Exploring 3d de Sitter gravity via holography

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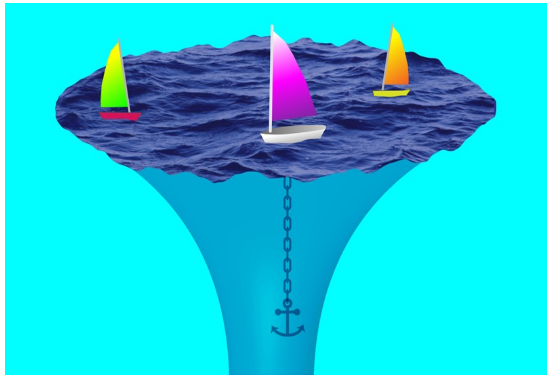

From viewpoint in “Physics” (APS)

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dS/CFT correspondence and de Sitter gravity

- dS/CFT correspondence should play important roles on understanding de Sitter (dS) quantum gravity as AdS/CFT does so
  - dS quantum gravity is important to understand the early universe
  - dS/CFT has not been understood yet compared with AdS/CFT understood as very few concrete examples are available
- A concrete example is given by higher-spin holography

Higher-spin gravity on dS_4 at the classical limit	⟺	3d $Sp(N)$ vector model at the large N limit

[Anninos-Hartman-Strominger ’11 (CQG ’17)]

  - Analytic continuation of duality between higher-spin gravity on AdS_4 and 3d $O(N)$ vector model [Klebanov-Polyakov PLB ’02]

dS/CFT correspondence

[Maldacena JHEP '03]

- A way to describe gravity theory on dS space is utilized wave functional of universe

$$\Psi_{\rm dS}[h,\phi_0] = \int \mathcal{D}g\mathcal{D}\phi \exp iS[g,\phi]$$

$$\text{with } g = h, \phi = \phi_0 \text{ at } t = t_\infty$$

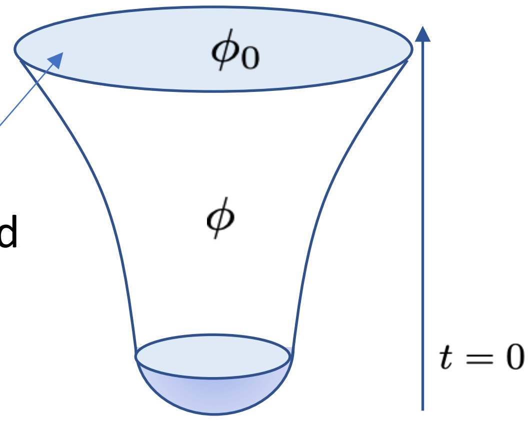
- The wave functional is identified as generating functional of correlation functions in dual CFT

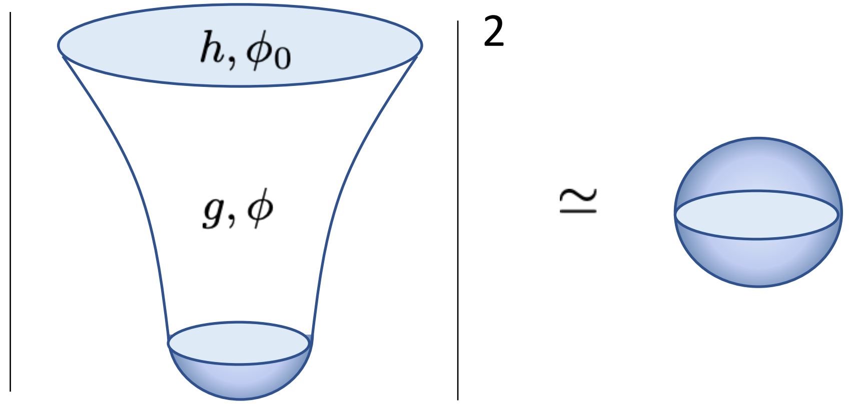
$$\Psi_{\rm dS}[\phi_0] = \left\langle \exp\left(\int d^d x \phi_0(x)\mathcal{O}(x)\right)\right\rangle$$

- In particular, gravity partition function is computed from square of wave functional

$$Z_{\rm G} = \int \mathcal{D}h\mathcal{D}\phi_0 |\Psi_{\rm dS}[h,\phi_0]|^2$$

Correlators are computed by dual Euclidean CFT





The aim of this talk

- Propose a dS/CFT correspondence involving 3d higher-spin gravity

Higher-spin dS_3 gravity ($\simeq SL(N)$ CS theory) at the classical limit (+ matters)	⟺	2d $SU(N)$ WZW model with large central charge (+ extra sectors)

- Evidence
  - The match of partition functions are shown
  - It is related to AdS_3 higher-spin holography via analytic continuation [Gaberdiel-Gopakumar PRD '11]
- Applications
  - Compute bulk correlators on dS_3 at late time
  - Determine allowable complex geometry as saddles of Chern-Simons gravity

Plan of this talk

- Introduction
- The match of partition functions
- Relation to Gaberdiel-Gopakumar duality
- Late-time bulk correlators
- Complex saddles of Chern-Simons gravity
- Conclusion

The match of partition functions

Our proposal and match of partition functions

- Our proposal

Higher-spin dS_3 gravity ($\simeq SL(N)$ CS theory) at the classical limit (+ matters)	⟺	2d $SU(N)$ WZW model with large central charge (+ extra sectors)

  - In the following we focus on the simplest case with N=2

- The match of partition functions
  - We compute the partition functions of 2d SU(2) WZW model on $\mathbb{S}^2$ and find agreement with gravity partition functions
  - We utilize Witten's method to compute the CFT partition functions on $\mathbb{S}^2$ [Witten CMP '89]

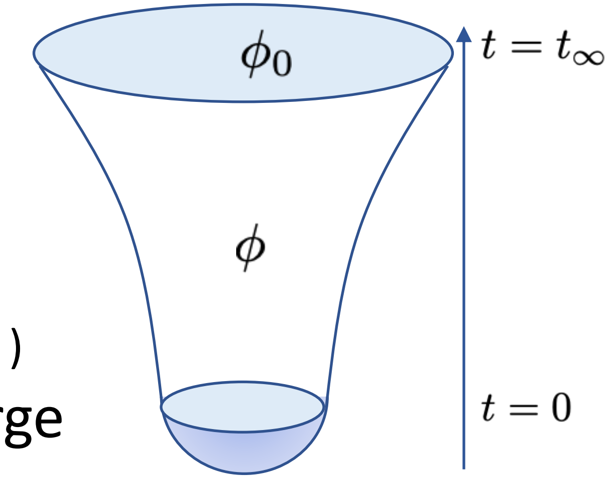
Central charge and the level of WZW model

- Virasoro symmetry appears near the future infinity with central charge [Strominger JHEP '01]

$$c = i\frac{3L_{\rm dS}}{2G_N} \equiv ic^{(g)}$$

Radius of de Sitter space

Newton constant (classical limit $G_N \to 0$)



  - Dual CFT is quite strange as it has pure imaginary central charge

- 2d SU(2) WZW model with level *k*

$$S = \frac{k}{2\pi}\int d^2z[g^{-1}dg\cdot g^{-1}dg] + k\,\Gamma_{\rm WZ},\;\; g\in {\rm SU}(2) \;\text{ with }\; c = \frac{3k}{k+2}$$

  - To reproduce the requirement from dual gravity, we consider a peculiar limit

$$k = -2 + i\,\frac{6}{c^{(g)}} + \mathcal{O}(c^{(g)-2})$$

Computations of partition function

- CFT computation
  - Partition function is given in terms of modular *S*-matrix of SU(2) WZW model [Witten CMP ' 89]

$$\mathcal{S}_j{}^l = \sqrt{\frac{2}{k+2}} \sin\left[\frac{\pi}{k+2}(2j+1)(2l+1)\right]$$

  - Vacuum partition function at the leading order in $1/c^{(g)}$ $\left(k = -2 + i\,\frac{6}{c^{(g)}} + \mathcal{O}(c^{(g)-2}) \right)$

$$|\mathcal{S}_0{}^0|^2 \simeq \exp\left(\frac{\pi c^{(g)}}{3}\right)$$

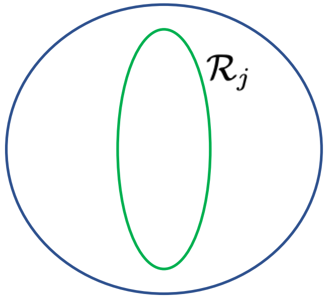
- Gravity computation
  - Definition of classical partition function

$$Z_G = \exp(-I_G)\,,\ I_G = -\frac{1}{16\pi G_N}\int d^3x\sqrt{g}(R - 2L_{\rm dS}^{-2})$$

  - Classical action on $\mathbb{S}^3$: $I_G = -\frac{\pi L_{\rm dS}}{2G_N} = -\frac{\pi c^{(g)}}{3}$ ⟸ Reproduces CFT computation!!

Wilson loop and bulk excitation

- Partition function on $\mathbb{S}^3$ with Wilson loop in rep. $\mathcal{R}_j$



$$= \mathcal{S}_0{}^j$$

  - The Wilson line on $\mathbb{S}^3$ ⇔ Operator in 2d WZW model [Witten CMP '89]
  - Conformal dimension of CFT operator

$$\Delta_j = \frac{2j(j+1)}{k+2} \equiv i\Delta^{(g)}$$

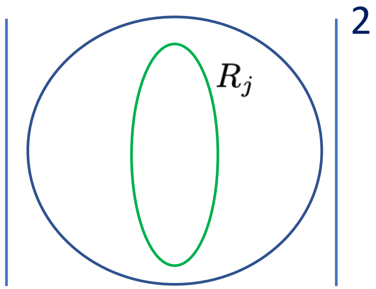
- dS/CFT map to bulk excitation energy

$$\Delta^{(g)} = L_{\mathrm{dS}} E_j$$

Partition function on Euclidean dS_3 black hole

[YH-Nishioka-Takayanagi-Taki PRL ’22;JHEP ’22]

- CFT computation
  - The modular *S*-matrix leads at the leading order in $1/c^{(g)}$



$$= |\mathcal{S}_0{}^j|^2 \simeq e^{\frac{\pi c^{(g)}}{3}\sqrt{1-8G_N E_j}}$$

- Gravity computation
  - Bulk excitation creates Euclidean dS_3 black hole

$$ds^2 = L_{\mathrm{dS}}^2\left[(1-8G_N E_j - r^2)d\tau^2 + \frac{dr^2}{1-8G_N E_j - r^2} + r^2 d\phi^2\right]$$

  - Classical action on the geometry

$$I_{\mathrm{G}} = -\frac{\pi}{3}c^{(g)}\sqrt{1-8G_N E_j}$$

⟵ Reproduces CFT computation!!

Relation to Gaberdiel-Gopakumar duality

Gaberdiel-Gopakumar duality for AdS_3

[Castro-Gopakumar-Gutperle-Raeymaekers JHEP ’12; Gaberdiel-Gopakumar JHEP ’12]
(see [Gaberdiel-Gopakumar PRD ’11] for original proposal)

- A version of Gaberdiel-Gopakumar duality

Higher-spin AdS_3 gravity (≃SL(N) CS theory) with matters at the classical limit ⟷ 2d coset model with large central charge $\frac{\mathrm{SU}(N)_k \times \mathrm{SU}(N)_1}{\mathrm{SU}(N)_{k+1}}$

Spins of gauge fields

$s = 2, 3, \ldots, N$

The coset describes analytic continuation of Virasoro-minimal model, which was shown to reduce to Liouville theory [Creutzig-YH JHEP ’21]

- The simplest case with N=2

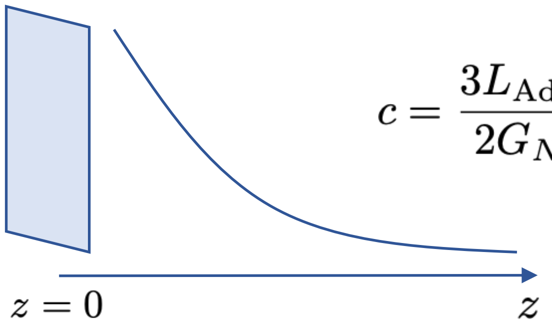
Einstein gravity on AdS_3 with matter at the classical limit ⟷ 2d coset model with large central charge $\frac{\mathrm{SU}(2)_k \times \mathrm{SU}(2)_1}{\mathrm{SU}(2)_{k+1}}$

Central charge and the level of coset model

- A version of Gaberdiel-Gopakumar duality

Einstein gravity on AdS_3 with matter at the classical limit ⟺ 2d coset model with large central charge $\frac{\mathrm{SU}(2)_k \times \mathrm{SU}(2)_1}{\mathrm{SU}(2)_{k+1}}$

- Comparison of central charge
  - Near the boundary of AdS_3 there appears Virasoro symmetry with central charge [Brown-Henneaux CMP '86]



$$c = \frac{3L_{\mathrm{AdS}}}{2G_N} \to \infty$$

  - The central charge of the coset is

$$c = 1 - \frac{6}{(k+2)(k+3)}$$

  - To have large central charge, we have to set

$$k = -2 - \frac{6}{c} + \mathcal{O}(c^{-2})$$

Analytic continuation from AdS_3 to dS_3

[YH-Nishioka-Takayanagi-Taki PRL '22;JHEP '22]

- Formally we can move from AdS_3 to dS_3 by replacing $L_{\rm AdS} \to iL_{\rm dS}$
- Gaberdiel-Gopakumar duality becomes

Einstein gravity on dS_3 with matter at the classical limit	⟷	2d coset model with imaginary central charge $\frac{SU(2)_k \times SU(2)_1}{SU(2)_{k+1}}$

- Comparison of central charge

$$c = 1 - \frac{6}{(k+2)(k+3)} = ic^{(g)}, \; c^{(g)} = \frac{3L_{\rm dS}}{2G_N} \to \infty \quad \longleftrightarrow \quad k \to -2 + i\frac{6}{c^{(g)}} + \mathcal{O}(c^{(g)-2})$$

At the leading order in $1/c^{(g)}$ only $SU(2)_k$ part dominates and the duality reduces to our proposal of dS_3/CFT_2 correspondence

Late-time bulk correlators

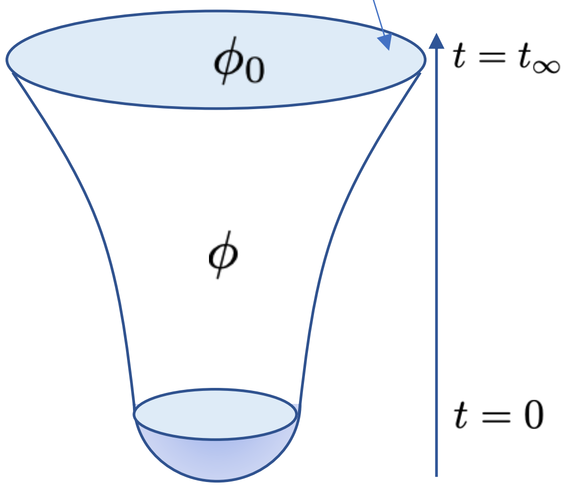
dS/CFT correspondence

[Maldacena JHEP '03]

- Wave functional of universe is proposed to be the same as generating functional of correlation functions in dual CFT

$$\Psi_{\rm dS}[\phi_0] = \left\langle \exp\left(\int d^d x \phi_0(x)\mathcal{O}(x)\right)\right\rangle$$
$$= \exp\left[\frac{1}{2}\int d^d x d^d y \langle \mathcal{O}(x)\mathcal{O}(y)\rangle \phi_0(x)\phi_0(y) + \cdots\right]$$

Correlators are computed by dual Euclidean CFT



- Late time correlators can be computed as expectation values

$$\langle \phi_0(\vec{k})\phi_0(-\vec{k})\rangle = \int \mathcal{D}\phi_0 |\Psi_{\rm dS}|^2 \phi_0(\vec{k})\phi_0(-\vec{k}) = -\frac{1}{2{\rm Re}\langle \mathcal{O}(\vec{k})\mathcal{O}(-\vec{k})\rangle}$$

$$\langle \phi_0(\vec{k}_1)\phi_0(\vec{k}_2)\phi_0(\vec{k}_3)\rangle = \int \mathcal{D}\phi_0 |\Psi_{\rm dS}|^2 \phi_0(\vec{k}_1)\phi_0(\vec{k}_2)\phi_0(\vec{k}_3) = \frac{2{\rm Re}\langle \prod_i \mathcal{O}(\vec{k}_i)\rangle}{\prod_i(-2{\rm Re}\langle \mathcal{O}(\vec{k}_i)\mathcal{O}(-\vec{k}_i)\rangle)}$$

Map from AdS to dS

- Dual CFT correlators are obtained by an analytic continuation of those dual to AdS but we should take care of phases during the procedure
  - Metric

$$ds^2_{\mathrm{AdS}} = L^2_{\mathrm{AdS}} \frac{d^2z + d^2\vec{x}}{z^2} \xrightarrow[z \to iy,\ L_{\mathrm{AdS}} \to -iL_{\mathrm{dS}}]{} ds^2_{\mathrm{dS}} = L^2_{\mathrm{dS}} \frac{-d^2y + d^2\vec{x}}{y^2}$$

  - Asymptotic behavior of bulk fields

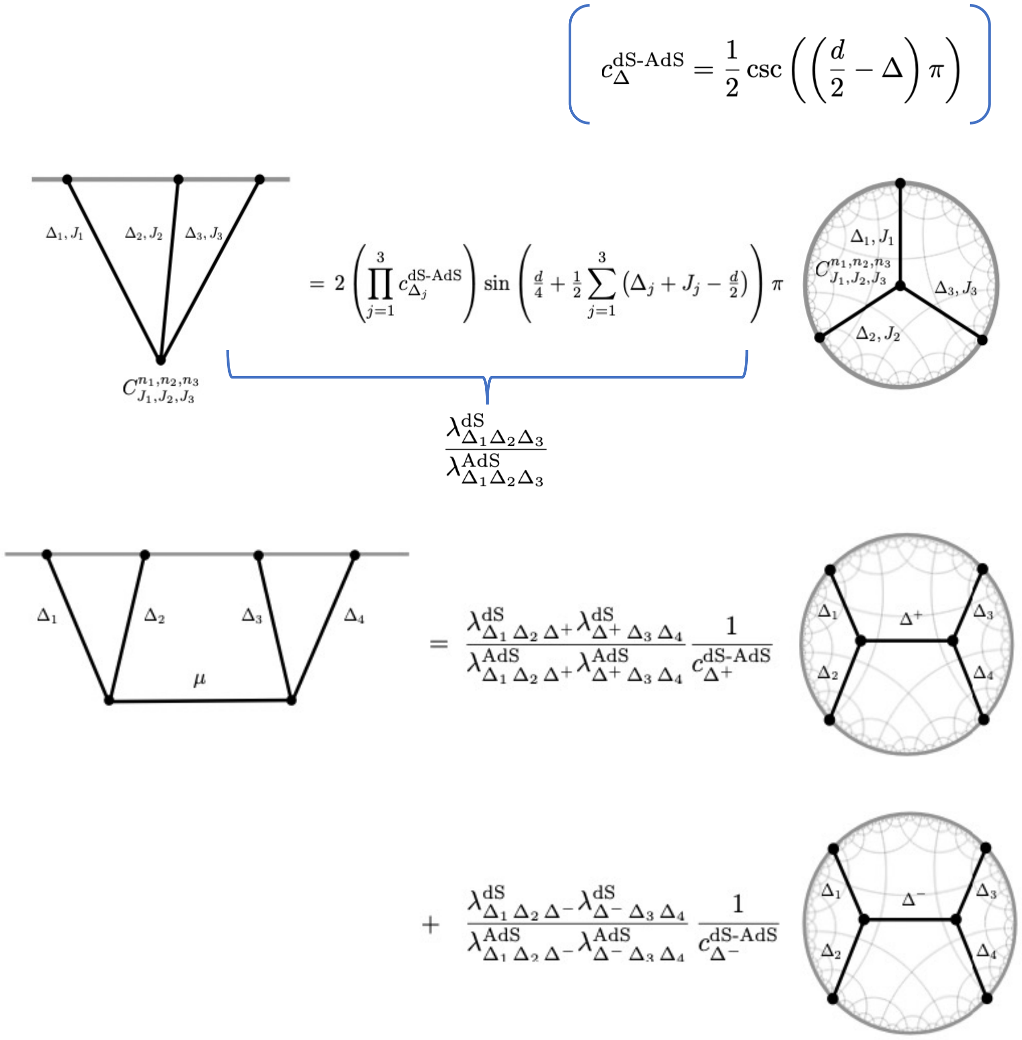
$$\phi^{\mathrm{AdS}}(z,\vec{x}) \sim \phi_+^{\mathrm{AdS}}(\vec{x}) z^{\Delta_+} + \phi_-^{\mathrm{AdS}}(\vec{x}) z^{\Delta_-} \quad (z \to +0) \xrightarrow[\phi_\pm^{\mathrm{AdS}} \to e^{-i\frac{\pi}{2}\Delta_\pm} \phi_\pm^{\mathrm{dS}}]{} \phi^{\mathrm{dS}}(y,\vec{x}) \sim \phi_+^{\mathrm{dS}}(\vec{x})(-y)^{\Delta_+} + \phi_-^{\mathrm{dS}}(\vec{x})(-y)^{\Delta_-} \quad (y \to -0)$$

  - Coupling to dual CFT operators

$$L^d_{\mathrm{AdS}} \int d^d\vec{x} \phi_\pm^{\mathrm{AdS}} \mathcal{O}^\pm_{\mathrm{AdS}} \xrightarrow[\mathcal{O}_\pm^{\mathrm{AdS}} \to e^{i\frac{\pi}{2}(\Delta_\pm - d)} \mathcal{O}_\pm^{\mathrm{dS}}]{} L^d_{\mathrm{dS}} \int d^d\vec{x} \phi_\pm^{\mathrm{dS}} \mathcal{O}^\pm_{\mathrm{dS}}$$

Bulk dS correlators at late time

- We computed bulk correlators from the dual CFT_2 by applying dS/CFT
  [Chen-YH PRL ’22; Chen-Chen-YH JHEP ’22]
  - Phases arise due to the analytic continuation and the square of wave functional
- Bulk Feynman diagram computations can be mapped from AdS to dS in the in-in formulation
  [Sleight-Taronna PRD ’21; JHEP ’21]
  (see also [Di Pietro-Gorbenko-Komatstu JHEP ’22])
  - Our results are consistent with those in the in-in formulation



Figures are taken from Sleight-Taronna JHEP ’21

Complex saddles of Chern-Simons gravity

Allowable complex geometry

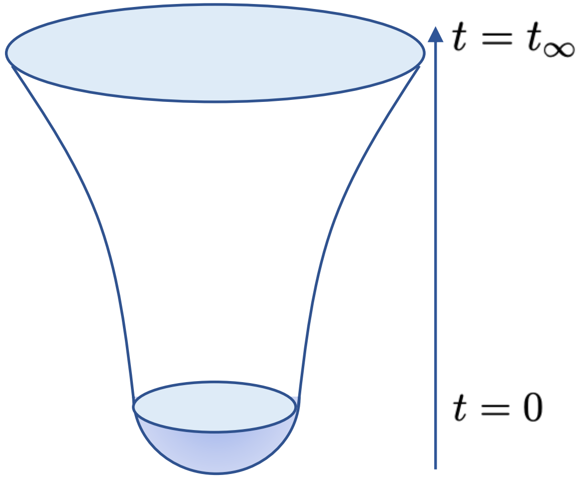
[Louko-Sorkin CQG '97;Kontsevich-Segal QJM '21;Witten'21]

- Sometimes useful to complexify metric like no-boundary proposal by Hartle and Hawking
- A complexified metric of S^{d+1}

$$ds^2 = \ell^2(\theta'(u)^2 du^2 + \cos^2\theta(u) d\Omega_d^2)$$

- Universe can start from $\theta = (n+1/2)\pi \ (n \in \mathbb{Z})$ thus there is a family of complex geometry labeled by *n*
- A criteria of *D*-dim. allowable geometry is

$$\mathrm{Re}\left(\sqrt{\det g} g^{i_1 j_1} \ldots g^{i_q i_q} F_{i_1 \ldots i_q} F_{j_1 \ldots j_q}\right) > 0, \ 0 \leq q \leq D$$



→ Only geometry with *n*=-1,0 are allowable and it is the same as the one by Hartle-Hawking

Complex saddles of Chern-Simons gravity

- We read off complex saddles of 3d Chern-Simons gravity via holography

$$S = S_{\mathrm{CS}}[A] - S_{\mathrm{CS}}[\tilde{A}],\ \ S_{\mathrm{CS}}[A] = -\frac{\kappa}{4\pi}\int \mathrm{tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right)$$

- The metric of dS_3 black hole is

$$ds^2 = L_{\mathrm{dS}}^2\left[(1-8G_N E_j - r^2)d\tau^2 + \frac{dr^2}{1-8G_N E_j - r^2} + r^2 d\phi^2\right]$$

- Large gauge transformation generates different geometry labeled by *n*

$$ds^2 = L_{\mathrm{dS}}^2\left[(1-8G_N E_j - r^2)\left(2n+1\right)^2 d\tau^2 + \frac{dr^2}{1-8G_N E_j - r^2} + r^2 d\phi^2\right]$$

- Gravity partition function is a sum of contributions from each saddles

$$Z_{\mathrm{dS}} = \sum_n \exp S_{\mathrm{GH}}^{(n)},\ \ S_{\mathrm{GH}}^{(n)} = \left(n+\frac{1}{2}\right)\frac{\pi L^2\sqrt{1-8G_N E}}{G_N}$$

Allowable geometry from dual CFT

[Chen-YH-Taki-Uetoko'23; in preparation]

- Dual CFT is given by Liouville theory with parameter *b* and the large central charge limit is realized by

$$c\,(\equiv ic^{(g)}) = 1 + 6(b + b^{-1})^2 \quad \Longrightarrow \quad b^{-2} = \frac{ic^{(g)}}{6} - \frac{13}{6} + \cdots$$

- dS_3 black hole is examined from 2-pt. function of heavy operator

$$\Psi_{\mathrm{dS}} \sim \langle V_\alpha(z_1) V_\alpha(z_2) \rangle$$

- Semi-classical expression of 2-pt. function can be read off from its exact result as [Harlow-Maltz-Witten'11]

$$|\langle V_\alpha(z_1) V_\alpha(z_2) \rangle| \sim e^{\frac{\pi}{6} c^{(g)} \sqrt{1-8G_N E}} - e^{-\frac{\pi}{6} c^{(g)} \sqrt{1-8G_N E}}$$

⟹ We should pick up saddles of CS gravity with *n*=-1,0 and the result reproduces the allowable geometry of Witten

Conclusion

Summary & future problems

- Summary
  - dS/CFT correspondence is proposed between classical higher-spin dS_3 gravity and 2d SU(N) WZW model with large central charge
  - Evidence is provided by comparing partition functions and relating to higher-spin AdS_3 holography
  - Late-time bulk correlators are computed via dS/CFT and the results are consistent with those obtained from the bulk in the in-in formulation
  - Allowable complex geometries of Chern-Simons gravity are read off from dual CFT correlators
- Future problems
  - Examine geometries dual to Liouville/Toda multi-point functions [Chen-YH-Taki-Uetoko'23; in preparation]
  - Generalize the analysis to other dS/CFT correspondence (e.g., higher dimensions, stringy realizations, etc.)