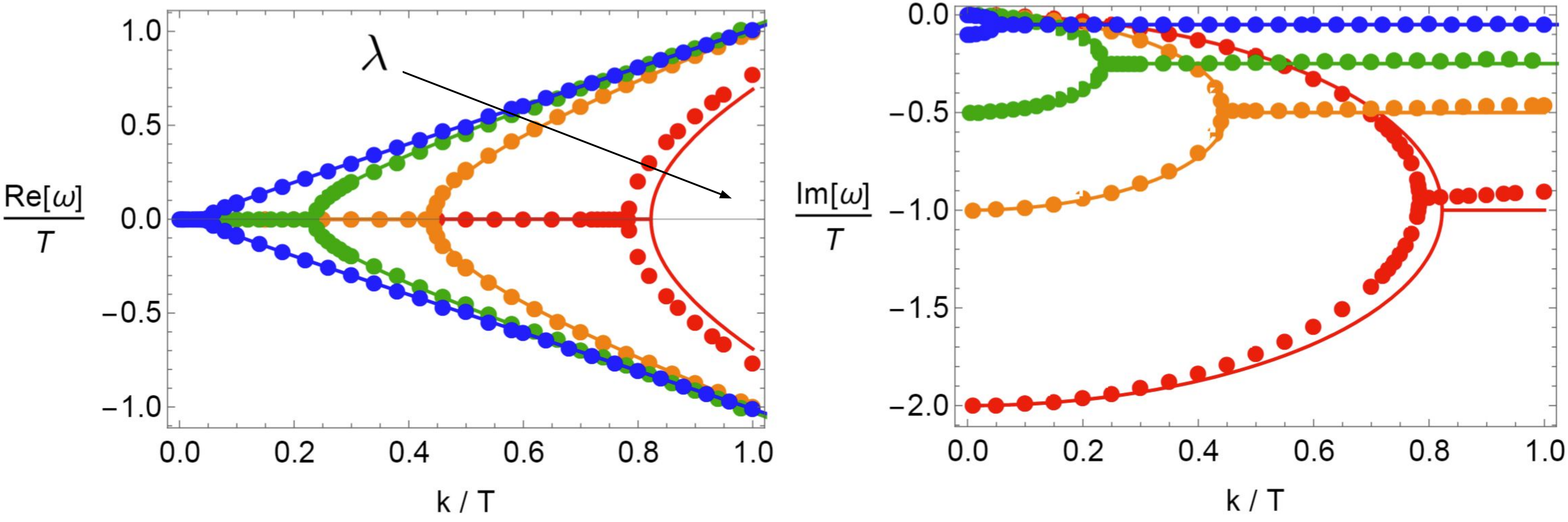
$$\omega = -i\,\frac{\sigma}{\epsilon_{\rm e}} - i\,\frac{\sigma}{\partial\rho/\partial\mu}\,k^2$$

Damped charge diffusion

We see the effects of polarization and screening (dynamical EM in matter)



# Screened EM waves




The photon is screened (skin effect).
Just solve it for real w and complex k
and back to standard textbooks results

Numerical data (QNMs)

1st order hydro

# Zero density and finite B

All dispersion relations remain of the same type but the coefficients are strongly modified by B^2

$$\omega = \pm v_{\rm ms}\, k - i\frac{\Gamma_{\rm ms}}{2}\, k^2$$

Magnetosonic waves

[sound mode carries magnetic flux now]

# Zero density and finite B

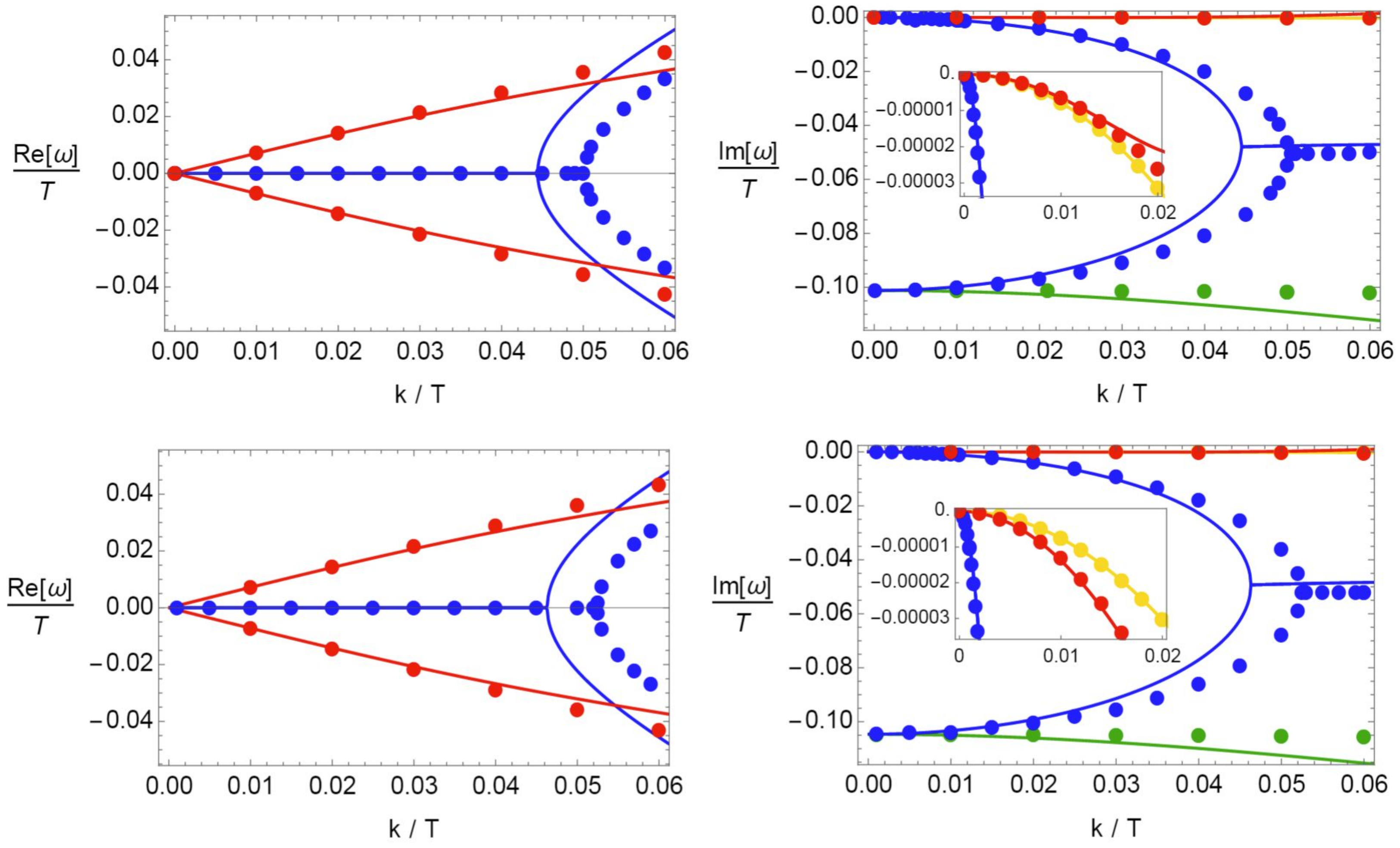



**Figure 2**. Dispersion relations of the lowest QNMs at zero density ($\mu/T = 0$) and $B/T^2 \neq 0$. Top and bottom panels are respectively for $B/T^2 = 0.5, 1$.

# Finite density and finite B

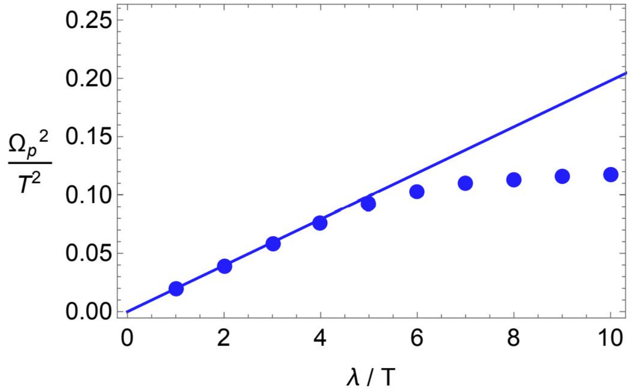
$$\omega = -i \frac{\left(\frac{\partial\rho}{\partial\mu}\right)_{T,B}(\epsilon+p)^2\ \sigma}{T\left[\left(\frac{\partial\epsilon}{\partial T}\right)_{\mu,B}\left(\frac{\partial\rho}{\partial\mu}\right)_{T,B}-\left(\frac{\partial\epsilon}{\partial\mu}\right)_{T,B}\left(\frac{\partial\rho}{\partial T}\right)_{\mu,B}\right](\rho^2+B^2\sigma)}\, k^2\,, \qquad \omega = -i\frac{\eta}{\mu_{\rm m}\,\rho^2}\,k^4\,.$$

- Longitudinal diffusive mode
- Shear diffusion becomes sub-diffusive
- EM waves and sound modes couple together (4 non-hydro modes)

$$\left[\omega\left(\omega+i\,\frac{\sigma}{\epsilon_{\rm e}}\right)-\Omega_p^2\right]^2 = \frac{B^2}{\epsilon_{\rm e}^2\,\mu_{\rm m}^2(\epsilon+p)^2}\left[\rho^2-\mu_{\rm m}^2\sigma^2(\rho^2-B^2)+\omega^2\left(2(\epsilon+p)(\sigma-i\epsilon_{\rm e}\omega)\right)\right]\,,$$

where $\Omega_p$ is the plasma frequency

$$\Omega_p^2 := \frac{\rho^2}{\epsilon_{\rm e}(\epsilon+p)}\,.$$



# Finite density and zero B

$$\left[\omega\left(\omega + i\,\frac{\sigma}{\epsilon_{\rm e}}\right) - \Omega_p^2\right]^2 = 0$$

$$\left(\sigma^2/\epsilon_{\rm e}^2 \gg 4\,\Omega_p^2\;;\;\text{small density}\right): \quad \omega = -\,i\,\frac{\epsilon_{\rm e}}{\sigma}\,\Omega_p^2\,, \qquad \omega = -i\,\frac{\sigma}{\epsilon_{\rm e}} + i\,\frac{\epsilon_{\rm e}}{\sigma}\,\Omega_p^2\,,$$

$$\left(\sigma^2/\epsilon_{\rm e}^2 \ll 4\,\Omega_p^2\;;\;\text{large density}\right): \quad \omega = \pm\,\Omega_p - i\,\frac{\sigma}{2\epsilon_{\rm e}}\,.$$

Small density : overdamped modes

Large density : underdamped modes

Finally, setting all the dissipative coefficients (e.g., $\sigma = 0$) to zero, one finds

sound waves → $$\omega^2 = \Omega_p^2 + v_s^2\,k^2\,, \qquad \omega^2 = \Omega_p^2 + \frac{k^2}{\epsilon_{\rm e}\,\mu_{\rm m}}\,.$$ ← EM waves

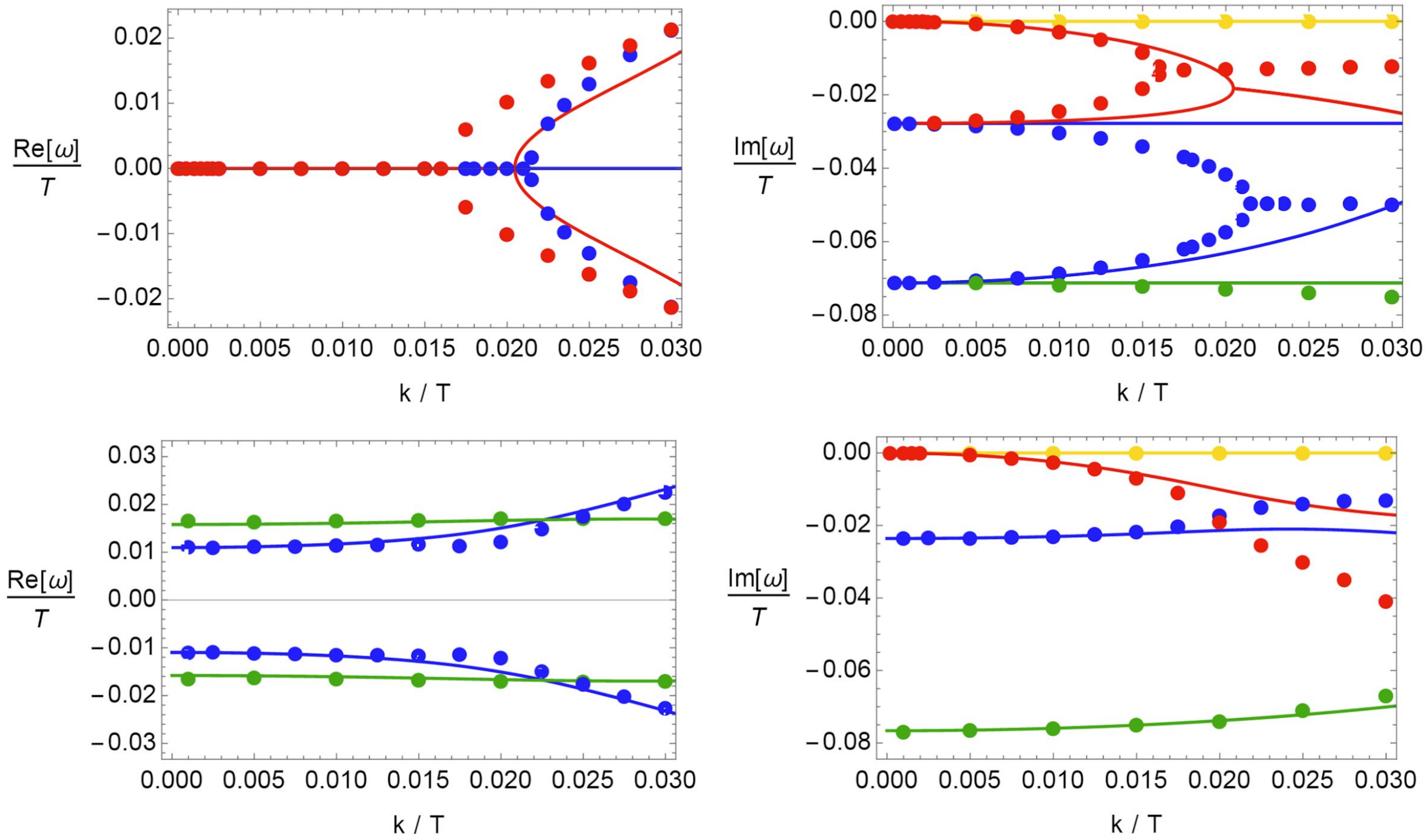



**Figure 3**. Dispersion relations of the lowest QNMs at finite density ($\mu/T = 0.5$). Top and bottom panels refer respectively to $B/T^2 = 0, 0.5$.

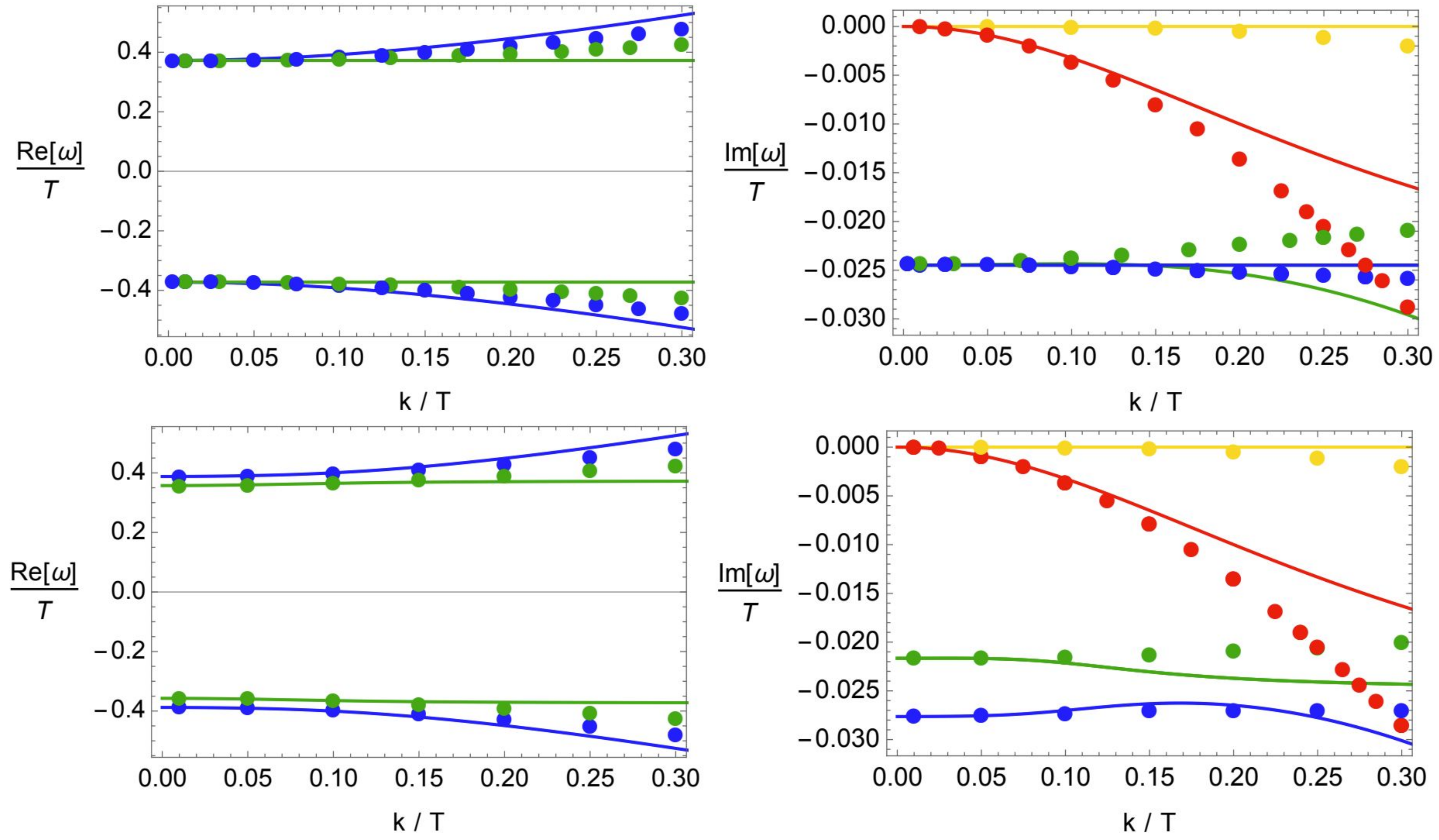



**Figure 4**. Lowest QNMs at finite density ($\mu/T = 5$). Top and bottom panels refer respectively to $B/T^2 = 0, 0.5$.

# Unscreened photon

$$\lambda \to 0 :$$

$$Z_{A_1}^{(L)} = 0$$

$$(\omega^2 - k^2) Z_{A_2}^{(L)} = 0$$

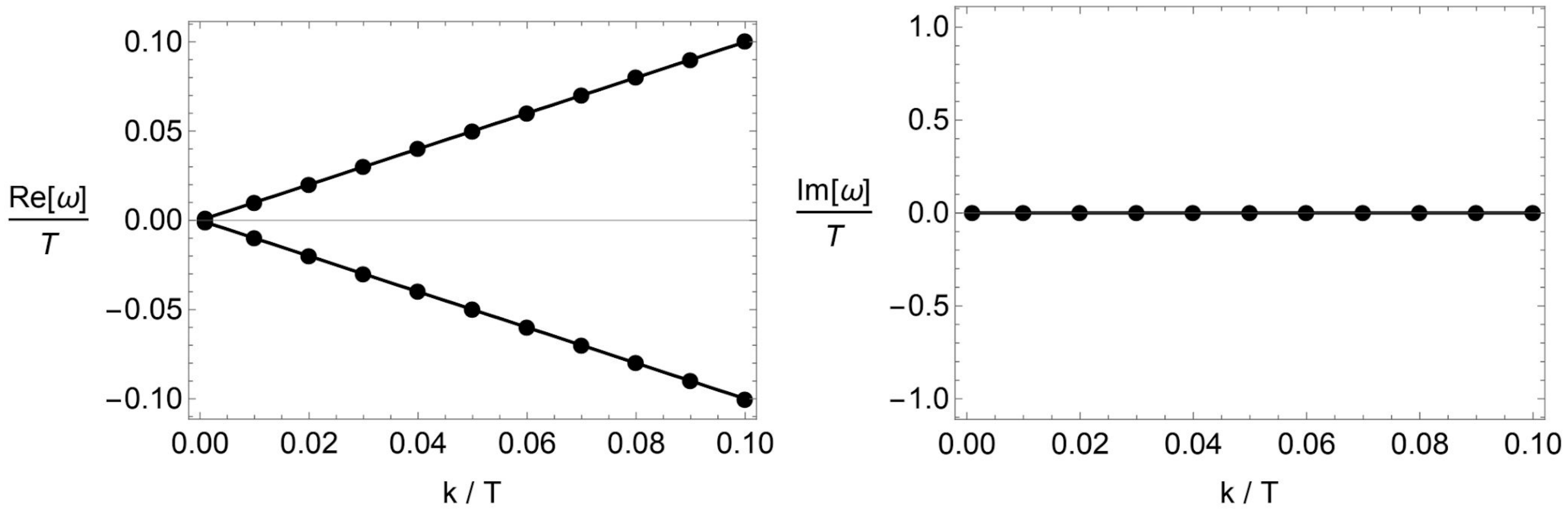


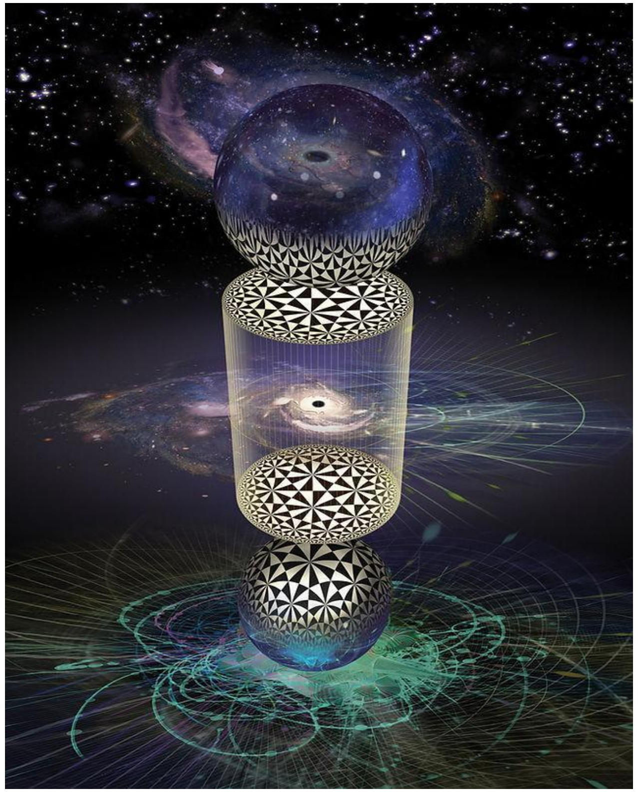

**Figure 15**. The emergent propagating photon at zero density, zero magnetic field and zero EM coupling $\lambda/T = 0$.

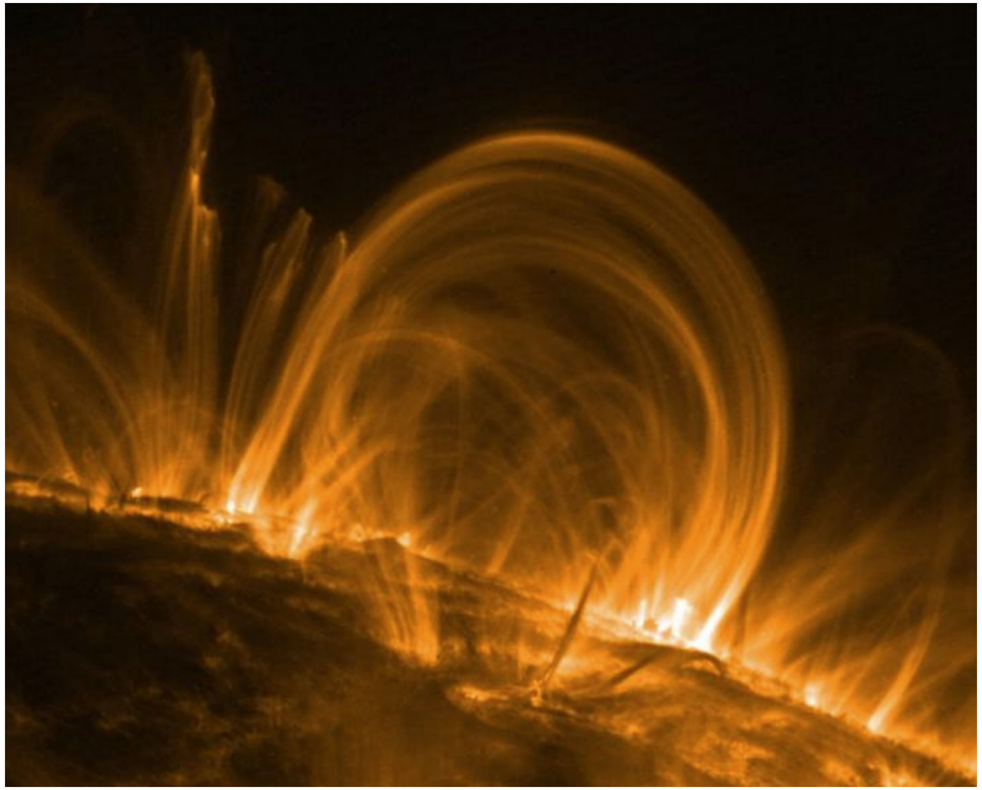
In this limit we are decoupling the photon from matter [no screening, no polarization]: emergent photon

This can be derived analytically [see paper]

# Summary of part I

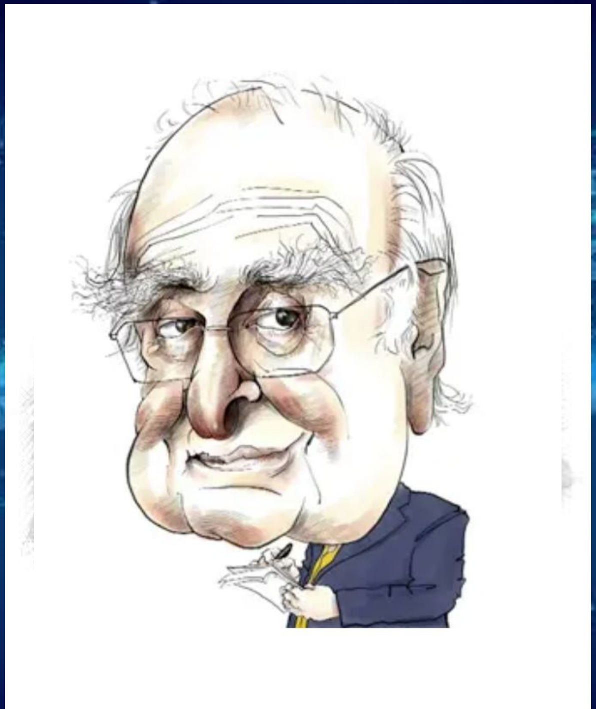
**Maxwell + mixed b.c.s.** = **magnetohydrodynamics**

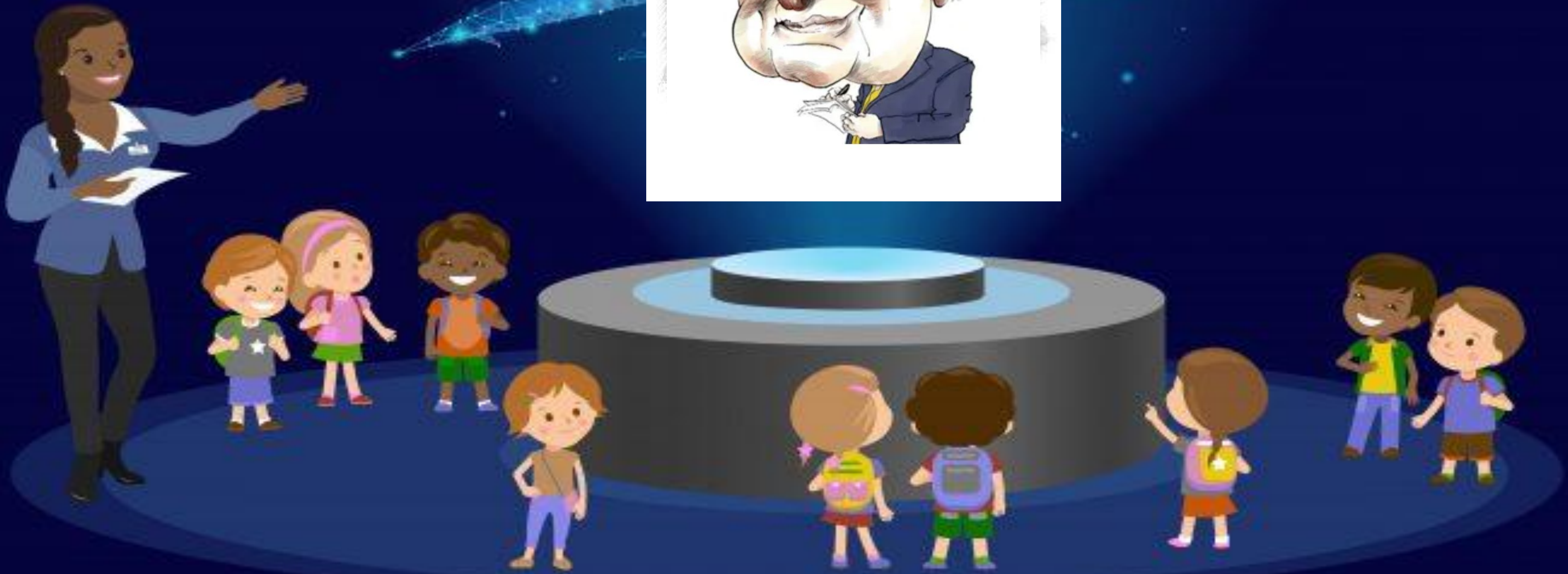




*[no need of fancy higher forms, sorry]*

# PART II





Building a Holographic Superconductor

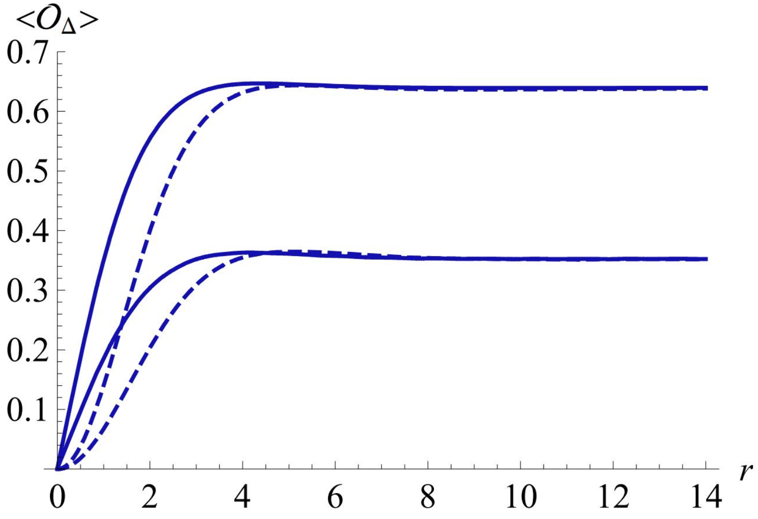
Sean A. Hartnoll, Christopher P. Herzog, and Gary T. Horowitz
Phys. Rev. Lett. **101**, 031601 – Published 14 July 2008

Close enough but not!

**That and 99% of holographic superconductor papers do not describe a superconductor !!**

# Making a real holographic superconductor

*[Many authors …]*




## Holographic Superconductor Vortices

Marc Montull, Alex Pomarol, and Pedro J. Silva
Phys. Rev. Lett. **103**, 091601 – Published 26 August 2009

## Holographic Meissner Effect

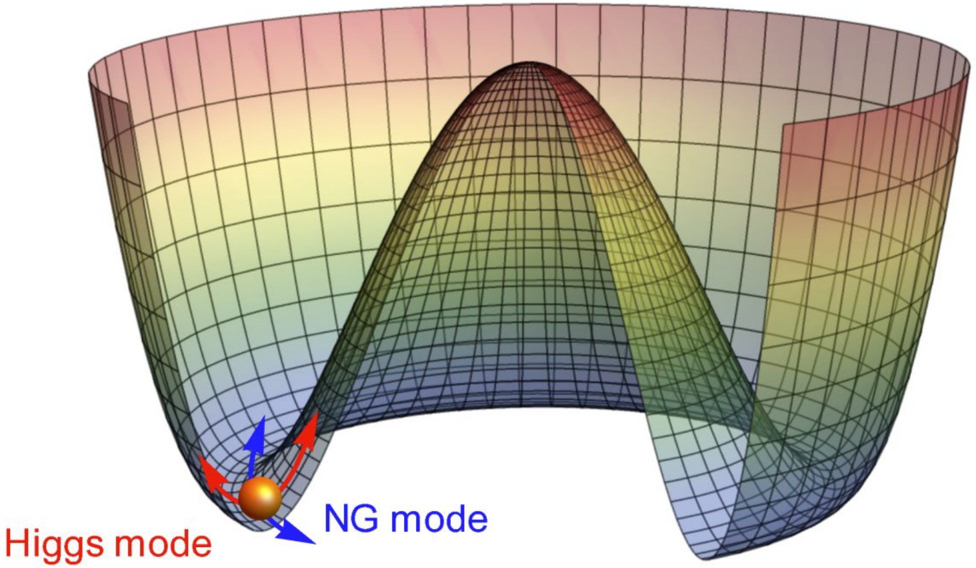
Makoto Natsuume, Takashi Okamura

$$e^2 \mathcal{J}_i = -\frac{1}{\lambda^2} \mathcal{A}_i \ ,$$

$$\partial_j \mathcal{F}^{ij} = e^2 \mathcal{J}^i \ .$$

| SUPERFLUID | SUPERCONDUCTOR |
| --- | --- |
| 1938 Kapitza | 1911 Onnes |
| Neutral particles | Charged particles (electrons) |
| Frictionless flow | Flow of current without resistance |
| Goldstone mode (second sound) | Massive gauge field – Meissner effect (Higgs mechanism) |

# Superfluid Ginzburg-Landau effective theory




Complex scalar order parameter

$$\Psi = |\Psi(r)|e^{i\theta(r)}$$

Phenomenological: $b = \beta\,(T - T_c)$

$$F\,[\Psi] = F_n\,(T) + \int \mathrm{d}^3 r\,\left[a\,|\nabla\Psi|^2 + b\,|\Psi|^2 + \frac{c}{2}\,|\Psi|^4\right]$$

Higgs mode: $\omega^2 = \dfrac{2|b|v^2}{a} + v^2 k^2$, Goldstone mode: $\omega^2 = v^2 k^2$

# Superfluid Hydrodynamics

Conservation law: $$\partial_\mu J^\mu = 0$$

Superfluid velocity: $$u_\phi = \frac{1}{\mu} \nabla \phi$$

Constitutive relation: $$J^\mu = \rho_n u^\mu + \rho_s u_\phi^\mu - \sigma_q T \Delta^{\mu\nu} \partial_\nu (\mu/T) + \dots$$

Josephson relation: $$\mu \partial_t u_\phi = -\nabla \mu + \xi \rho_s \nabla \nabla \cdot u_\phi + \dots$$

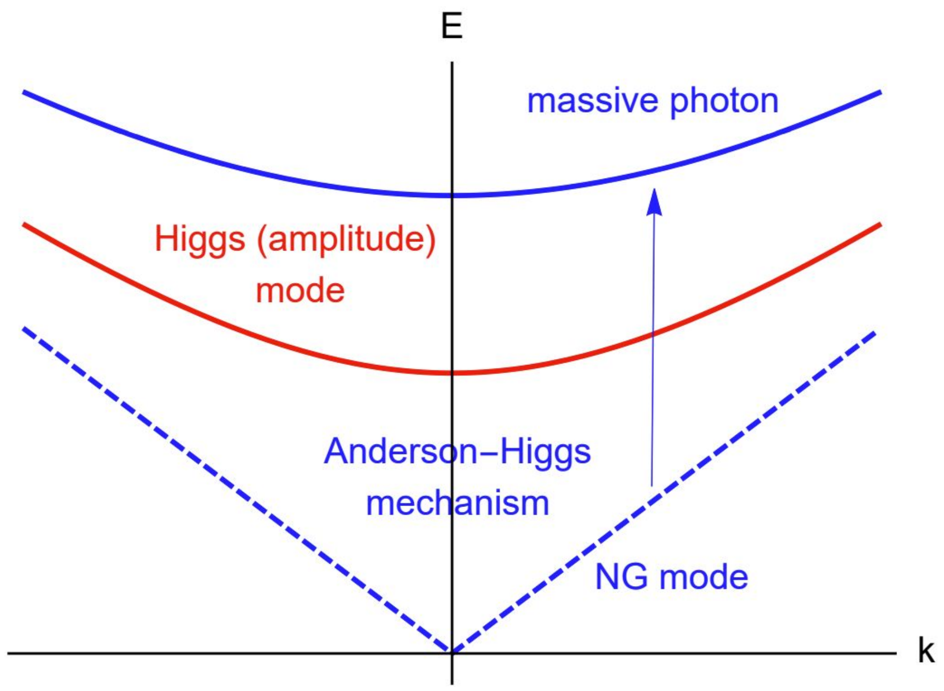
- Sound mode → $v^2 = \frac{\rho_s}{\mu \chi_{\rho\rho}}$
- Infinite conductivity → $\sigma = \sigma_q + \frac{i}{\omega} \frac{\rho_s}{\mu}$
- Extended with momentum-energy conservation
- Matching exactly the holographic superfluid model

→ arXiv:2107.08802

# Superconductor Ginzburg-Landau effective theory

$$\mathcal{L} = a\left(\partial_\mu + i\tilde{q}A_\mu\right)\Psi\;\left(\partial^\mu - i\tilde{q}A^\mu\right)\Psi^* - b\,|\Psi|^2 - \frac{c}{2}\,|\Psi|^4 - \frac{1}{4\lambda}F^2$$

$$|\Psi| = \Psi_0 + \phi\,, \quad (\phi \ll \Psi_0) \qquad \mathcal{L} = a\,\partial_\mu\phi\;\partial^\mu\phi - 2|b|\,\phi^2 + \frac{b^2}{2c} + \boxed{a\,\tilde{q}^2\,\Psi_0^2\,A_\mu A^\mu} - \frac{1}{4\lambda}F^2$$




(1) No Goldstone !

(2) Massive photon:

$$\partial_\mu F^{\mu\nu} + \frac{1}{\lambda_{GL}^2}A^\nu = 0\,, \qquad \lambda_{GL}^2 := \frac{v^2}{2a\,q^2\Psi_0^2\,\lambda}$$

Anderson-Higgs mechanism

# Holographic superconductor

Same bulk action as for the holographic superfluid model

$$S_{\text{bulk}} = \int \mathrm{d}^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |D\Psi|^2 - M^2|\Psi|^2\right]$$

but different boundary conditions !!

$$\delta J^{x\,(L)}_{\text{ext}} = -\frac{\omega}{\lambda}Z^{(L)}_{A_x} - \frac{1}{e^2}\frac{\omega}{\omega^2 - k^2}Z^{(S)}_{A_x}\,, \qquad \delta J^{y\,(L)}_{\text{ext}} = -\frac{\omega^2 - k^2}{\lambda}Z^{(L)}_{A_y} - \frac{1}{e^2}Z^{(S)}_{A_y}$$

Already considered in several papers (seminal works: 0906.2396, 1005.1776)

Our focus: collective excitations!

# Transverse collective modes

$$\omega\,(\omega + i\,\tilde{\sigma}) = \tilde{v}^2 k^2 + m^2\,, \qquad \tilde{v}^2 := 1 - \lambda\,\chi_{BB}$$

EM wave in matter

massive

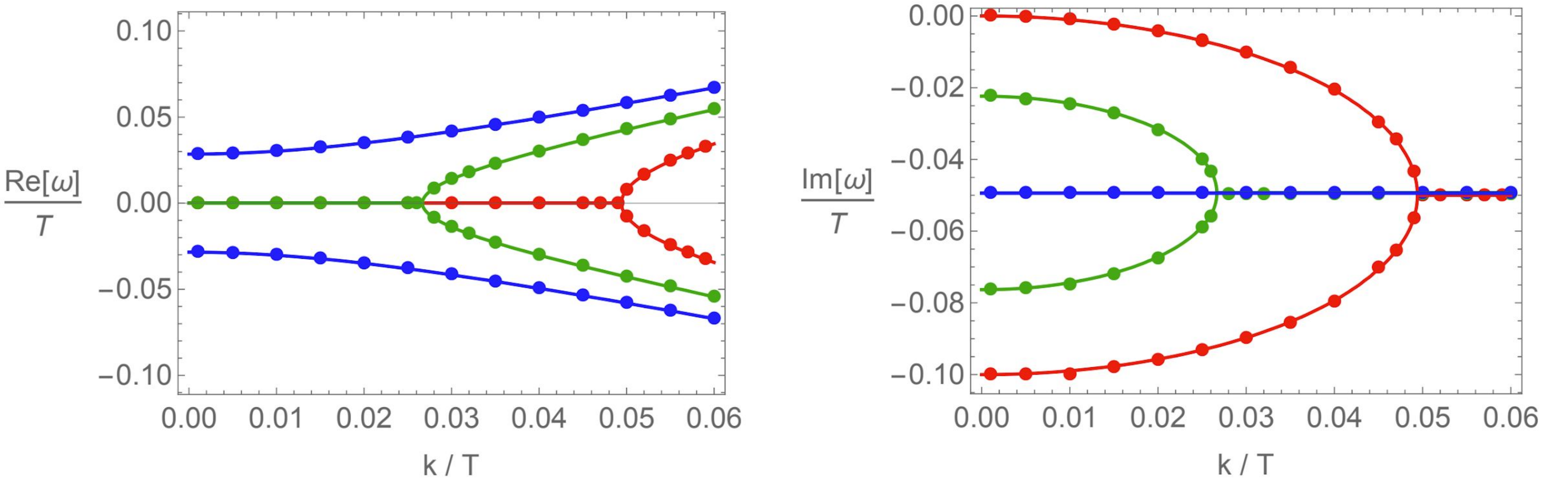
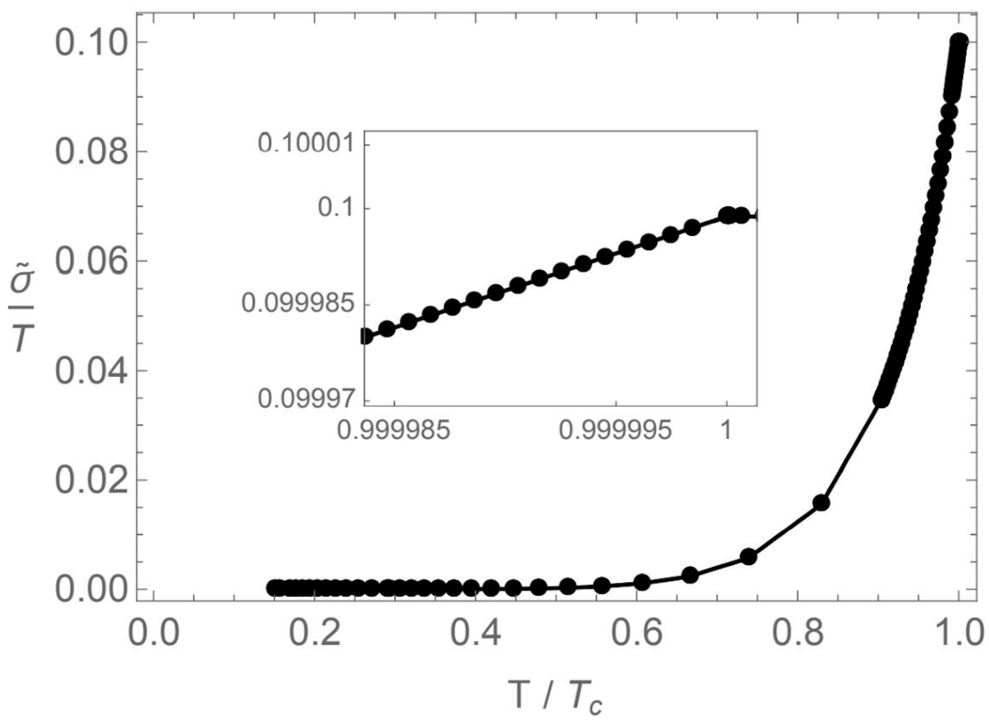
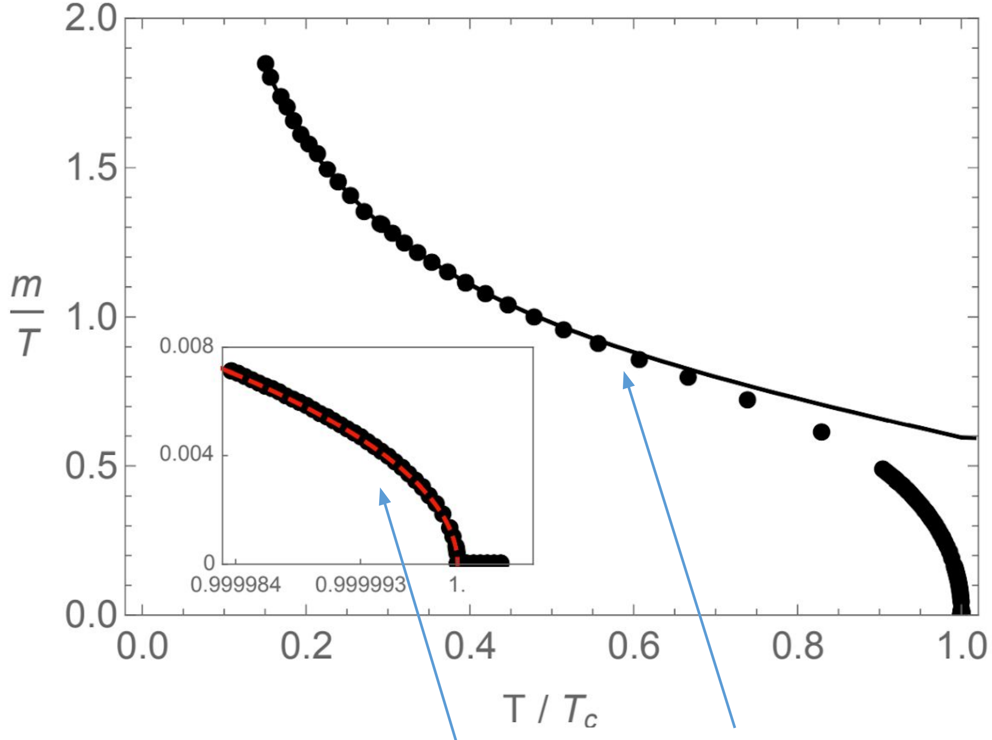



**Figure 4**. The dispersion relation of the lowest collective modes in the transverse sector for different values of the reduced temperature $T/T_c = (1, 0.999, 0.998)$ (red, green, blue). Symbols represent

# Transverse collective modes




$$\tilde{\sigma} = \sigma_0 \lambda$$

$$\sigma_0 := \lim_{\omega \to 0} \mathrm{Re}[\sigma(\omega)]$$

$$m = \Omega_p := \sqrt{\lambda \frac{\rho^2}{\epsilon + p}}$$

$$m = \alpha \sqrt{1 - T/T_c}$$

# Perturbative analytical results

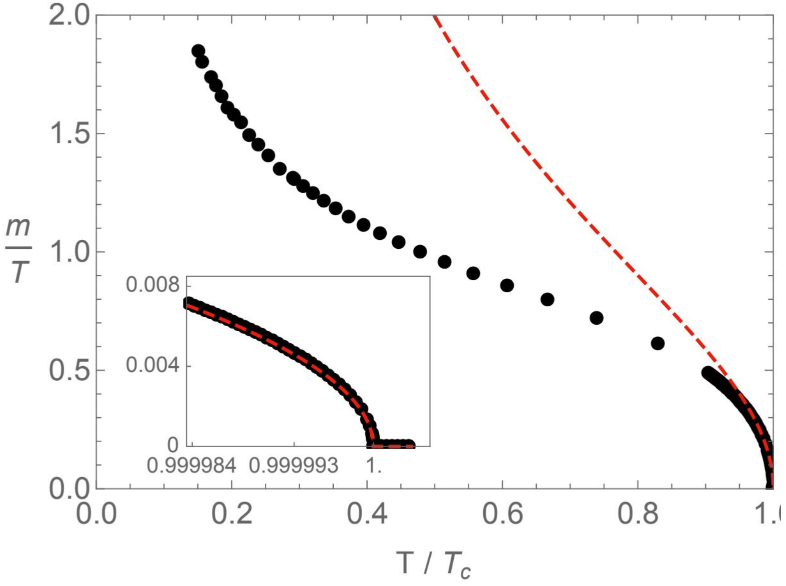
**Holographic Meissner Effect**

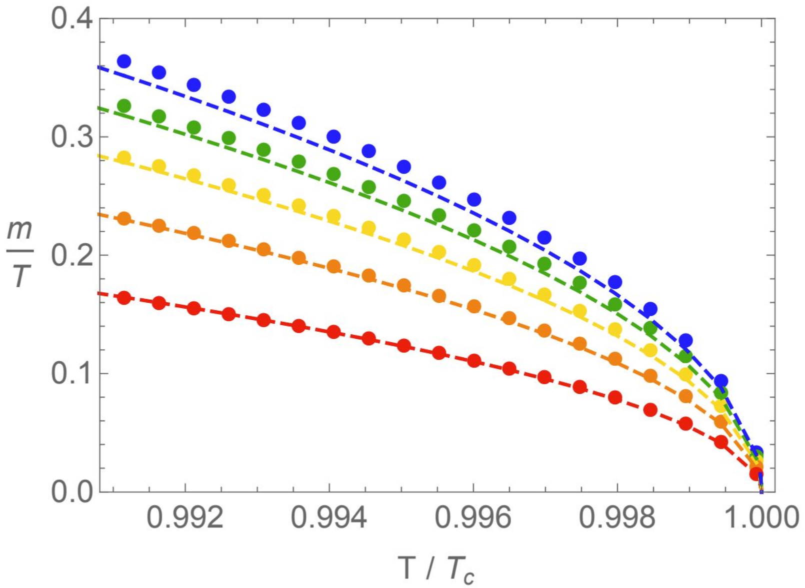
Makoto Natsuume, Takashi Okamura

$$m = \frac{1}{\lambda_{\text{holo}}} = \sqrt{\frac{2\lambda}{1+\lambda} I}$$


$$I := \int_0^1 \mathrm{d}z \left(\frac{\psi(z)}{z}\right)^2$$

holography ⟷ GL theory



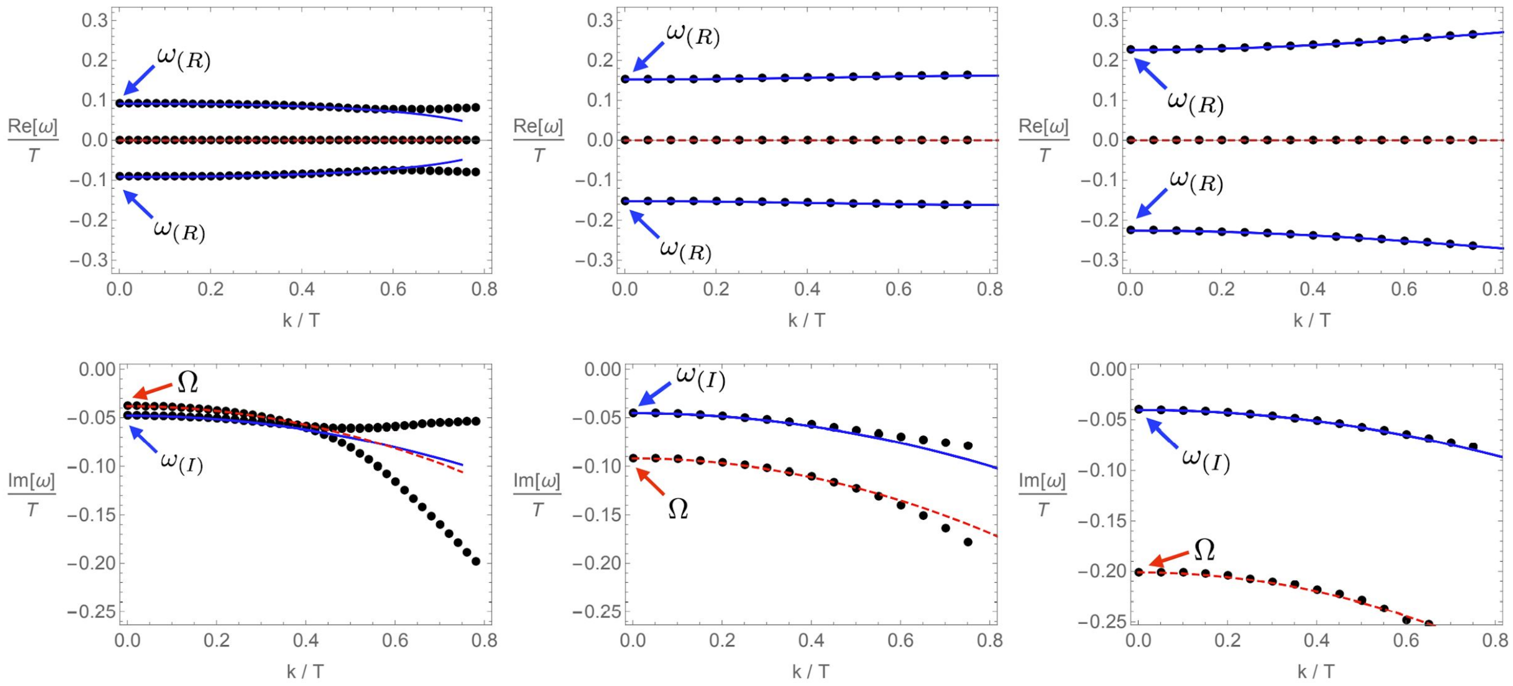



# Longitudinal collective modes

For small wave-vector (very phenomenological)

$$\omega\left(\omega + i\,\underline{\tilde{\sigma}} + i\,\Gamma\,k^2\right) = \mathcal{V}^2\,k^2 + \underline{m^2}\,, \qquad \omega + i\,\Omega + i\,D_\Omega\,k^2 = 0$$

Superfluid sound mode — Superfluid amplitude mode



# The fate of the superfluid sound mode

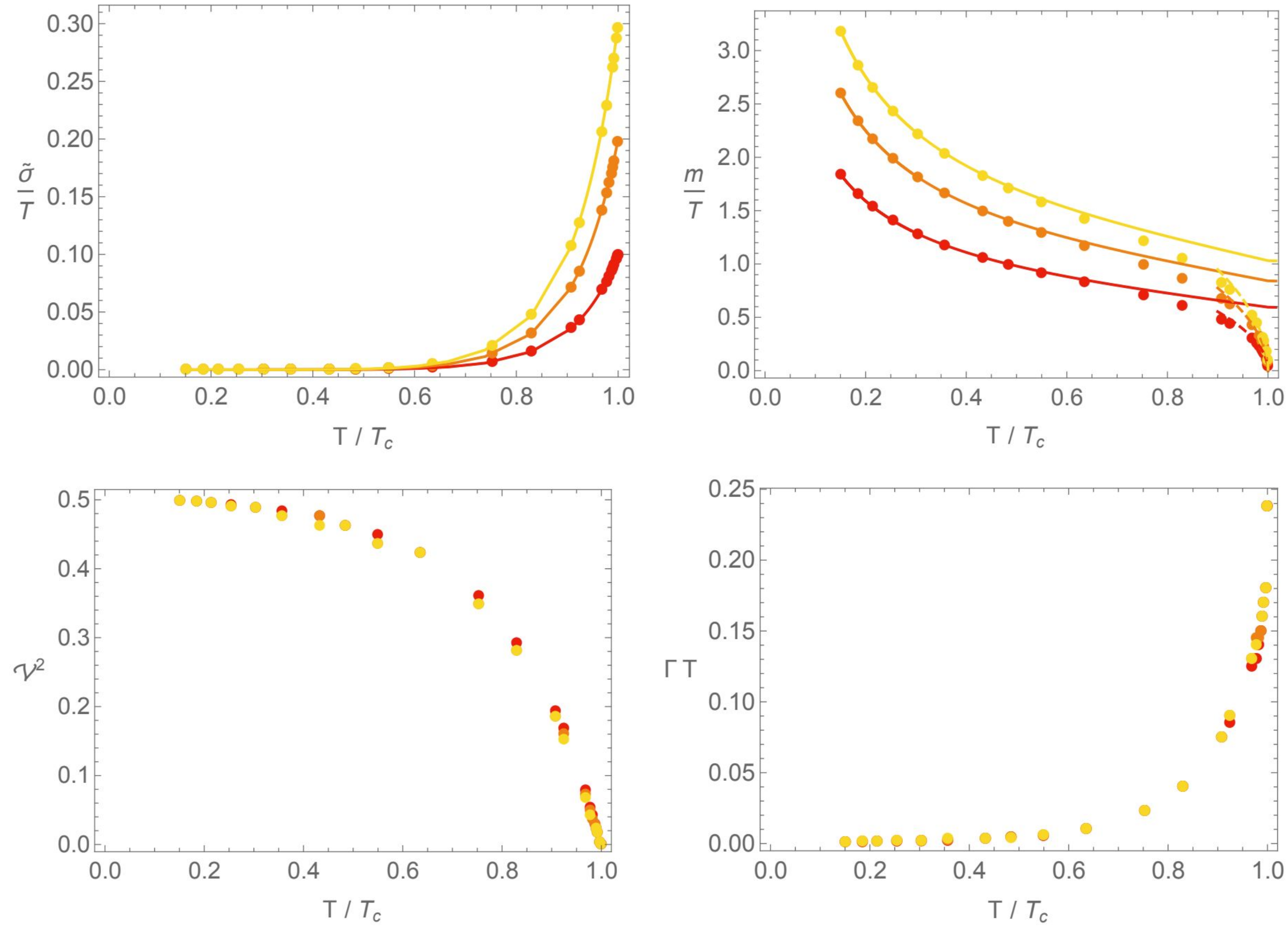



**Figure 13**. Coefficients appearing in the dispersion relation (6.2) as a function of the reduce temperature for $\lambda = 0.1, 0.2, 0.3$ (red, orange, yellow). Solid lines represent Eq.(5.11) for $\tilde{\sigma}$ and

$$\omega\left(\omega + i\,\tilde{\sigma} + i\,\Gamma\,k^2\right) = \mathcal{V}^2\,k^2 + m^2$$

- m is the same as for the transverse mode
- Sigma is the same as for the transverse mode
- Gamma and v do not depend on the Coulomb interaction and they correspond exactly to the speed and the attenuation constant of the superfluid sound mode (0903.2209)

# The Higgs mode

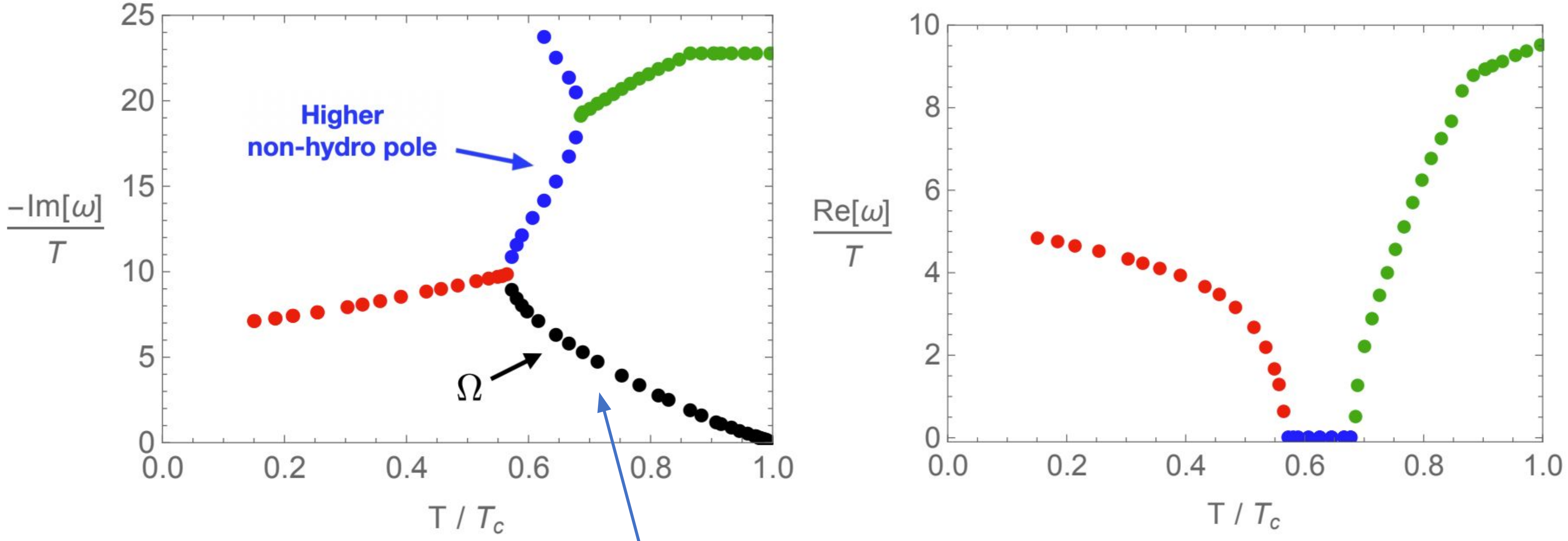
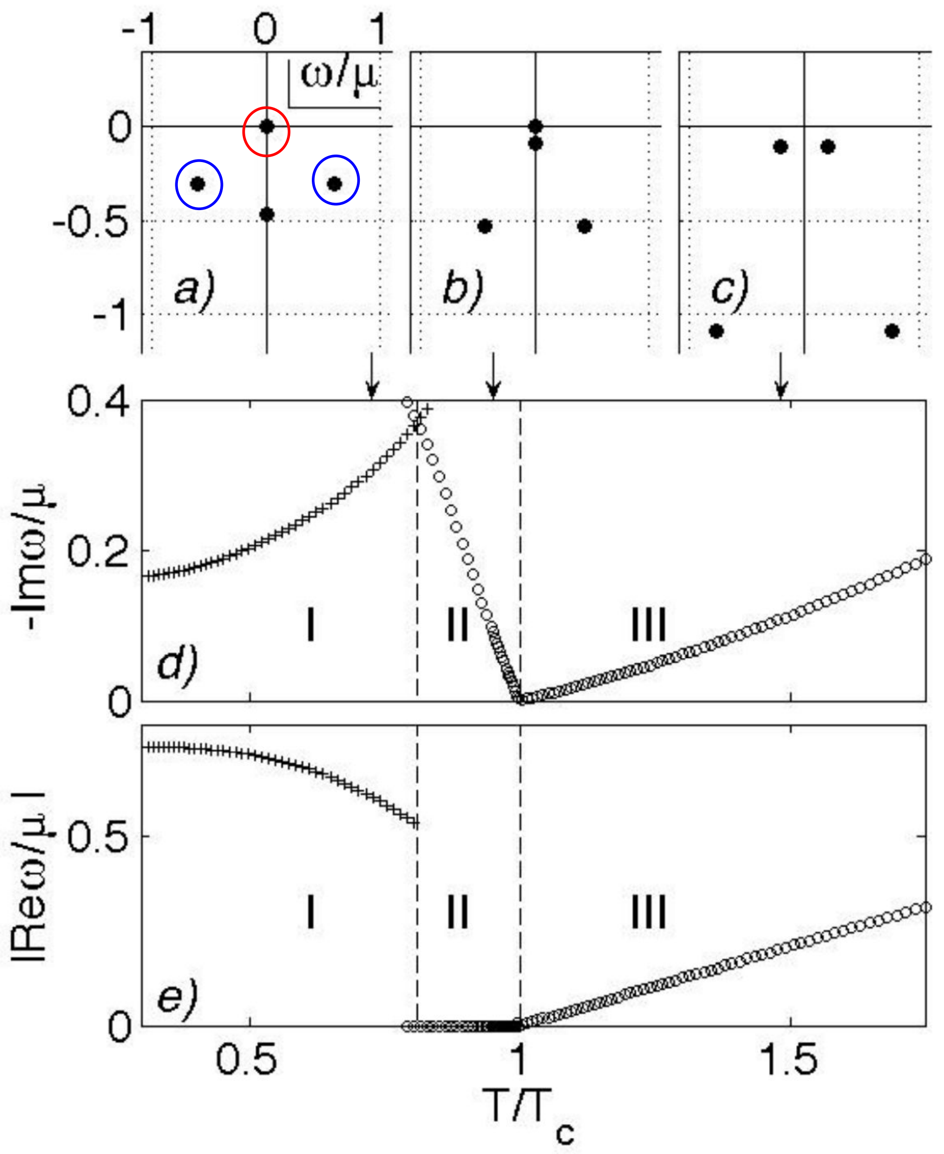



**Figure 15**. The dynamics of the Higgs mode as a function of the reduce temperature. The black dots encode the fluctuations of the amplitude of the order parameter close to the critical point.

Amplitude mode

# VS holographic superfluids




Results from 1207.4194

Overdamped to underdamped crossover but totally different dynamics !

(1) The massive mode at low T is a non-hydro pole which becomes soft: ○

(2) Always a hydro mode: ○

# Higgs mode at low temperature

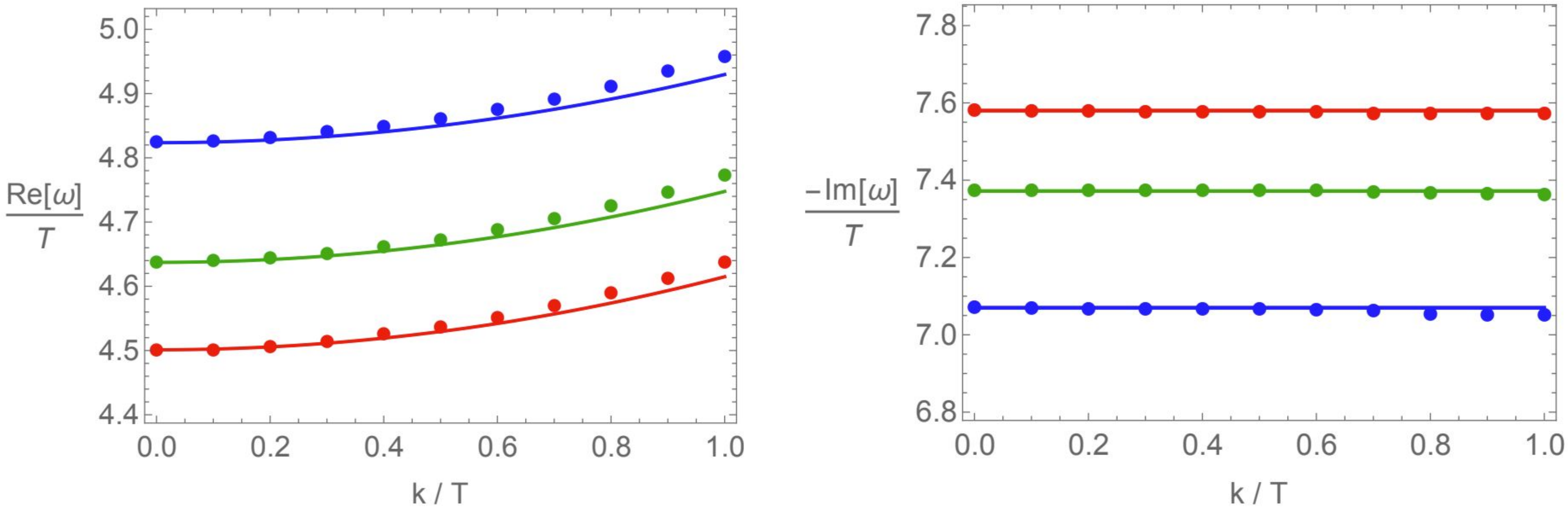
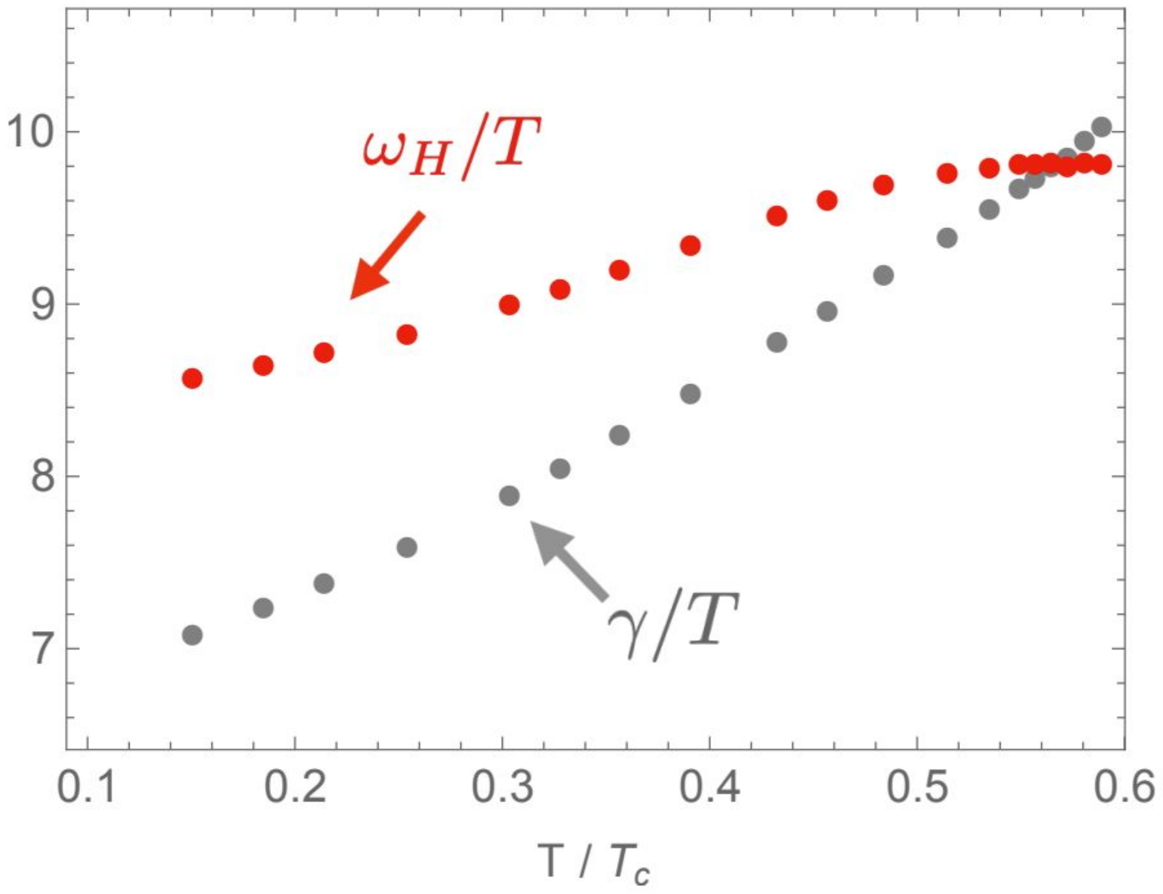



**Figure 18**. The dispersion relation of the Higgs mode at low temperature: $T/T_c = (0.15, 0.21, 0.25)$ (blue, green, red). **Left:** Re$[\omega]$ vs. $k$. **Right:** Im$[\omega]$ vs. $k$. The solid lines are the predictions from GL theory, Eq.(6.9).

# Overdamped to underdamped crossover

$$\omega^2 = \omega_H^2 + v^2 k^2 - i2\gamma\omega\,, \qquad \omega_H^2 := \frac{2|b|v^2}{a}$$

$$\omega = -i\gamma \pm \sqrt{\omega_H^2 - \gamma^2}\,.$$



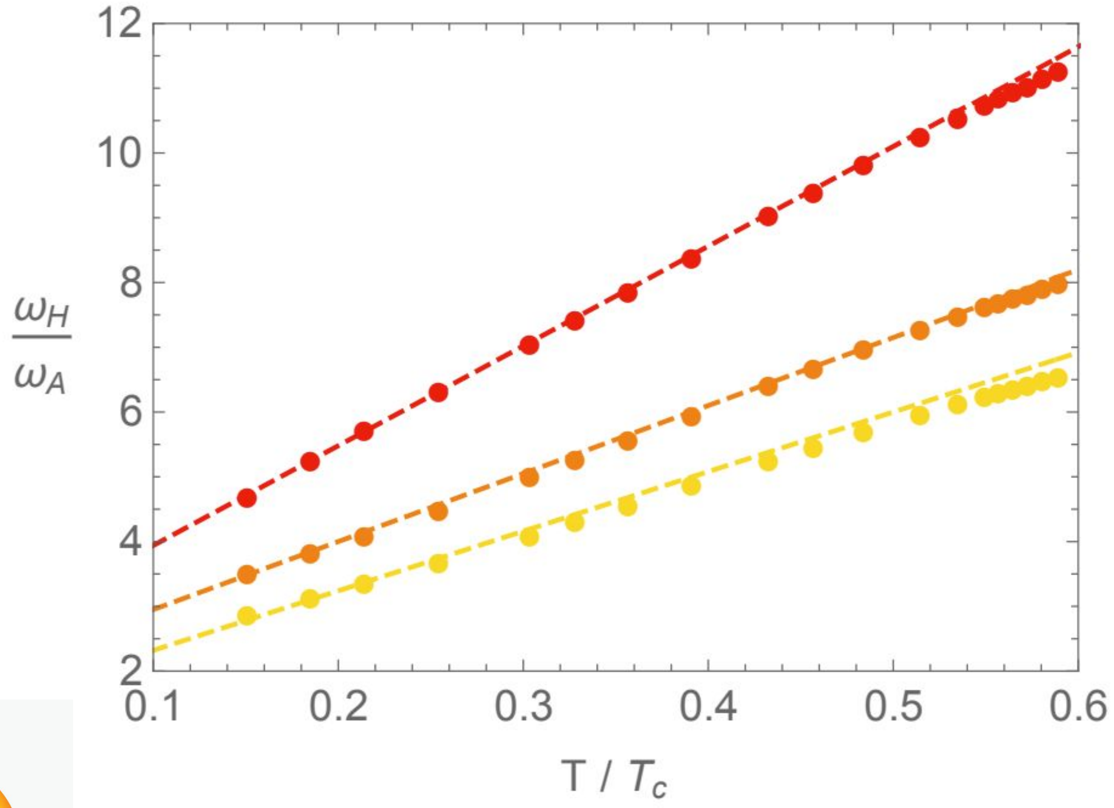

Crossover around T=Tc/2

[Interesting aside: dissipative term does seem to vanish at T=0, backreaction ?]

# Higgs mass

$$\kappa_{GL} \sim \frac{m_{\rm H}}{m_{\rm A}} \sim \frac{1}{\sqrt{\lambda}}$$

The GL scaling is recovered

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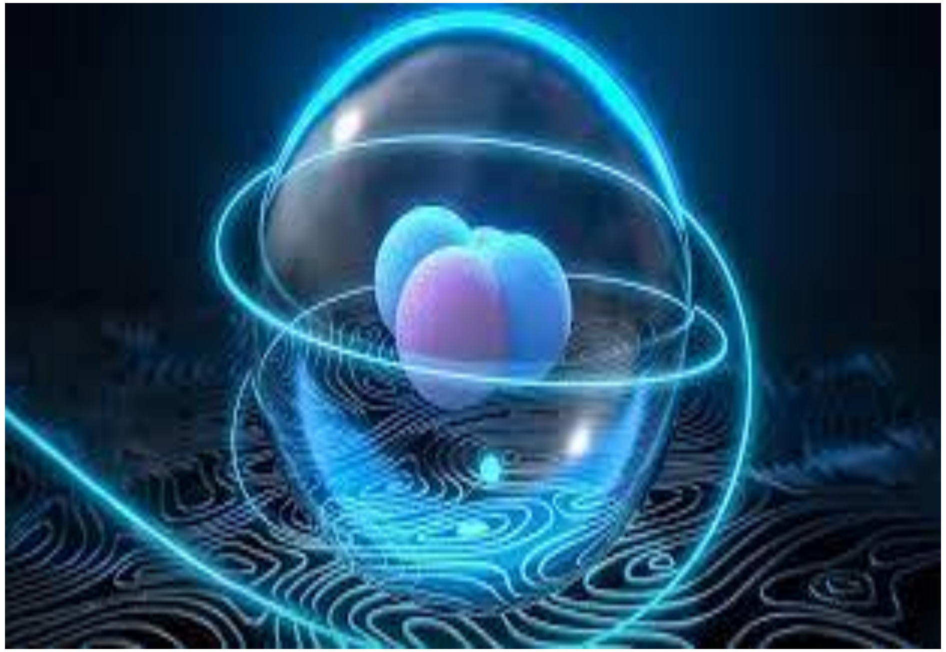
$$\left.\frac{\omega_H}{2\Delta}\right|_{T\approx 0.15T_c} \approx 0.162$$

Much lower than

$$\omega_H = 2\Delta$$

# Summary of part II

**Hol. superfluid + mixed b.c.s.** **=** **Hol. superconductor**




Gauge symmetries in the boundary field theory

Dynamical gauge fields at the boundary

Many applications (cond-mat, plasma physics, QCD, MHD, cosmology)

Relation with bulk higher-form symmetries


Thank you for listening!

more to come...

# First application

Building a Holographic Superconductor

Sean A. Hartnoll, Christopher P. Herzog, and Gary T. Horowitz
Phys. Rev. Lett. **101**, 031601 – Published 14 July 2008

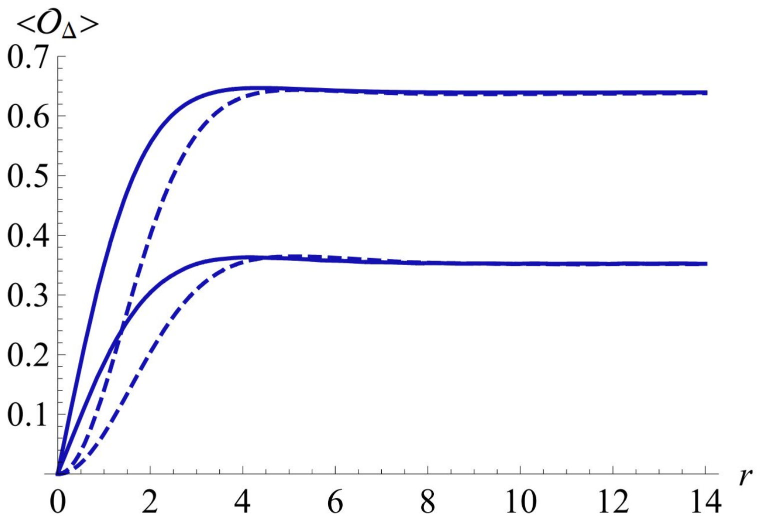
## Making a real holographic superconductor (2009)

*[Many authors …]*

*Example.*

Holographic Superconductor Vortices

Marc Montull, Alex Pomarol, and Pedro J. Silva
Phys. Rev. Lett. **103**, 091601 – Published 26 August 2009



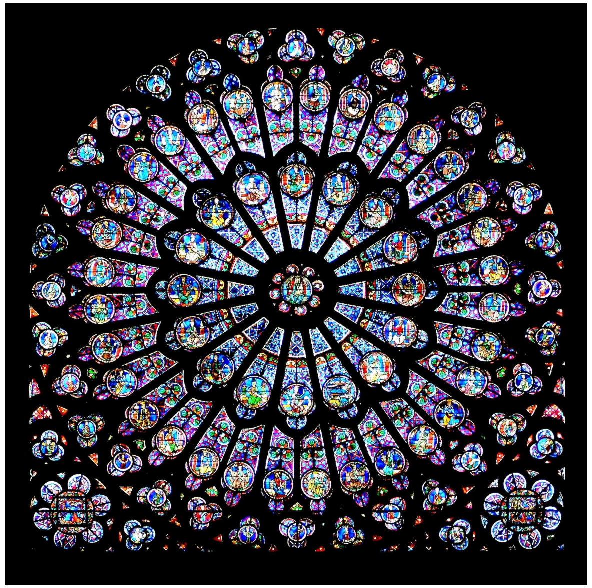
# Recent application

**High Energy Physics - Theory**

*[Submitted on 15 Dec 2017 (v1), last revised 26 Nov 2018 (this version, v4)]*

## Holographic Plasmons

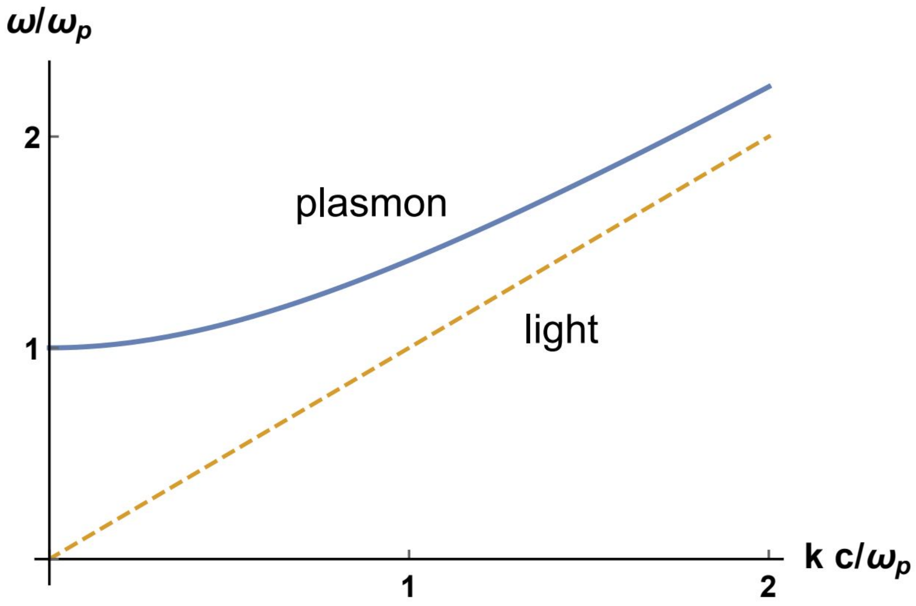
Ulf Gran, Marcus Tornsö, Tobias Zingg



Maxwell equation
Continuity equation
(Drude model)

$$\epsilon(\omega) = 1 + \frac{4\pi i \sigma(\omega)}{\omega}$$

$$\boxed{\omega^2 = \omega_p^2 + c^2 k^2}$$



more to come...

# See also

## Holographic Meissner Effect

Makoto Natsuume, Takashi Okamura

The holographic superconductor is the holographic dual of superconductivity, but there is no Meissner effect in the standard holographic superconductor. This is because the boundary Maxwell field is added as an external source and is not dynamical. We show the Meissner effect analytically by imposing the semiclassical Maxwell equation on the AdS boundary. Unlike in the Ginzburg-Landau (GL) theory, the extreme Type I limit cannot be reached even in the $e \to \infty$ limit where $e$ is the $U(1)$ coupling of the boundary Maxwell field. This is due to the bound current which is present even in the pure bulk Maxwell theory. In the bulk 5-dimensional case, the GL parameter and the dual GL theory are obtained analytically for the order parameter of scaling dimension 2.

$$e^2 \mathcal{J}_i = -\frac{1}{\lambda^2} \mathcal{A}_i \ ,$$
$$\partial_j \mathcal{F}^{ij} = e^2 \mathcal{J}^i \ .$$

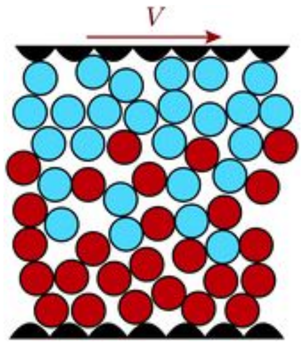
$$\delta_{g_{\mu\nu}} S_{\mathbf{m}} = \frac{1}{2}\int \mathrm{d}^3x \, \sqrt{-g}\, T^{\mu\nu}_{\mathbf{m}}\, \delta g_{\mu\nu} \,, \quad \delta_{A_\mu} S_{\mathbf{m}} = \int \mathrm{d}^3x \, \sqrt{-g}\, J^{\mu}_{\mathbf{m}}\, \delta A_{\mu} \,.$$

$$\begin{aligned} T^{\mu\nu}_{\mathrm{m}} &= T^{\mu\nu} - T^{\mu\nu}_{\mathrm{EM}} = \epsilon_{\mathrm{m}}\, u^\mu u^\nu + p_{\mathrm{m}}\, \Delta^{\mu\nu} - M^{\mu\gamma}_{\mathrm{m}}\, F^{\nu}_{\ \gamma} + \Pi^{\mu\nu} \\ J^{\mu}_{\mathrm{m}} &= J^{\mu} - J^{\mu}_{\mathrm{EM}} \ = \rho_{\mathrm{m}}\, u^\mu + \nabla_\nu M^{\mu\nu}_{\mathrm{m}} + \nu^\mu \,, \end{aligned}$$

$$\begin{aligned} \left(\frac{\partial p}{\partial B}\right)_{T,\mu} &= \left(\frac{\partial p_{\mathrm{m}}}{\partial B}\right)_{T,\mu} - \frac{B}{\lambda} = \chi_{BB} B - \frac{B}{\lambda} \,, \\ \left(\frac{\partial \epsilon}{\partial B}\right)_{T,\mu} &= \left(\frac{\partial \epsilon_{\mathrm{m}}}{\partial B}\right)_{T,\mu} + \frac{B}{\lambda} = -2\chi_{BB} B + \frac{B}{\lambda} \,, \end{aligned}$$

# Zero density and zero B

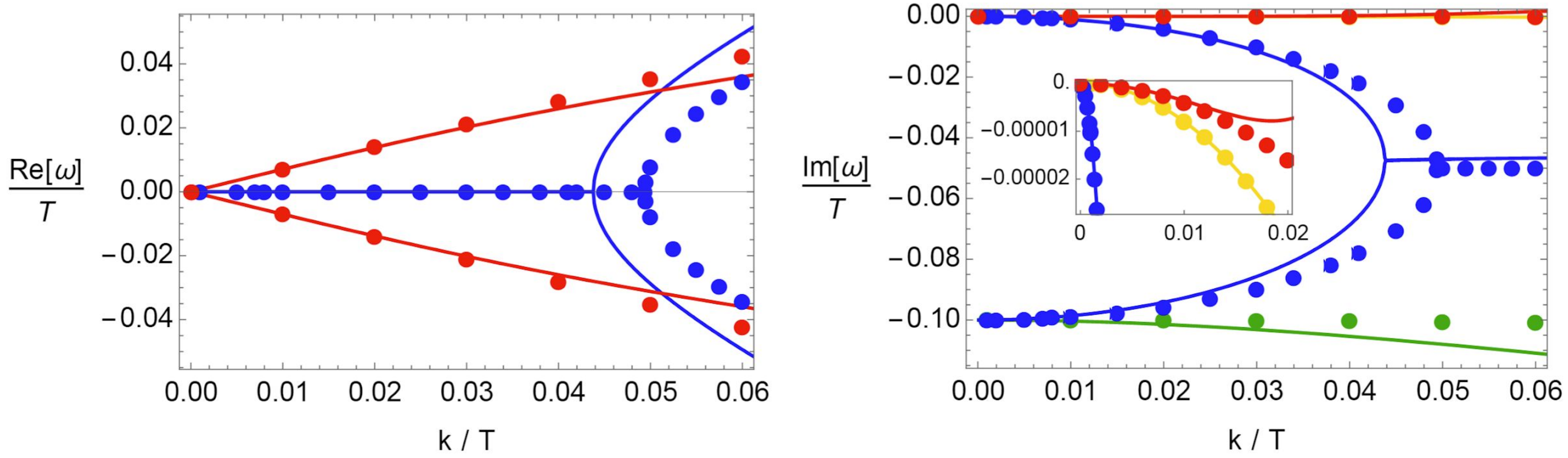
$$\omega = \pm v_s\, k - i\frac{\Gamma_s}{2}\,k^2\,, \qquad \omega = -i\frac{\eta}{\epsilon + p}\,k^2$$




where $v_s^2 = \partial p/\partial\epsilon$ and $\Gamma_s = \eta/(\epsilon + p)$.

The stress tensor sector is “trivial” since decouples

# Zero density and zero B




sound waves

shear diffusion

EM waves

damped charge diffusion

# Hydrodynamics breakdown

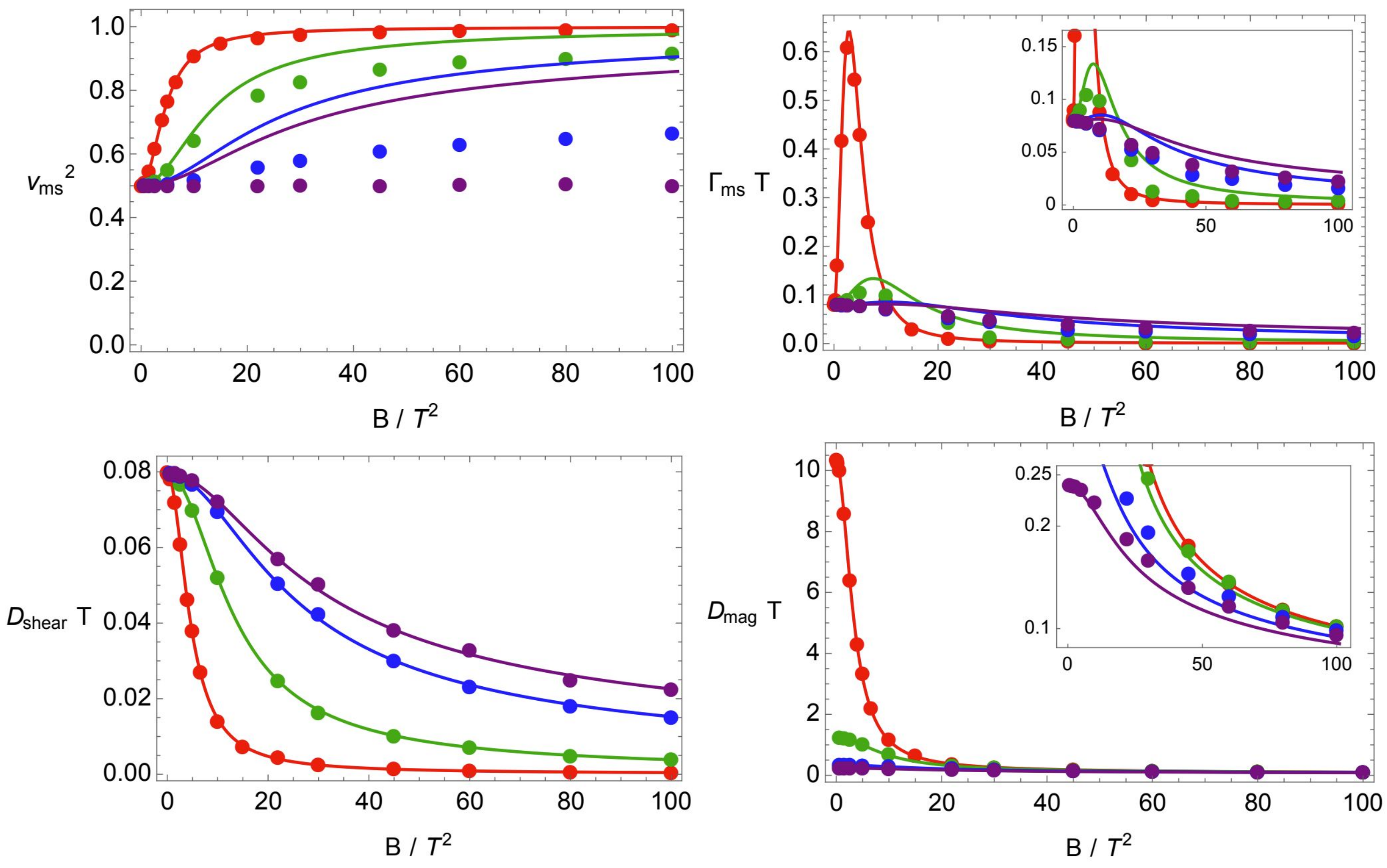



**Figure 16**. **Top:** Speed and attenuation constant of magnetosonic waves. **Bottom:** The diffusion constants of shear and magnetic diffusion. The colors correspond to $\lambda/T = (0.1, 1, 10, 1000)$ (red, green, blue, purple). The insets are a zoom in the low magnetic field regime.

We see a failure of the hydro description only at large magnetic field together with large EM coupling.

We suspect the problem is the EM coupling

$$\frac{1}{\lambda}\nabla_\nu F^{\mu\nu} = J^\mu_{\rm free} + J^\mu_{\rm bound} + J^\mu_{\rm ext}\,, \tag{2.18}$$

in which $J^\mu_{\rm free} := \rho_{\rm m} u^\mu + \nu^\mu$ and $J^\mu_{\rm bound} := \nabla_\nu M^{\mu\nu}_{\rm m}$. $J^\mu_{\rm free}$ refers to the current of free charges while $J^\mu_{\rm bound}$ incorporates the polarization effects. **We can decompose the polarization tensor $M^{\mu\nu}$ and $\mathcal{H}^{\mu\nu}$ with respect to fluid velocity as**

$$\begin{aligned} M^{\mu\nu}_{\rm m} &= P^\mu u^\nu - P^\nu u^\mu - \epsilon^{\mu\nu\rho} u_\rho M\,, \\ \mathcal{H}^{\mu\nu} &= u^\mu D^\nu - u^\nu D^\mu - \epsilon^{\mu\nu\rho} u_\rho H\,, \end{aligned} \tag{2.19}$$

and can also be identified with $M^{\mu\nu}_{\rm m} = 2\partial p_{\rm m}/\partial F_{\mu\nu}$, $\mathcal{H}^{\mu\nu} = 2\partial p/\partial F_{\mu\nu}$. In (3+1) dimensions [50], the magnetization $M$ in Eq.(2.19) becomes the magnetic polarization vector $M^\mu$. The electric polarization vector $P^\mu$ and the magnetization $M$ are associated with the electric field $E^\mu$ and magnetic field $B$ via the susceptibilities $(\chi_{EE}\,, \chi_{BB})$, i.e.,

$$P^\mu = \chi_{EE} E^\mu\,, \qquad M = \chi_{BB} B\,, \tag{2.20}$$

with

$$\chi_{EE} = 2\frac{\partial p_{\rm m}}{\partial E^2}\,, \qquad \chi_{BB} = 2\frac{\partial p_{\rm m}}{\partial B^2}\,. \tag{2.21}$$

**The physical meaning of $D^\mu$ and $H$ are the electric displacement vector and the magnetic $H$-field. This can be seen by re-writing Eq. (2.18) in terms of $\mathcal{H}^{\mu\nu}$**

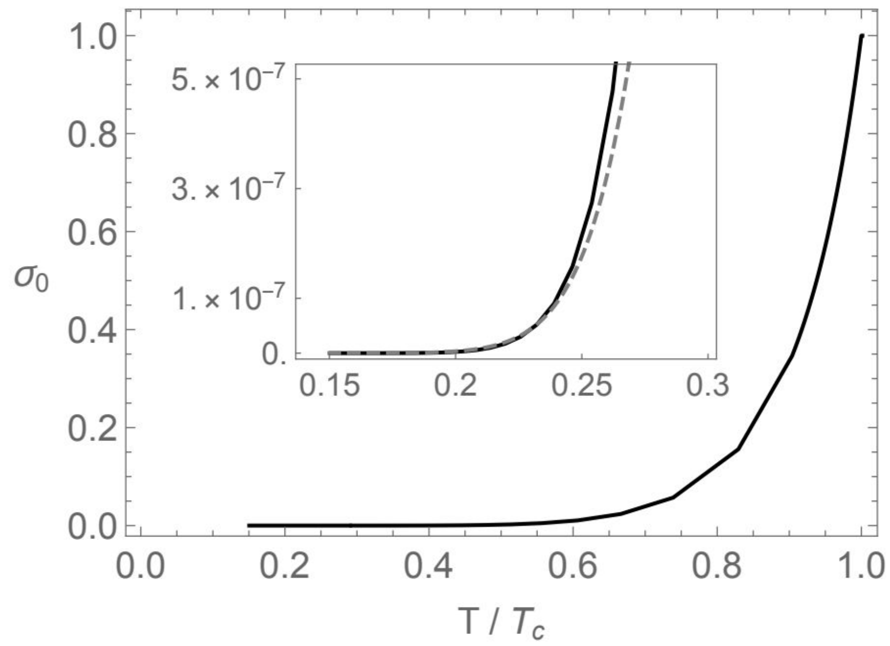
$$\nabla_\nu \mathcal{H}^{\mu\nu} = J^\mu_{\rm free} + J^\mu_{\rm ext}\,. \tag{2.22}$$

Eq.(2.20) implies that $D^\mu$ and $H$ are also proportional to the electric and magnetic field $E^\mu$ and $B$ via the following relations

$$D^\mu = \frac{1}{\lambda}E^\mu + P^\mu = \epsilon_{\rm e} E^\mu\,, \qquad H = \frac{1}{\lambda}B - M = \frac{1}{\mu_{\rm m}}B\,, \tag{2.23}$$

in which we have defined the electric permittivity $\epsilon_{\rm e}$ and the magnetic permeability $\mu_{\rm m}$. Using all the previous identities and definitions, we finally arrive at the following identities

$$\chi_{EE} = \epsilon_{\rm e} - \frac{1}{\lambda}\,, \qquad \chi_{BB} = \frac{1}{\lambda} - \frac{1}{\mu_{\rm m}}\,, \tag{2.24}$$




the superconducting energy gap $\Delta$ via

$$\sigma_0 \sim e^{-\Delta/T}, \qquad \Delta := \sqrt{\langle O_2 \rangle}/2,$$