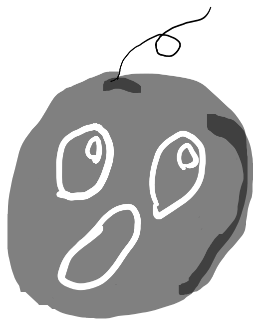
2023 International Conference on Holography, String Theory and Spacetime in Da Nang

ENTANGLEMENT AT THE SOFT HAIR HORIZON



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PLB, Vol 820 (2021) 136578 (arXiv: 2103.00516)

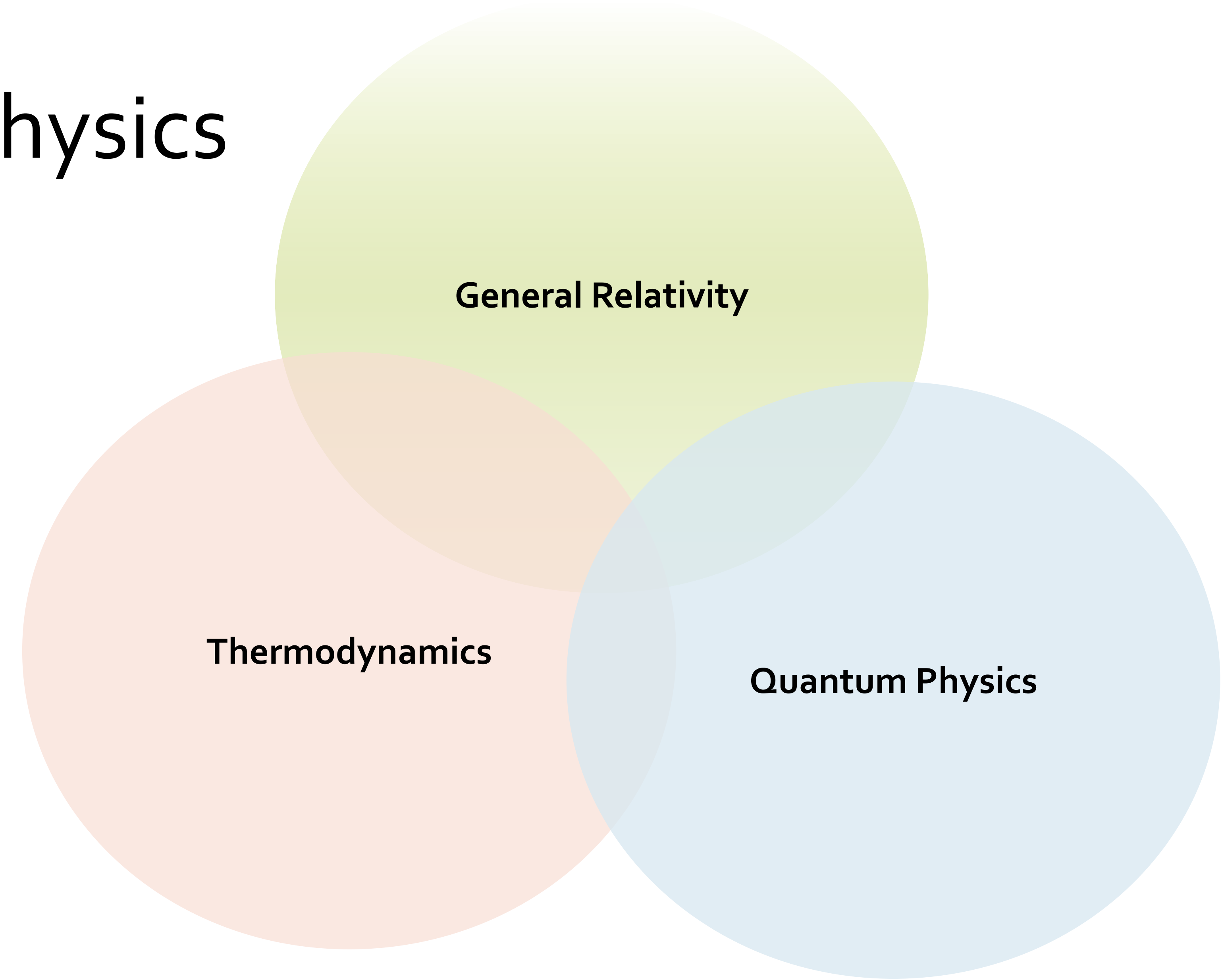
PLB, Vol 833 (2022) 137385 (arXiv: 2206.02390) w/ Sayid Mondal

Outline

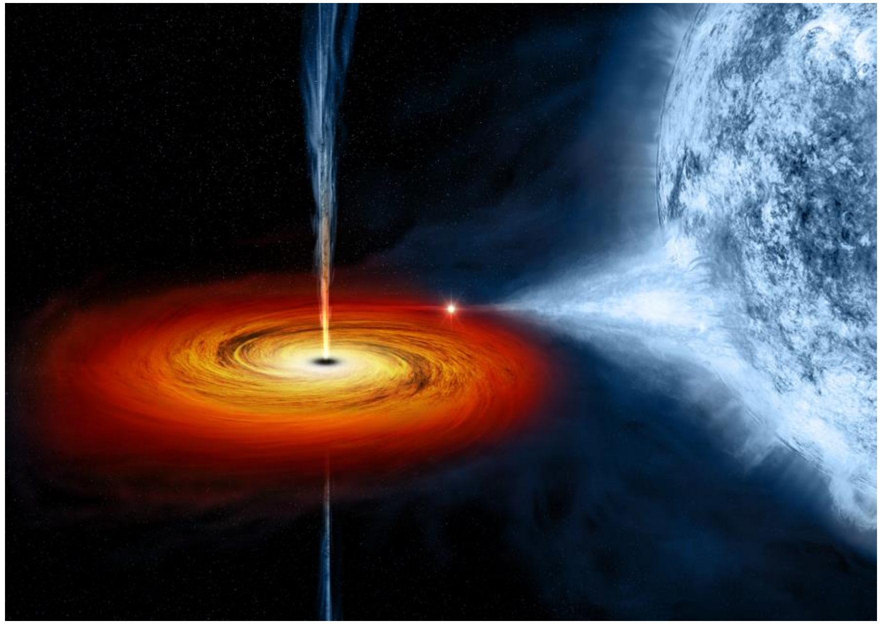
- Part I Introduction
- Part II Hawking radiation as tunneling
- Part III Hawking radiation as stimulated emission
- Part IV Hairy horizon and non-equilibrium
- Part V Entanglement at hairy horizon

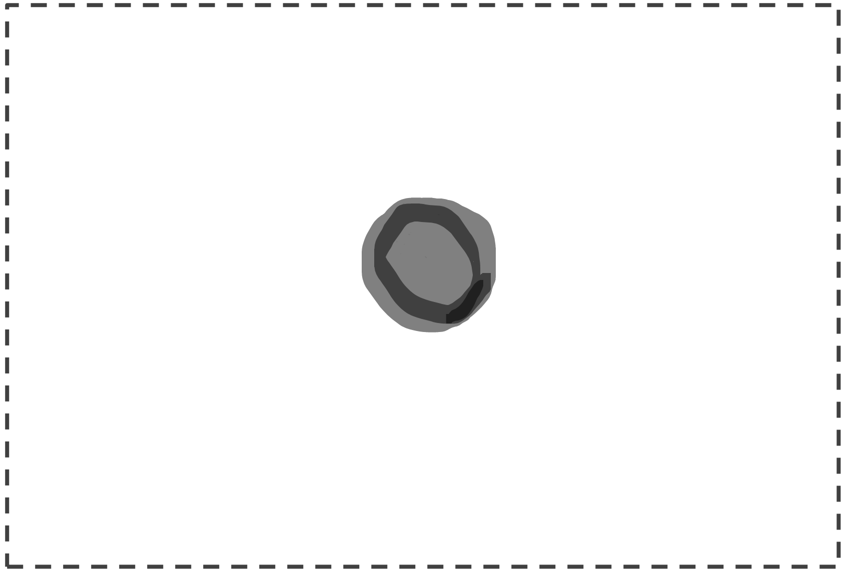
Part I
Introduction

Black holes physics



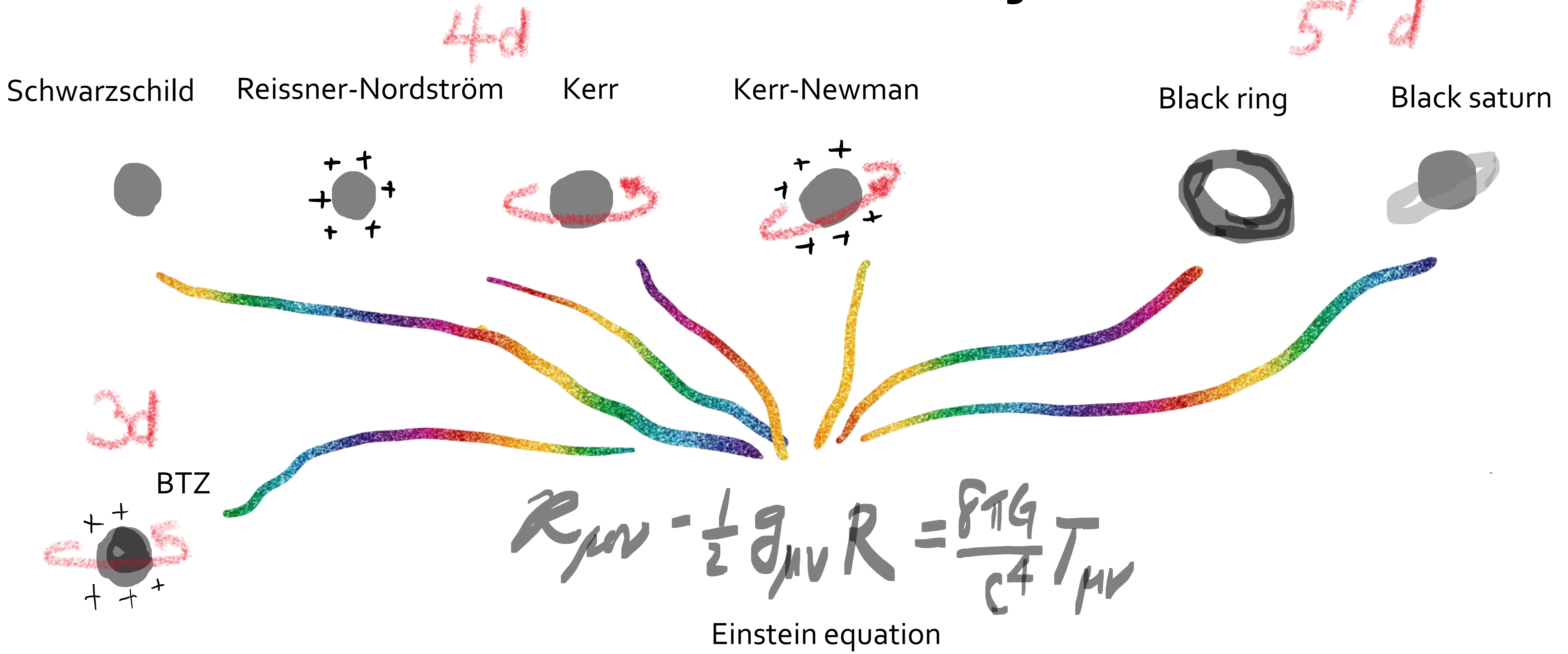
Realistic v.s. Ideal black holes





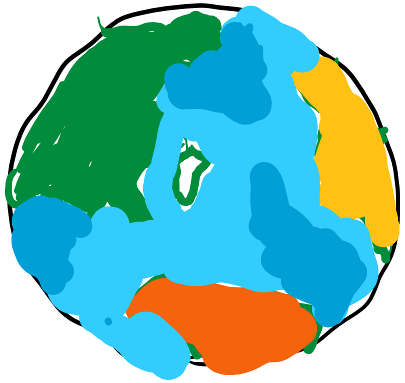
Reflective or periodic boundary condition

Black holes are relativistic objects

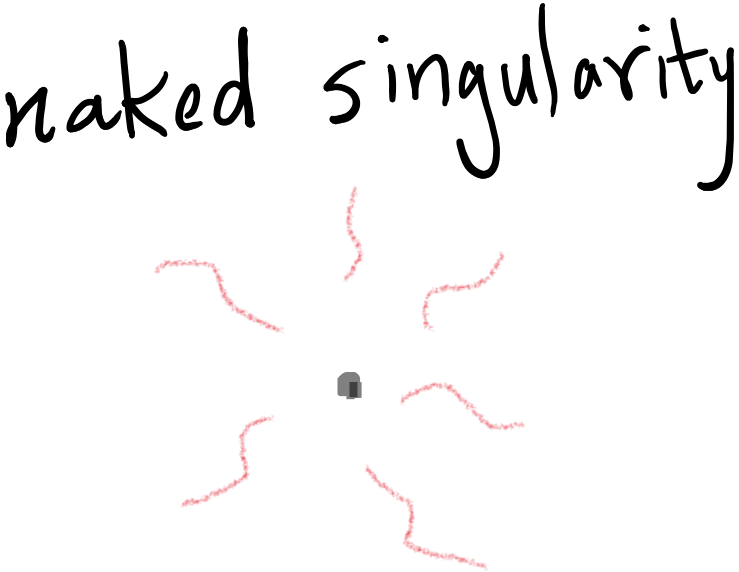


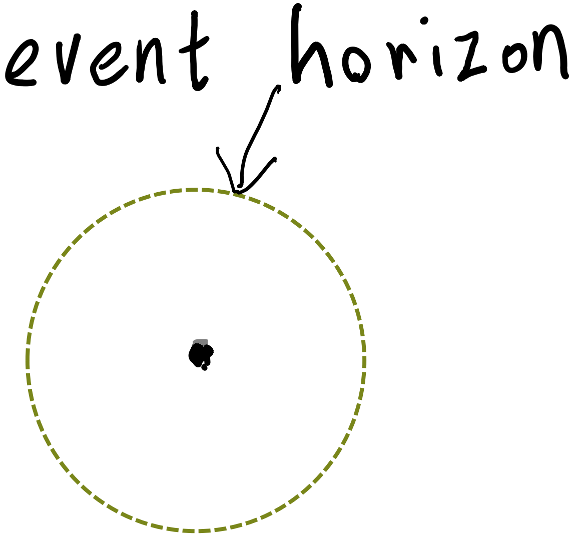
No hair theorem

- All black hole solutions are completely determined by three observables: mass (M), charge (Q), and angular momentum (J).

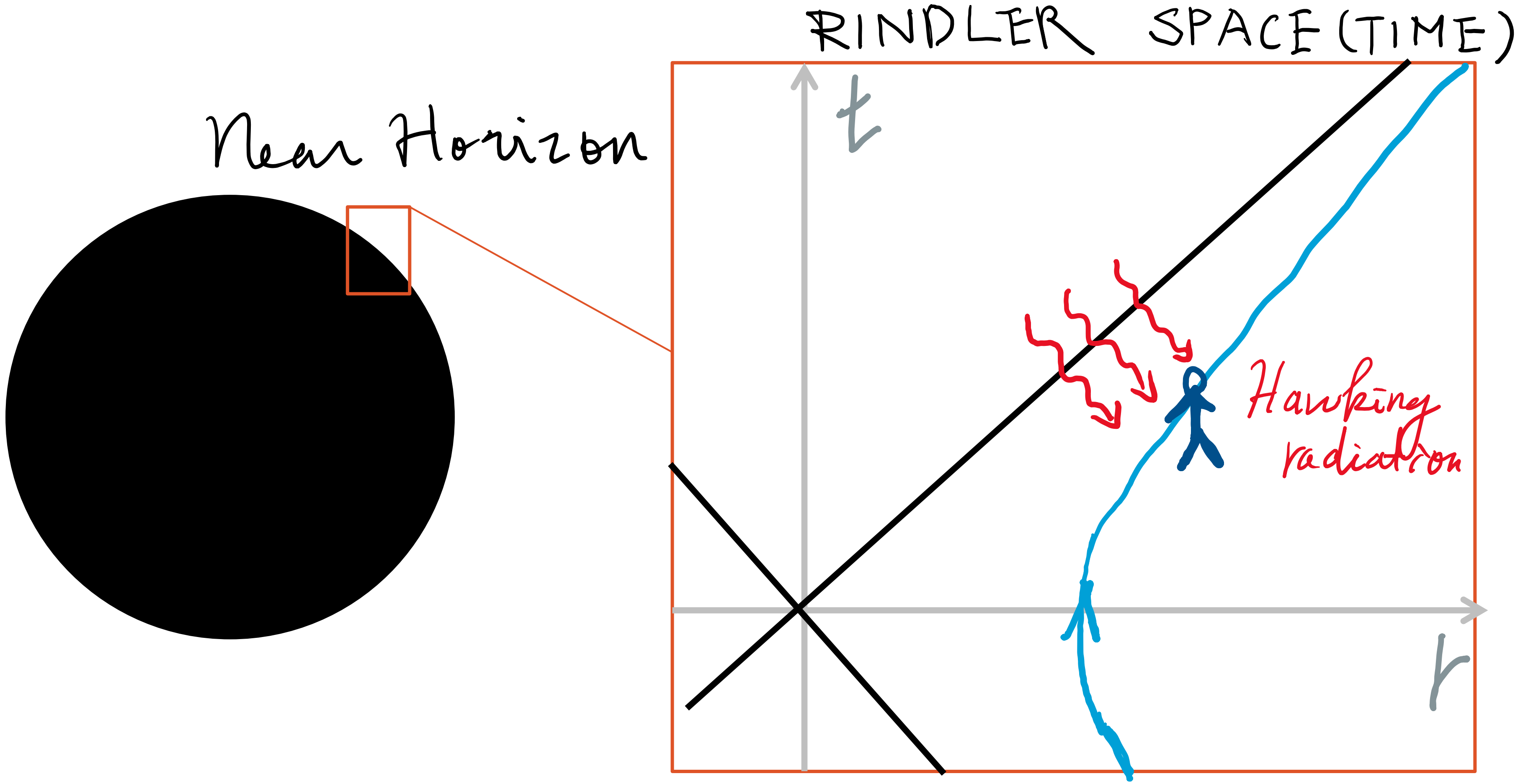


Cosmic censorship conjecture





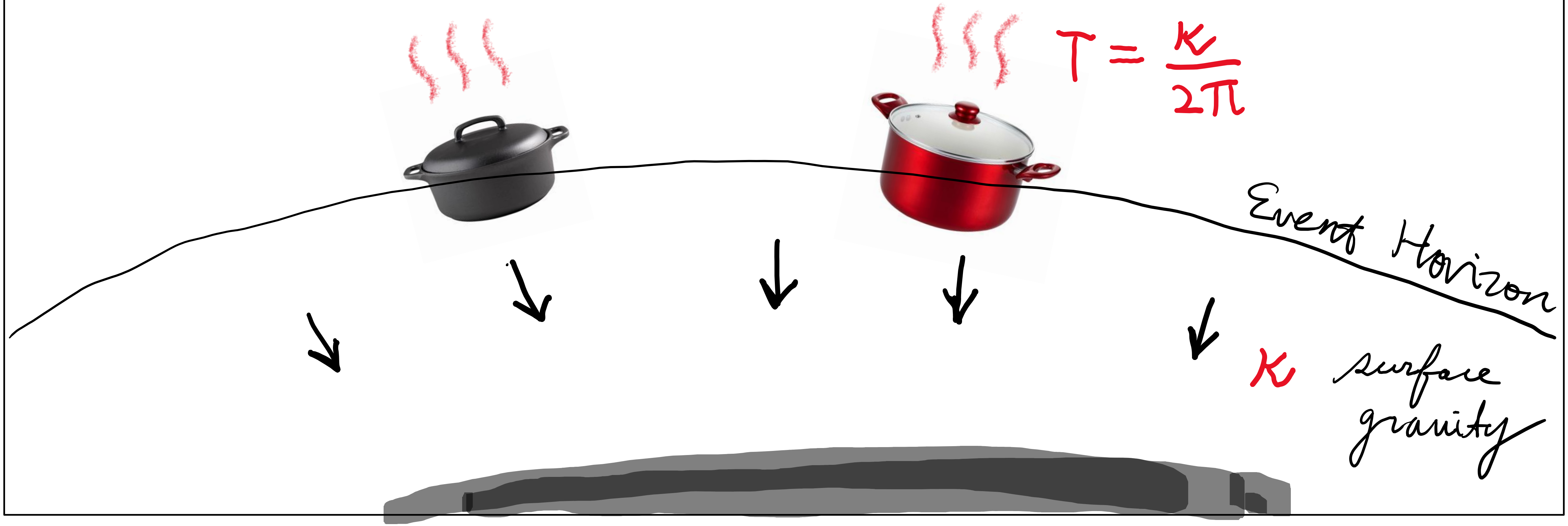
Black holes are thermal objects



$$T = 0.00000006169\,K \left(\frac{M_\odot}{M}\right)$$

Zeroth law of black holes thermodynamics

- Zeroth law: if two systems are each in thermal equilibrium with a third system, they are in thermal equilibrium with each other (same temperature)



First law of black holes thermodynamics

- First law: the system's internal energy changes as work, heat or particles enter/leave the system, respecting the law of conservation of energy.

$$dM = \frac{\kappa}{8\pi} dA + \cdots$$

horizon area

First law of black holes thermodynamics

- First law: the system's internal energy changes as work, heat or particles enter/leave the system, respecting the law of conservation of energy.

$$dM = \frac{\kappa}{2\pi}\, d\frac{A}{4} + \Omega\, dJ + \Phi\, dQ$$

TdS

Ω: angular velocity

Φ: electrostatic potential

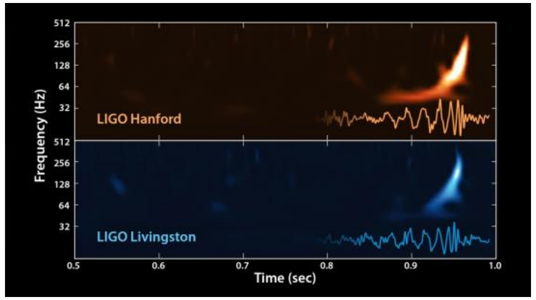
Second law of black holes thermodynamics

- Second law: the entropy always increases in an irreversible process

$$S = \frac{A}{4}$$

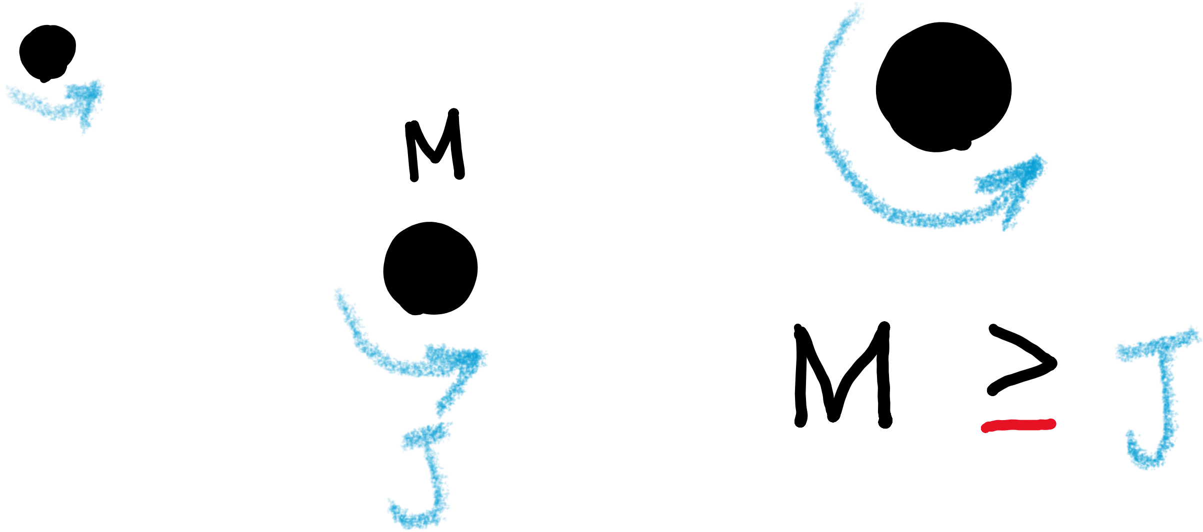
$$\Delta A \geq 0$$

Black hole merger



Third law of black holes thermodynamics

- Third law: the system can not reach absolute zero at finite steps. (the entropy approaches a constant, most likely zero, as temperature goes to zero)



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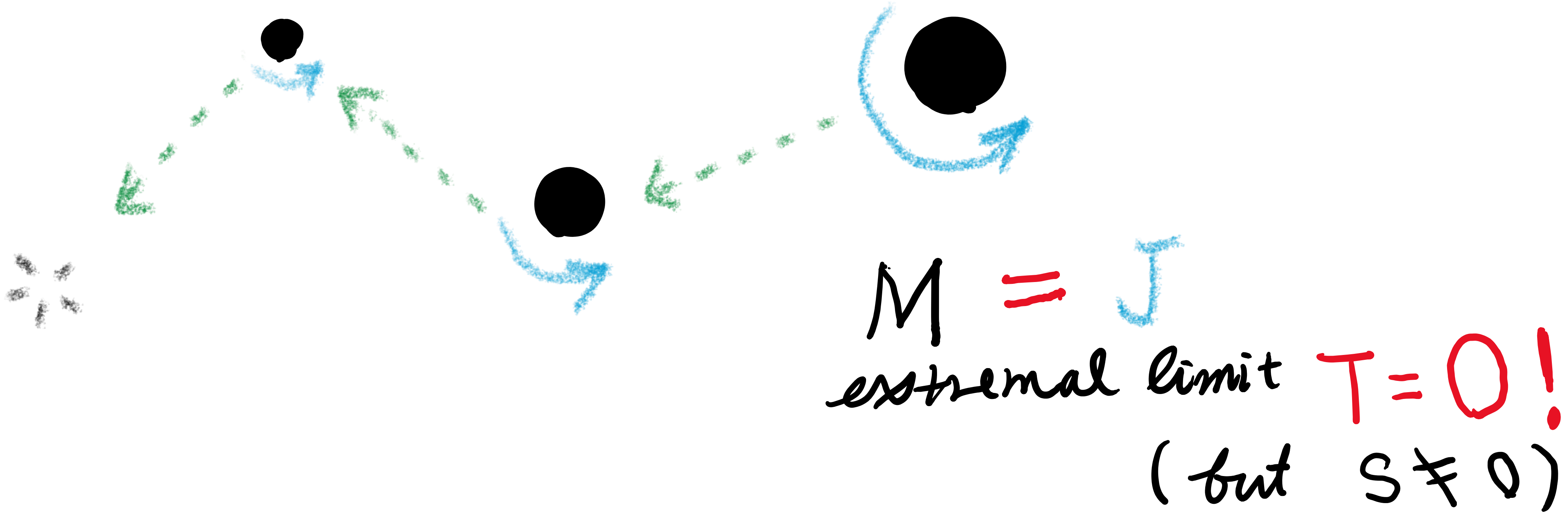
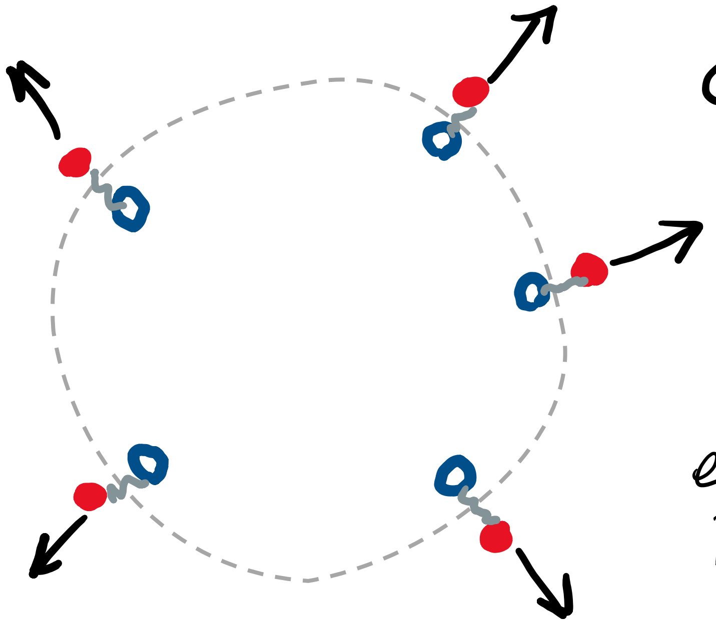


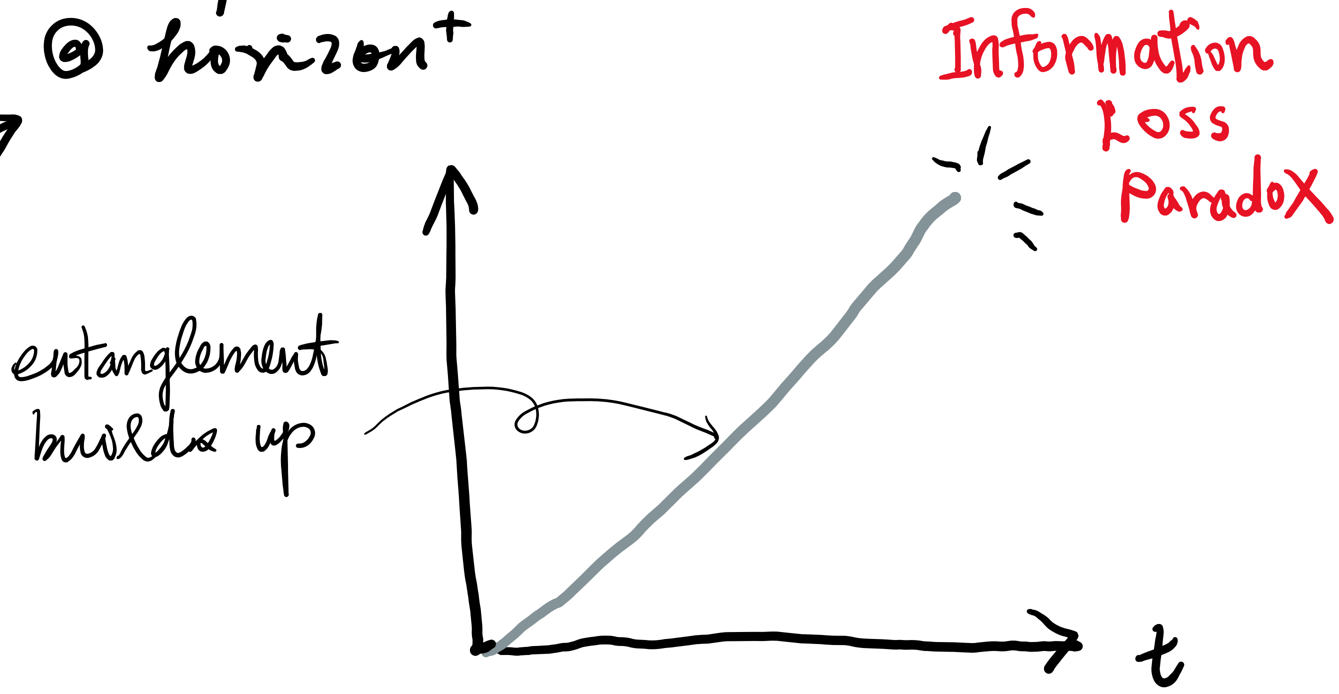
Black holes are quantum objects



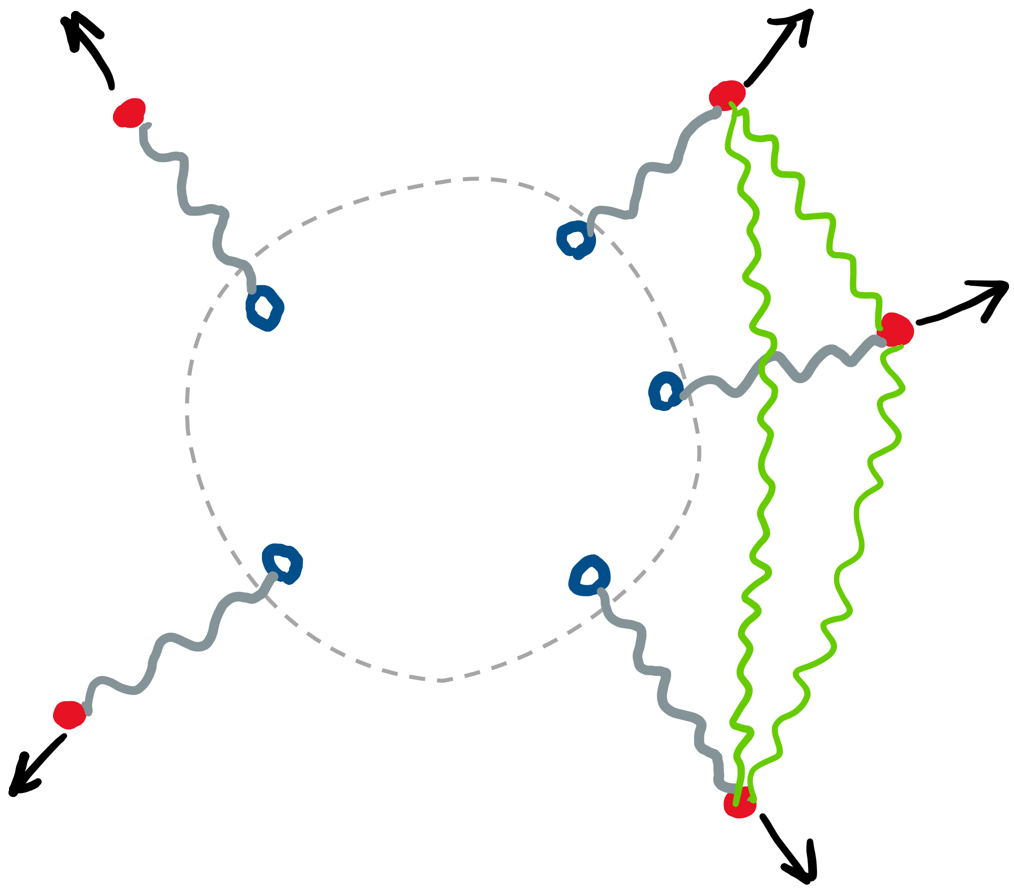
Hawking Radiation

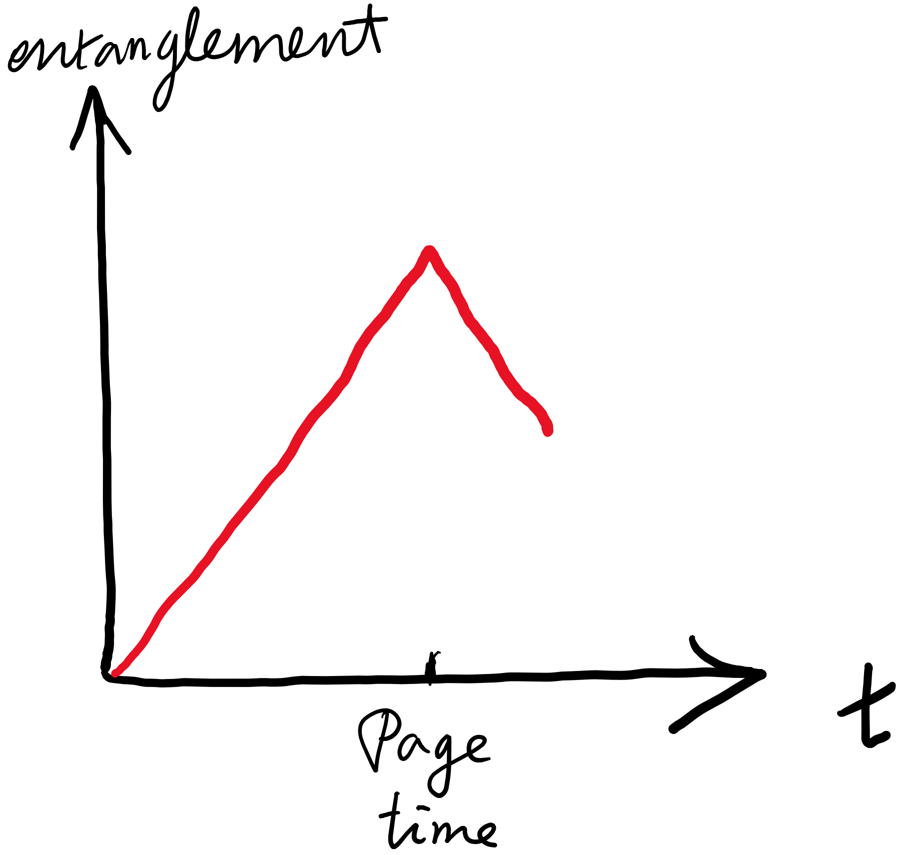
||

Pair production @ horizon$^+$



Black holes are quantum objects





Part II
Hawking radiation as tunneling

Thermal v.s. non-thermal (featured)

- Parikh-Wilczek regarded Hawking radiation as a tunneling process (with back reaction or energy conservation) and derived the tunneling rate: [PW, PRL 2000]

$$\Gamma \sim e^{-8\pi\omega(M-\omega/2)} = \underbrace{e^{-\omega/T_H}}_{\text{Boltzmann factor}} e^{+4\pi\omega^2}$$

- The tunneling rate is composed of a thermal part and non-thermal part. This suggests radiation contains more features than just temperature (determined by mass M).

1. Painlevé coordinates : $ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + 2\sqrt{\frac{2M}{r}}\,dt\,dr + dr^2 + r^2 d\Omega_2^2$

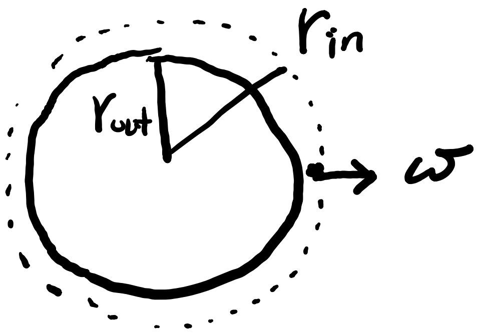
2. radial null geodesics : $\dot{r} = \pm 1 - \sqrt{\frac{2M}{r}}$

3. Backreaction : $M \longrightarrow M-\omega$

4. W.K.B. approximation : $\mathrm{Im}\,S = \mathrm{Im}\int_{r_{in}}^{r_{out}} p_r\,dr = \mathrm{Im}\int_{M}^{M-\omega}\int_{r_{in}}^{r_{out}} \frac{dr}{\dot{r}}\,dH$

$$= \mathrm{Im}\int_{2M}^{2(M-\omega)}\int_{M}^{M-\omega} \frac{dM'}{1-\sqrt{\frac{2M'}{r}}}\,dr$$

$$= 4\pi\omega\left(M-\frac{\omega}{2}\right)$$

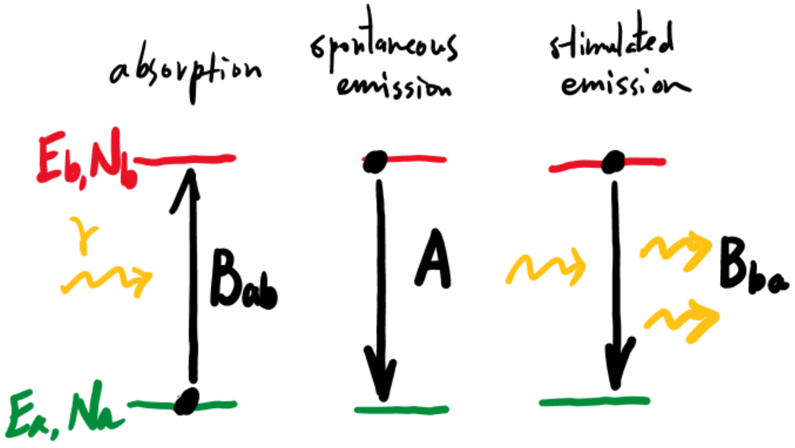


Energy Conservation = no information loss

- The exponent is simply the change of Bekenstein-Hawking entropy as the (Schwarzschild) black hole loses a bit of energy/mass ω via tunneling. The conservation of entropy/information may help resolve the notorious information loss paradox [Zhang-Cai-You-Zhan, PLB 2009; Kyung Kiu Kim-W, PLB 2014; Kuwakino-W, JHEP 2015] or reveal the existence of remnant [Li Xiang, PLB 2007; Yi-Xin Chen, Kai-Nan Shao, PLB 2009]
- Or not [Mathur, CQG 2009], see also Firewall [AMPS, JHEP 2013]
- We have two observations here:
    1. Microscopic degrees of freedom to carry information are still unclear.
    2. PW tunneling rate can be derived without concept of spacetime [Braunstein-Patra, PRL 2011]
- *Is a quantum mechanical model of Hawking radiation with PW tunneling feature possible?*

Part III
Hawking radiation as stimulated emission

Hawking radiation as stimulated emission



$$\rho(\omega) = \frac{A}{(N_a/N_b)B_{ab} - B_{ba}}$$

(a) Each state has degeneracy $g_{a(b)} \sim e^{\frac{\alpha}{4}\mathcal{A}(\beta M_{a(b)})}$

(b) Dof are located somewhere at or outside horizon

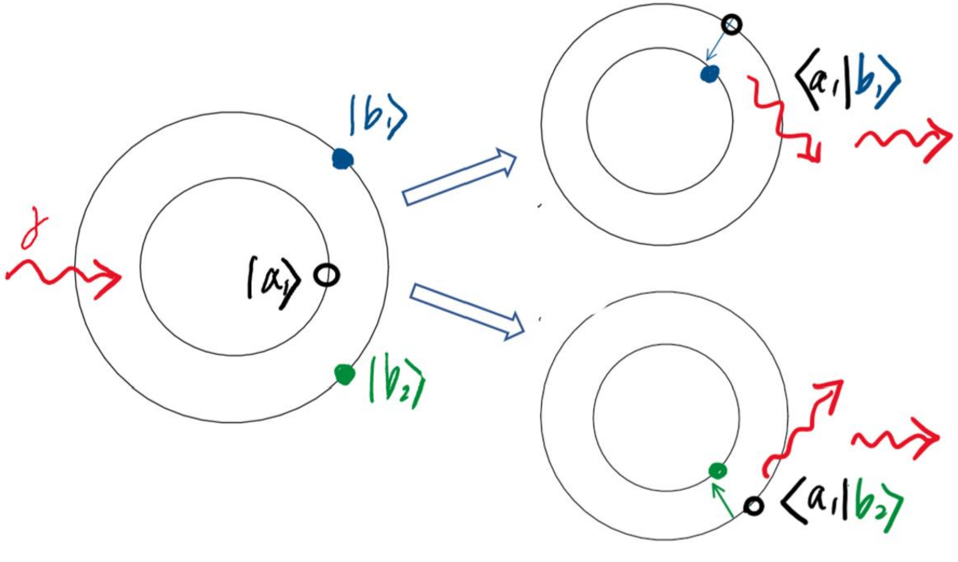


FIG. 1. (Left) Degenerate excited states $|b_1\rangle$ and $|b_2\rangle$ are stimulated by a photon in the cavity. (Right) The stimulated emission may have different feature depending on which transition $\langle a_1|b_1\rangle$ or $\langle a_1|b_2\rangle$ occurs.

Stimulated emission = PK tunneling rate

the proportionality coefficients α and β will be determined shortly. At the large black hole limit where $1/M \ll \omega \ll M$, equation (2) can be cast into

$$\rho(\omega) \simeq (A/B_{ba})e^{-\omega/T_H}\, e^{\alpha\frac{\pi}{16}C(M,\omega)}, \tag{4}$$

here

$$C(M,\omega) = (\tfrac{2}{\beta^2} + \tfrac{8}{\beta} - 32\beta - 32\beta^2)M\omega + (\tfrac{3}{\beta^2} + \tfrac{8}{\beta} + 16\beta^2)\omega^2 + \mathcal{O}(\omega^3). \tag{5}$$

We remark the choices of coefficients α and β as follows:

- To recover the Boltzmann factor, we choose $\beta = 1/2$ such that the leading term in function $C(M,\omega)$ vanishes. This suggests those degrees of freedom are seated at the horizon.[5]
- To reproduce the Parikh-Wilczek nonthermal spectrum, we further choose $\alpha = 2$. This implies that the degeneracy at each energy level is twice amount of the black hole entropy, for $S_{BH} = A/4$.

* Isotropic metric is used to calculate area

$$ds^2 = -\frac{(1-M/2r)^2}{(1+M/2r)^2}dt^2 + (1+M/2r)^4(dr^2 + r^2 d\Omega^2).$$

Part IV
Hairy horizon and non-equilibrium

Supertranslation and soft hair

- In the Jaynes-Cummings model of cavity-black holes, we may promote uneven coupling g_{ij} to some angle-dependent function.
- Asymptotc symmetry of asymptotic flat space (AFS) = infinite-dimensional Bondi-Burg-Metzner-Sachs (BMS) group, i.e. Lie(BMS4) = SO(1,3) $\ltimes$ S ; S is supertranslation generated by asymptotic Killing vector, determined by arbitrary function f [Bondi;Burg-Metzner; Sachs 62]

$$\varsigma = f\partial_u - \frac{1}{r}\left(D^z f\,\partial_z + D^{\bar z} f\,\partial_{\bar z}\right) + D^z D_z f\,\partial_r + \cdots$$

where $f(z,\bar z)$ is arbitrary function of z and $\bar z$

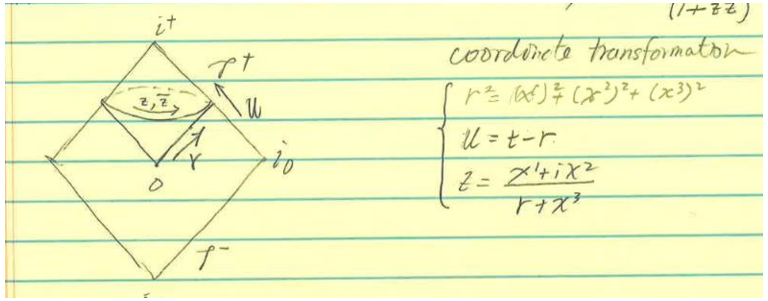
$$\begin{aligned} ds^2 &= -du^2 - 2dudr + 2r^2\gamma_{z\bar z}dzd\bar z \\ &\quad + \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + rC_{\bar z\bar z}d\bar z^2 + D^z C_{zz}dudz + D^{\bar z}C_{\bar z\bar z}dud\bar z \\ &\quad + \frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)dudz + c.c. + \ldots, \end{aligned}$$

m_B : Bondi mass aspect

N_z : angular momentum aspect

C_{zz} : gravitational wave w/ Bondi news tensor $N_{zz} \equiv \partial_u C_{zz}$

D_z : covariant derivative on S^2



Schwarzschild black hole with soft hair horizon

- Weinberg's soft gravitons is manifestly Goldstone bosons of spontaneously broken supertranslation. [Weinberg 65; Lysov-Mitra-Strominger 15] This soft dof's might contribute to black hole entropy. [Hawking-Perry-Strominger 16]

- Apply supertranslation to SSBH solution in isotropic metric, with hair function C(z) [Compere-Long 2016]

stereographic map $z = e^{i\phi} \cot \frac{\theta}{2}$

$$ds^2 = -\frac{(1-\frac{M}{2\rho_s})^2}{(1+\frac{M}{2\rho_s})^2}dt^2 + (1+\frac{M}{2\rho_s})^4\left(d\rho^2 + \left(\left((\rho-E)^2+U\right)\gamma_{AB} + (\rho-E)C_{AB}\right)dz^A dz^B\right)$$

$$\rho_s = \sqrt{(\rho - C - C_{0,0})^2 + ||\mathcal{D}C||^2}.$$

$$C_{AB}(\theta,\phi) \equiv -(2D_A D_B - \gamma_{AB}D^2)C,$$

$$U(\theta,\phi) \equiv \frac{1}{8}C_{AB}C^{AB},$$

$$E(\theta,\phi) \equiv \frac{1}{2}D^2 C + C - C_{(0,0)}.$$

Tunneling through hairy horizon

- Apply PT tunneling method along a fixed angular direction, we obtain tunneling rate per solid angle

$$\frac{d\Gamma}{d\Omega} = \frac{1}{2\pi}\mathrm{Im}S_{out}$$

$$\mathrm{Im}S_{out} = Im\int_0^{\omega}\int_{\rho_{in}}^{\rho_{out}} \frac{(\rho_s + \frac{(M-\omega')}{2})^3}{(\rho_s - \frac{(M-\omega')}{2})\rho_s^2} d\rho d(-\omega').$$

- This implies a distribution of entropic density

$$\frac{dS}{d\Omega} = M\sqrt{M^2 - 4||\mathcal{D}C||^2} + 4||\mathcal{D}C||^2 \ln\left\{M + \sqrt{M^2 - 4||\mathcal{D}C||^2}\right\}$$

- At limit of small ||DC||/M, the entropy receives log correction from soft hair

$$S = 4\pi M^2 + \underbrace{16\pi\overline{||\mathcal{D}C||^2}\ln M}_{\text{logarithmic correction}} - \frac{41}{2}\pi\overline{||\mathcal{D}C||^4}\frac{1}{M^2} + \cdots$$

Non-equilibrium thermodynamics

- We may interpret the hairy correction as non-equilibrium perturbation (if we place the black hole in a box with thermal bath)

$$S^{neq} = \underbrace{4\pi M^2}_{S^{eq}} + \underbrace{\alpha(M)||\mathcal{D}C||^2}_{\text{non-equilibrium perturbation}} + \cdots ,$$

- One may define a non-equilibrium temperature away from Hawking temperature, which reflects the uneven surface gravity near horizon

$$\frac{1}{\Theta} = \frac{\partial S^{neq}}{\partial M} = \frac{1}{T_H} + \frac{16\pi}{M}||\mathcal{D}C||^2 + \cdots$$

Gedanken exp to measure Θ

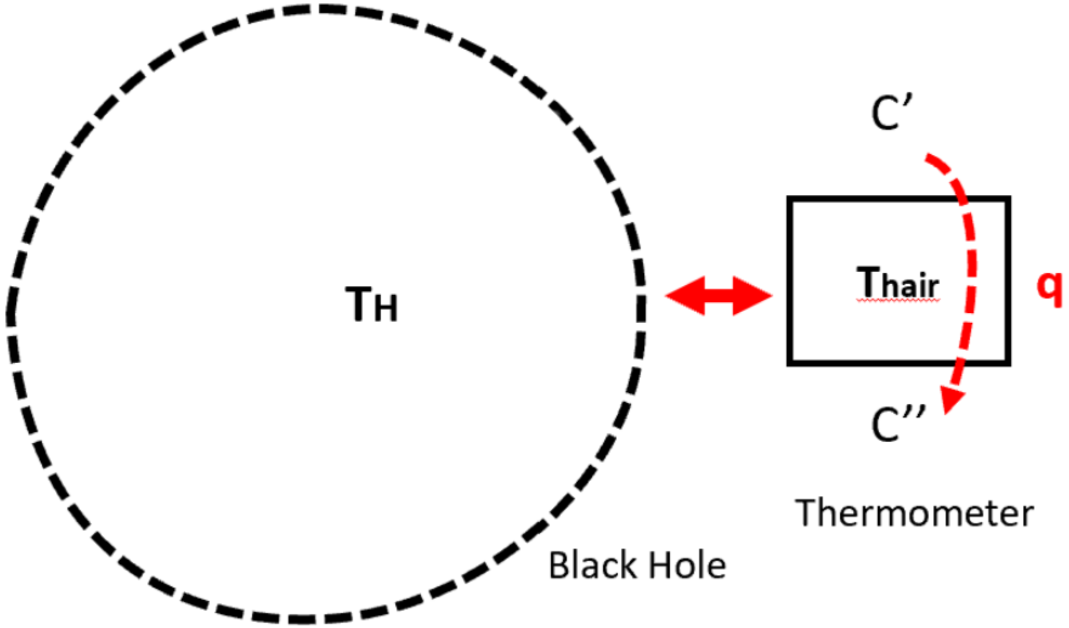


FIG. 1: A Gedanken experiment of thermometer placed near a hairy black hole to measure non-equilibrium temperature. Although the heat exchange between the black hole and the thermometer is balanced, T_{hair} is different from the equilibrium Hawking temperature T_H due to the transverse heat flow q driven by the difference of hair functions C' and C'' across the thermometer.

Part V
Entanglement at hairy horizon

Alice falls into a hairy black hole

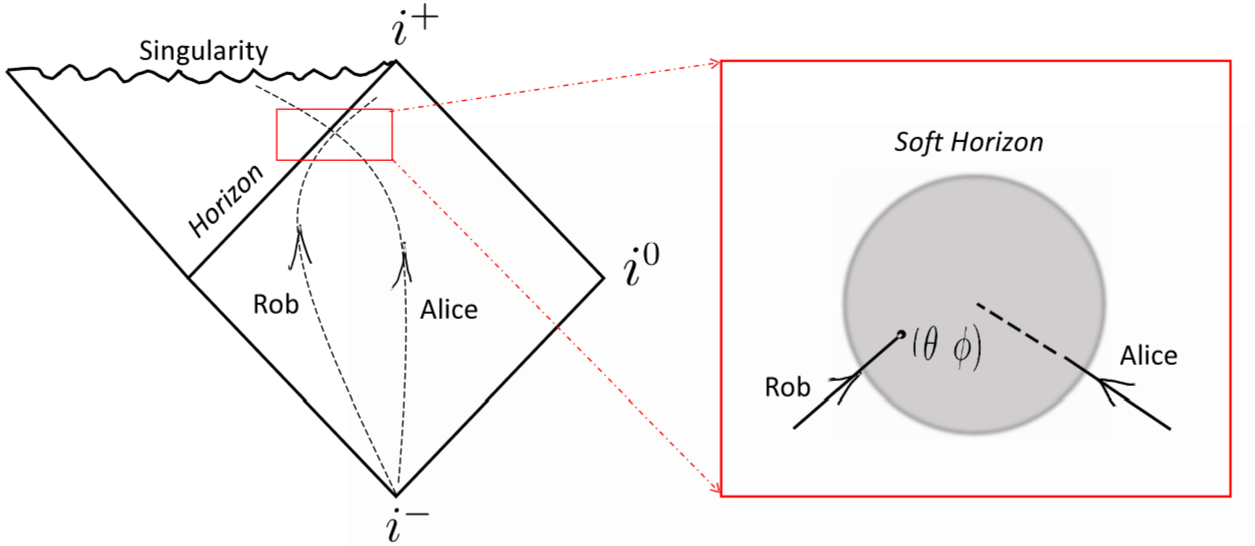


Figure 1: *A Gedanken scenario that Alice and Rob share an entangled qubit at past timelike infinity i^-. Alice then free-falls towards the black hole while Rob hangs around at the horizon. The degradation of entanglement might depend on Rob's angular location if the horizon were supertranslated by soft charges.*

A hairy black hole forms by a shock wave

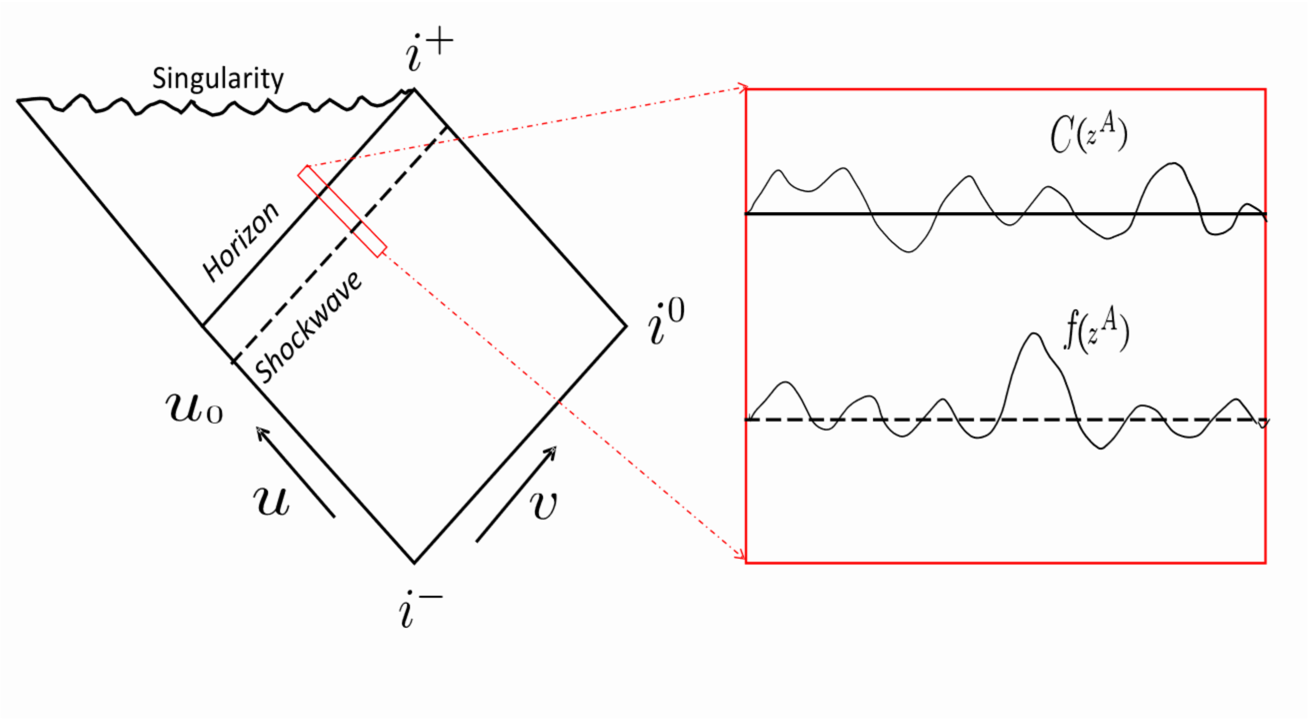


Figure 2: *A black hole is formed by a shock wave where the soft hair function $C(z^A)$ is point-wisely mapped to the waveform factor $f(z^A)$ via the eq. (14).*

A shock wave with arbitrary waveform

$$\begin{aligned} \mathrm{d}s^2 &= -\mathrm{d}u\ \mathrm{d}v + f(z^A)\delta(u-u_0)\,\mathrm{d}u^2 + r^2\gamma_{AB}\mathrm{d}z^A\mathrm{d}z^B, \\ &= -\mathrm{d}u\ \mathrm{d}\hat{v} - \Theta(u-u_0)\,\partial_B f(z^A)\mathrm{d}u\ \mathrm{d}z^B + r^2\gamma_{AB}\mathrm{d}z^A\mathrm{d}z^B. \end{aligned}$$

$$\hat{v} = v - \Theta(u-u_0)\, f(z^A),$$

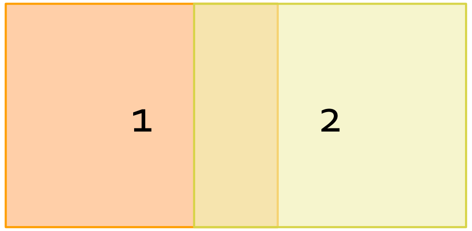
Near horizon geometry of hairy black hole

$$ds^2 \simeq \underbrace{-x^2dt^2 + \kappa^{-2}dx^2}_{\text{Rindler space}} + \underbrace{\cdots}_{\text{Sphere part}},$$

$$\kappa = \frac{\sqrt{M^2 - 4||\mathcal{D}C||^2}}{4M^2}.$$

Point-wise mapping between hair and form

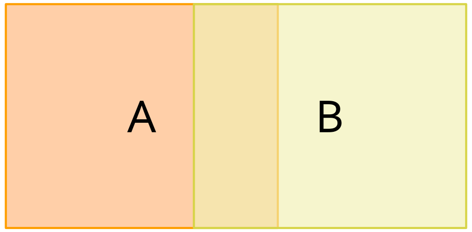
$$\frac{\kappa_1-\kappa_2}{\kappa_1+\kappa_2}\sqrt{\frac{-\hat{v}_2{}^*}{u_1^*\kappa_1\kappa_2}}\frac{2}{M^2}\partial_{z^A}||\mathcal{D}C||^2 = \partial_{z^A} f.$$



Mutual information v.s. Negativity

$$I(A:B) = S_A + S_B - S_{AB},$$

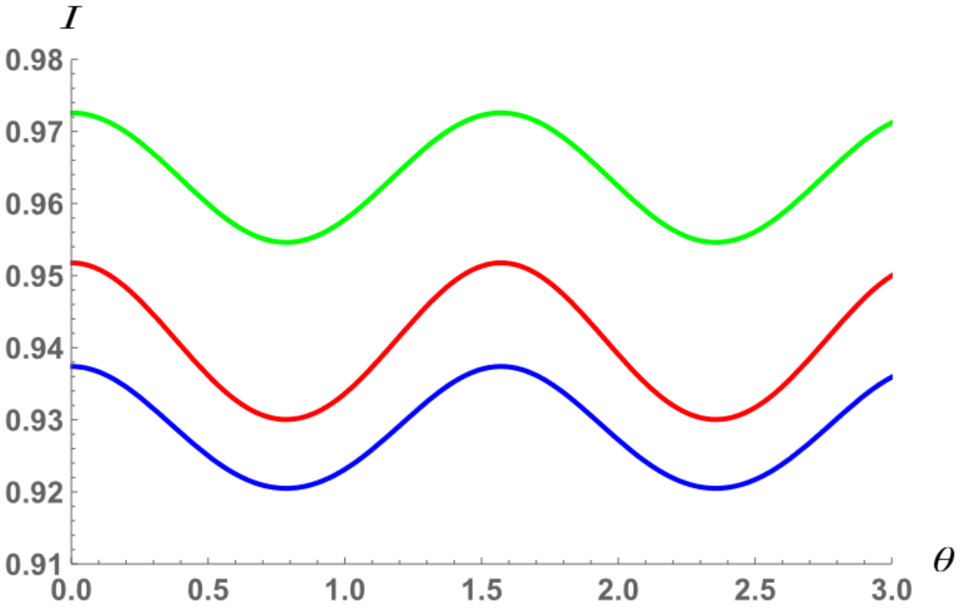
$$S_A = -\mathrm{Tr}(\rho_A \log \rho_A).$$



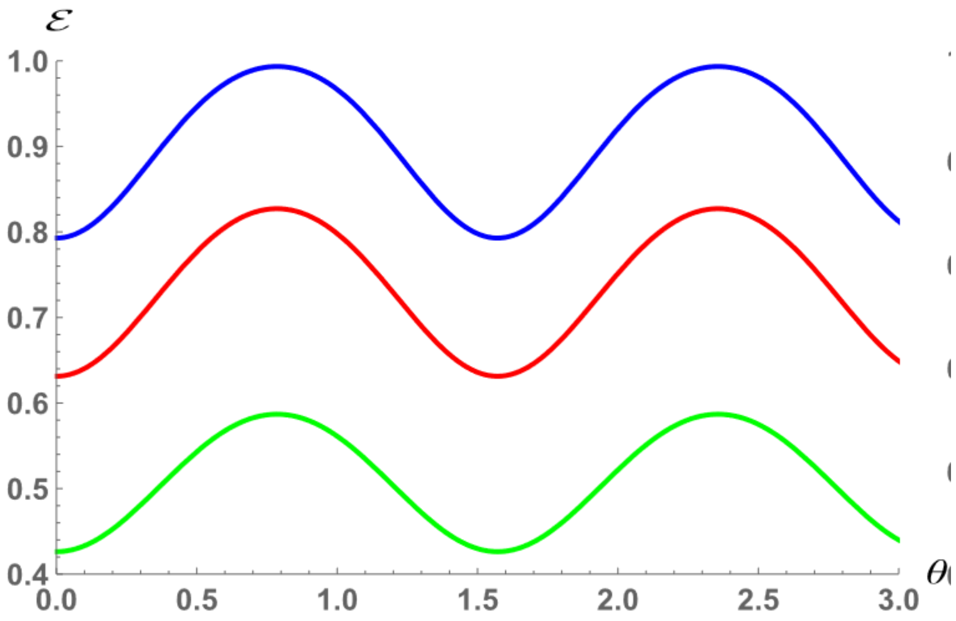
$$\mathcal{E} = \log \mathrm{Tr}|\rho^{T_B}| = \log(1 + 2\sum_{\lambda_i<0} |\lambda_i|),$$

where λ_i are the negative eigenvalues of the matrix ρ^{T_B}. The partial transposed reduced density matrix ρ^{T_B} of ρ is obtained by exchanging the subsystem B's qubit as $|m\ n\rangle\langle p\ q| \rightarrow |m\ q\rangle\langle m\ n|$.

Mutual information/Negativity variation



(a)



(a)

$\epsilon' = 0.3$ *for different black hole mass* $M = .03$ *(green curve),* $M = .05$ *(red curve) and* $M = .07$ *(blue curve).*

Thank You