Hairy AdS black holes with Robin boundary conditions

Takaaki Ishii (Rikkyo University)

to appear

w/ Tomohiro Harada, Takuya Katagiri, Norihiro Tanahashi

24 Feb 2023, 6th Holography workshop @ Danang

This can be also considered as

Hairy AdS black holes with mixed boundary conditions

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Setup

Holographic superconductivity: breaking of U(1) gauge symmetry by Higgs mechanism in AdS

[Hartnoll-Herzog-Horowitz]

"Classic" action ($8\pi G_N=1$)

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}\left(R-2\Lambda\right)-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-|D\phi|^2-m^2|\phi|^2\right)$$

- Scalar mass: $m^2 = -2$

Ansatz

$$\mathrm{d}s^2 = -\left(1+r^2\right)f(r)e^{-\chi(r)}\mathrm{d}t^2+\frac{\mathrm{d}r^2}{\left(1+r^2\right)f(r)}+r^2\mathrm{d}\mathcal{M}_2^2$$

$$A = A_t(r)\mathrm{d}t, \qquad \phi = \phi(r)$$

- Goal: find solutions with $\phi \neq 0$.

Popular assumptions

We often use **Poincare AdS** and **Dirichlet boundary conditions.**

- Poincare AdS: dual to flat space field theory $\mathrm{d}M_2^2 = \mathrm{d}x^2 + \mathrm{d}y^2$
- Dirichlet: $\phi = \frac{\phi_1}{r} + \frac{\phi_2}{r^2} + \cdots$ with $\phi_1 = 0$ in AdS boundary (r→∞)

<u>Outcomes</u>

- $\phi \neq 0$ in low temperature, finite charge density
- Spontaneous symmetry breaking $\langle O_2 \rangle \sim \phi_2 \neq 0$
- Interpretation: superconductivity/superfluidity

This talk

Other boundary conditions are available.

Often used

- Asymptotically **Poincare AdS**
- **Dirichlet** b.c. for the scalar field ϕ

Options

- Poincare AdS (flat), **global AdS (sphere)**, hyperbolic
- Dirichlet, Neumann, **Robin (mixed b.c.)**

Preceding work

Perturbations of global RNAdS by complex scalar with **Robin b.c.** were studied.

[Katagiri-Harada, 2006.10301]

- Quasinormal modes → (superradiant) instability

This talk: Backreacted solutions.

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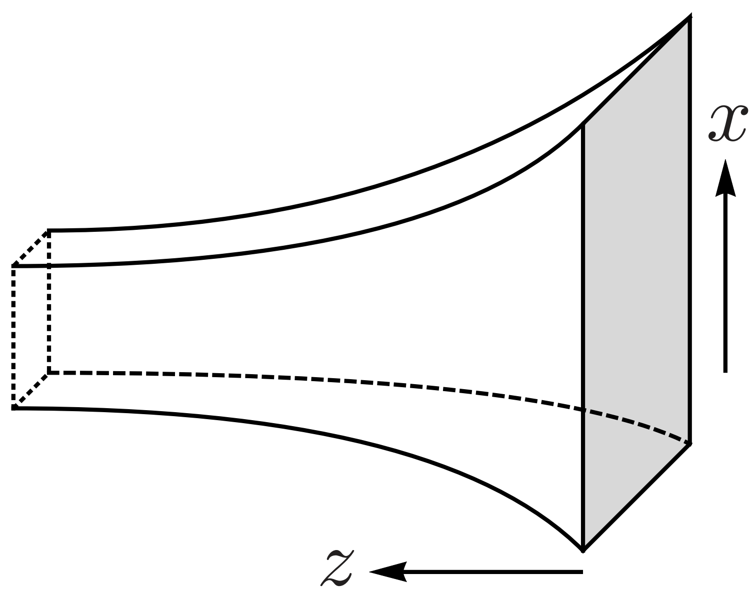
Poincare or global AdS

Choices for AdS boundary topology: **Poincare AdS** (flat boundary) or **global AdS** (spherical boundary).



Poincare AdS

Global AdS

This talk: We consider asymptotically **global AdS**

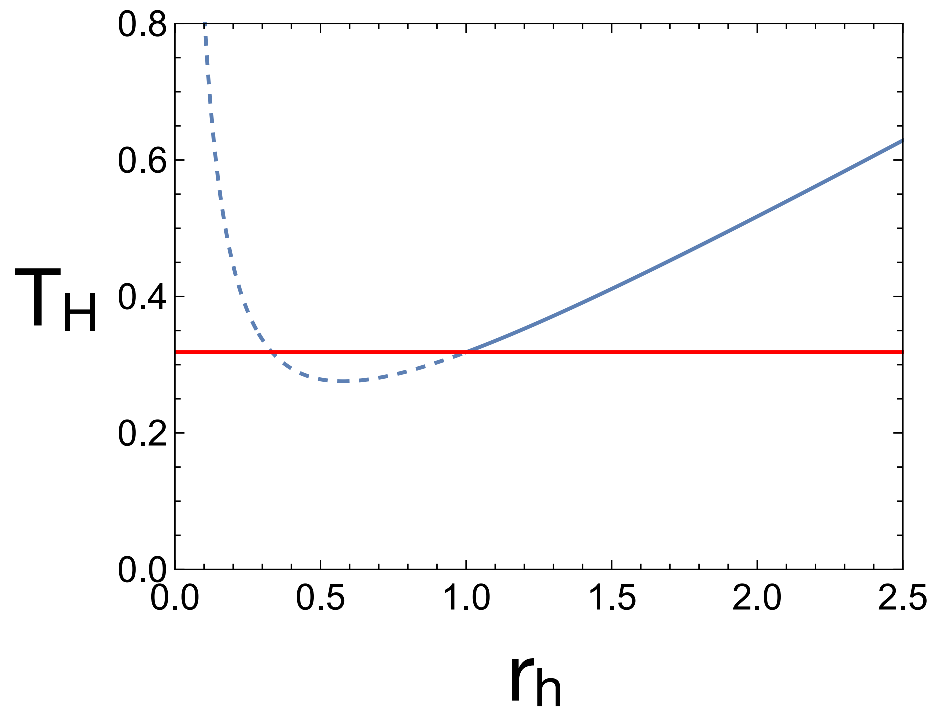
Hawking-Page phase transition

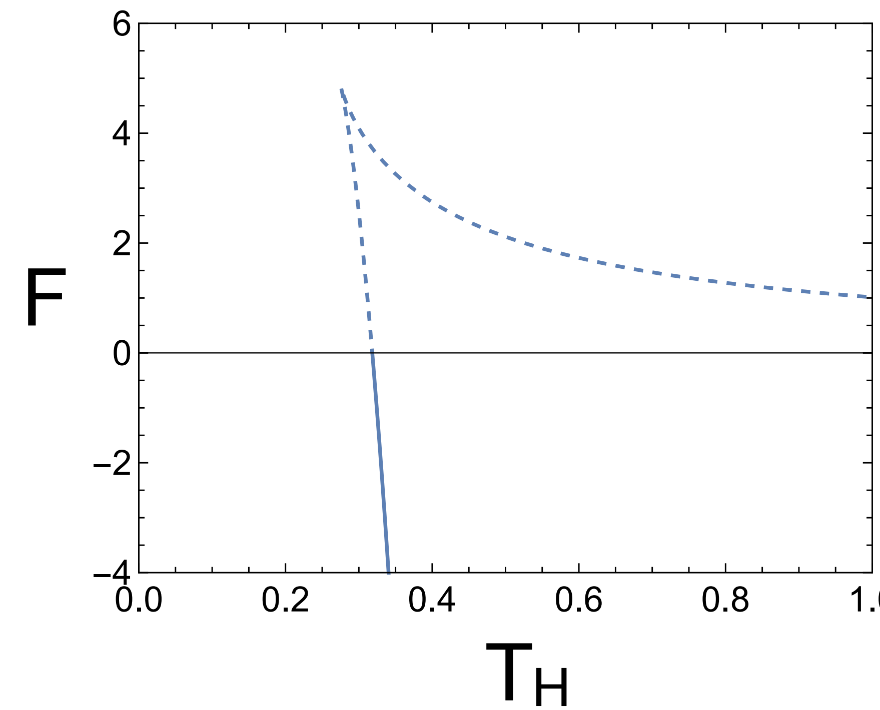
In global AdS, first order phase transition is discussed between horizonless and BH geometries.

[Hawking-Page,Witten]

- The dominant phase is determined by free energies.

<u>E.g. Thermal AdS vs Schwarzschild AdS BH</u>

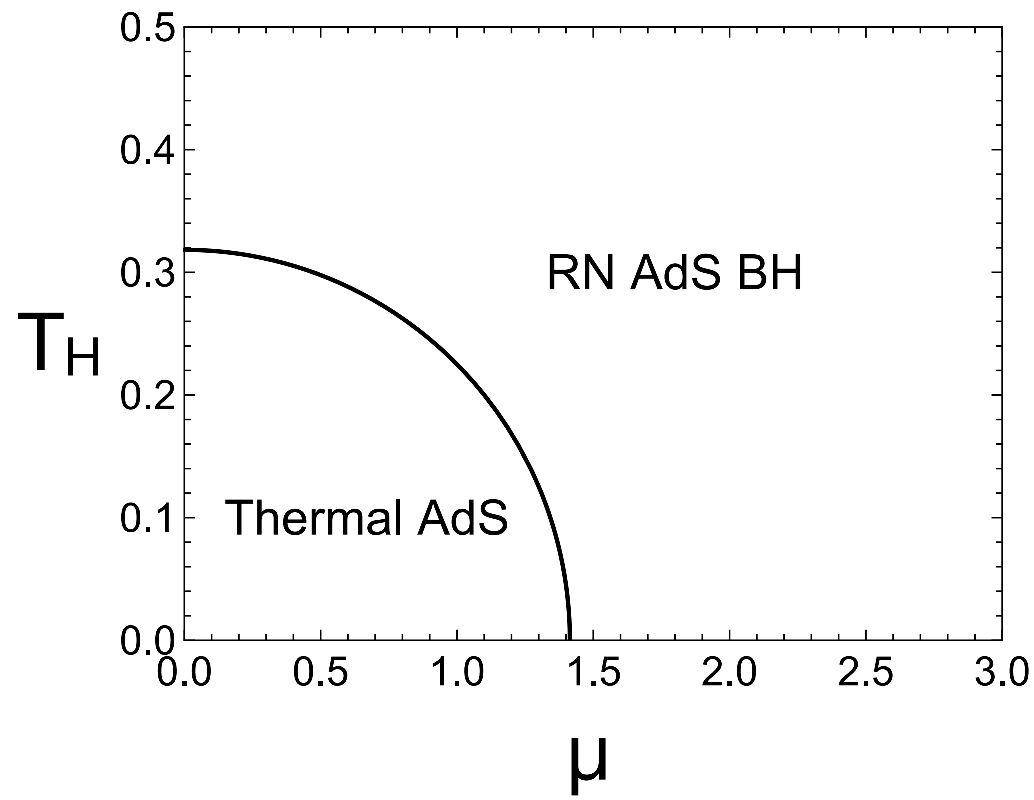




Charged AdS BH

We write phase diagram **in grand canonical ensemble.**
(μ: chemical potential)

[Chamblin-Emparan-Johnson-Myers]



This talk: We also include **charged scalar with Robin boundary conditions.**

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Scalar boundary conditions

We have several options for the **asymptotic behavior** of the scalar ϕ in r→∞, if $-9/4 \leq m^2 \leq -5/4$.

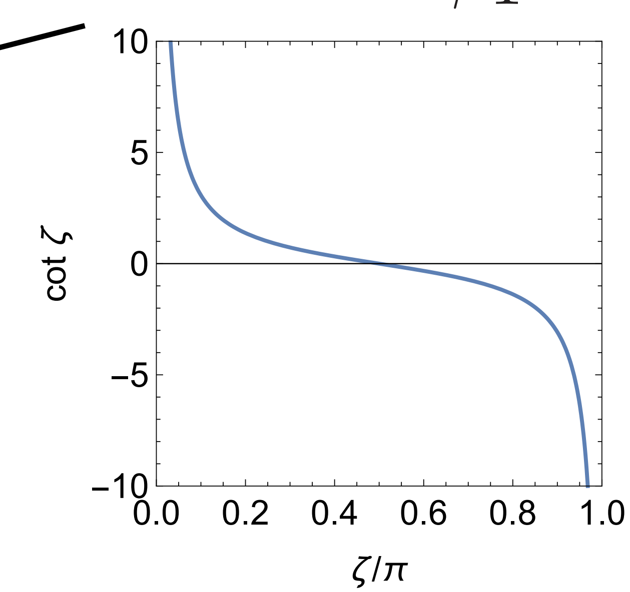
[Witten]

- Asymptotic behavior: $\phi = \dfrac{\phi_1}{r} + \dfrac{\phi_2}{r^2} + \cdots$

$$\cot\zeta = \frac{\phi_2}{\phi_1} \quad (0 \leqq \zeta < \pi)$$

<u>Boundary conditions</u>

- Dirichlet $\phi_1 = 0$ ($\zeta = 0$)
- Neumann $\phi_2 = 0$ ($\zeta = \pi/2$)
- **This talk: Robin $\phi_1 \neq 0$, $\phi_2 \neq 0$**



Hairy solutions ($\phi \neq 0$) in global AdS

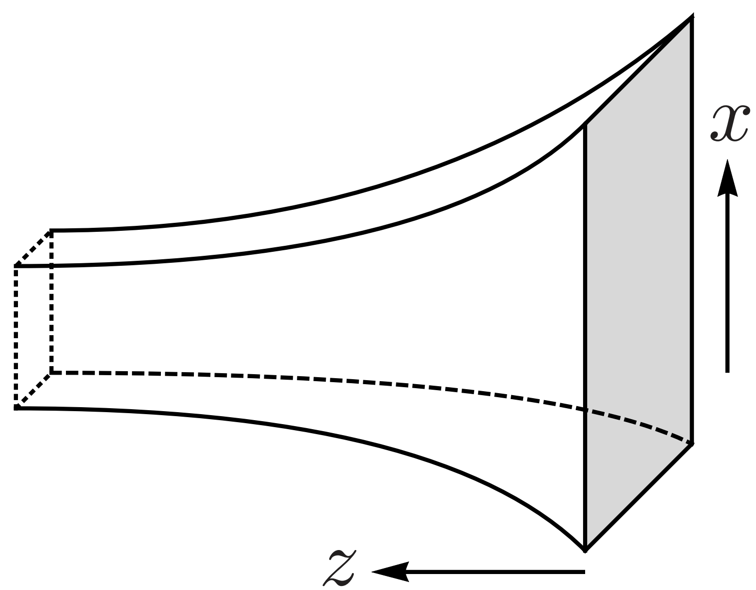
Global AdS	$\mu=0$	$\mu>0$
Horizonless	Neutral boson star	Charged boson star
With BH horizon	Neutral hairy black hole	**Charged Hairy black hole**

this work

- Parameters: (μ, T_H, ζ, q) (q: gauge coupling)

What are boson stars?

In global AdS, **nonzero-scalar horizonless solutions** can be constructed.



Poincare AdS

Global AdS

- Such solutions are called **boson stars.**

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Holographic renormalization

To obtain the expression of free energy (grand potential), we do **holographic renormalization for Robin b.c.**

[e.g. Skenderis]

- This is a systematic way to obtain such quantities.

3 steps to Robin b.c.

1. Dirichlet: standard addition of counterterms
2. Neumann: Legendre transform from Dirichlet
3. **Robin: modification of Neumann**

Dirichlet theory

On-shell action diverges, and we make it finite by **adding covariant counterterms.**

[e.g. Skenderis]

Renormalized action

$$S^D_{\rm ren} = \lim_{\rm cutoff \to \infty} (S_{\rm bulk} + S_{\rm ct})$$

Scalar source and expectation value

$$\Phi_D = \sqrt{2}\phi_1 = 0, \quad \langle O_2 \rangle = \sqrt{2}\phi_2$$

Variation of action

$$\delta S^D_{\rm ren} = \int {\rm d}^3 x \sqrt{-h} \left(\frac{1}{2} \langle T^{ij} \rangle \delta h_{ij} + \langle J^i \rangle \delta \Psi_i + \langle O_2 \rangle \delta \Phi_D \right)$$

Neumann theory

Legendre transform of Dirichlet theory by a finite boundary term is Neumann theory.

[Witten, Papadimitriou]

Legendre transform by finite boundary term

$$S_{\rm ren}^{N} = S_{\rm ren}^{D} + S_{\rm LT} \qquad S_{\rm LT} = -2 \int {\rm d}^3x \sqrt{-h}\, \phi_1 \phi_2$$

Scalar source and expectation value

$$\Phi_N = -\sqrt{2}\phi_2 = 0, \quad \langle O_1 \rangle = \sqrt{2}\phi_1$$

Variation of action

$$\delta S_{\rm ren}^{N} = \int {\rm d}^3x \sqrt{-h} \left(\frac{1}{2} \langle T^{ij} \rangle \delta h_{ij} + \langle J^i \rangle \delta \Psi_i + \langle O_1 \rangle \delta \Phi_N \right)$$

Double trace deformation

Robin boundary conditions appear as **modification of Neumann theory by a double trace term.**

[Witten, Papadimitriou]

Additional finite term for consistent variation

$$S_{\rm ren}^R = S_{\rm ren}^N + S_{\rm Dtr} \\ = S_{\rm ren}^D + S_{\rm LT} + S_{\rm Dtr}$$

$$S_{\rm Dtr} = \cot\zeta \int {\rm d}^3x\sqrt{-h}\,\phi_1^2$$

Scalar source and expectation value

$$\Phi_R = -\sqrt{2}\,(\phi_2 - \phi_1\cot\zeta) = 0, \quad \langle O_1\rangle = \sqrt{2}\phi_1$$

Variation of action

$$\delta S_{\rm ren}^R = \int {\rm d}^3x\sqrt{-h}\left(\frac{1}{2}\langle T^{ij}\rangle\delta h_{ij} + \langle J^i\rangle\delta\Psi_i + \langle O_1\rangle\delta\Phi_R\right)$$

Grand potential

We evaluate the phase structure in **grand canonical ensemble** by comparing the grand potential.

Total energy (BH mass)

$$\mathcal{E}_R = \mathcal{E}_N + \mathcal{E}_{\mathrm{Dtr}} = \mathcal{E}_D + \mathcal{E}_{\mathrm{LT}} + \mathcal{E}_{\mathrm{Dtr}}$$

$$\mathcal{E} = \int d\Omega_2 \langle T_{tt} \rangle$$

Grand potential

$$\Omega_R = \Omega_N + \Omega_{\mathrm{Dtr}} = \Omega_D + \Omega_{\mathrm{LT}} + \Omega_{\mathrm{Dtr}}$$

$$\Omega = \mathcal{E} - T_{\mathrm{H}} \mathcal{S}_{\mathrm{BH}} - \mu \mathcal{Q}$$

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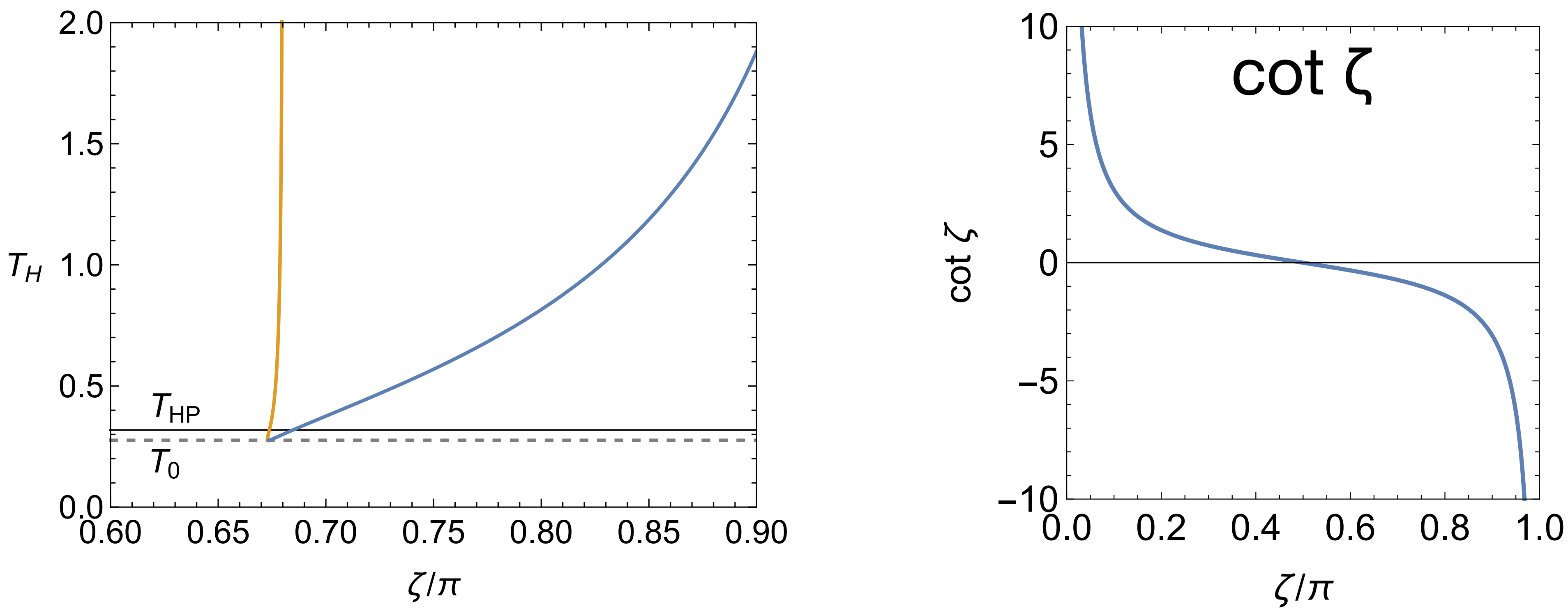
5-1. Neutral case ($\mu=q=0$)

	$\phi=0$	$\phi\neq0$
No horizon	Thermal AdS	Robin boson star
With BH horizon	Schwarzschild AdS BH	Neutral hairy Robin BH

- We compare free energies of these 4 solutions and determine the phase diagram in the (ζ, T_H)-plane.

Instability for forming scalar hair

The Robin boundary conditions make neutral scalar unstable to become nonzero in **ζ/π>0.67.**

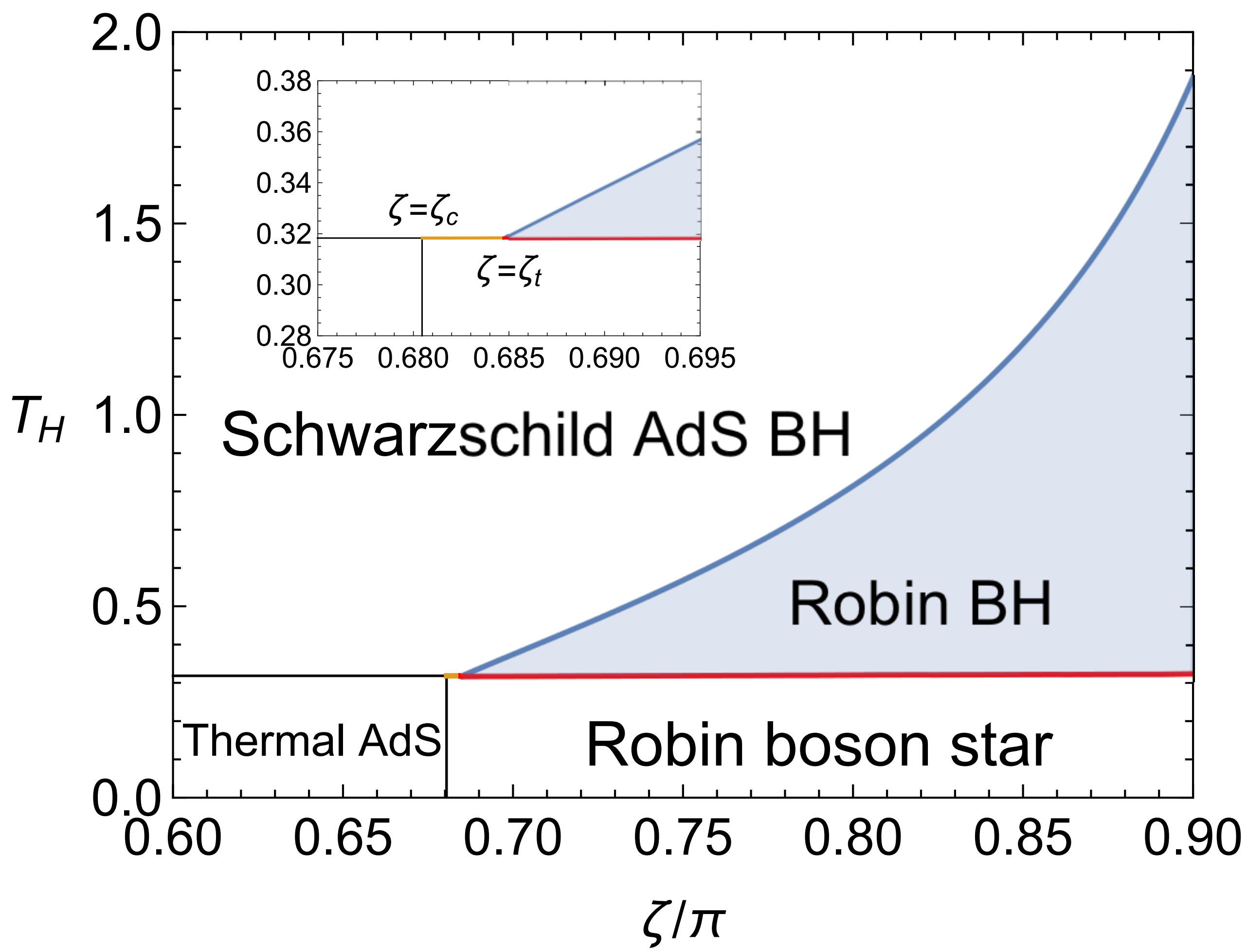


- Figure: location of the onset of instability
- Hairy BH to the right of each curve.

(Orange: small BH branch, Blue: large BH branch)

Neutral phase diagram

We compare free energies between 4 different solutions: scalar field ϕ=0 or ϕ≠0 (Robin), and BH or horizonless



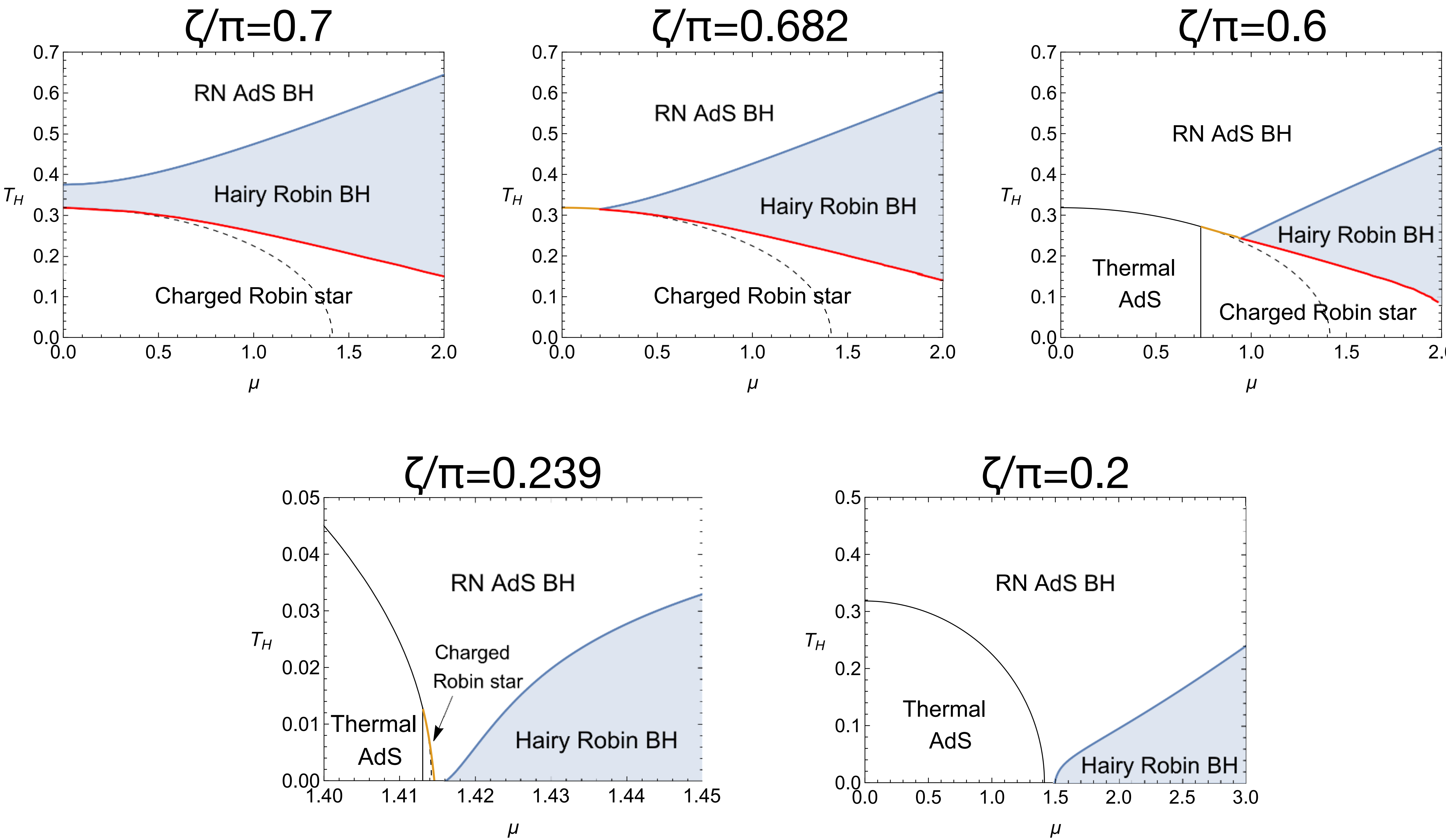
- Blue line: 2nd order, scalar hair
- Red: 1st order Hawking-Page
- Orange: 1st order Hawking-Page

5-2. Charged case $(\mu, T_H, \zeta, q) \neq 0$

	$\phi=0$	$\phi\neq0$
No horizon	Thermal AdS	Charged Robin star
With BH horizon	RNAdS BH	Charged Hairy Robin BH

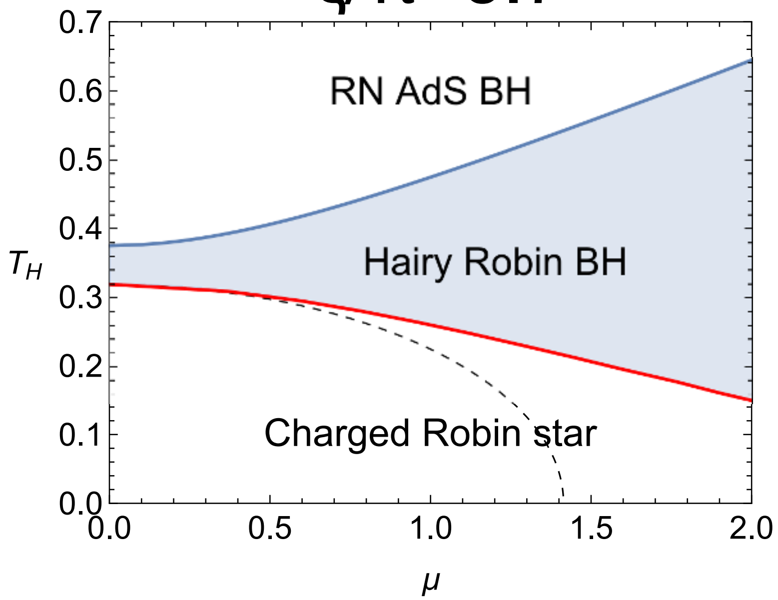
- We compare grand potentials of these 4 solutions and determine the phase diagram in the (μ, T_H)-plane.

Charged phase diagrams (q=1)

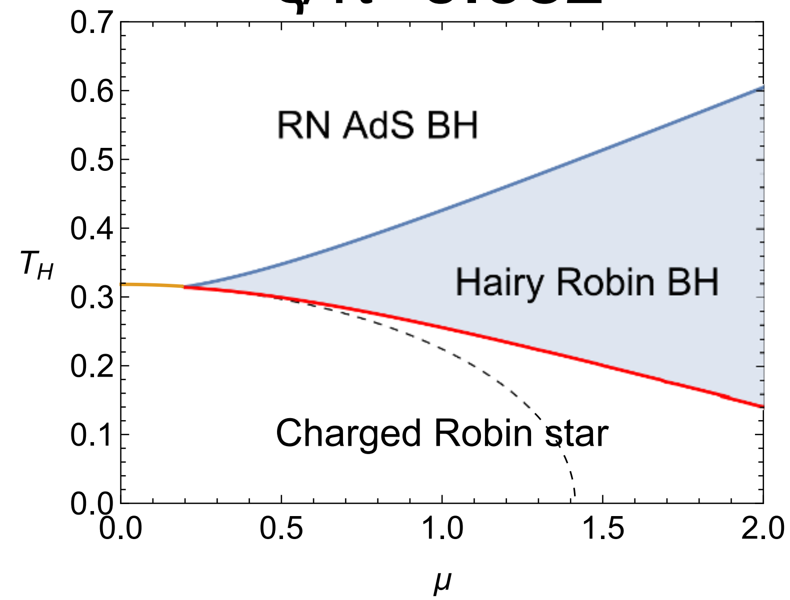


Large ζ

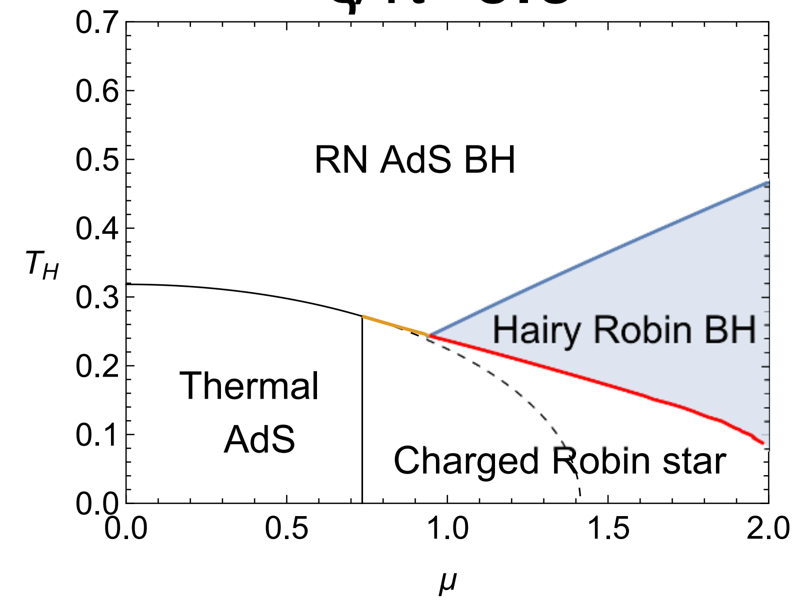
ζ/π=0.7



ζ/π=0.682

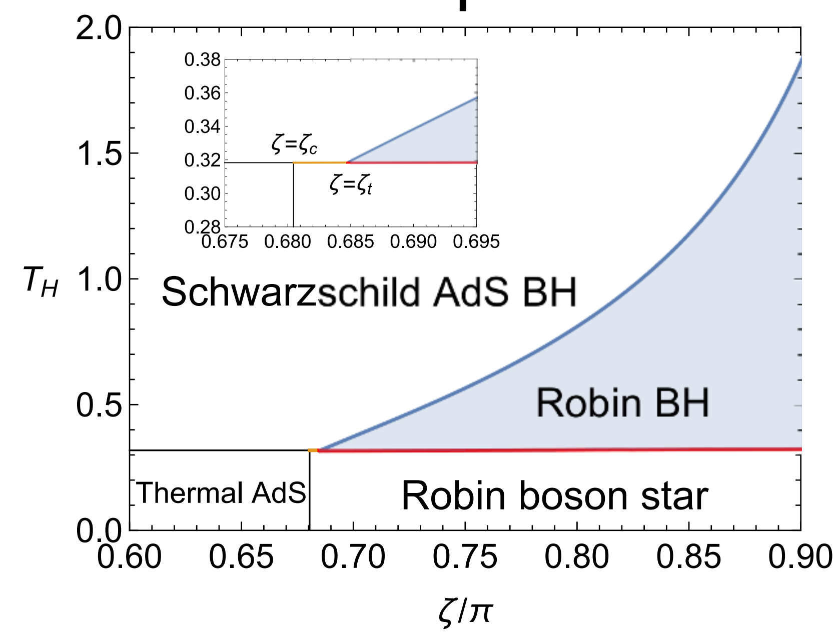


ζ/π=0.6

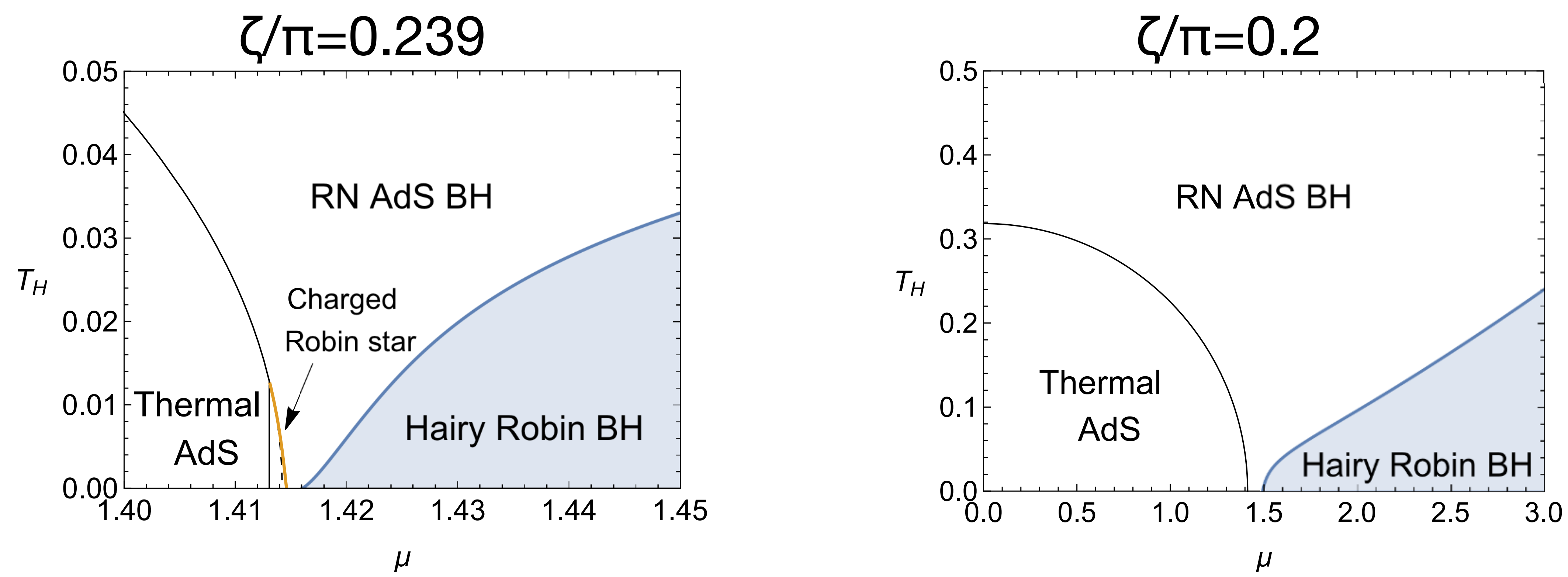


- Blue line: 2nd order, scalar hair
- Red: 1st order Hawking-Page
- Orange: 1st order Hawking-Page

c.f. μ=0



Small ζ at small coupling q



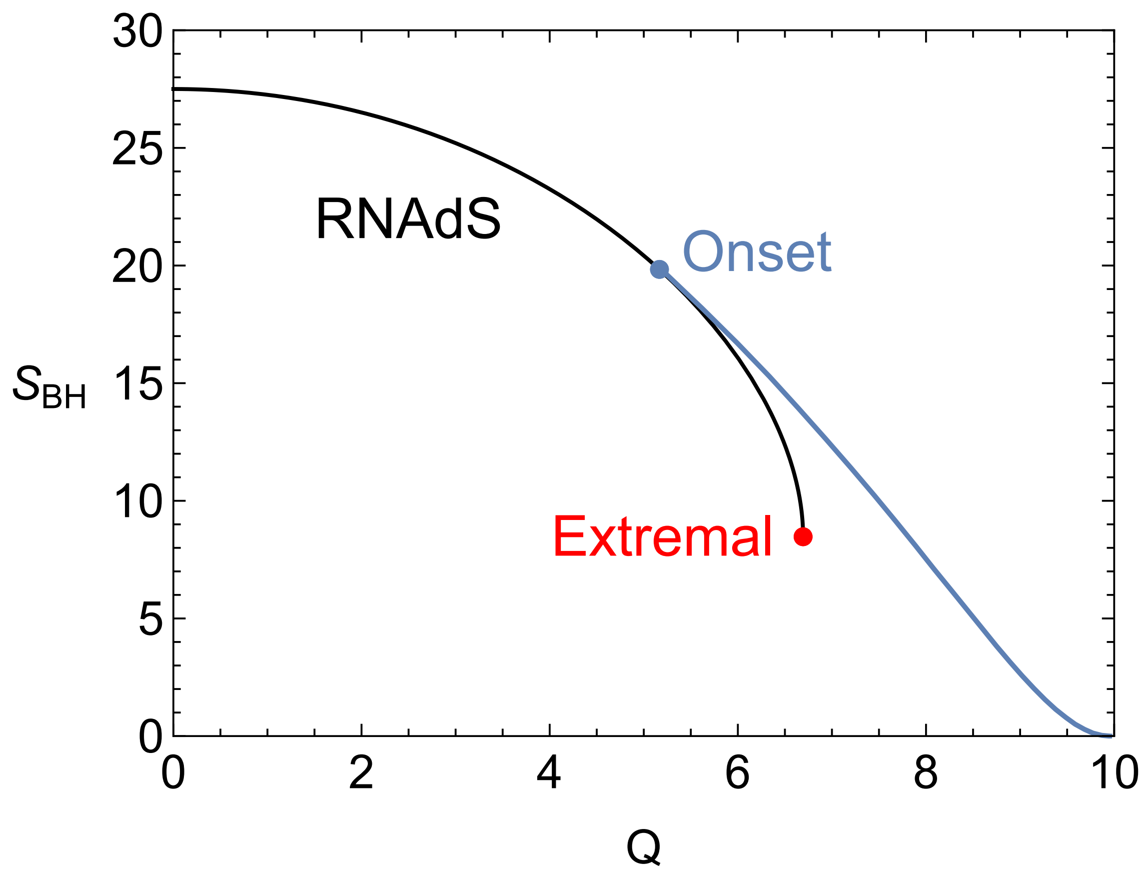
- **When the gauge coupling is small as q=1,** extremal BH can survive as thermodynamically dominant phase.

- But these structures are not observed if q is large ($q>\sqrt{2}$).

Extra: Microcanonical ensemble

For solutions at the same (E,Q), a **hairy BH has higher entropy** than a zero-scalar BH.

Microcanonical, ζ/π=0.6, q=1



Summary

We constructed hairy BH in global AdS when we use **Robin boundary conditions.**

We studied the **phase diagram in the (μ,T_H) plane** when (ζ,q) is varied.

Future direction How much can we utilize Robin (mixed) boundary conditions?