Cubic Closed String Field Theory on a Double Layer

Taejin Lee

Kangwon National University
KOREA

taejin@kangwon.ac.kr

Danang, Feb. 24, 2023

Contents

1. Introduction
2. Review: Witten's Cubic Open String Field Theory
3. Open String Field Theory in the Proper-Time Gauge
4. Closed String Field Theory in the Proper-Time Gauge
5. Closed Cubic Closed String Field Theory (Extension of Witten's) No-Go Theorem?
6. Closed String Theory on a Double Layer
7. The Neumann Functions for Closed String
8. Construction of Identity and Two-String Vertex for Closed String
9. Construction of Cubic Closed String Vertex
10. Three-Graviton Scattering Amplitude
11. Discussions and Conclusions
12. References:
    1. TL, PLB **768**, 248 (2017)
    2. TL, PLB **796**, 196 (2019)
    3. TL, arXiv:2201.09632 (2022)

Witten's Cubic Open String Field Theory

$$S_{\text{open}} = \int \text{tr}\left(\Psi * Q\Psi + \frac{2g}{3}\Psi * \Psi * \Psi\right), \tag{1}$$

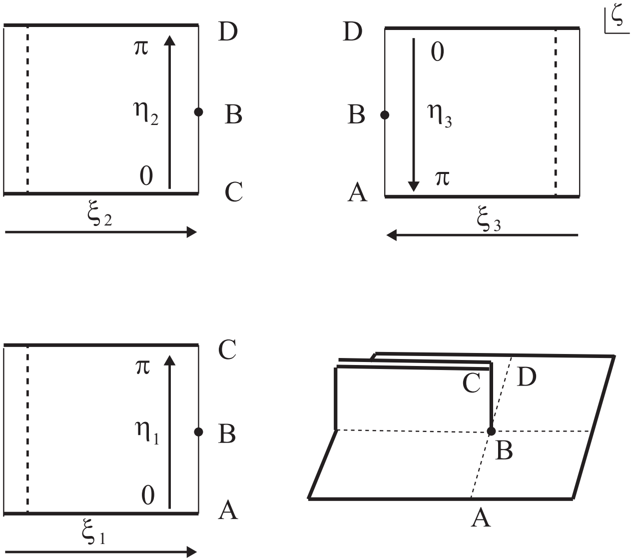
Star product between the string field operators

$$\begin{aligned}(\Psi_1 * \Psi_2)[X(\sigma)] &= \int \prod_{\frac{\pi}{2}\le\sigma\le\pi} DX^{(1)}(\sigma) \prod_{0\le\sigma\le\frac{\pi}{2}} DX^{(2)}(\sigma) \\ &\prod_{\frac{\pi}{2}\le\sigma\le\pi} \delta\left[X^{(1)}(\sigma) - X^{(2)}(\pi-\sigma)\right] \Psi_1[X^{(1)}(\sigma)]\Psi_2[X^{(2)}(\sigma)].\end{aligned} \tag{2}$$

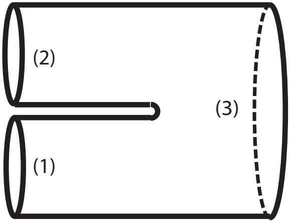
BRST gauge transformation

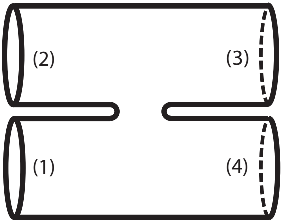
$$\delta\Psi = Q * \epsilon + \Psi * \epsilon - \epsilon * \Psi. \tag{3}$$

Witten's Cubic Open String Field Theory

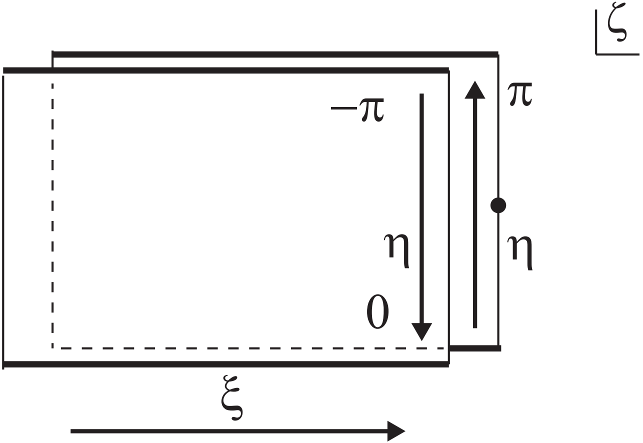


Closed String Field Theory in the Proper-Time Gauge

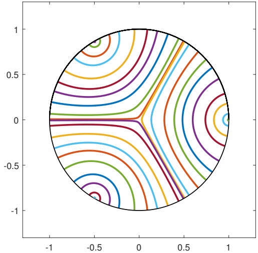


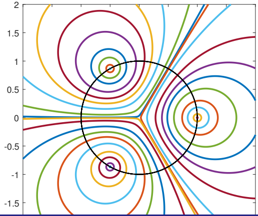


Closed String Theory on a Double Layer



Closed String Theory on a Double Layer : ω-complex plane (unit disk)





Closed Cubic Closed String Field Theory (Extension of Witten's)

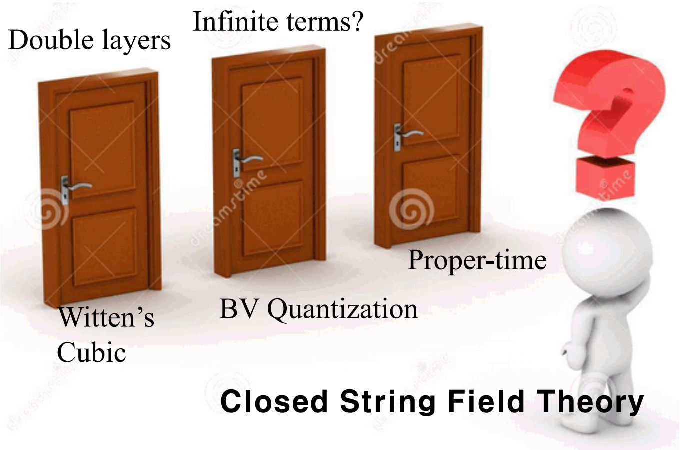
No-Go Theorem? Nucl. Phys. B 336, 185 (1990)

Argument of Sonoda and Zwiebach: Symmetric BRST inv. cubic term cannot generate a quartic BRST inv. & gauge covariant term. Claim: **"Covariant (closed) string field theory cannot be cubic"** .

Some exceptional cases, including Witten's Cubic Open String Field Theory.
Resolution:

1. Asymmetric cubic interaction. (proper-time gauge)
2. Double Copies of Witten's cubic open string field theory ⇒ Closed String Field Theory with Double Layers.

Closed String Field Theory - 3 Ways to Go



Closed String Theory on a Double Layer: Over-lapping conditions

Star product between two string field operators may be defined as

$$
\begin{aligned}
(\Psi_1 * \Psi_2)\,[X(\sigma)]_{\mathrm{closed}} = \int \prod_{-\pi\le\sigma\le-\frac{\pi}{2}} \prod_{\frac{\pi}{2}\le\sigma\le\pi} DX^{(1)}(\sigma) \\
\prod_{0\le\sigma\le\frac{\pi}{2}} \prod_{-\frac{\pi}{2}\le\sigma\le 0} DX^{(2)}(\sigma) \prod_{-\pi\le\sigma\le-\frac{\pi}{2}} \prod_{\frac{\pi}{2}\le\sigma\le\pi} \delta\left[X^{(1)}(\sigma) - X^{(2)}(\pi-\sigma)\right] \\
\Psi_1[X^{(1)}(\sigma)]\Psi_2[X^{(2)}(\sigma)],
\end{aligned}
\tag{4}
$$

The Neumann Functions for Closed String: Vertex Operator

Polyakov Path Integral

$$
\begin{aligned}
\mathcal{A} &= \int \prod_{r,n} dP_{r,n} \Psi_r[P_r] W(P_{r,n}, p_r), & \text{(5a)} \\
W(P_{r,n}, p_r) &= \int D[X] \exp\Bigg(i \sum_r \int_{\partial M_r} P^\mu_r(\eta_r) X_\mu(\xi_r, \eta_r) d\eta_r \\
&\quad - \sum_r \int_{M_r} \mathcal{L}\, d\xi_r d\eta_r \Bigg), & \text{(5b)} \\
\mathcal{L} &= \frac{1}{2\pi} \left\{ \left(\frac{\partial X^\mu}{\partial \xi_r}\right)^2 + \left(\frac{\partial X^\mu}{\partial \eta_r}\right)^2 \right\}. & \text{(5c)}
\end{aligned}
$$

Vertex Operator and Polyakov String Path Integral

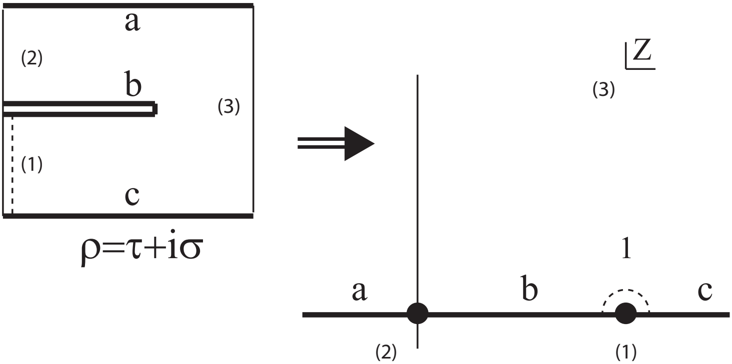
Strategy of Calculation of Vertex Operator

1. Construct the covariant closed string field theory
2. Rewrite the scattering amplitudes generated by the closed string field theory by using the Polyakov string path integral
3. Re-express the Polakov string path integrals in terms of the oscillator operators
4. Identify the Fock space (operator) representations of the string field theory vertices
5. Choose appropriate external string states, corresponding to the various particles and evaluate the scattering amplitudes.

Fock space representations of Overlapping functionals are not vertex operators!

Open String Field Theory in the Proper-Time Gauge (TL)

Mapping of the three-string vertex diagram onto the upper half complex plane



not symmetric, no BRST

Vertex Operator - proper-time gauge: KLT-Double Copy

$$
\begin{aligned}
\mathcal{A}[1,2,3] \quad = \quad & g\langle\{\mathbf{k}^{(r)}\}| \\
& \exp\left\{\sum_{r,s}\Big(\sum_{n,m\ge1}\frac{1}{2}\bar{N}^{rs}_{nm}\frac{\alpha^{(r)\dagger}_n}{2}\cdot\frac{\alpha^{(r)\dagger}_m}{2}+\sum_{n\ge1}\bar{N}^{rs}_{n0}\frac{\alpha^{(r)\dagger}_n}{2}\cdot\frac{p^{(s)}}{2}\Big)\right\} \\
& \exp\left\{\tau_0\sum_r\frac{1}{\alpha_r}\left(\frac{1}{2}\left(\frac{p^{(r)}}{2}\right)^2-1\right)\right\} \\
& \exp\left\{\sum_{r,s}\Big(\sum_{n,m\ge1}\frac{1}{2}\bar{N}^{rs}_{nm}\frac{\tilde{\alpha}^{(r)\dagger}_n}{2}\cdot\frac{\tilde{\alpha}^{(r)\dagger}_m}{2}+\sum_{n\ge1}\bar{N}^{rs}_{n0}\frac{\tilde{\alpha}^{(r)\dagger}_n}{2}\cdot\frac{p^{(s)}}{2}\Big)\right\} \\
& \exp\left\{\tau_0\sum_r\frac{1}{\alpha_r}\left(\frac{1}{2}\left(\frac{p^{(r)}}{2}\right)^2-1\right)\right\}|0\rangle.
\end{aligned}
$$

Neumann Functions for Closed String

$$
\begin{aligned}
G(\rho_r, \rho'_s) &= \ln |z - z'| \\
&= -\delta_{rs} \left\{ \sum_{n=1} \frac{1}{2n} \left(\omega_+^{-n} \omega'^n_- + \omega_+^{*-n} \omega'^{*n}_- \right) - \max(\xi, \xi') \right\} \\
&\quad + \bar{C}^{rs}_{00} + \sum_{n=1} \left(\bar{C}^{rs}_{n0} \omega^n_r + \bar{C}^{rs}_{-n0} \omega^{*n}_r \right) + \sum_{m=1} \left(\bar{C}^{rs}_{0m} \omega'^m_s + \bar{C}^{rs}_{0-m} \omega'^{*m}_s \right) \\
&\quad + \sum_{n,m\geq 1} \left\{ \bar{C}^{rs}_{nm} \omega^n_r \omega'^m_s + \bar{C}^{rs}_{-nm} \omega^{*n}_r \omega'^m_s + \bar{C}^{rs}_{n-m} \omega^n_r \omega'^{*m}_s \right. \\
&\quad \left. + \bar{C}^{rs}_{-n-m} \omega^{*n}_r \omega^{*\prime m}_s \right\}, \qquad (6) \\
\omega_r &= e^{\zeta_r} = e^{\xi_r + i\eta_r}, \quad \omega'_s = e^{\xi'_r + i\eta'_s}, \\
(\omega_+, \omega_-) &= \begin{cases} (\omega_r, \omega'_s), & \text{for } \xi_r \geq \xi'_s \\ (\omega'_s, \omega_r), & \text{for } \xi_r \leq \xi'_s \end{cases} \qquad (7)
\end{aligned}
$$

Neumann Functions for Closed String

SC mapping in Witten's cubic string theory:

$$e^{-\zeta_r} = \frac{a_r}{(z_r - Z_r)} + \sum_{n=0} c_n^{(r)} (z_r - Z_r)^n. \tag{8}$$

- $\bar{C}^{rs}_{00} = \ln |Z_r - Z_s|$, for $r \neq s$.
- $\bar{C}^{rr}_{00} = \ln a_r$.
- From reality condition of the Green's function $G(\rho_r, \rho'_s) = G(\rho_r, \rho'_s)^*$, $\bar{C}^{rs}_{nm} = \bar{C}^{rs*}_{-n-m}$.
- $\bar{C}^{rs}_{n0} = \bar{C}^{sr}_{0n} = \frac{1}{2}\bar{N}^{rs}_{n0} = \frac{1}{2n}\oint_{Z_r}\frac{dz}{2\pi i}\frac{1}{z-Z_s}e^{-n\zeta_r(z)}, \quad n \geq 1.$
- $\bar{C}^{rs}_{n-m} = \bar{C}^{rs}_{-nm} = 0.$
- $\bar{C}^{rs}_{nm} = \bar{C}^{rs}_{-n-m} = \frac{1}{2}\bar{N}^{rs}_{mn} = \frac{1}{2nm}\oint_{Z_r}\frac{dz}{2\pi i}\oint_{Z_s}\frac{dz'}{2\pi i}\frac{1}{(z-z')^2}e^{-n\zeta_r(z)-m\zeta'_s(z')}$, $n, m \geq 1$. (Neumann function for open string).

Three-Graviton Scattering Amplitude (proper-time)

$\mathcal{A}_{[3-graviton]}$ is precisely the three-graviton interaction term which may be obtained from the Einstein's gravity action.

$$\begin{aligned}\mathcal{A}_{[3-graviton]} \quad = \quad & \kappa \int \prod_{i=1}^{3} dp^{(i)} \delta\left(\sum_{i=1}^{3} p^{(i)}\right) \\ & h_{\mu_1\nu_1}(p^{(1)}) h_{\mu_2\nu_2}(p^{(2)}) h_{\mu_3\nu_3}(p^{(3)}) \\ & \left\{\eta^{\mu_1\mu_2} p^{(1)\mu_3} + \eta^{\mu_2\mu_3} p^{(2)\mu_1} + \eta^{\mu_3\mu_1} p^{(3)\mu_2}\right\} \\ & \left\{\eta^{\nu_1\nu_2} p^{(1)\nu_3} + \eta^{\nu_2\nu_3} p^{(2)\nu_1} + \eta^{\nu_3\nu_1} p^{(3)\nu_2}\right\}\end{aligned}$$

where $\kappa = \frac{g}{2^7 \cdot 3} = \sqrt{32\pi G_{10}}$.

Gravity $=$ Gauge $\times$ Gauge

$$S = \langle\Phi|\mathcal{K}\Phi\rangle + \frac{g}{3}\left(\langle\Phi|\Phi\circ\Phi\rangle + \langle\Phi\circ\Phi|\Phi\rangle\right).$$

Classical equation of motion with a D-brane source, J_D

$$\mathcal{K}\Phi + g\Phi\circ\Phi = J_D.$$

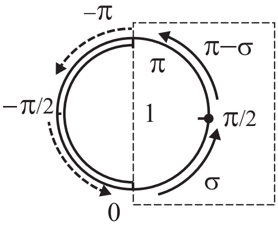
Perturbative solution in the weak field limit

$$\Phi = \frac{1}{\mathcal{K}}J_D - \frac{g}{\mathcal{K}}\left\{\frac{1}{\mathcal{K}}J_D \circ \frac{1}{\mathcal{K}}J_D\right\} + \cdots$$

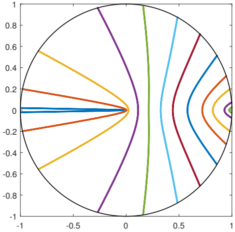
Construction of Identity Vertex for Closed String

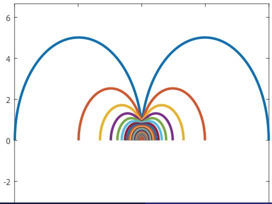
$$I[X(\sigma)] = \langle X(\sigma)|I\rangle = \prod_{\frac{\pi}{2}\le\sigma\le\pi} \prod_{-\pi\le\sigma\le-\frac{\pi}{2}} \delta\left(X(\sigma) - X(\pi-\sigma)\right) \tag{9}$$

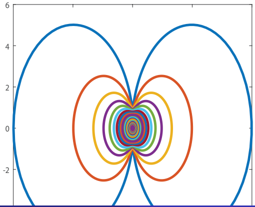
$$\omega \quad = \quad \left(\frac{1+ie^{\zeta}}{1-ie^{\zeta}}\right)^2, \quad z = -i\frac{\omega-1}{\omega+1}, \quad -\pi \le \eta \le \pi. \tag{10}$$



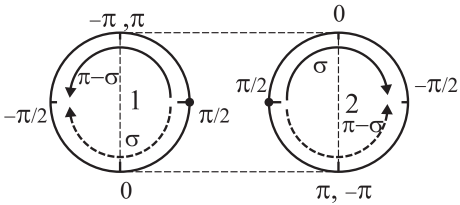
Construction of Identity Vertex for Closed String: ω and z - complex planes

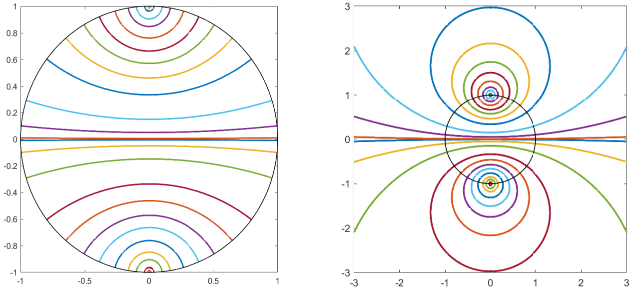




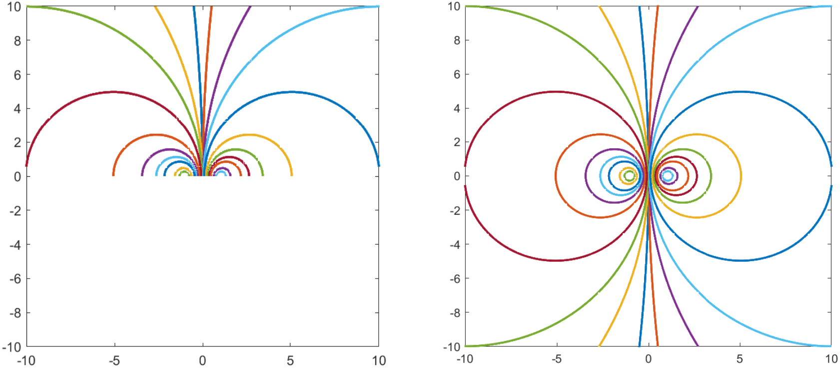


Two-String Vertex for Closed String: ω-complex plane



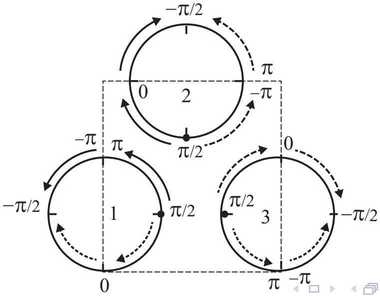


Two-String Vertex for Closed String: z-complex plane

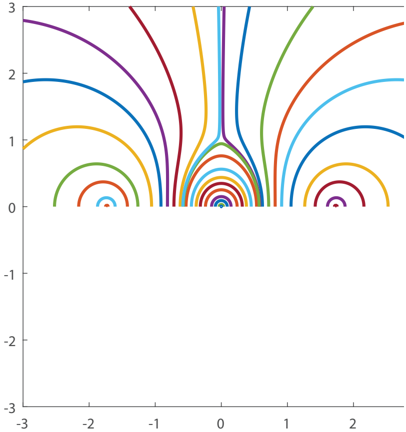


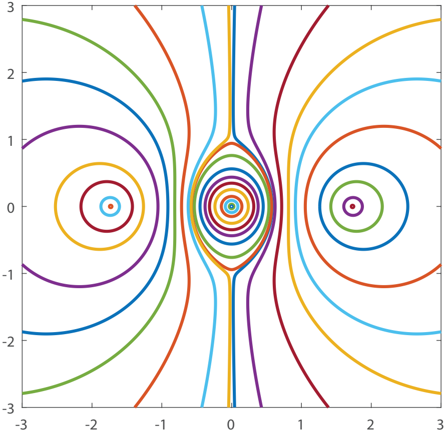
Cubic Vertex for Closed String

$$(\Psi_1 * \Psi_2)\,[X(\sigma)]_{\text{closed}} = \int \prod_{-\pi\le\sigma\le-\frac{\pi}{2}} \prod_{\frac{\pi}{2}\le\sigma\le\pi} DX^{(1)}(\sigma)$$
$$\prod_{0\le\sigma\le\frac{\pi}{2}} \prod_{-\frac{\pi}{2}\le\sigma\le 0} DX^{(2)}(\sigma) \prod_{-\pi\le\sigma\le-\frac{\pi}{2}} \prod_{\frac{\pi}{2}\le\sigma\le\pi} \delta\left[X^{(1)}(\sigma) - X^{(2)}(\pi-\sigma)\right]$$
$$\Psi_1[X^{(1)}(\sigma)]\Psi_2[X^{(2)}(\sigma)], \tag{11}$$



Cubic Vertex for Closed String: z-complex plane





Cubic Vertex and Cubic Gauge Coupling

$$|V^{\text{Closed}}_{[3]}[1,2,3]|0\rangle = \exp\left\{2\sum_{r=1}^{3}\ln\frac{8}{3}\left(\frac{\left(p^{(r)}\right)^2}{2}-1\right)\right\}\prod_{r<s}|Z_r - Z_s|^{2p^{(r)}\cdot p^{(s)}}$$
$$\exp\left\{\sum_{r,s}\Big(\sum_{n,m\geq 1}\frac{1}{2}\bar{N}^{rs}_{nm}\frac{\alpha^{(r)\dagger}_n}{2}\cdot\frac{\alpha^{(s)\dagger}_m}{2}+\sum_{n\geq 1}\bar{N}^{rs}_{n0}\frac{\alpha^{(r)\dagger}_n}{2}\cdot\frac{p^{(s)}}{2}\Big)\right\}$$
$$\exp\left\{\sum_{r,s}\Big(\sum_{n,m\geq 1}\frac{1}{2}\bar{N}^{rs}_{nm}\frac{\tilde{\alpha}^{(r)\dagger}_n}{2}\cdot\frac{\tilde{\alpha}^{(r)\dagger}_m}{2}+\sum_{n\geq 1}\bar{N}^{rs}_{n0}\frac{\tilde{\alpha}^{(r)\dagger}_n}{2}\cdot\frac{p^{(s)}}{2}\Big)\right\}|0\rangle \tag{12}$$

Cubic Vertex and Cubic Gauge Coupling

$$|\Psi_1, \Psi_2, \Psi_3\rangle = \prod_{r=1}^{3} \left\{ h_{\mu\nu}(p^r) \alpha_{-1}^{(r)\mu} \tilde{\alpha}_{-1}^{(r)\nu} \right\} |0,\rangle \tag{13}$$

three-graviton-scattering amplitude $\mathcal{A}_{[3]}$:

$$\begin{aligned} \mathcal{A}_{[3]} &= \kappa \int \prod_{i=1}^{3} dp^{(i)} \delta\left(\sum_{i=1}^{3} p^{(i)}\right) h_{\mu_1\nu_1}(p^{(1)})\, h_{\mu_2\nu_2}(p^{(2)})\, h_{\mu_3\nu_3}(p^{(3)}) \\ & \left\{ \eta^{\mu_1\mu_2} p^{(1)\mu_3} + \eta^{\mu_2\mu_3} p^{(2)\mu_1} + \eta^{\mu_3\mu_1} p^{(3)\mu_2} \right\} \\ & \left\{ \eta^{\nu_1\nu_2} p^{(1)\nu_3} + \eta^{\nu_2\nu_3} p^{(2)\nu_1} + \eta^{\nu_3\nu_1} p^{(3)\nu_2} \right\} \end{aligned} \tag{14}$$

where $\kappa = \frac{\sqrt{3}g}{2^{18}} = \sqrt{32\pi G_{10}}$. This is precisely the three-graviton-scattering amplitude of the Einstein gravity

Conclusions and Discussions

1. Breaking of BRST symmetry. Gauge invariance of Witten's cubic string (open/closed) string theory. Witten's extended BRST invariance may not be fundamental. (not imposed by constraints)
2. Double copy interpretation of four-graviton scattering amplitude
3. Classical solutions with *Dp*-brane sources
   $\Rightarrow$ Construction of black hole solutions and *AdS* spaces in the string field theory $\Rightarrow$ Double copy interpretation
4. One-loop closed string field theory and its double copy interpretation.
5. Double copy extensions to the BRST ghost sector .
6. Extensions to Super-symmetric theories