A simple approach to gravitational collapse into black hole

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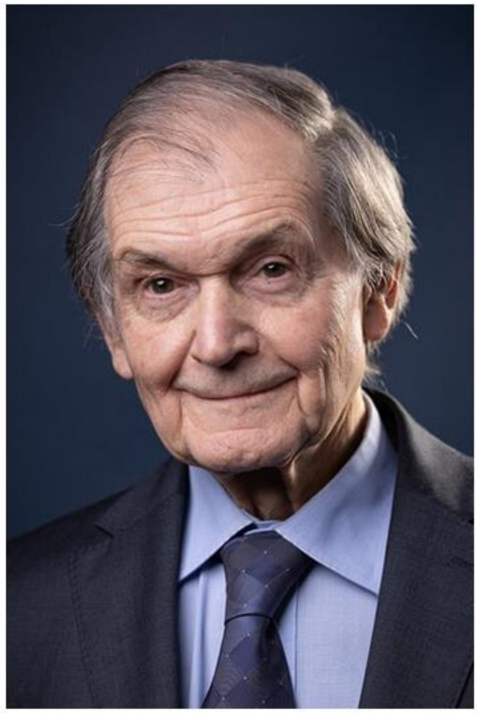
Refs. SY To appear in IJMPD

SY Phys.Lett.B 834 (2022) 137418

The Nobel Prize in Physics 2020



Roger Penrose

Prize share: 1/2

Reinhard Genzel

Prize share: 1/4

Andrea Ghez

Prize share: 1/4

The Nobel Prize in Physics 2020 was divided, one half awarded to Roger Penrose "for the discovery that black hole formation is a robust prediction of the general theory of relativity", the other half jointly to Reinhard Genzel and Andrea Ghez "for the discovery of a supermassive compact object at the centre of our galaxy"

https://www.nobelprize.org/prizes/physics/2020/summary/

The goal is to “understand Penrose figure”

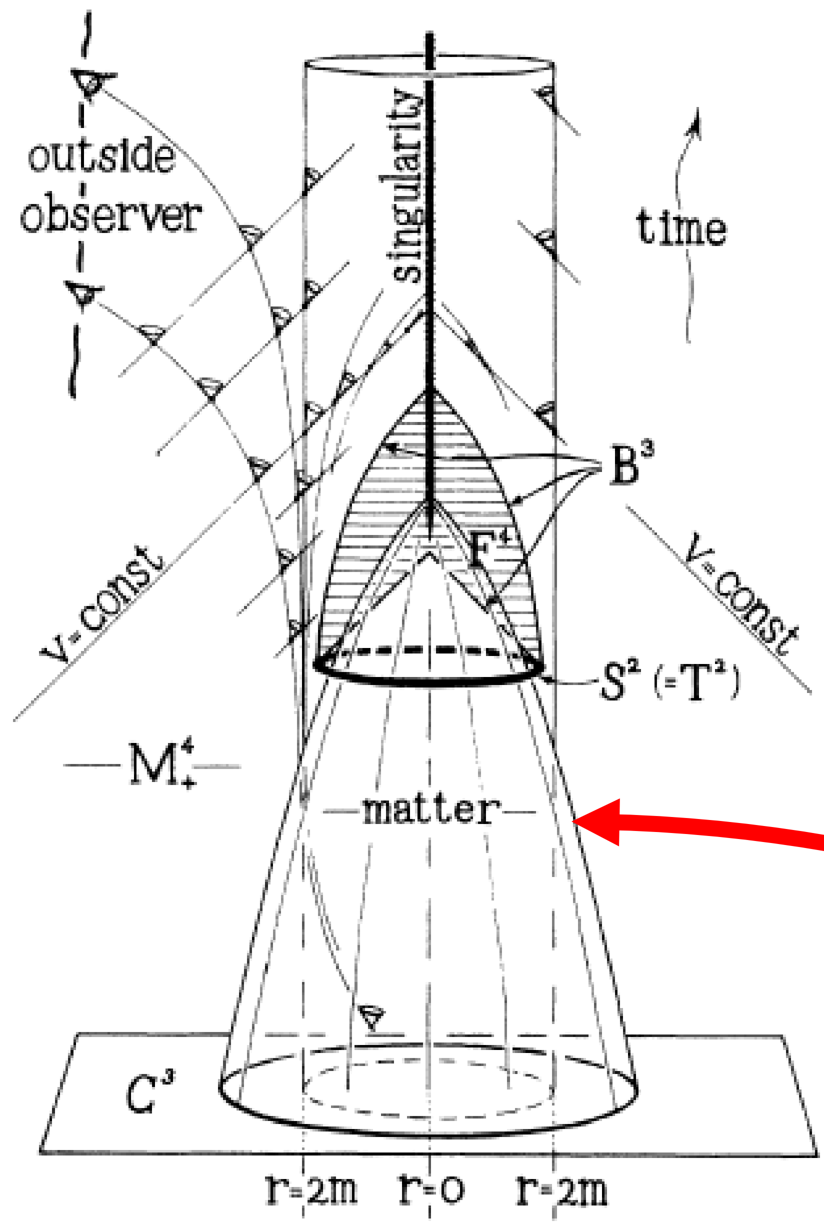


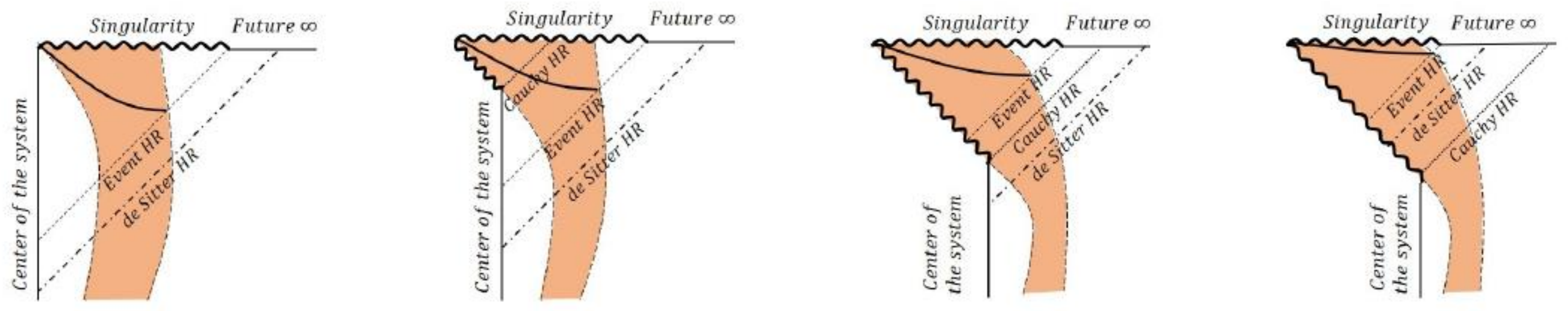
FIG. 1. Spherically symmetrical collapse (one space dimension surpressed). The diagram essentially also serves for the discussion of the asymmetrical case.

?

[Penrose ‘65]

<u>Q.</u> How do you robustly determine the orbit of infalling matter? Is there any obstruction?

Possible local causal structure of Gravitational Collapse (GC)



(a) Emergent black hole.

(b) Emergent local naked singularity 1.

(c) Emergent local naked singularity 2.

(d) Emergent global naked singularity.

In earlier study, GC happens with emergent naked singularity!

[Chirstodoulou ’84, ’91, ‘94] [Dwivedi-Joshi ’89, ‘92] [Shapiro-Teukolsky ’92]

⬇

It is important to construct an (analytic) solution describing GC without naked singularity!

How to find an analytic solution of GP?

2 approaches

Prepare each solution for the 2 regions separated by infalling fluid. Then

① "Junction/Jump/Matching condition"

connect them at their boundary smoothly.

[Darmois '27] [O'Brien-Synge '52] [Lichnerowicz '55] [Israel '58] cf. [Bonner-Vickers '81]

Well studied, general, but technical, not simple.

② "Interpolating method"

find a good coordinate system interpolating them.

[Misner-Thorne-Wheller '73] [Landau-Lifschits '75] [Adler-Bjorken-Chen-Liu '05]

Not well studied, heuristic, but simple sometimes with clear whole picture.

Plan

✓ 1. Introduction

2. (A)dS + infalling matter -> (A)dSBH

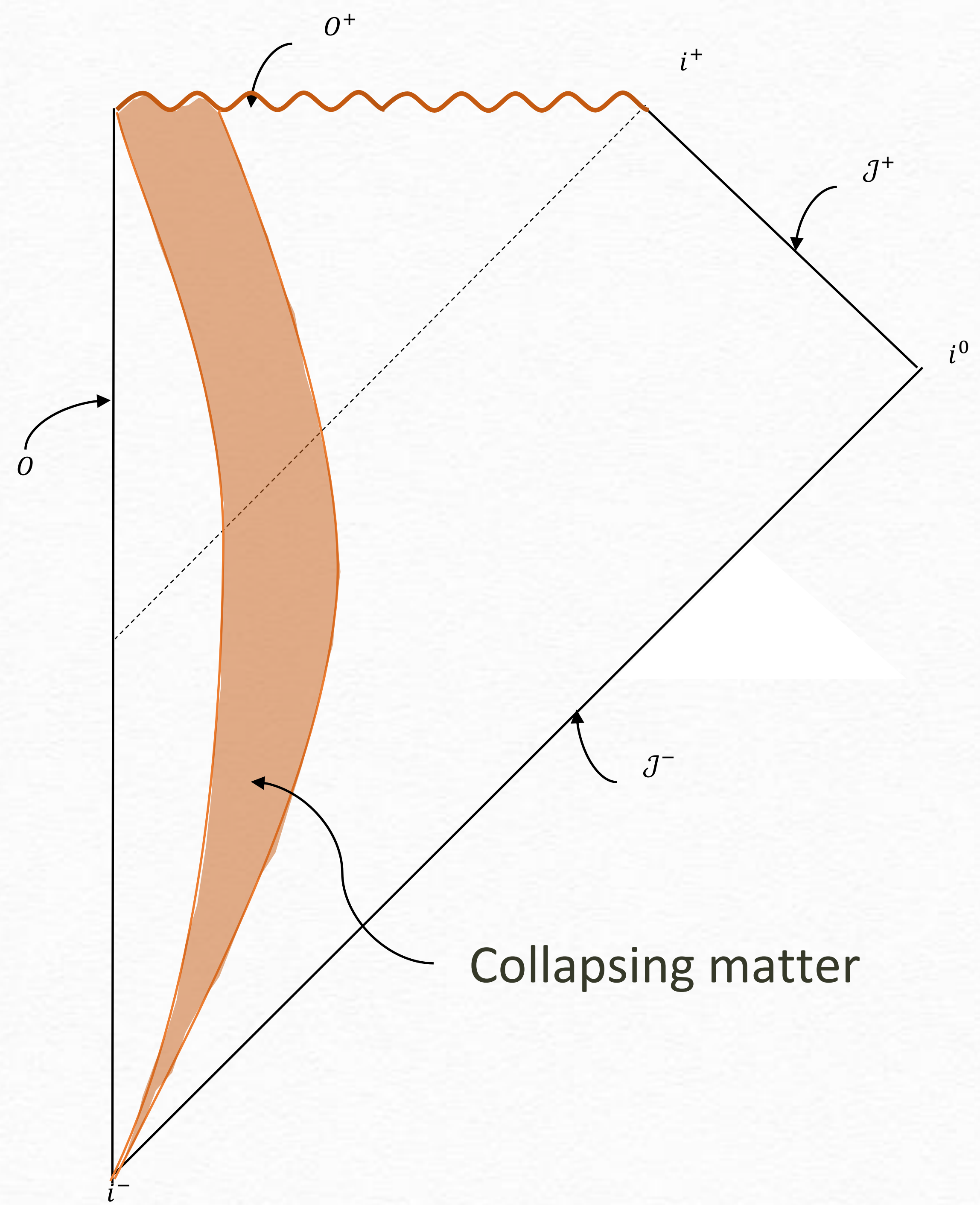
3. FLRW + infalling matter -> dSBH

4. Summary and future direction

(A)dS + infalling fluid -> (A)dS BH

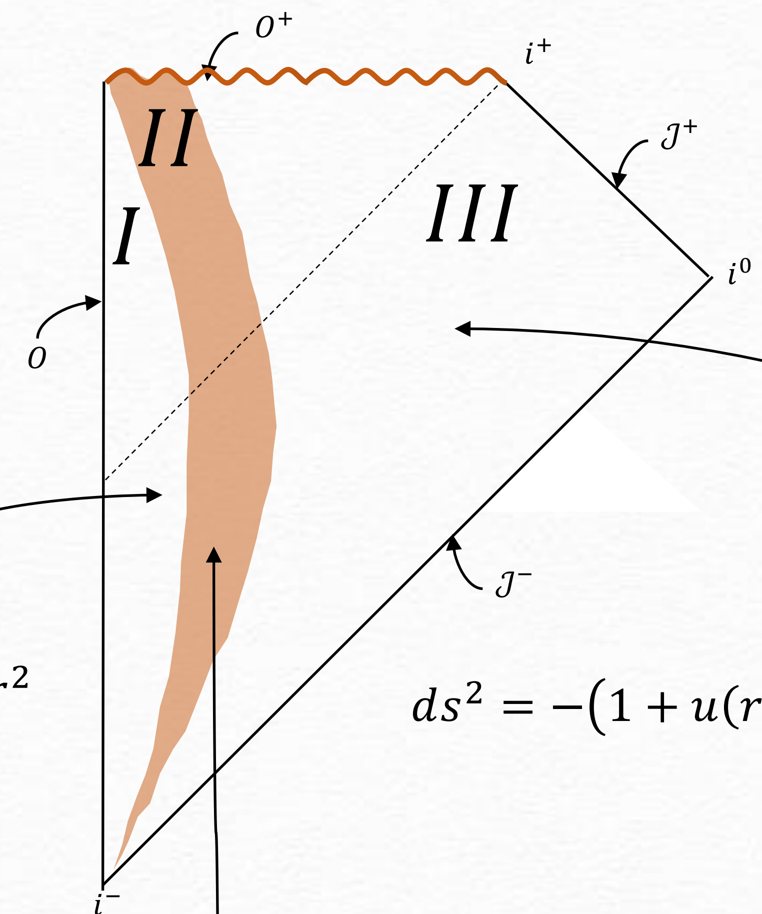
[SY, To appear in IJMPD]

GC of spherical matter into BH



<u>Q.</u> Can you construct an analytic solution for this process?

Interpolating method



I. (A)dS metric

$$ds^2 = -\big(1+u(r)\big)(dx_s^0)^2 + \frac{1}{1+u(r)}dr^2 + r^2\tilde{g}_{ij}dx^i dx^j$$

$$u(r) = -\frac{2\Lambda r^2}{(d-2)(d-1)}$$

$$m(t,r) = 0$$

III. Cottler BH metric

$$ds^2 = -\big(1+u(r)\big)(dx_s^0)^2 + \frac{1}{1+u(r)}dr^2 + r^2\tilde{g}_{ij}dx^i dx^j$$

$$u(r) = -\frac{2\Lambda r^2}{(d-2)(d-1)} - \frac{m}{r^{d-3}}$$

$$m(t,r) = m$$

$$t = x_s^0 - \int dr\,\Big(\frac{\sqrt{-u(r)}}{1+u(r)}\Big)$$

II. Interpolating metric

Cf.[Adler-Bjorken-Chen-Liu ’05]

Extended Eddington-Finkelstein coordinates

$$ds^2 = -\big(1+u(t,r)\big)dt^2 - 2u(t,r)dtdr + \big(1-u(t,r)\big)dr^2 + r^2\tilde{g}_{ij}dx^i dx^j$$

$$u(t,r) = -\frac{2\Lambda r^2}{(d-2)(d-1)} - \frac{m(t,r)}{r^{d-3}}$$

Constraint of stress energy tensor

In the interpolating region with infalling fluid,

$$ds^2 = -\big(1+u(t,r)\big)dt^2 - 2u(t,r)dtdr + (1-u(t,r))dr^2 + r^2\tilde{g}_{ij}dx^i dx^j$$

The Einstein equation **fixes** the form of energy momentum tensor

$$T_{\mu\nu} = \frac{1}{8\pi G_N}\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu}\right)$$

$$T^t{}_t = \frac{d-2}{16\pi G_N}\frac{\partial_r(r^{d-3}\delta u)}{r^{d-2}},$$

$$T^t{}_r = \frac{d-2}{16\pi G_N}\frac{\partial_t u}{r} = -T^r{}_t,$$

$$T^r{}_r = \frac{d-2}{16\pi G_N}\left(\frac{\partial_r(r^{d-3}\delta u)}{r^{d-2}} - 2\frac{\partial_t u}{r}\right),$$

$$T^i{}_j = \frac{\delta^i{}_j}{16\pi G_N}\frac{(\partial_r - \partial_t)^2(r^{d-3}\delta u)}{r^{d-3}},$$

Read off macroscopic physical quantities

$$T^\mu{}_\nu = \rho v^\mu v_\nu + q^\mu v_\nu + v^\mu q_\nu + P^\mu{}_\nu,$$

$$P^a{}_b = p_\perp h^a{}_b, \quad P^i{}_j = p_\parallel \delta^i{}_j \qquad h_{\mu\nu} = g_{\mu\nu} + v_\mu v_\nu$$

$$a,b = t,r \qquad i,j = \varphi,\theta,\ldots$$

$$\rho = \frac{d-2}{16\pi G_N}\frac{\partial_r m(t,r)}{r^{d-2}},$$

$$q^\mu = \frac{d-2}{32\pi G_N}\frac{\partial_t m(t,r)}{r^{d-2}}\begin{pmatrix} u \\ -(2+u) \\ \vec{0}\end{pmatrix},$$

$$p_\perp = \frac{d-2}{16\pi G_N}\left(\frac{-\partial_r m(t,r)}{r^{d-2}} + 2\frac{\partial_t m(t,r)}{r^{d-2}}\right),$$

$$p_\parallel = \frac{1}{16\pi G_N}\frac{(\partial_r - \partial_t)^2(-m(t,r))}{r^{d-3}}.$$

Normalization condition: $v_\mu v^\mu = -1$

$(v^\mu) = (v^t, v^r, \vec{0})$

Request to be well-defined for all region

$$v^\mu = \left(1 - \frac{u}{2}, \frac{u}{2}, \vec{0}\right)$$

Input of fluid information

① Impose **shell structure** for fluid configuration.

$$m(t,r) = m\,\theta(r)\,F(\frac{t+h(r)}{\Delta})$$

m : Total mass parameter of infalling matter

Δ : (Time) width of infalling matter

$h(r)$: function describing the orbit of infalling matter

$F(x)$: function labeling matter layer

② Impose an **equation of state** for fluid near interface.

$$p(r) = w\rho(r) \qquad @\ F = 1-\epsilon$$

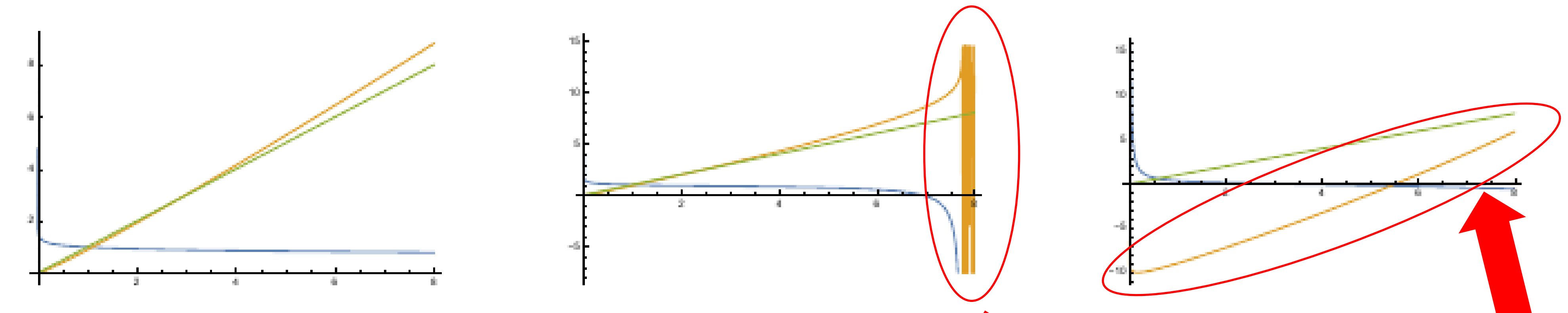
$$p(r) = p_{\parallel}(r) + v p_{\perp}(r)$$

Q. Does this solution describe a desired GC?

A. Not always. It may not be physical!

Unphysical solution

The function h(r) which is supposed to be the orbit of infalling fluid



(a) $w_{\parallel} = 0.1, y = 0.1.$ (b) $w_{\parallel} = 0.1, y = 1.2.$ (c) $w_{\parallel} = 0.95, y = 0.2.$

Figure 2: The blue, yellow and green curves depict the functions $h'(r), h(r), r$, respectively, where the horizontal axis is the r coordinate, with parameters chosen as shown.

Shell crossing singularity! **??**

The unphysical formal solutions should be excluded!

What are the physical requirements?

(0) (Energy condition) The local energy density be non-negative:

$$\rho \geq 0 \ \Leftrightarrow \ \partial_r m(t,r) \geq 0 \ \Leftrightarrow \ F'\left(\frac{t+h(r)}{\Delta}\right) h'(r) \geq 0$$

(i) (Absence of shell crossing singularity) The orbit should be monotonically infalling:

$$h'(r) > 0 \qquad \text{for gravitational collapse}$$

(ii) The emergent infinite redshift surface monotonically increase:

$$h'(r) < \frac{\Delta}{m} r^{d-4}\left(d-3-\frac{2\Lambda}{d-2}r^2\right)(F^{-1})'\left(\frac{r^{d-3}}{m}\left(1-\frac{2\Lambda}{(d-2)(d-1)}r^2\right)\right)$$

(iii) (Flux energy condition) The energy flux vector be causal outside the event horizon:

$$\left(\partial_r m(t,r)\right)^2 \geq \left(\partial_t m(t,r)\right)^2 \quad \Rightarrow \quad h'(r) \geq 1 \qquad \text{for } r > r_{EH}$$

The radius of emergent event horizon

Comment

$d = 3$

$$\Rightarrow \quad h'(r) < \frac{\Delta}{m}(-2\Lambda r)(F^{-1})'\left(\frac{1}{m}(1-\Lambda r^2)\right) \quad \Rightarrow$$

No GC solution unless $\Lambda < 0$

Cf. [BTZ ’92][Ida ’00]

Gravitational collapse or evaporation

<u>(0)</u> (Energy condition) $F'\left(\frac{t+h(r)}{\Delta}\right) h'(r) \geq 0$

Either

$$F'\left(\frac{t+h(r)}{\Delta}\right) \geq 0$$
$$h'(r) \geq 0$$

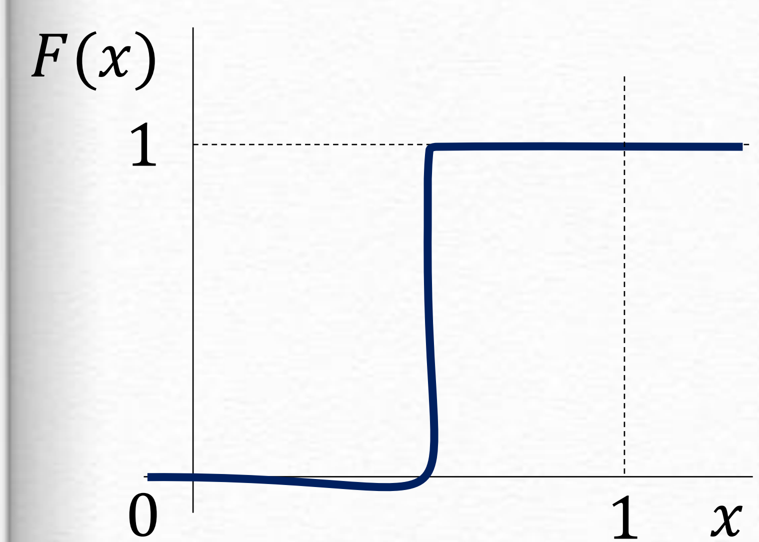
Gravitational collapse

$$F'\left(\frac{t+h(r)}{\Delta}\right) \leq 0$$
$$h'(r) \leq 0$$

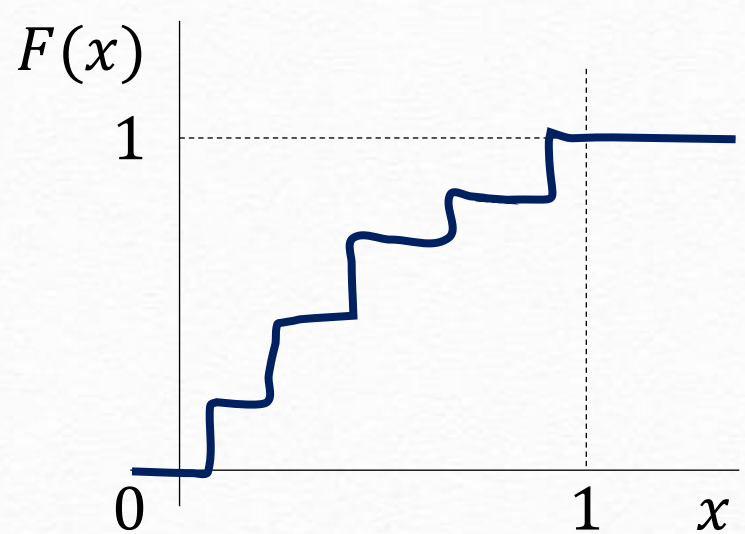
Gravitational evaporation

(= white hole evaporation)

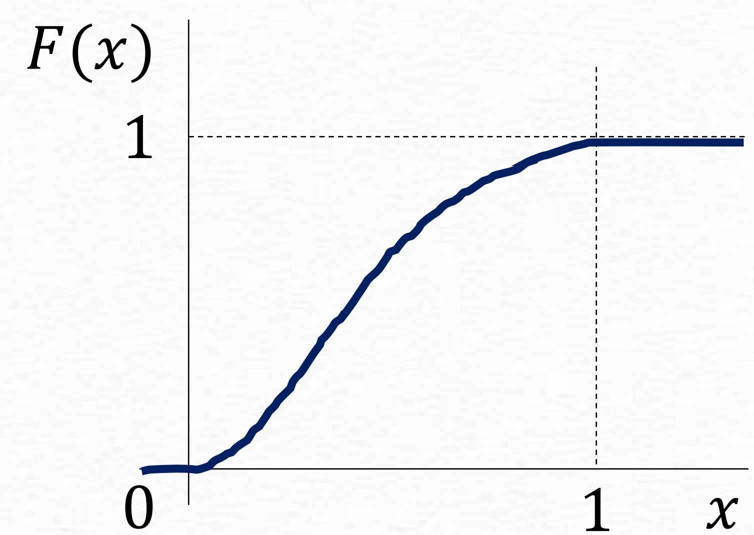
$$F(x) = \begin{cases} 0 & x < 0 \\ F(x) & 0 < x < 1 \\ 1 & 1 < x \end{cases}$$



A thin matter shell



Sequence of thin matter shells



Thick matter shells

Sufficient for analytic solution?

m : Mass

Δ : Width

$F(x)$: Distribution

$p_{\parallel}(r) + v p_{\perp}(r) = w\rho(r)$

: EoS near interface

INPUT → GR system described by EF coordinates

OUTPUT → $h(r)$: Orbit

$$h'(r) = 1 + \frac{(\bar{w}-\bar{v})\left[(y\bar{w})^{\bar{w}}\Gamma(2-\bar{w})J_{-\bar{w}}(2x) - \{\bar{w}(1-\bar{w}) + (y\bar{w})^{\bar{w}}\bar{v}\}\Gamma(\bar{w})J_{\bar{w}}(2x)\right]}{x\left[(y\bar{w})^{\bar{w}}\Gamma(2-\bar{w})J_{1-\bar{w}}(2x) + \{\bar{w}(1-\bar{w}) + (y\bar{w})^{\bar{w}}\bar{v}\}\Gamma(\bar{w})J_{\bar{w}-1}(2x)\right]}.$$

$\bar{v} = 2(d-2)v, \bar{w} = (d-2)(v+w)$ $\quad y = \frac{F''(1)}{F'(1)\Delta}$

Parameter regions to satisfy the physical conditions with $\Delta \gg 1$

① $\bar{v}, |\bar{w}|, y \ll 1, \bar{v}y > 0$ ② $|\bar{v}|, |\bar{w}| \gg 1, y \ll 1, \bar{v}y > 0, \left|\frac{\bar{v}}{\bar{w}}\right| \sim 1$

Analytic solution describing Penrose figure

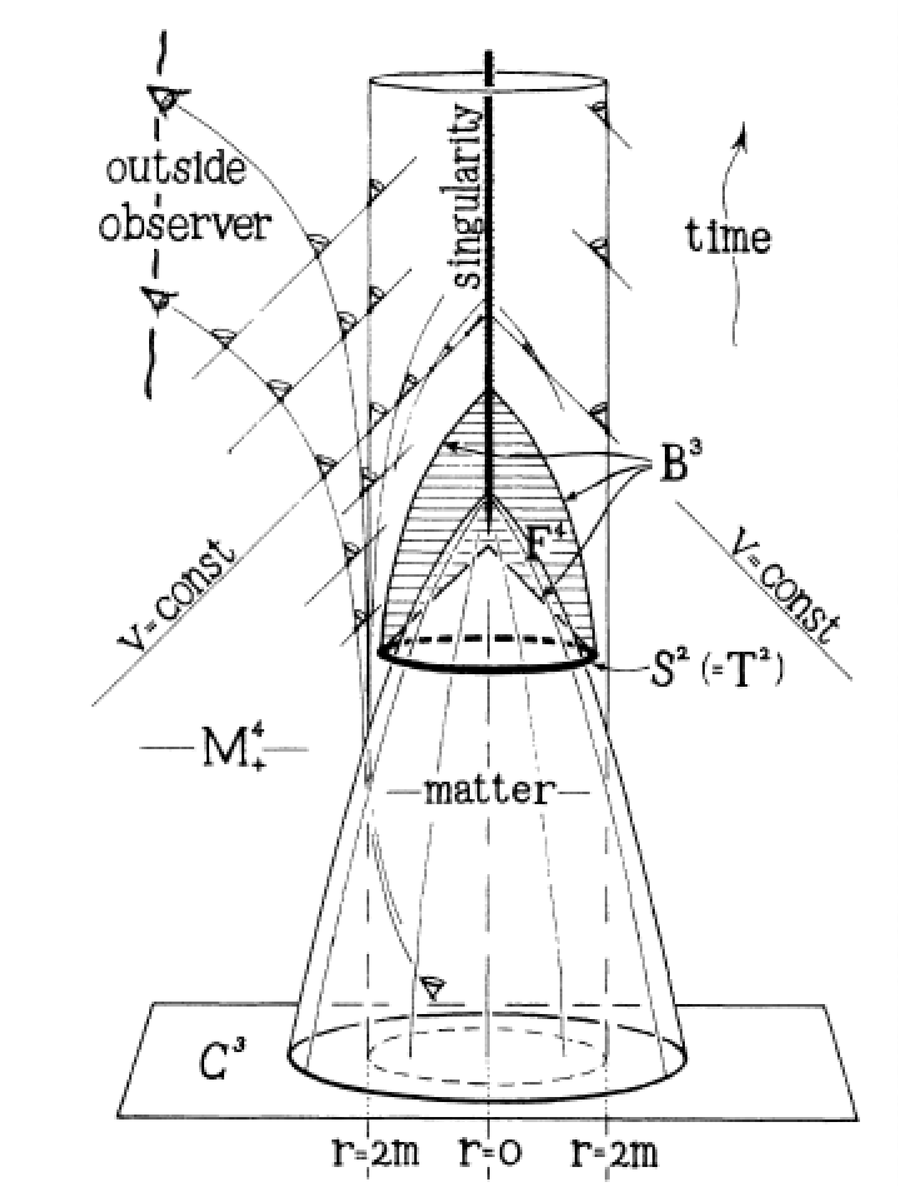
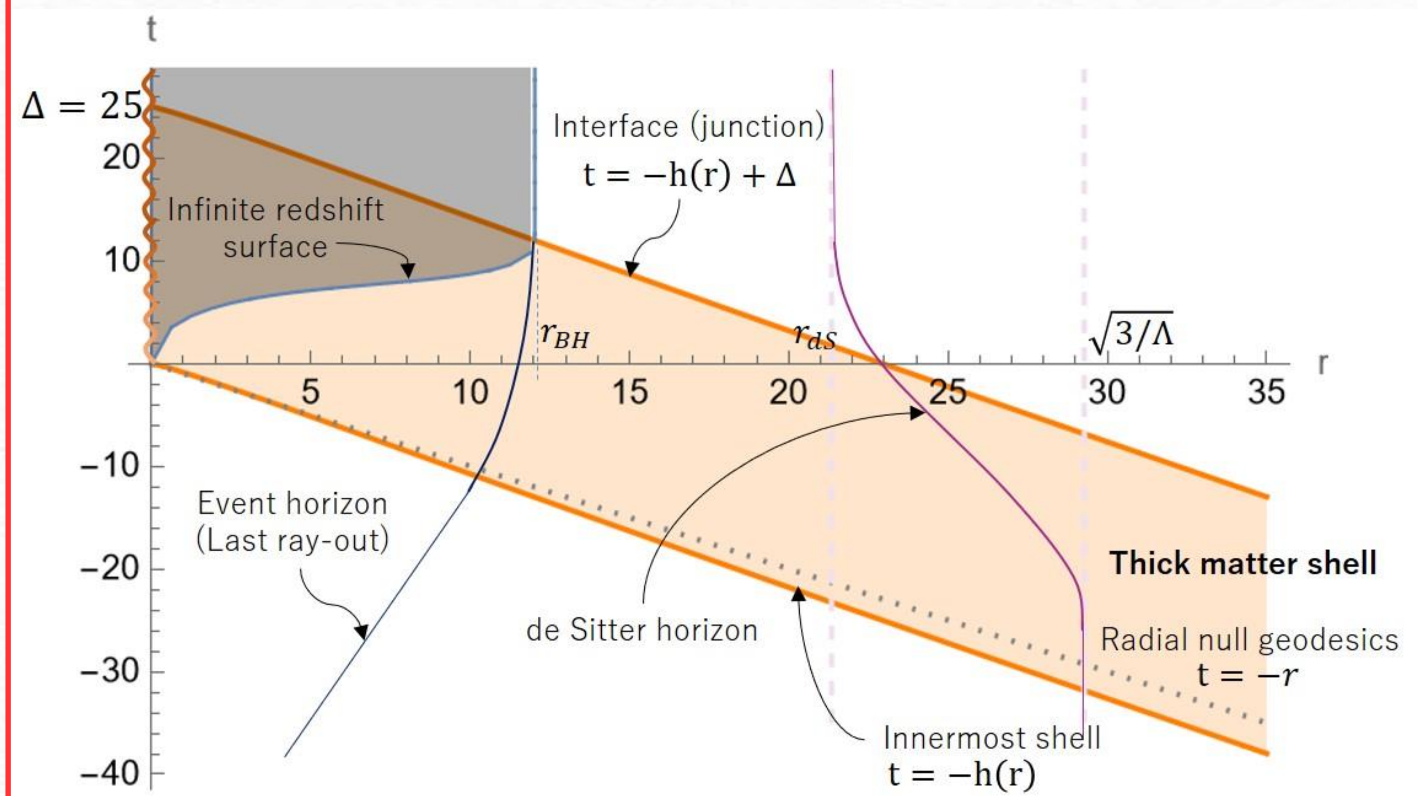


FIG. 1. Spherically symmetrical collapse (one space dimension surpressed). The diagram essentially also serves for the discussion of the asymmetrical case.

[Penrose '65]



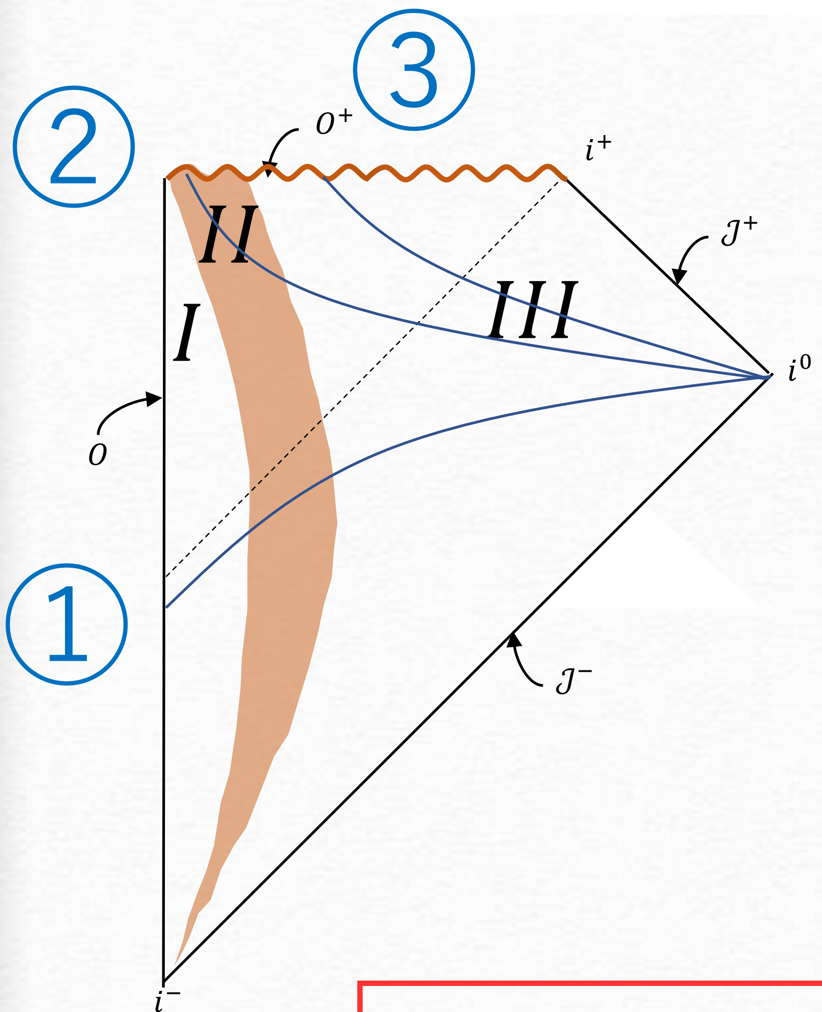
[SY, To appear in IJMPD]

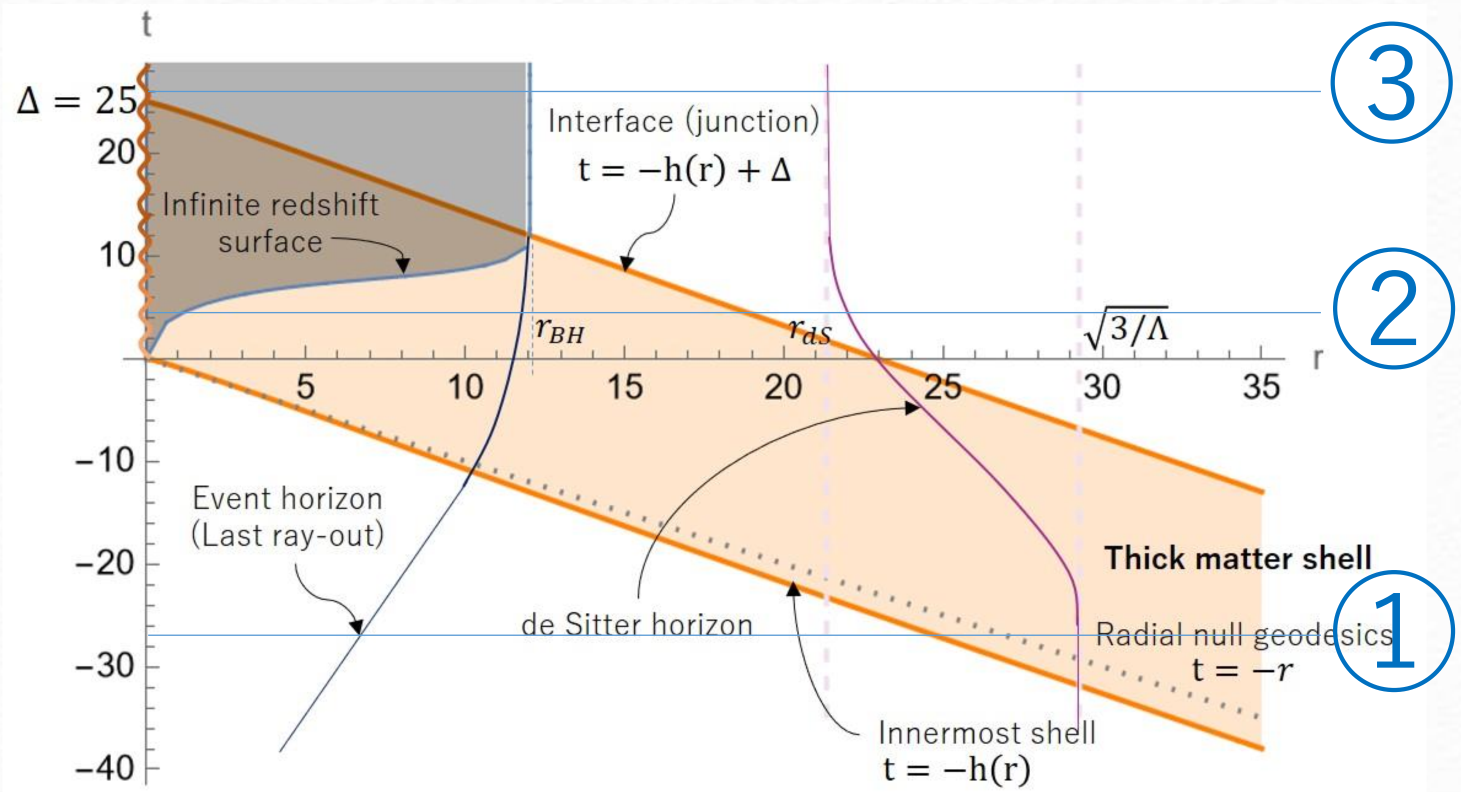
The Penrose figure

The Finkelstein diagram

Total energy conservation under GC

Time	t	$t<0$	$0 \le t < \Delta$	$\Delta \le t$
Energy of collapsing matter	M_1	M	$M(1-F(\frac{t}{\Delta}))$	0
Mass of black hole	M_2	0	$MF(\frac{t}{\Delta})$	M
Total energy	E	M	M	M





$$E=\int_{\Sigma_t} d^{d-1}\vec{x}\,\sqrt{|g|}\,T^0_\mu(t,\vec{x})\,n^\mu(t,\vec{x})$$

[Aoki-Onogi-SY '20] Cf.[Fock '59]

Plan

✓ 1. Introduction

✓ 2. (A)dS + infalling matter -> (A)dSBH

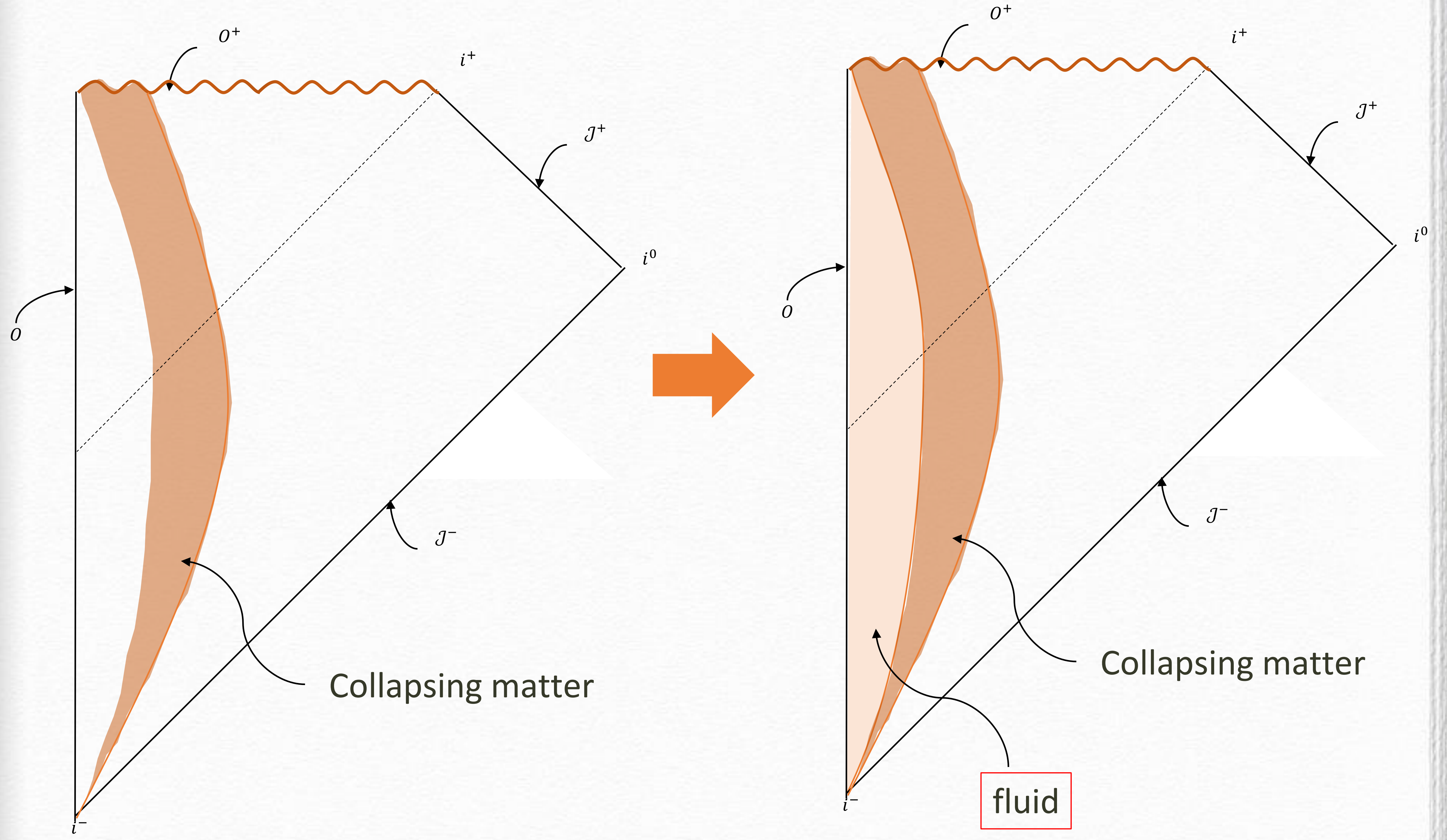
3. FLRW + infalling matter -> dSBH

4. Summary and future direction

FLRW universe + infalling fluid -> dS BH

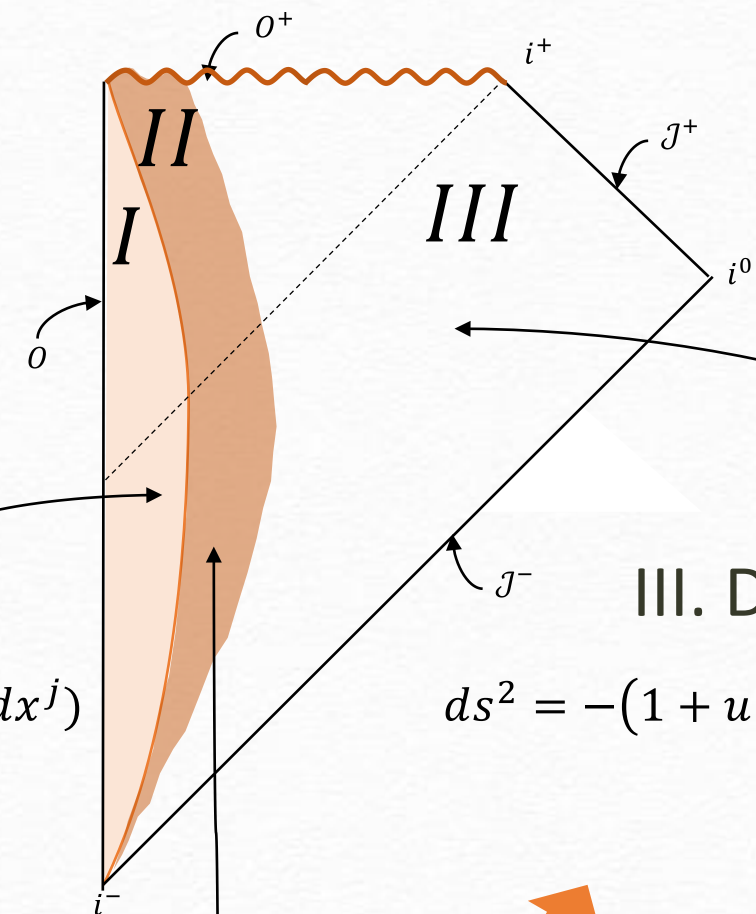
[SY, Phys.Lett.B 834 (2022) 137418]

Gravitational collapse of matter with fluid inside?



<u>Q.</u> Can you construct an analytic solution in homogeneous isotropic universe?

Interpolating method



I. FLRW metric

$$ds^2 = -(dx^0)^2 + a(x^0)^2(d\check{r}^2 + \check{r}^2\,\tilde{g}_{ij}dx^i dx^j)$$

$$r = a(x^0)\check{r}$$

$$t = x^0$$

$$\psi = -H(t)r$$

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

III. De Sitter Schwarzschild BH

$$ds^2 = -(1+u(r))(dx_s^0)^2 + \frac{1}{1-u(r)}dr^2 + r^2\,\tilde{g}_{ij}dx^i dx^j$$

$$u(r) = -\frac{2\Lambda r^2}{(d-2)(d-1)} - \frac{m}{r^{d-3}}$$

$$t = x_s^0 - \int dr\,\left(\frac{-\sqrt{-u(r)}}{1+u(r)}\right)$$

$$\psi = -\sqrt{-u(r)}$$

$$H_0^2 = \frac{2\Lambda}{(d-1)(d-2)}$$

[Adler-Bjorken-Chen-Liu ’05]

II. Interpolating metric

Gullstrand-Painleve coordinates (metric)

$$ds^2 = -(1+\psi(t,r)^2)dt^2 + 2\psi(t,r)dtdr + dr^2 + r^2\,\tilde{g}_{ij}dx^i dx^j$$

Constraint of stress energy tensor

In the interpolating region with infalling fluid,

$$ds^2 = -\left(1+\psi(t,r)^2\right)dt^2 + 2\psi(t,r)dtdr + dr^2 + r^2\,\tilde{g}_{ij}dx^i dx^j$$

The Einstein equation **fixes** the form of energy momentum tensor

$$T_{\mu\nu} = \frac{1}{8\pi G_N}\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu}\right)$$

$$H_0^2 = \frac{2\Lambda}{(d-1)(d-2)}$$

$$T^t{}_t = \frac{d-2}{16\pi G_N}\frac{(r^{d-3}(-\psi^2+H_0^2r^2))'}{r^{d-2}},$$

$$T^t{}_r = \frac{d-2}{16\pi G_N}\frac{2\psi\dot{\psi}}{r}, \qquad T^r{}_t = 0,$$

$$T^r{}_r = \frac{d-2}{16\pi G_N}\left(\frac{(r^{d-3}(-\psi^2+H_0^2r^2))'}{r^{d-2}} + \frac{2\dot{\psi}}{r}\right),$$

$$T^i{}_j = \frac{\delta^i_j}{16\pi G_N}\left(\frac{(r^{d-3}(-\psi^2+H_0^2r^2))''}{r^{d-3}} + 2\frac{(r^{d-3}\dot{\psi})'}{r^{d-3}}\right),$$

Read off macroscopic physical quantities

$$T_{\mu\nu} = \rho u_\mu u_\nu + P_{\mu\nu} \qquad u_\mu = (-1,0,\vec{0})$$

$$u_\mu u^\mu = -1$$

$$\rho = -\frac{d-2}{16\pi G_N}\frac{(r^{d-3}(-\psi^2+H_0^2r^2))'}{r^{d-2}},$$

$$p_\perp = \frac{d-2}{16\pi G_N}\left(\frac{(r^{d-3}(-\psi^2+H_0^2r^2))'}{r^{d-2}} + \frac{2\dot{\psi}}{r}\right),$$

$$p_\parallel = \frac{1}{16\pi G_N}\frac{(r^{d-3}(-\psi^2+H_0^2r^2))'' + 2(r^{d-3}\dot{\psi})'}{r^{d-3}}.$$

$$\overline{g_{\mu\nu}} = g_{\mu\nu} + u_\mu u_\nu$$

$$P_{ab} = p_\perp\,\overline{g_{ab}} \qquad a,b = t,r$$

$$P_{ij} = p_\parallel\,\overline{g_{ij}} \qquad i,j = \varphi,\theta,\ldots$$

Region I: perfect fluid + dark energy

→ The metric is equivalent to the FLRW one if

$$\psi(t,r) = -H(t)r$$

Fluid inside is a perfect fluid. ⬇

$$p_{\parallel} = p_{\perp} = \frac{d-2}{16\pi G_N}\Big(-(d-1)\big(H(t)^2 - H_0^2\big) - 2\dot{H}(t)\Big) =: p$$

Impose an equation of state: $p = w\rho$

⬇

$$H(t) = H_0 \coth\left(\frac{H_0}{2}(w+1)(d-1)(t-t_0)\right)$$

$$\rightarrow a(t) = C_1\left(\sinh\left(\frac{H_0}{2}(w+1)(d-1)(t-t_0)\right)\right)^{\frac{2}{(w+1)(d-1)}}$$

Region II: Thick shell of fluid + dark energy

$$\psi(t,r) = -s\sqrt{\frac{m(t,r)}{r^{d-3}} + H(t,r)^2 r^2}$$

$$m(t,r) = m_0\,\theta(r)\,F\left(\frac{t-h(r)}{\Delta}\right)$$

$$H(t,r) = H_0^2 + \delta H(t)^2 \Theta\left(\frac{t-h(r)}{\Delta}\right)$$

$$\delta H(t)^2 = H(t)^2 - H_0^2$$

INPUT → m_0 : Total mass parameter of infalling matter

→ Δ : (Time) width of infalling matter

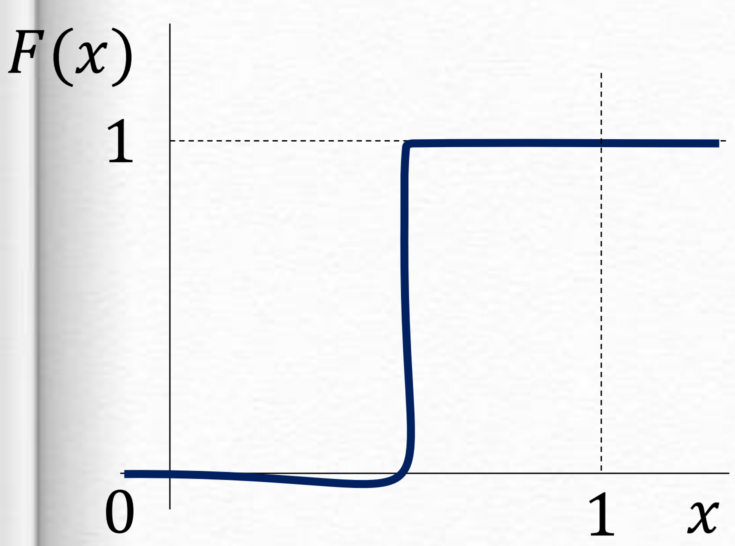
$h(r)$: function describing the orbit of infalling matter → **OUTPUT**

→ $F(x)$: upslope function labeling matter distribution

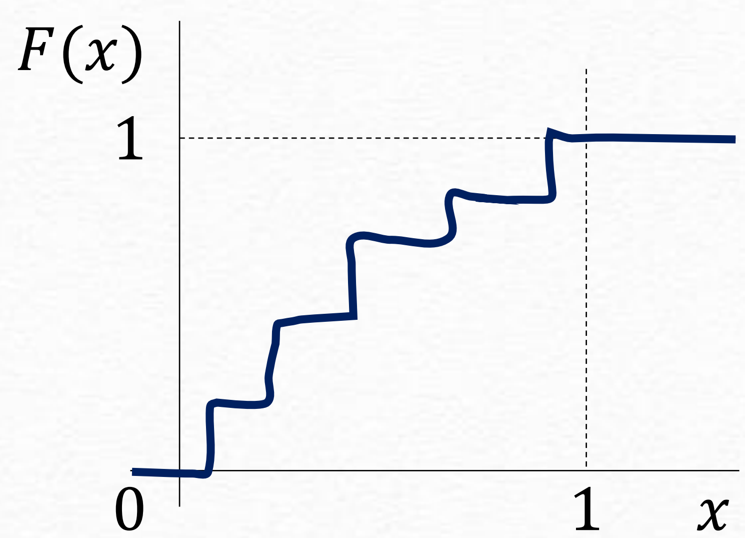
$\Theta(x)$: downslope function

$$F(x) = \begin{cases} 0 & x < 0 \\ F(x) & 0 < x < 1 \\ 1 & 1 < x \end{cases}$$

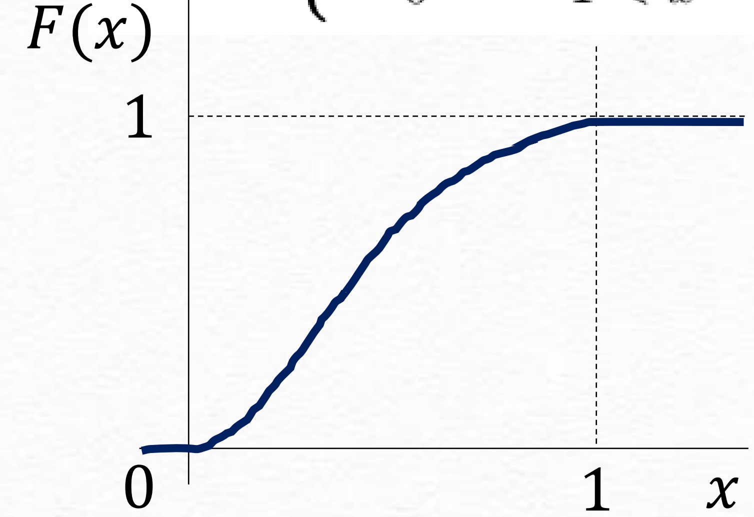
$$\widetilde{\Theta}(x) = \begin{cases} 1 & x < 0 \\ \Theta(x) & 0 < x < 1 \\ 0 & 1 < x \end{cases}$$



A thin matter shell



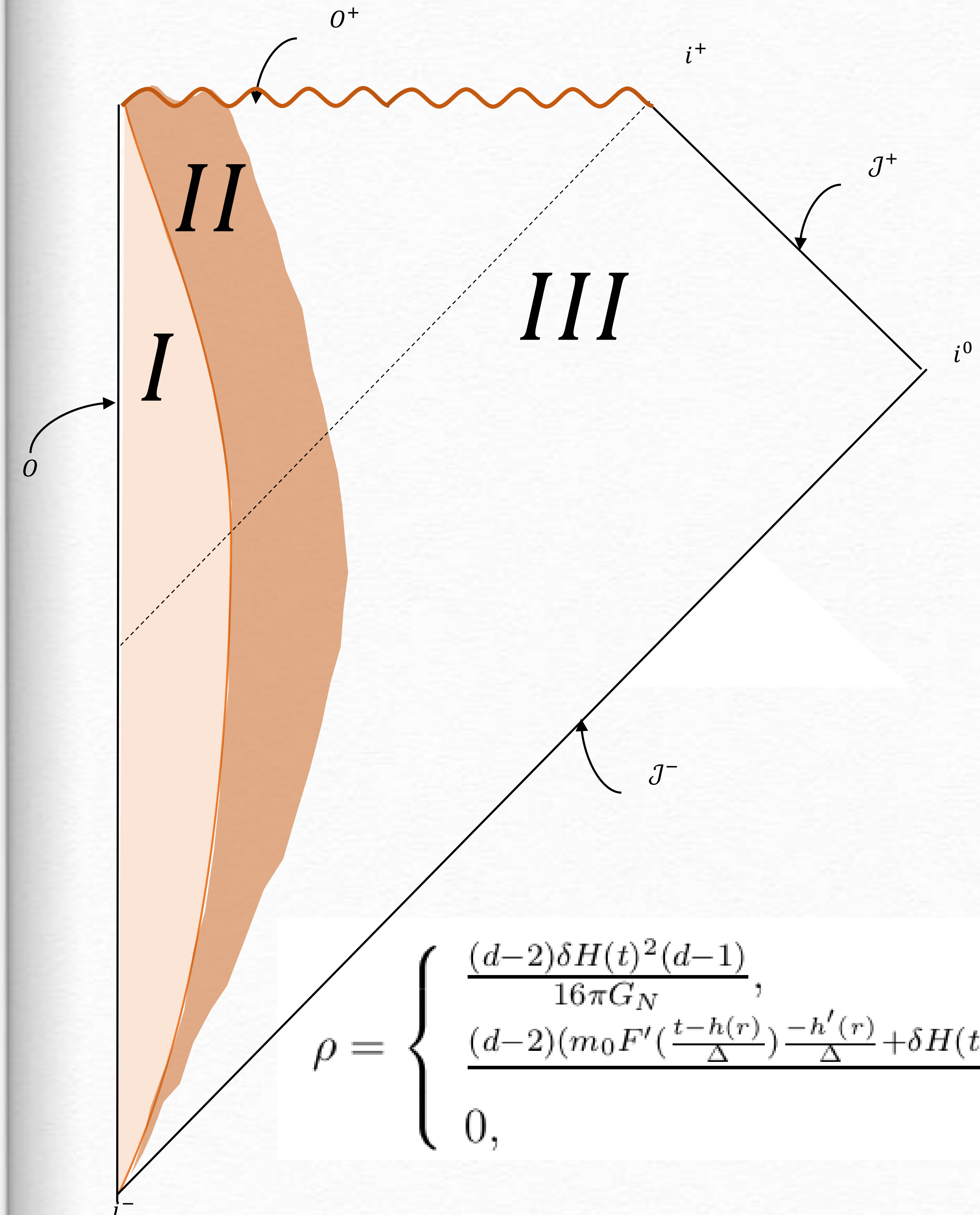
Sequence of thin matter shells



Thick matter shells

Continuity condition

->The density & perpendicular component of pressure be continuous at interface.



$$\rho^I = \rho^{II} \qquad \rho^{II} = \rho^{III}$$

$$p_\perp^I = p_\perp^{II} \qquad p_\perp^{II} = p_\perp^{III}$$

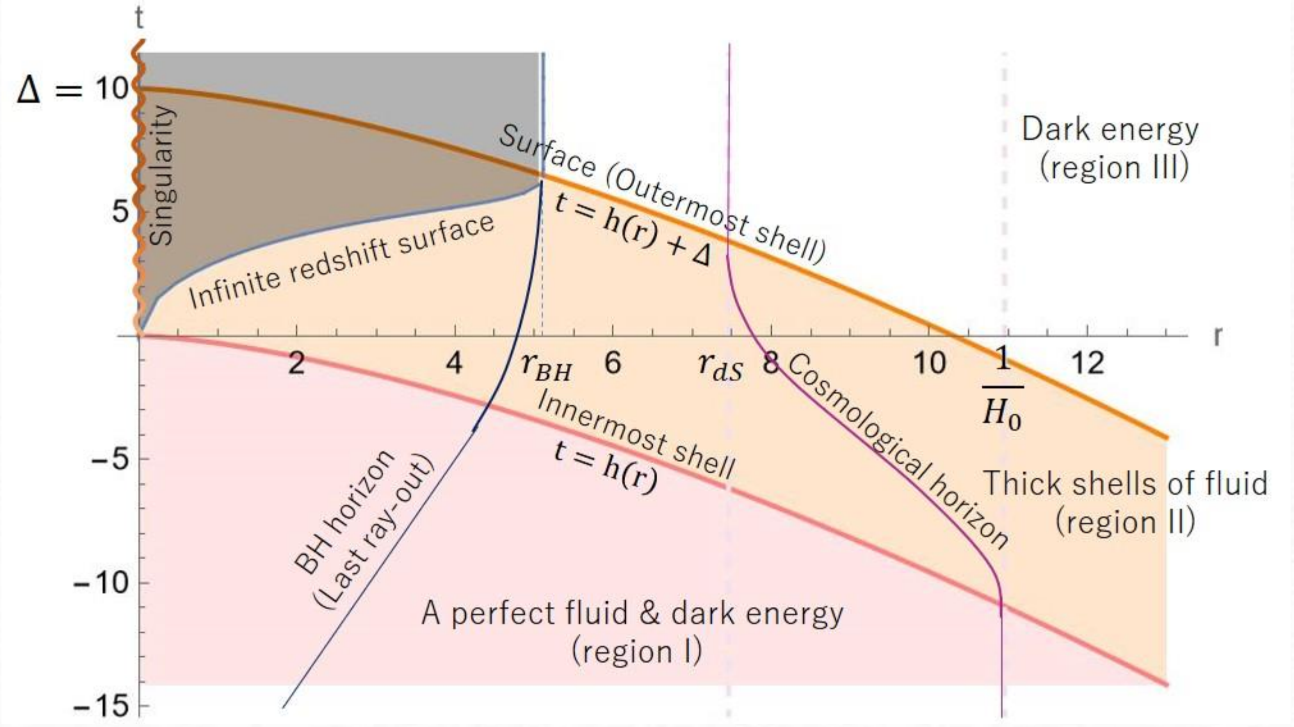
$$p_\perp = -\rho + \frac{d-2}{16\pi G_N}\frac{2\dot{\psi}}{r}$$

$$\rho = \begin{cases} \frac{(d-2)\delta H(t)^2(d-1)}{16\pi G_N}, & \text{region I} \\ \frac{(d-2)(m_0 F'(\frac{t-h(r)}{\Delta})\frac{-h'(r)}{\Delta} + \delta H(t)^2\{\Theta'(\frac{t-h(r)}{\Delta})\frac{-h'(r)}{\Delta} r^{d-1} + \Theta(\frac{t-h(r)}{\Delta})(d-1)r^{d-2}\})}{16\pi G_N r^{d-2}}, & \text{region II} \\ 0, & \text{region III} \end{cases}$$

Analytic solution

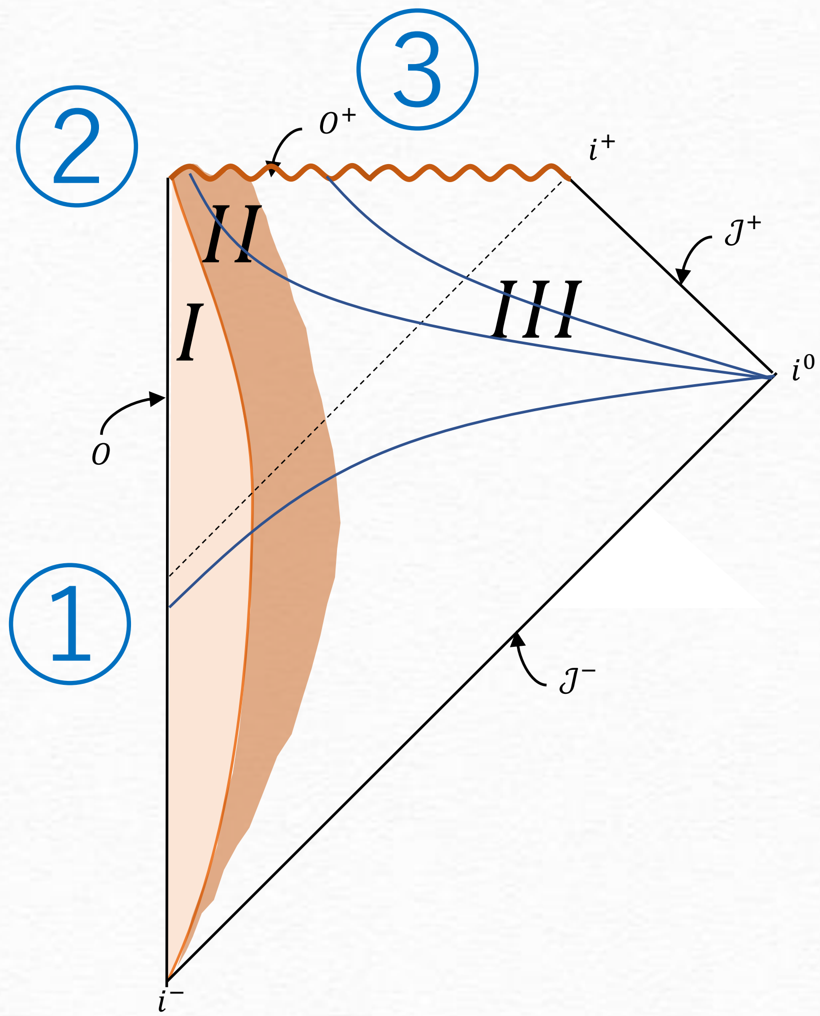
$$h(r) = \begin{cases} -\frac{2}{H_0(w+1)(d-1)}\operatorname{arcsinh}\left(\sqrt{\frac{\alpha}{m_0}}H_0 r^{\frac{d-1}{2}}\right), & h'(r) < 0 \\ -\Delta + \frac{2}{H_0(w+1)(d-1)}\operatorname{arcsinh}\left(\sqrt{\frac{\alpha}{m_0}}H_0 r^{\frac{d-1}{2}}\right), & h'(r) > 0 \end{cases}$$

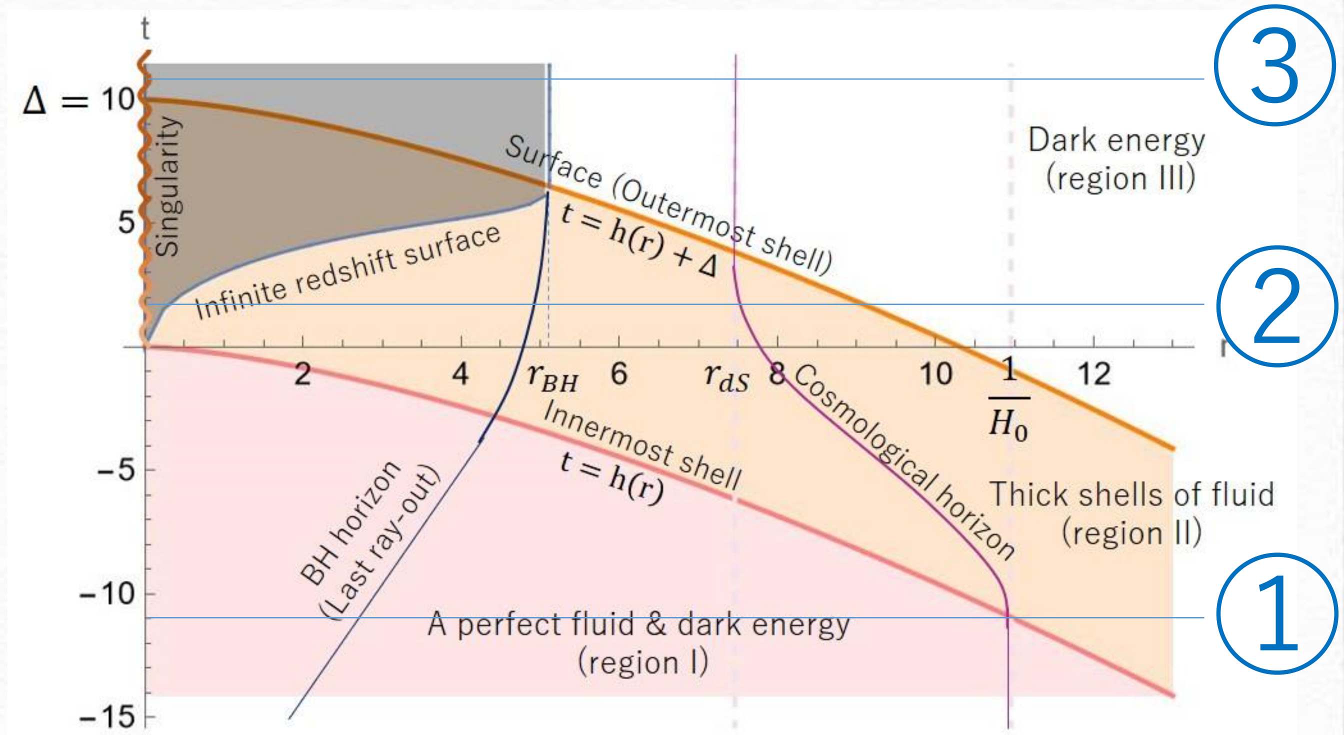
$$\alpha := -\lim_{x\to\sigma}\frac{\Theta'(x)}{F'(x)}$$



$$w = 0.1, \Delta = 10, m_0 = 4, \Lambda = 0.025,\ F(x) = \sin^2\frac{\pi x}{2}, \Theta(x) = \cos^2\frac{\pi x}{2}$$

Total energy conservation under GC





$$E = \int_{\Sigma_t} d^{d-1}\vec{x}\,\sqrt{|g|}T^0_\mu(t,\vec{x})n^\mu(t,\vec{x})$$

[Aoki-Onogi-SY '20] cf. [Fock '59]

Plan

✓ 1. Introduction

✓ 2. (A)dS + infalling matter -> (A)dSBH

✓ 3. FLRW + infalling matter -> dSBH

4. Summary and future direction

Summary

① Investigated **gravitational collapse of shells of fluid with spherical symmetry** without any radiation described by the Einstein gravity with cosmological constant.

② A key was to finding a good coordinate system to describe the interpolating region such as **Eddington-Finkelstein coordinates** interpolating (A)dS and Cottler BH and **Gullstrand-Painleve coordinates** interpolating FLRW and de-Sitter BH.

③ Imposing some **physical conditions** (**energy conditions, absence of shell crossing singularity, monotonic increase of infinite redshift surface, continuity condition**), the orbit equation of the infalling matter was determined.

④ Adopting the definition presented in the previous paper, the total energy is independent of the given time evolution.

Future direction

- How about other types of BH?
- Include radiation?