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Spindles and new (0,2) SCFTs

Matthew M. Roberts
APCTP

JHEP **10** (2022) 067 [arXiv:2207.06427]
with Igal Arav (Amsterdam), Jerome P. Gauntlett (Imperial), and Christopher Rosen (U. Crete)

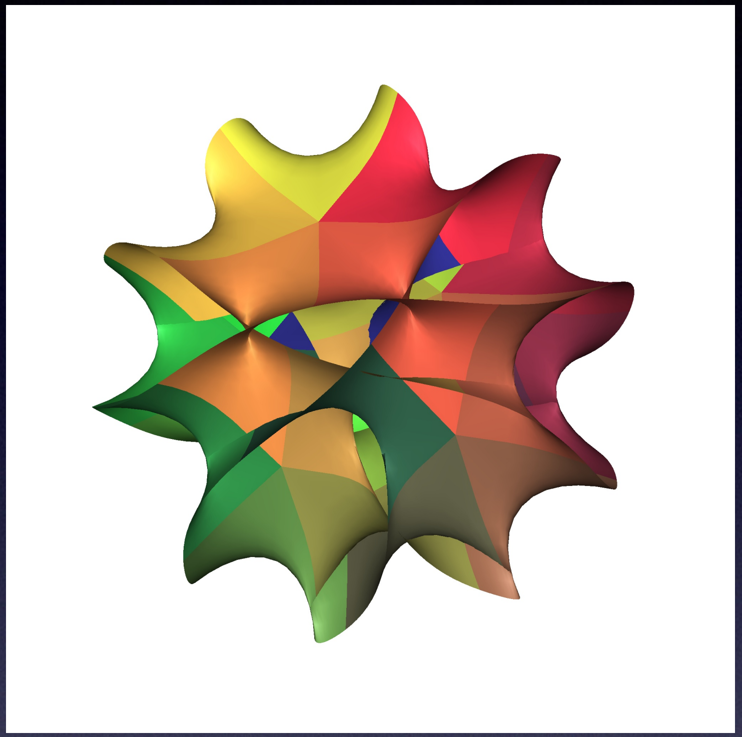
2023/02/22, 6th International Conference on Holography, String Theory and Spacetime in Da Nang

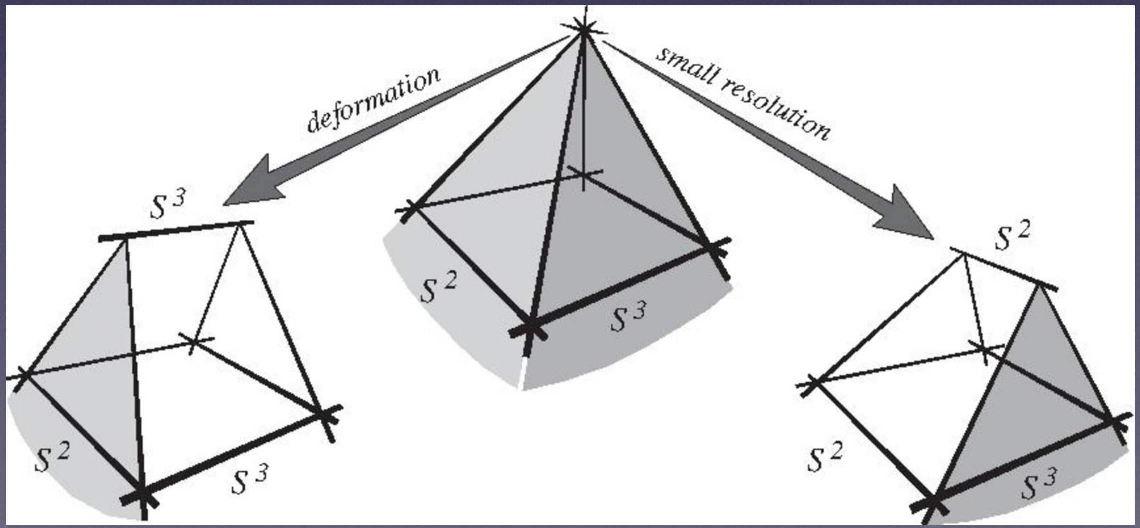
Outline

- What is a spindle?
    - Spindles in string theory
- The Leigh-Strassler theory
    - Wrapping Leigh-Strassler on a spindle

Compactifications and defects

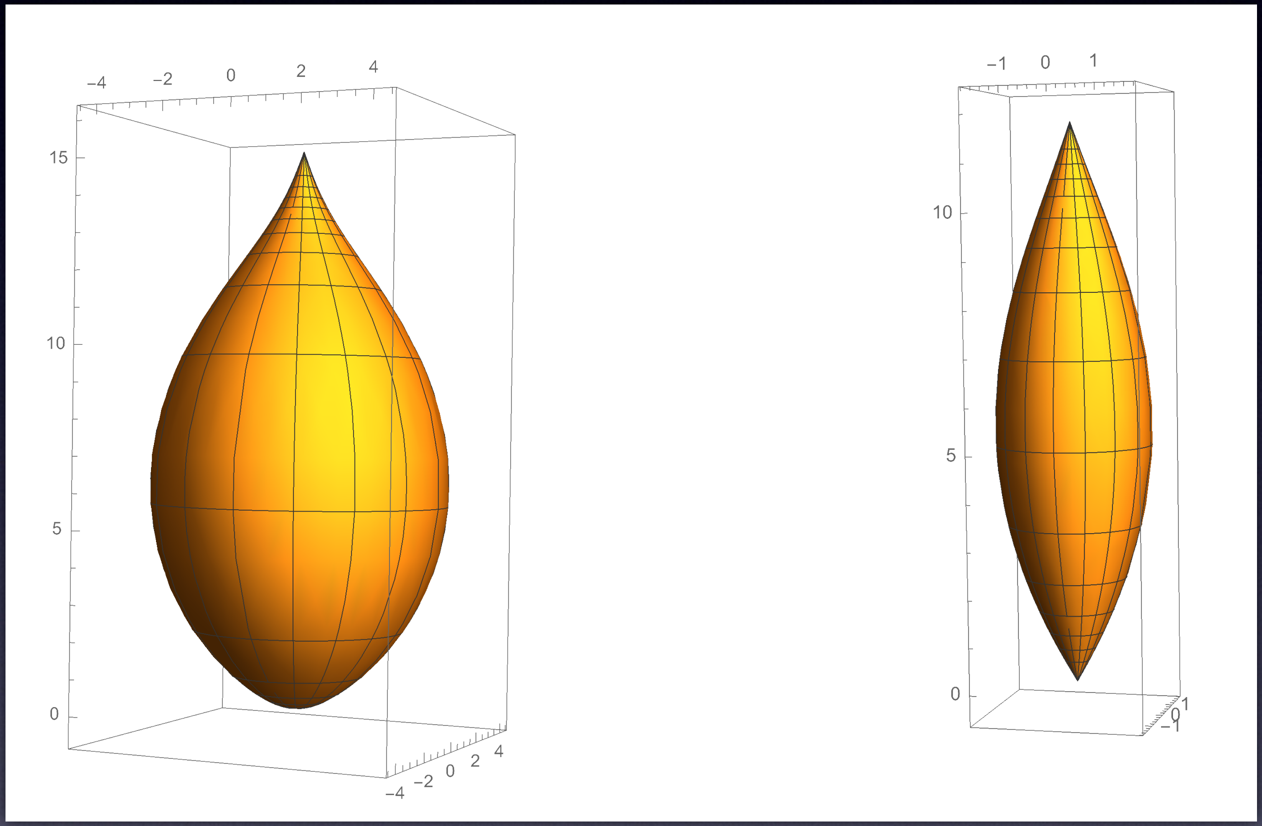
- String theory predicts too many dimensions, so a great deal of time is spent trying to "hide" them by compactifications
- We also study lower dimensional QFTs via compactifications of higher dimensional theories
- We usually ask for these small manifolds to be smooth, but sometimes allow "acceptable" singularities (defects)
- SUSY as a guiding principle - strong evidence that *all* non-SUSY compactifications are unstable (swampland)





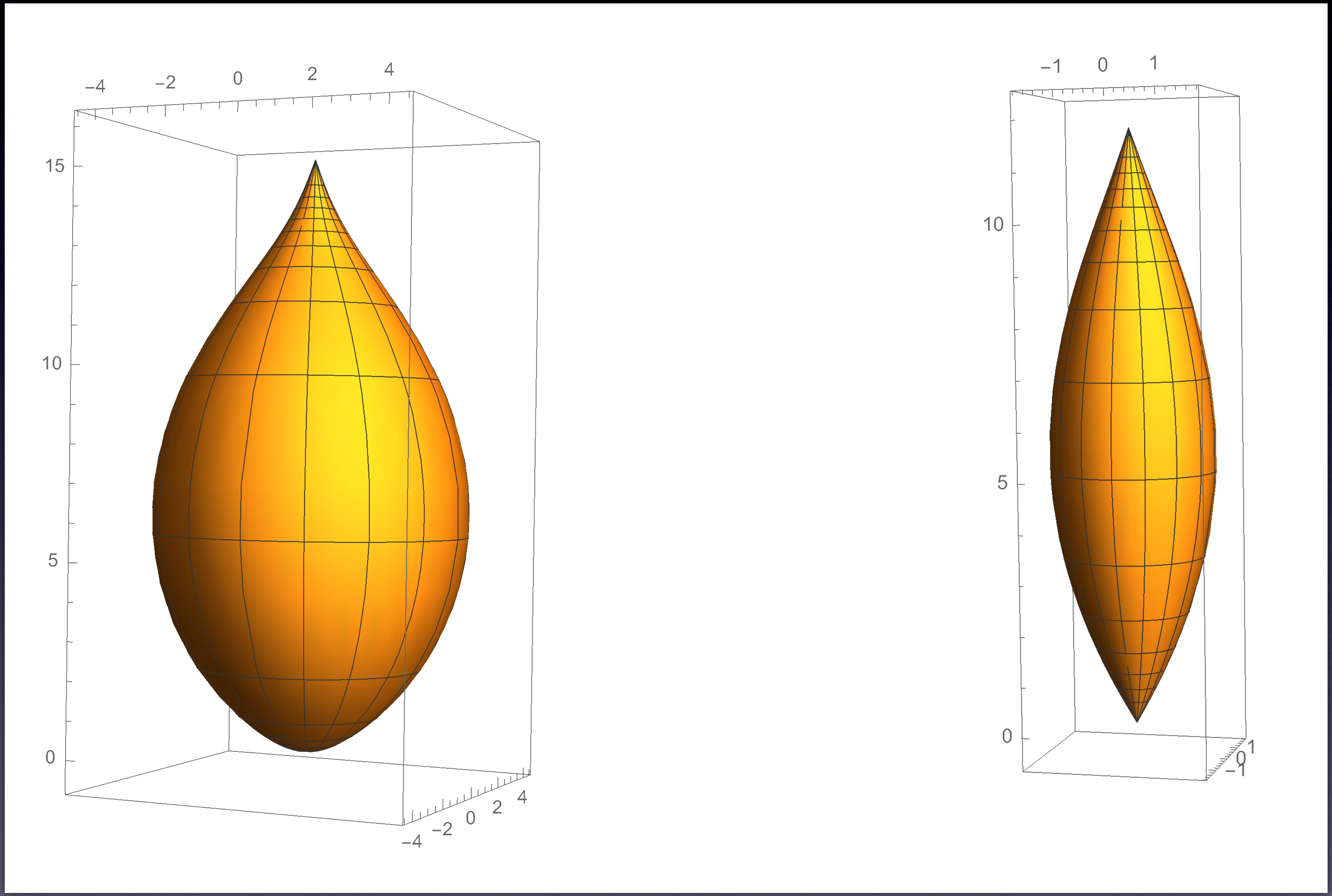
What is a spindle?

- Droplet geometry
- Topological S^2 with conical deficits
- Orbifold (locally $\mathbb{R}^n/\Gamma$)
- Not a “good” orbifold (no smooth cover), but can be thought of as two $\mathbb{C}/Z_{n_\pm}$ glued together



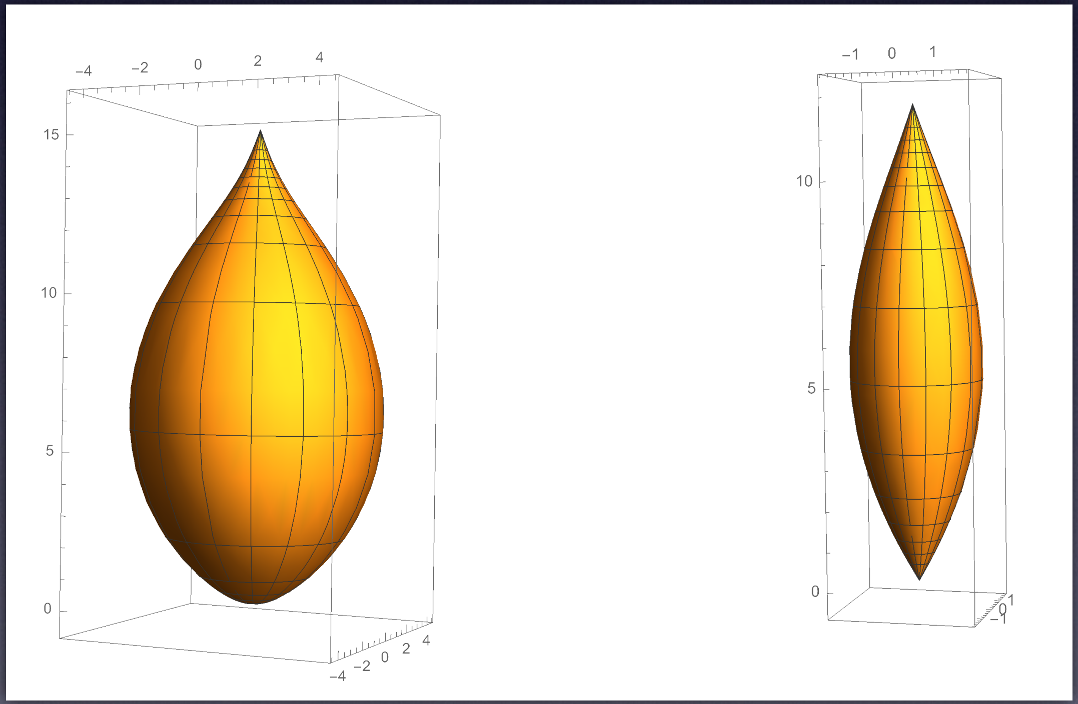
- Canonical example:
- $\mathbb{WCP}_{[n_+,n_-]} = \{[z_1, z_2] : (z_1, z_2) \sim (\lambda^{n_+} z_1, \lambda^{n_-} z_2)\}$
- Poles at $z_1, z_2 = 0$: when $n_\pm \neq 1$, conical defect, $\theta \sim \theta + 2\pi/n_\pm$
- No constant curvature metric (except trivial case)

- $\mathbb{WCP}_{[n_+,n_-]} = \{[z_1, z_2] : (z_1, z_2) \sim (\lambda^{n_+} z_1, \lambda^{n_-} z_2)\}$



$\mathbb{WCP}_{[3,1]}$ $\mathbb{WCP}_{[3,2]}$

- General axisymmetric spindle:
- $ds^2 = d\theta^2 + h(\theta)^2 d\phi^2, \ \phi \sim \phi + 2\pi$
  - $h(\theta_\pm) = 0$
  - $h'(\theta_\pm) = \pm \dfrac{1}{n_\pm}$



Spindles and bundles

- Why spindles?
  - Generalization of S^2 for compactifications
- Can we trust them in string theory?
  - If we have gauge fields, connections of an "orbibundle", the *total space* can be regular
- New class of compactifications at our disposal

S^3 and spindles

- Hopf fibration of S^3: T^2 over interval
  - $ds^2 = d\theta^2 + \cos^2\theta \sin^2\theta d\bar{\mu}^2 + (d\bar{\nu} + \sin^2\theta d\bar{\mu})^2$
- We can view as $U(1)$ bundle over round S^2:
- $$N_\phi = 1, \qquad \chi = 2$$
  - Spindles are a simple generalization of this

- Round S^3: $ds^2 = d\theta^2 + \cos^2\theta d\phi_1^2 + \sin^2\theta d\phi_2^2$
  - Pick new torus basis:
    $\phi_1 = n_+\psi_2,\ \phi_2 = n_-\psi_2 + \psi_1/n_+$
  - $\mathbb{WCP}_{[n_+,n_-]}$: $(z_1, z_2) \to (\lambda^{n_+}z_1, \lambda^{n_-}z_2)$
    action generated by $V = n_+\partial_{\phi_1} + n_-\partial_{\phi_2} = \partial_{\psi_2}$
- $ds^2 = d\theta^2 + F(\theta)d\psi_1{}^2 + G(\theta)(d\psi_2 + A)^2$
- $$N_\phi = \int \frac{dA}{2\pi} = \frac{1}{n_+n_-}, \quad \chi = \int \frac{\omega}{2\pi} = \frac{n_+ + n_-}{n_+n_-}$$

Orbibundles and spindles

- In general, when fluxes are appropriately quantized in orbibundle sense, the total space is regular
- $$g \int \frac{F}{2\pi} = \frac{p}{n_+ n_-}, \qquad p \in \mathbb{Z}$$
- Connections for global symmetries are *explicitly* geometric in a KK sense ($SO(6)_R \to S^5$ fibration)
- When we have orbibundle spindles, the uplifted 10d solution is totally regular

Holographic SCFTs on spindles

- Wrapping on a spindle: looking for 5d SUSY solutions of the form $AdS_3 \times \Sigma_{\text{spindle}}$
  - dual to compactifying a 4d SCFT on a spindle
  - (Much harder problem: Flows from AdS_5 with conformal boundary $\mathbb{R}^{(1,1)} \times \Sigma_{\text{spindle}}$)
- We will consider a minimally SUSY case:
- The 4d $\mathcal{N} = 1$ Leigh-Strassler SCFT

SUSY compactification on S^2

- We can preserve SUSY on curved spaces by "twisting":
- $\delta\psi_\mu = 0 = D_\mu\epsilon = \left(\partial_\mu + \frac{1}{4}\gamma^{ab}\omega_{ab\mu} - i\frac{g}{2}A_\mu\right)\epsilon$
- Fix a chirality on S^2, $\gamma^{12}\epsilon = \pm i\epsilon$,
- Balance geometric curvature $R^{(2)}$ against gauge field curvature $F^{(2)}$
- $A \pm \omega_{12} = 0$, $\epsilon = \text{const}.$, $N_\phi = \pm\chi$
- Charged fields reduce to LLL on S^2

Leigh-Strassler fixed point

- Think of maximal $\mathcal{N} = 4$ SYM in $\mathcal{N} = 1$ language:
  - Three chirals, one vector
- Add a mass to one chiral only, conjectured to flow to strongly coupled LS fixed point
  - Lorentz invariant flow well studied holographically
  - $a^{\mathcal{N}=4} = N^2/4 > a^{LS} = 27N^2/128$

Bulk ingredients

- Key ingredients (from 5d perspective)
- For LS flow:
  - 1 Cx. Scalar: $\Delta = 3$, $tr(\lambda\lambda + \mathcal{O}(X^3))$
  - 2 Re. Scalars: $\Delta = 2$, $tr(X^2)$
- For spindles:
  - 3 gauge fields: $U(1)^3 \subset SU(4)$

Bulk ingredients

- Useful basis for gauge fields:
- $U(1)_R : A_1 + A_2 + A_3,\ D\epsilon = \nabla\epsilon - i\frac{g}{2}(A_1 + A_2 + A_3)\epsilon,$
- $U(1)_B : A_1 + A_2 - A_3,\ D\phi = \partial\phi - ig(A_1 + A_2 - A_3)\phi,$
- $U(1)_F : A_1 - A_2$, neither ϵ nor ϕ are charged

Δ=3

Δ=2

The bosonic part of the Lagrangian, in a *mostly minus* signature, is given by

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{4}R + \tfrac{1}{2}(\partial\varphi)^2 + \tfrac{1}{8}\sinh^2 2\varphi\,(D\theta)^2 + 3(\partial\alpha)^2 + (\partial\beta)^2 - \mathcal{P} \\ &- \tfrac{1}{4}\left[e^{4\alpha-4\beta}F^{(1)}_{\mu\nu}F^{(1)\mu\nu} + e^{4\alpha+4\beta}F^{(2)}_{\mu\nu}F^{(2)\mu\nu} + e^{-8\alpha}F^{(3)}_{\mu\nu}F^{(3)\mu\nu}\right] \\ &+ \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma\delta}F^{(1)}_{\mu\nu}F^{(2)}_{\rho\sigma}A^{(3)}_{\delta}\,, \end{aligned} \tag{2.1}$$

U(1)³

where

$$D\theta \equiv d\theta + g\left(A^{(1)} + A^{(2)} - A^{(3)}\right)\,. \tag{2.2}$$

The scalar potential $\mathcal{P}$ is given by

$$\mathcal{P} = \frac{g^2}{8}\left[\left(\frac{\partial W}{\partial\varphi}\right)^2 + \frac{1}{6}\left(\frac{\partial W}{\partial\alpha}\right)^2 + \frac{1}{2}\left(\frac{\partial W}{\partial\beta}\right)^2\right] - \frac{g^2}{3}W^2\,, \tag{2.3}$$

where W is the "superpotential" defined by

$$W = -\frac{1}{4}\left[\left(e^{-2\alpha-2\beta} + e^{-2\alpha+2\beta} - e^{4\alpha}\right)\cosh 2\varphi + \left(e^{-2\alpha-2\beta} + e^{-2\alpha+2\beta} + 3e^{4\alpha}\right)\right]. \tag{2.4}$$

LS vacuum: $e^{6\alpha} = 2,\ e^{2\varphi} = 3,\ \beta = 0$

$$\Delta^{IR} = 1 + \sqrt{7},\ 2 + \sqrt{7},\ 3 + \sqrt{7}$$

Plan of attack

- Ansatz: $ds^2 = e^{2V(\theta)}d(AdS_3)^2 + d\theta^2 + h(\theta)^2 d\phi^2$
- $A^{(i)} = a_i(\theta)d\phi$, $\varphi = \varphi(\theta)$ etc.
- We will study BPS equations of 5d theory to look for SUSY spindle backgrounds (total SUSY is 2d (0,2)):
  - Labelled by (n_+, n_-) and fluxes $g\int \frac{F_i}{2\pi} = \frac{p_i}{n_+ n_-}$
  - With uplift to 10d, we confirm the total space is regular

- Since chiral mass is *charged* under $U(1)_B$, we tune flux $p_B = p_1 + p_2 - p_3 = 0$
  - Mass is constant over spindle in UV - Assumption! If $p_B \neq 0$, then mass term must vanish at one/both poles
- We allow $p_R = p_1 + p_2 + p_3 \neq 0, \;\; p_F = p_1 - p_2 \neq 0$
- BPS equations: coupled, first order ODEs
  - Technical details in our paper

- SUSY spindles have *two* compactification classes:
  - Twist (the usual case):
  - $$g\int\frac{F_R}{2\pi}=\frac{p_1+p_2+p_3}{n_+n_-}=\pm\frac{n_++n_-}{n_+n_-}=\pm\chi$$
  - $\gamma^{12}\epsilon=\pm\, i\epsilon$ everywhere on spindle
  - Simple generalization of smooth twist

- “Anti-twist” (new, doesn’t occur on S^2):
- $$g\int \frac{F_R}{2\pi} = \frac{p_1 + p_2 + p_3}{n_+ n_-} = \pm \frac{n_+ - n_-}{n_+ n_-}$$
    - $\gamma^{12}\epsilon\,|_{N.P.} = \pm\, i\epsilon\,|_{N.P.}, \qquad \gamma^{12}\epsilon\,|_{S.P.} = \mp\, i\epsilon\,|_{S.P.}$
    - ϵ *rotates* from chiral to anti-chiral between the two poles of the spindle
- $[\cos\xi\gamma^{12} + \sin\xi\gamma^{1}]\epsilon = +\,i\epsilon, \;\; \xi\,|_{N.P.} = 0, \; \xi\,|_{S.P.} = \pi$

Homework: staring at these for a few months

with these Killing spinors can be written in the form

$$
\begin{aligned}
f^{-1}\xi' &= gW\cos\xi + 2\kappa e^{-V}\,,\\
f^{-1}V' &= \frac{g}{3}W\sin\xi\,,\\
f^{-1}\alpha' &= -\frac{g}{12}\partial_\alpha W\sin\xi\,,\\
f^{-1}\beta' &= -\frac{g}{4}\partial_\beta W\sin\xi\,,\\
f^{-1}\varphi' &= -\frac{g}{2}\frac{\partial_\varphi W}{\sin\xi}\,,\\
f^{-1}\frac{h'}{h} &= \frac{1}{\sin\xi}\Big(2\kappa e^{-V}\cos\xi + \frac{gW}{3}(1+2\cos^2\xi)\Big)\,,
\end{aligned}
\tag{3.10}
$$

along with the two constraint equations

$$
\begin{aligned}
(s-Q_z)\sin\xi &= -\frac{1}{2}gWh\cos\xi - \kappa h e^{-V}\,,\\
\frac{g}{2}\partial_\varphi W\cos\xi &= \partial_\varphi Q_z\sin\xi h^{-1}\,.
\end{aligned}
\tag{3.11}
$$

Furthermore the field strength components in the orthonormal frame are given by

$$
\begin{aligned}
e^{2\alpha-2\beta}F^{(1)}_{34} &= -\frac{g}{12}[4W-\partial_\alpha W+3\partial_\beta W]\cos\xi - \kappa e^{-V}\,,\\
e^{2\alpha+2\beta}F^{(2)}_{34} &= -\frac{g}{12}[4W-\partial_\alpha W-3\partial_\beta W]\cos\xi - \kappa e^{-V}\,,\\
e^{-4\alpha}F^{(3)}_{34} &= -\frac{g}{6}[2W+\partial_\alpha W]\cos\xi - \kappa e^{-V}\,.
\end{aligned}
\tag{3.12}
$$

Results

- Solutions are “nearly analytic”: system not quite integrable, but can analytically extract almost all data
- *All* LS spindles are in “anti-twist” class:
  - $$g\int \frac{F_R}{2\pi} = \frac{p_1 + p_2 + p_3}{n_+ n_-} = \pm \frac{n_+ - n_-}{n_+ n_-},$$
  - Compare with $\chi(\Sigma) = \dfrac{n_+ + n_-}{n_+ n_-}$: *not* topological twist!
  - In $\mathcal{N} = 4$ SYM wrappings, both twist and anti-twist occur

- Other fluxes: $p_B = 0$ by fiat
- Regular solutions exist only when $0 \leq 2|p_F| < |n_+ - n_-|$
- We can calculate the central charge *analytically*
- $$c^{LS} = \frac{3(n_+ - n_-)\left[(n_+ - n_-)^2 - 4p_F^2\right]\left[3(n_+ - n_-)^2 + 4p_F^2\right]}{32n_+n_-\left[(n_+ - n_-)^2(n_+^2 + n_+n_- + n_-^2) + 4n_+n_-p_F^2\right]}N^2$$

- "Minimal flux" case: $p_F = p_B = 0$,
- Only $p_R = \dfrac{n_+ - n_-}{4n_+n_-} \neq 0$
- $c = \dfrac{4(n_+ - n_-)^3}{3n_+n_-(n_+^2 + n_+n_- + n_-^2)} a_{LS}, \qquad a_{LS} = \dfrac{27N^2}{128}$

Comments

- Antitwist S^2 case, $n_+ = n_- = 1$, is *excluded* ($c = 0$)
- We can also calculate c_{2D} via c-extremisation (reducing 4d anomaly polynomial on spindle), with exact agreement!
- In field theory, c-ext. works for twist *and* anti-twist
  - *Supergravity* tells us twist case doesn't happen (reality conditions on solution)

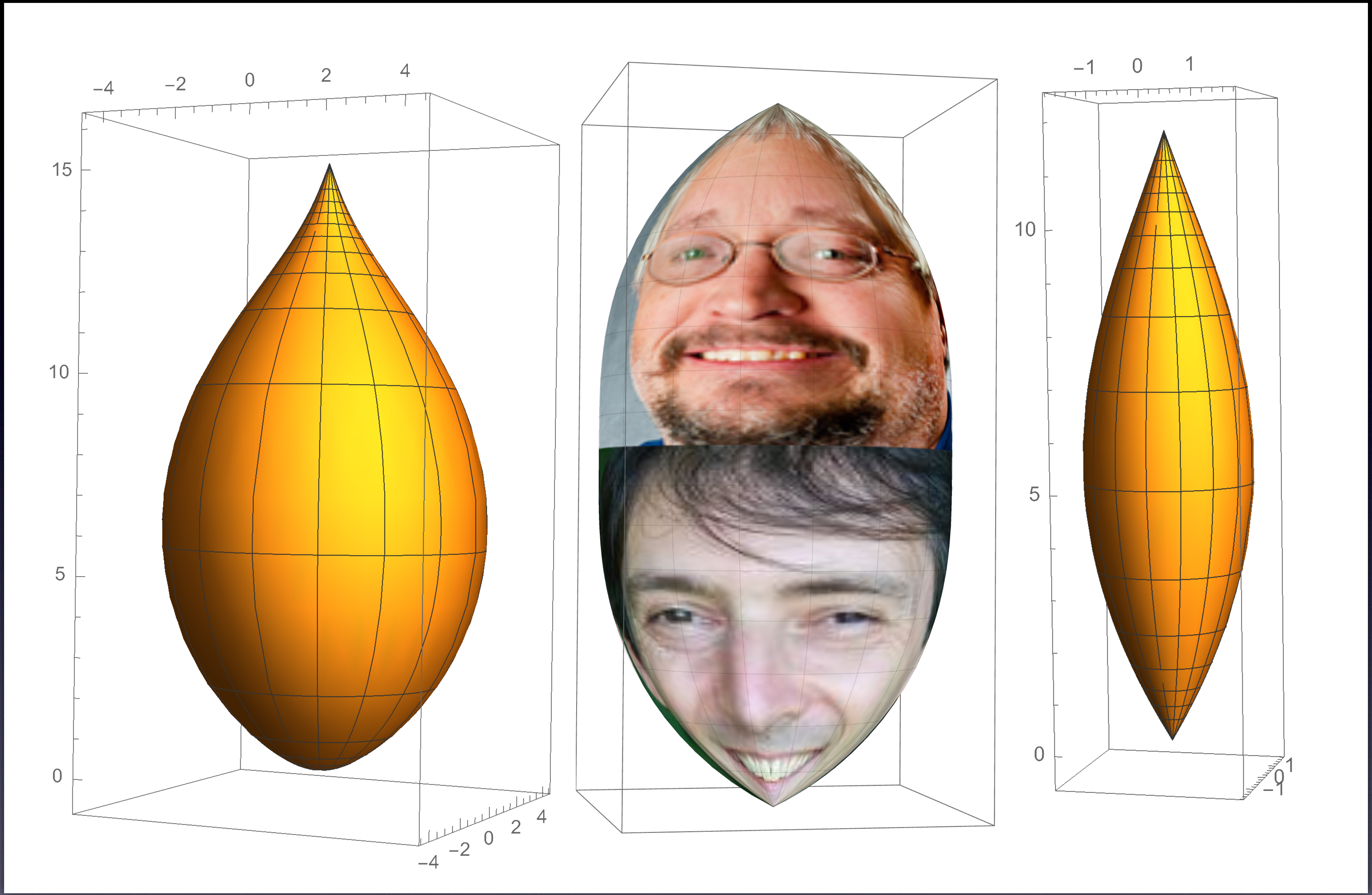
- One can consider $\mathcal{N} = 4$ SYM wrapped on same spindle with fluxes:
- If we constrain $p_B = 0$, again *only* anti-twist!
- $$c^{\mathcal{N}=4} = \frac{3(n_+ - n_-)((n_+ - n_-)^2 - 4p_F^2)}{2n_+n_-(5n_+^2 + 5n_-^2 + 6n_+n_- - 4p_F^2)}N^2$$
- One can show $c^{\mathcal{N}=4} > c^{LS}$ for *all* allowed fluxes
  - Flows should exist between these SCFTs!

Conclusions

- Constructed holographic dual of the $\mathcal{N} = 1$ Leigh-Strassler theory compactified on a spindle
  - Uplift is regular once flux appropriately quantized
  - Solutions only exist in "anti-twist" class, $N_\phi \neq \chi$, a *new* SUSY compactification method
  - Bulk calculation of c_{2D} matches QFT extremisation
  - First construction of spindles built with hyper 'plets

Open questions

- BPS spectrum of new (0,2) fixed points?
- Spindle pairs with $c^{UV} > c^{IR}$: can we construct flows between them by turning on LS deformation *on* the spindle?
- Where are the asymptotically AdS_5 solutions?
- Anti-twist: can't be section of CY, what is brane construction of anti-twist spindle wrapping?



Thank you