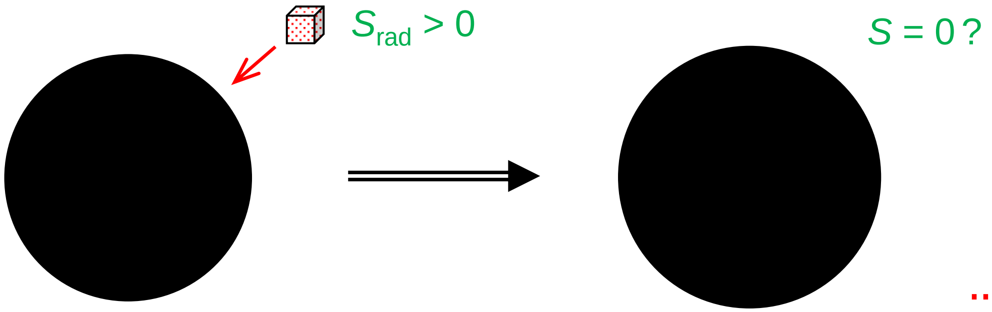
Black Hole and de Sitter Microstructures from a Semiclassical Perspective

Yasunori Nomura

UC Berkeley; LBNL; Kavli IPMU

What happens if matter falls into a black hole?



... $\Delta S < 0$!?

A proposal

[Bekenstein, 1973]

The entropy of a BH is proportional to its horizon area.

$$S_{\text{BH}} = \frac{A}{4G_{\text{N}}}$$

Note: $G_{\text{N}} = \ell_{\text{P}}^2 \sim (10^{-33}\ \text{cm})^2$ → huge entropy

Indeed, $\Delta\left(\frac{A}{4G_{\text{N}}} + S_{\text{matter}}\right) \geq 0$

Does this make sense?

$$\frac{A}{4G_{\text{N}}} = 4\pi G_{\text{N}} M^2$$

$$\frac{\partial S}{\partial E} = \frac{1}{T}$$

M

→ finite temperature

Doesn’t a BH only absorb stuff?

Black holes radiate [Hawking, 1974]

The horizon is “smooth.”

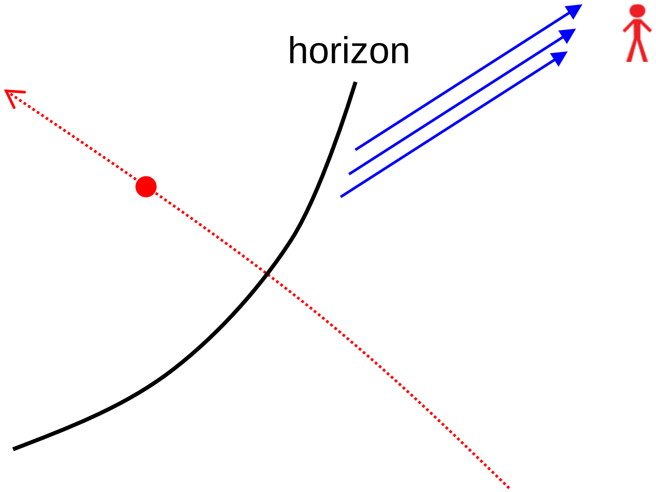
$\Downarrow$ Quantum mechanical effect

Hawking temperature

There must be radiation corresponding to $T_{\mathrm{H}} \sim \dfrac{1}{8\pi M G_{\mathrm{N}}}$.

BHs are thermodynamic objects.

→ Spacetime is composed of microscopic d.o.f.s!



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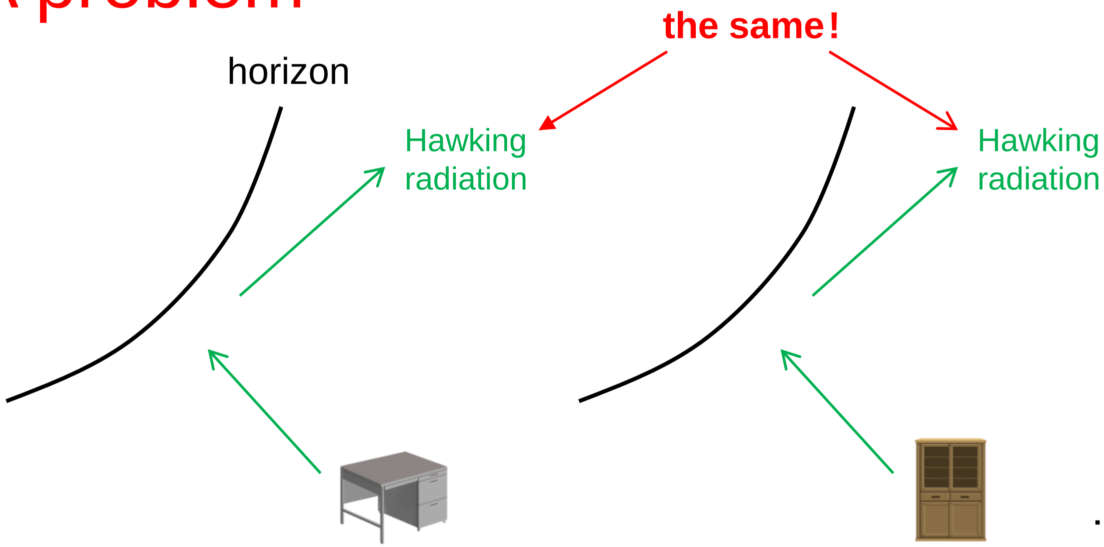
A problem


the same!

horizon

Hawking radiation

Hawking radiation

The time evolution
is **not** one-to-one!
(not unitary)

… (the original form of)
BH information problem

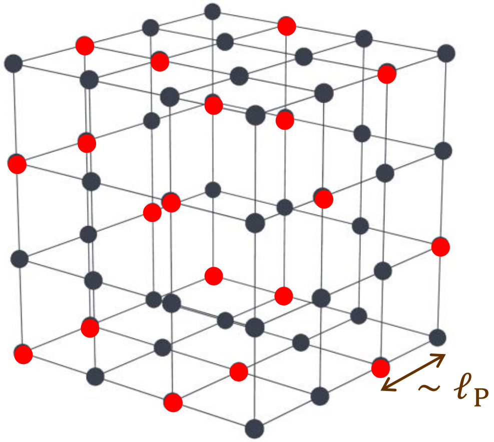
Holography

A clue comes from the BH physics itself.

A BH is the highest entropy state of the region,

and still $S \propto \frac{A}{\ell_\mathrm{P}^2}$

Strange!

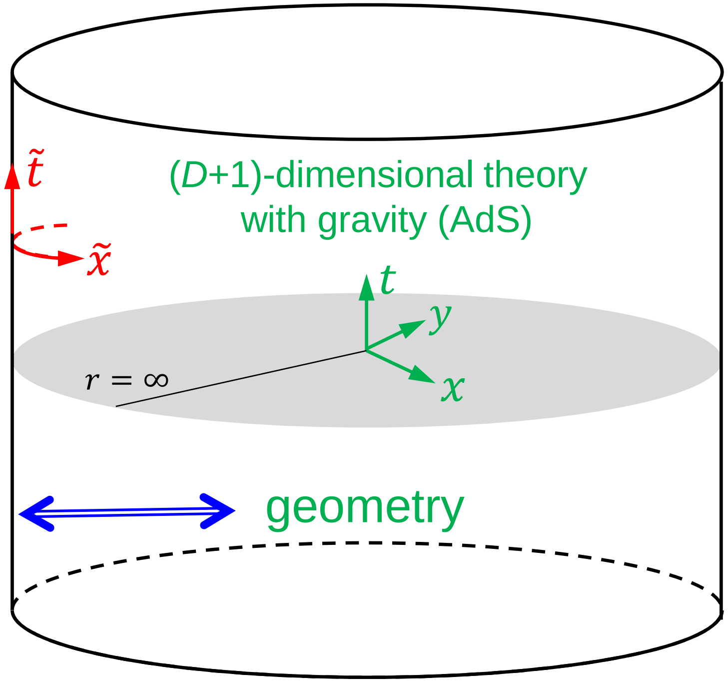


$$S \sim \ln 2^{V/\ell_\mathrm{P}^3} \propto \frac{V}{\ell_\mathrm{P}^3} \ggg \frac{A}{\ell_\mathrm{P}^2}$$

The concept that spacetime exits down to $\sim\ell_\mathrm{P}$ is an illusion!

→ suggests that there is a formulation of quantum gravity in spacetime **one less dimension** than the naïve one.

AdS/CFT correspondence [Maldacena, 1997]

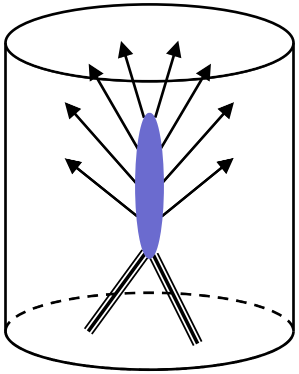


D-dimensional theory
without gravity (CFT)

(*D*+1)-dimensional theory
with gravity (AdS)

quantum information ⟷ geometry

BH evolution **must be** unitary.

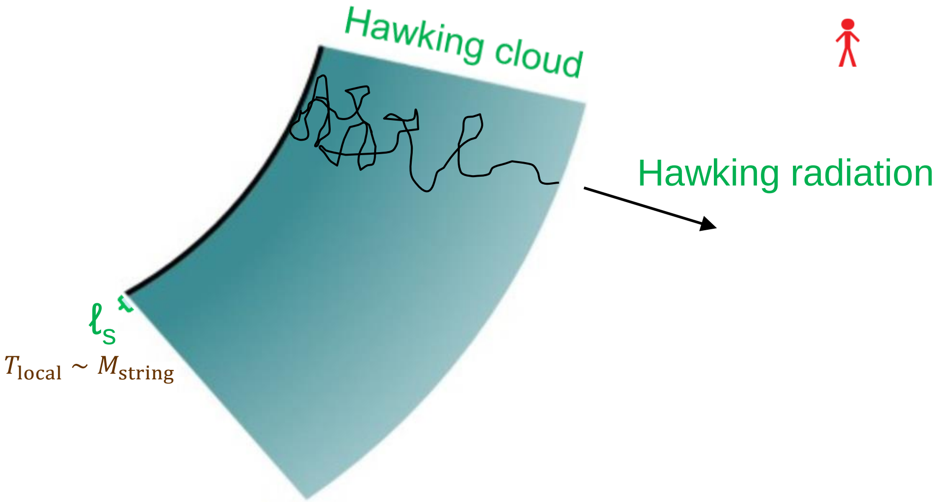


=

A process in non-gravitational
(unitary) theory

… problem “solved”

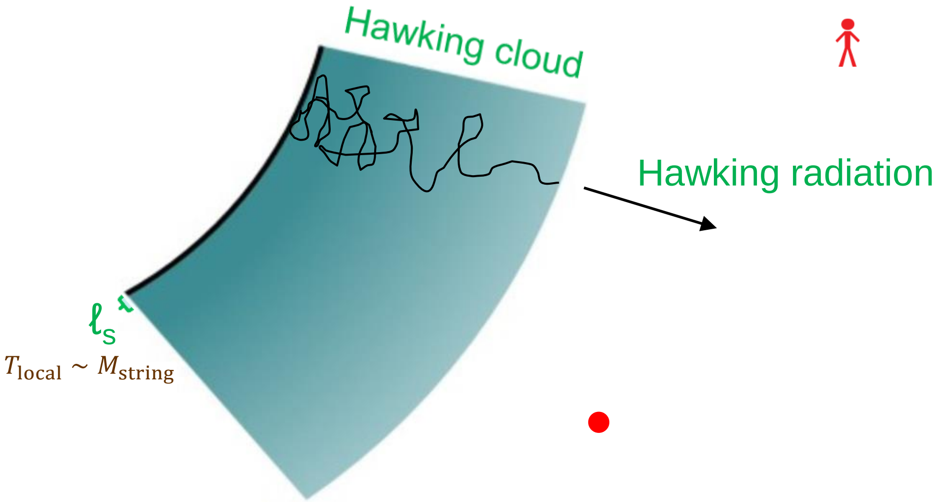
BH at the quantum level



The horizon behaves
as the surface of regular material.

… no issue with unitarity

BH at the quantum level



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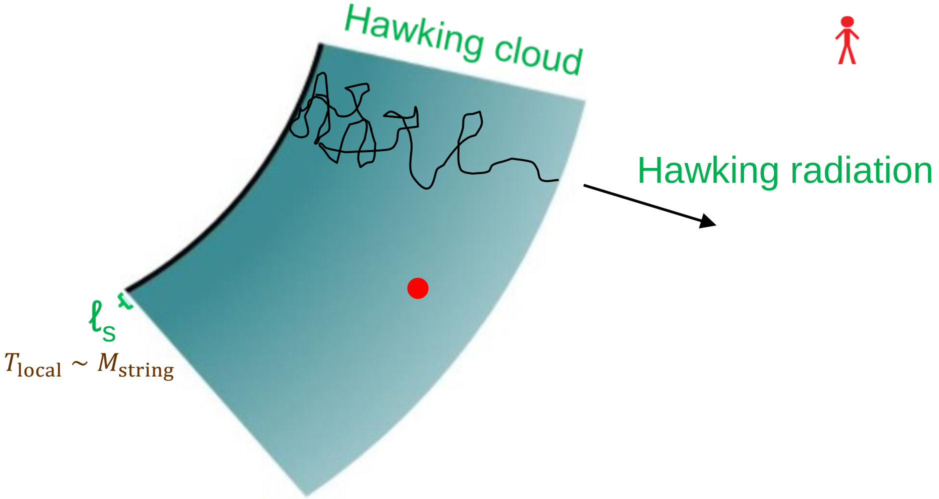

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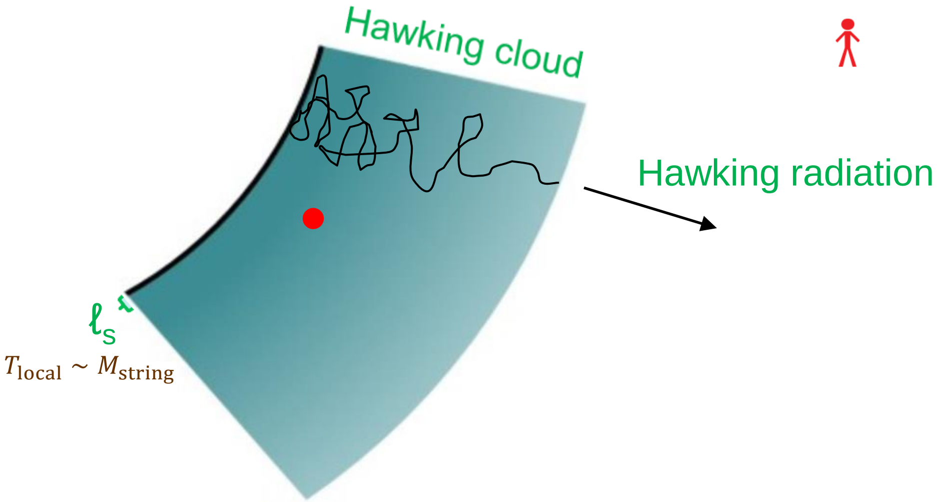

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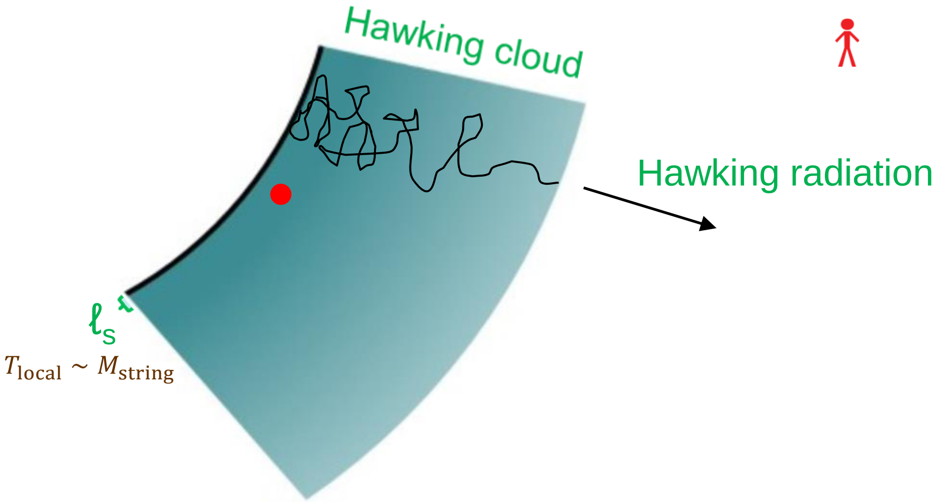

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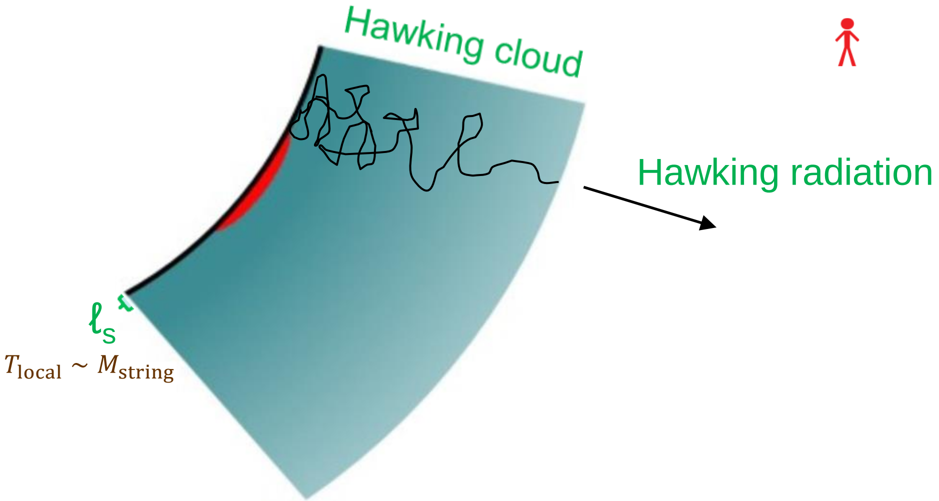

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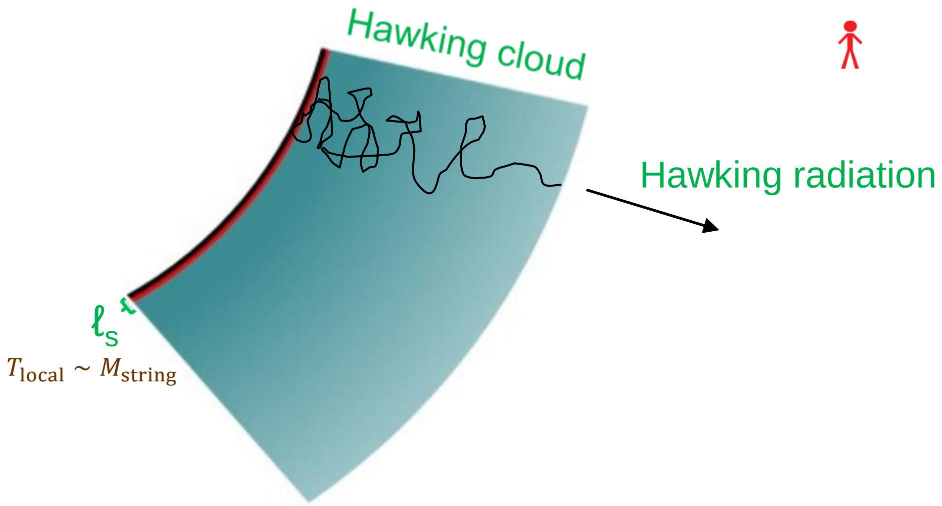

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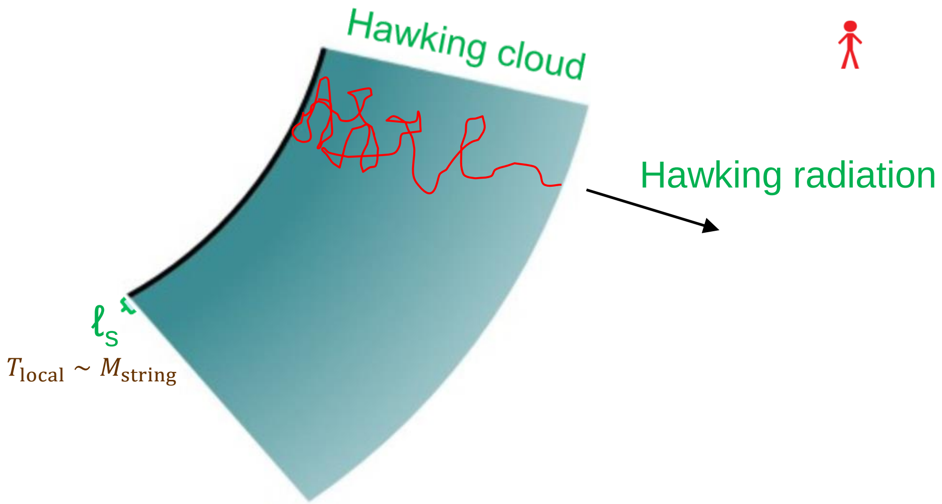

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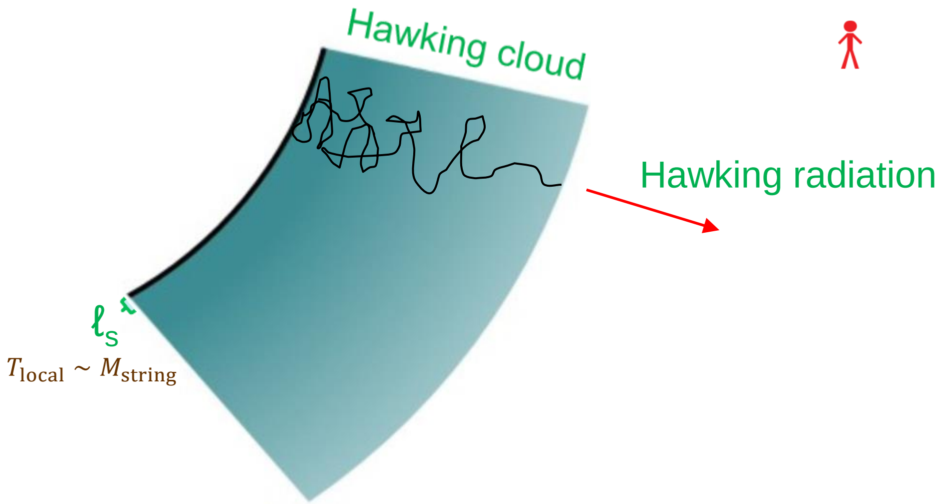

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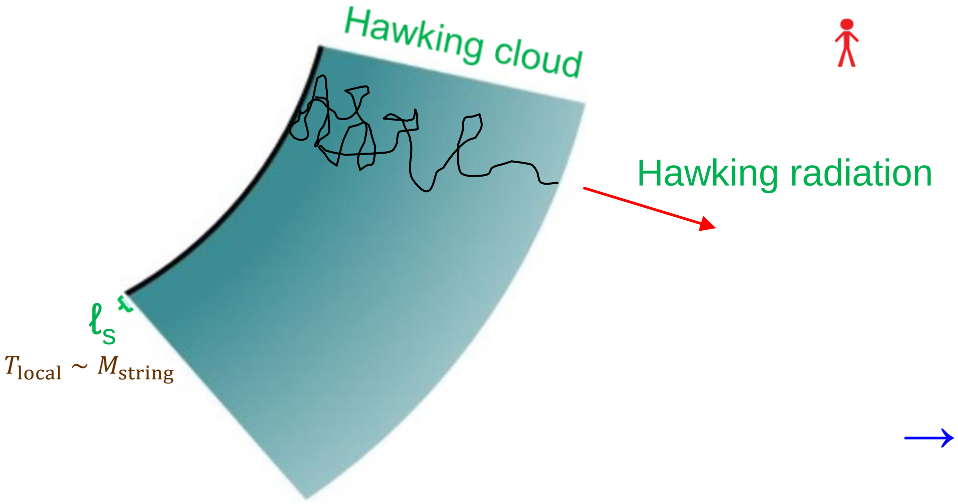

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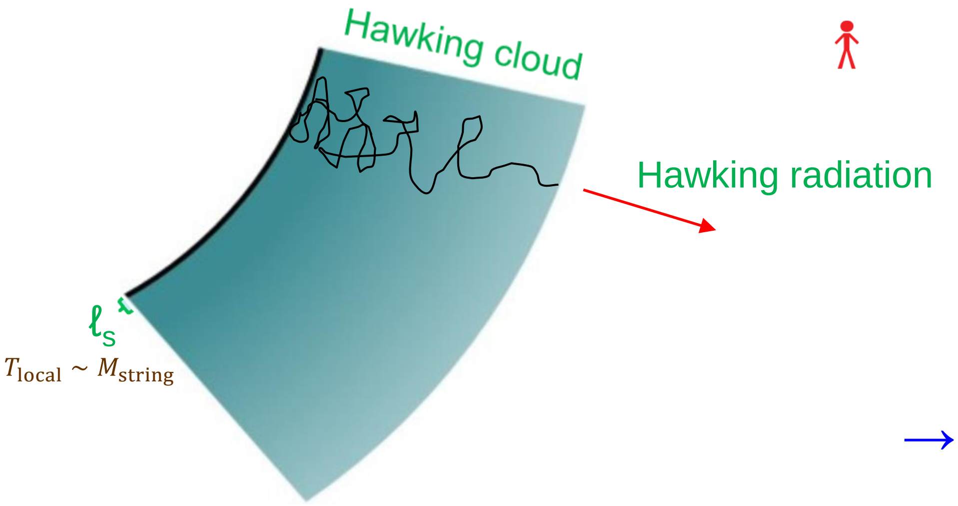
→ What about the interior?

BH at the quantum level



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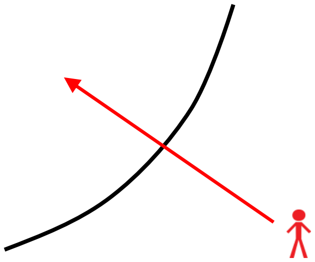
→ What about the interior?

Alternatively



$\xrightarrow[\text{Hawking's analysis}]{}$ information loss

→ What was wrong with Hawking's analysis?

Claim I :

In quantum gravity, a system with a BH (horizon) accommodates two very different descriptions.

These two descriptions, however, are physically equivalent.

Claim II :

Each description makes only one of QM (unitarity) and GR (interior spacetime) manifest.

Nevertheless, the theory is consistent with both; the properties of the one not chosen arise dynamically through subtle effects.

⟹ We will discuss each of these two descriptions and relation between them, including their complementary nature.

…..

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C. Murdia, Y.N., K. Ritchie, “Black hole and de Sitter microstructures from a semiclassical perspective,” arXiv:2207.01625

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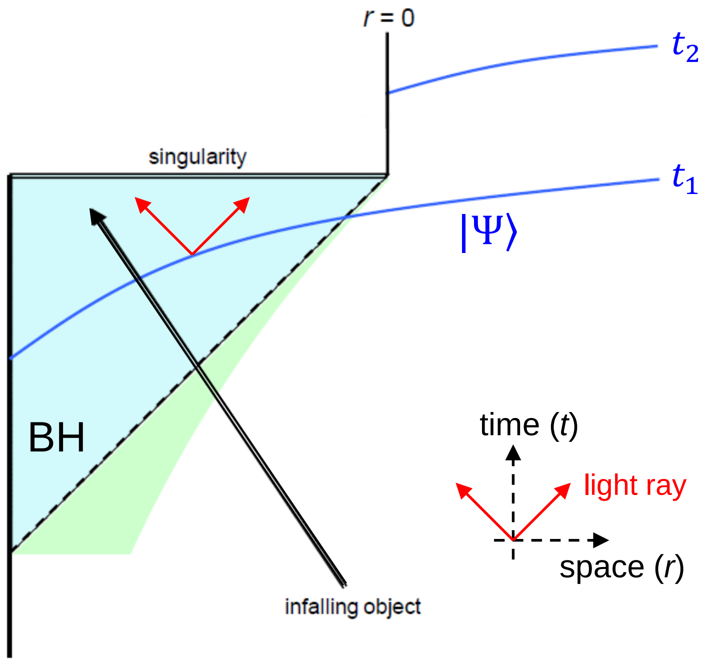
Picture based on Global Spacetime

— replica wormholes & QES prescription —

Start with “global spacetime”



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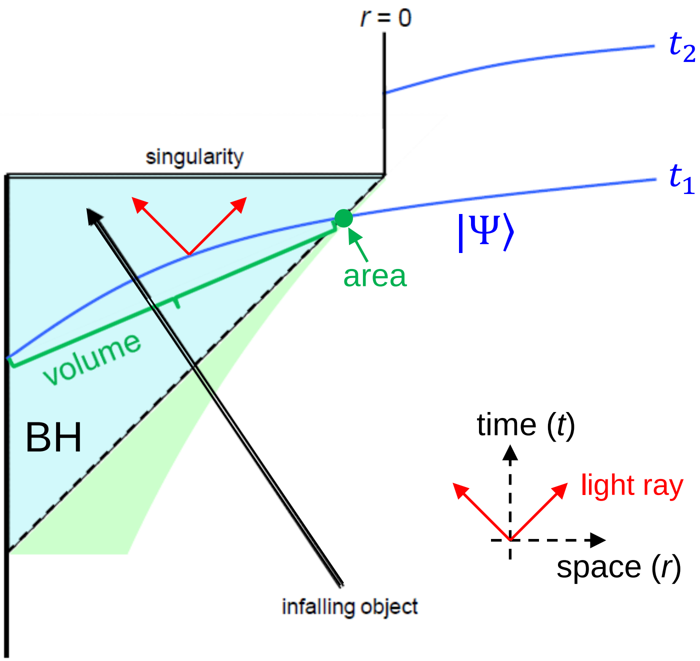

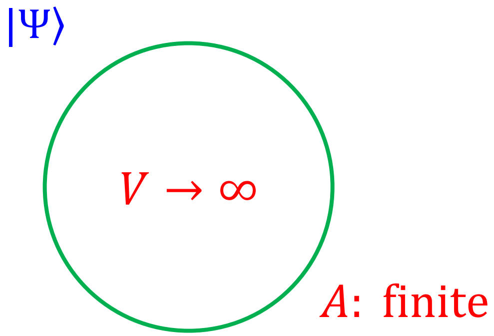

… at odds with $S = \frac{A}{4\ell_P^2}$

Hugely redundant!

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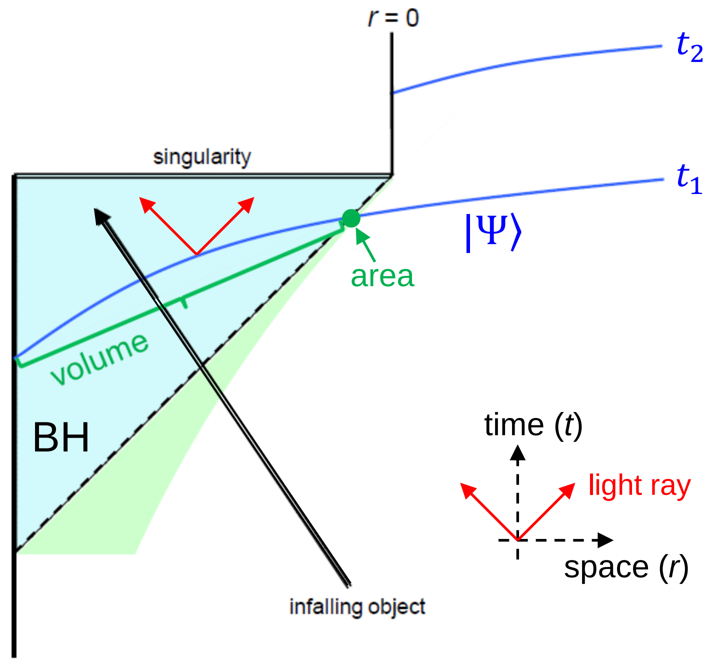

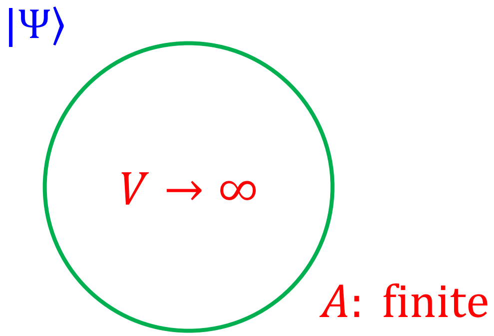

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Hugely redundant!

$$\langle\Psi_1|\Psi_2\rangle = 0 \quad \longrightarrow \quad \langle\Psi_1|\Psi_2\rangle \sim e^{-\frac{S}{2}}$$

semiclassical
(QFT in curved spacetime)

quantum gravity

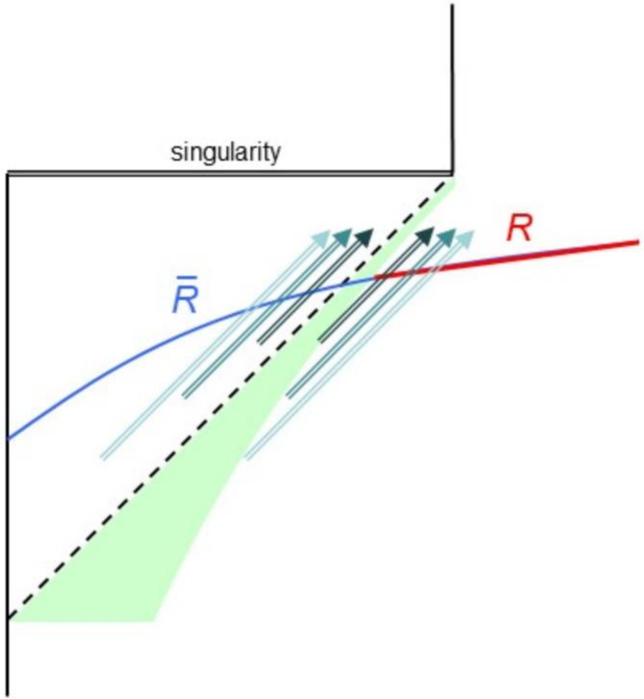
… only e^S independent states

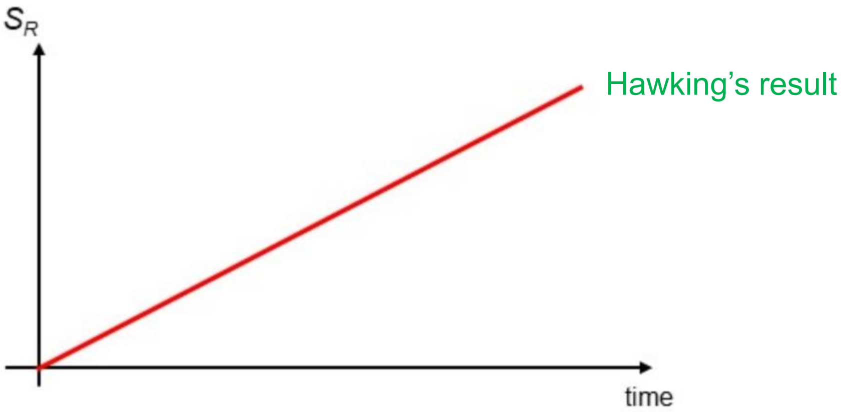
$$|\Psi\rangle = \sum_{i=1}^{e^S} c_i|\psi_i\rangle \qquad c_i \sim e^{-\frac{S}{2}}$$

$$\langle\Psi_1|\Psi_2\rangle = \sum_{i=1}^{e^S} c_{1,i}^* \, c_{2,i} \sim e^{\frac{S}{2}} e^{-S} \sim e^{-\frac{S}{2}}$$

$\to e^{e^S}$ approximately orthogonal states

Unitarity of Hawking evaporation

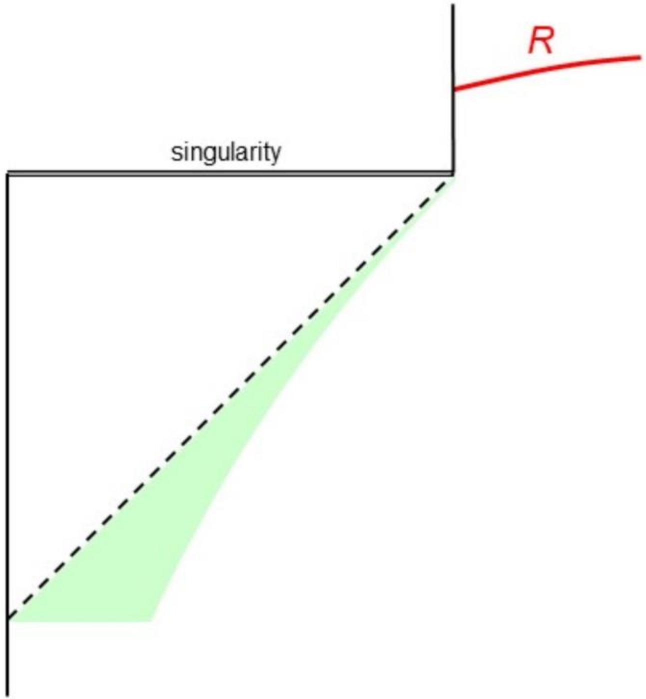


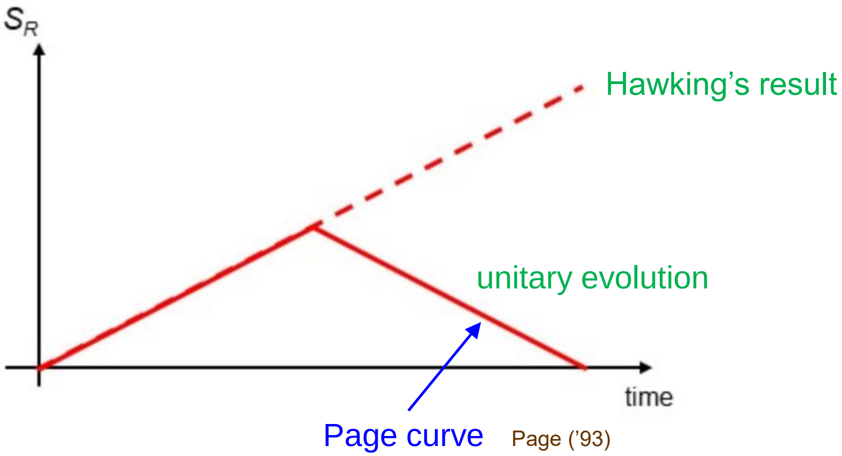


$$S_R = -\mathrm{Tr}[\rho_R \ln \rho_R] \qquad (\rho_R = \mathrm{Tr}_{\bar{R}}|\Psi\rangle\langle\Psi|)$$

~ the # of EPR particles in R whose partners are in $\bar{R}$

Unitarity of Hawking evaporation





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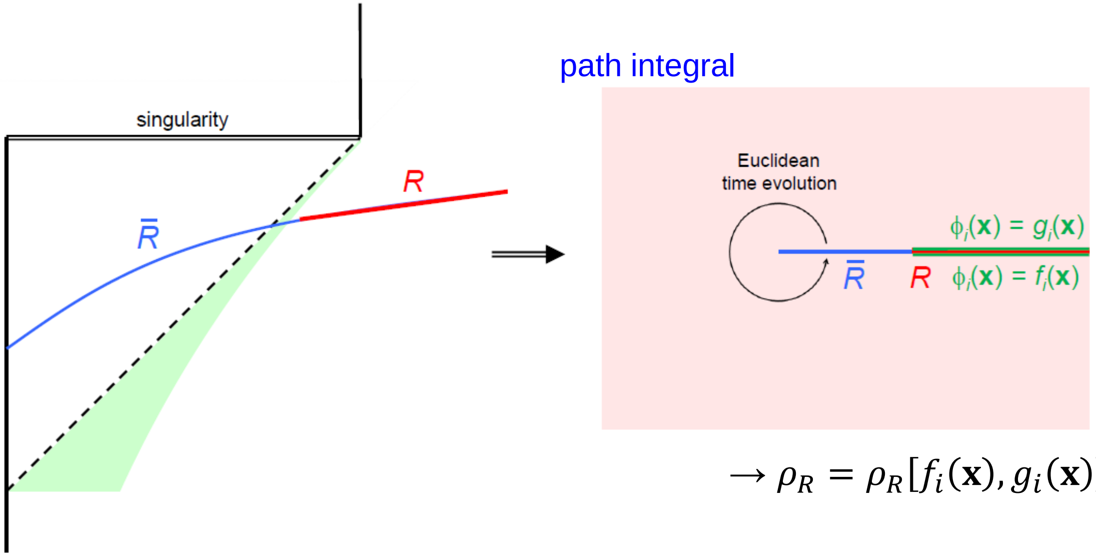
→ How to get this curve?

Page curve from replica wormholes

Penington, Shenker, Stanford, Yang ('19);
Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini ('19)



$\rightarrow \rho_R = \rho_R[f_i(\mathbf{x}), g_i(\mathbf{x})]$ (~ coefficient of $|g_i(\mathbf{x})\rangle\langle f_i(\mathbf{x})|$)

$$\boxed{S_R \equiv -\mathrm{Tr}[\rho_R \ln \rho_R] \;=\; \lim_{n\to 1} \frac{1}{1-n} \ln \mathrm{Tr}[\rho_R^n]}$$

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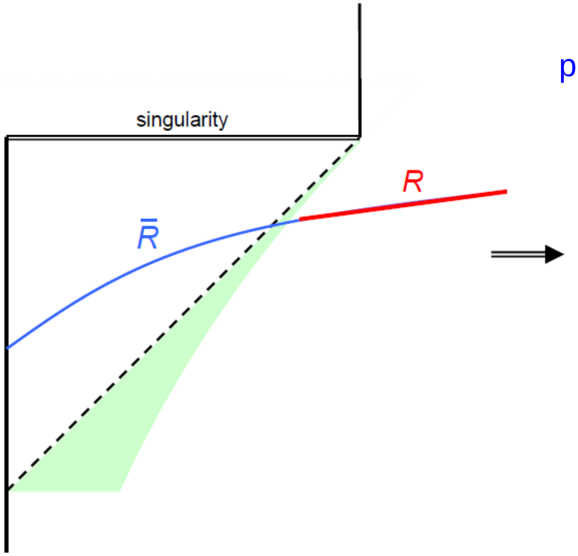
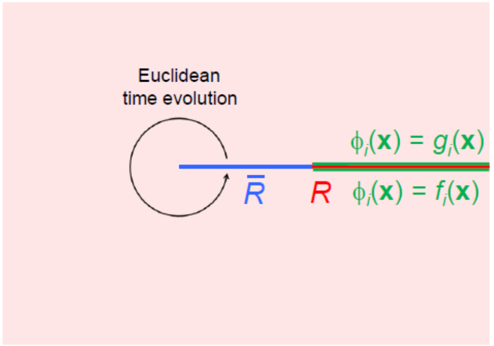


path integral



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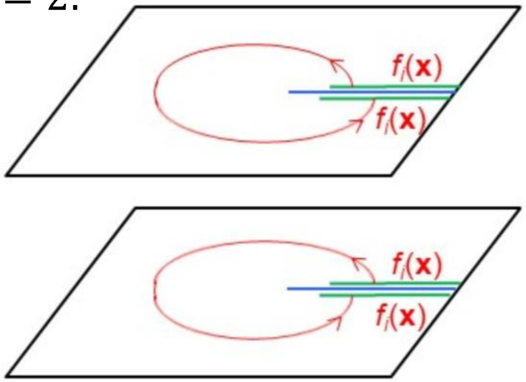
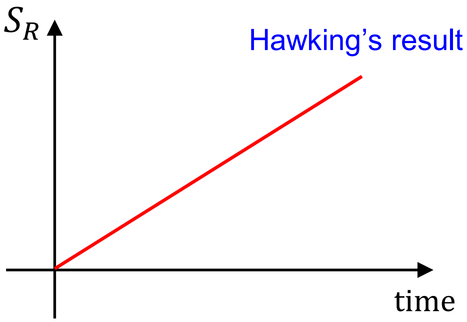
$n = 2$:





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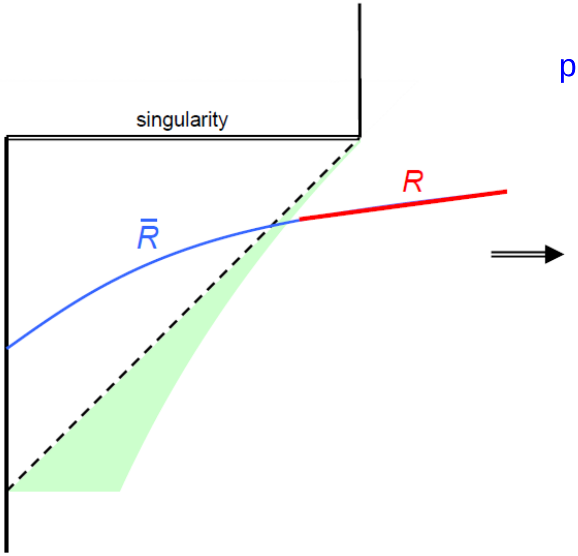

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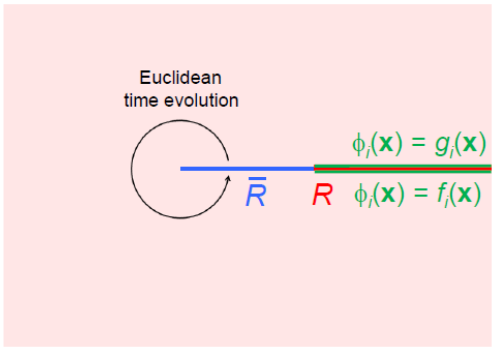

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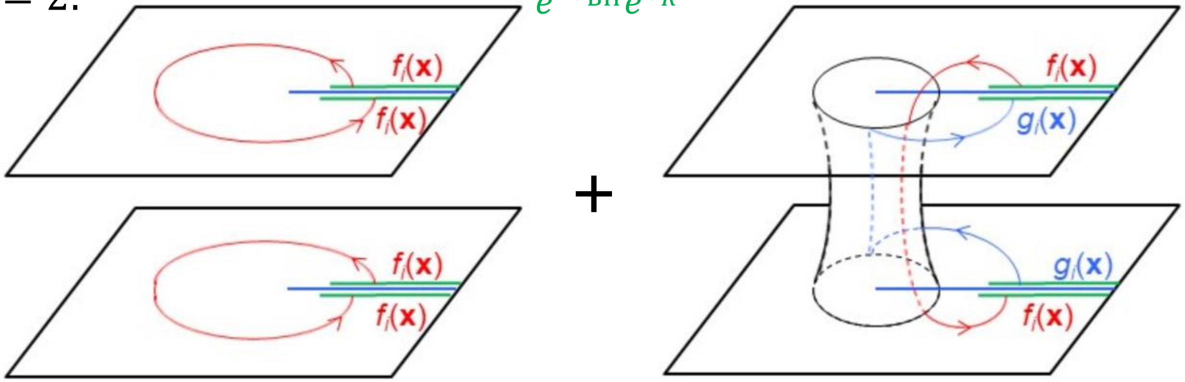
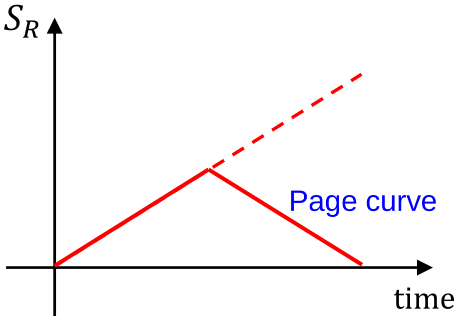
$n = 2$:

$e^{2S_{\mathrm{BH}}}e^{S_R}$ + $e^{S_{\mathrm{BH}}}e^{2S_R}$

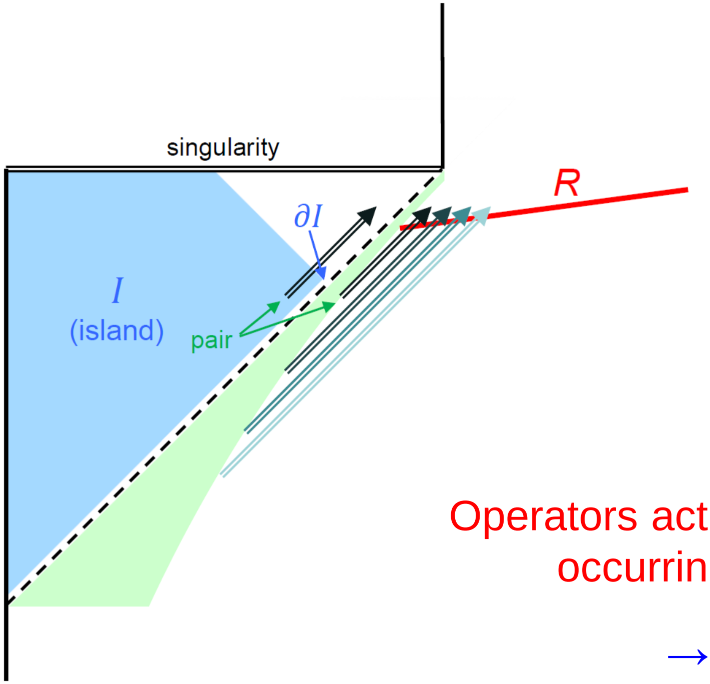


replica wormhole (nonperturbative effect)



Quantum extremal surface (QES) prescription

Penington ('19); Almheiri, Engelhardt, Marolf, Maxfield ('19); …



Entanglement entropy of radiation in *R* is given by

$$S_R = \underset{\partial I}{\text{min ext}} \left[\frac{\text{Area}(\partial I)}{4G_\text{N}} + S_{\text{von Neumann}}(R \cup I) \right]$$

∂I: quantum extremal surface (QES)

Operators acting on *R* can represent physics occurring in I (entanglement wedge reconstruction)

→ Hawking radiation emitted is **not** independent of the interior d.o.f.s!

…; Maldacena, Susskind ('13); …

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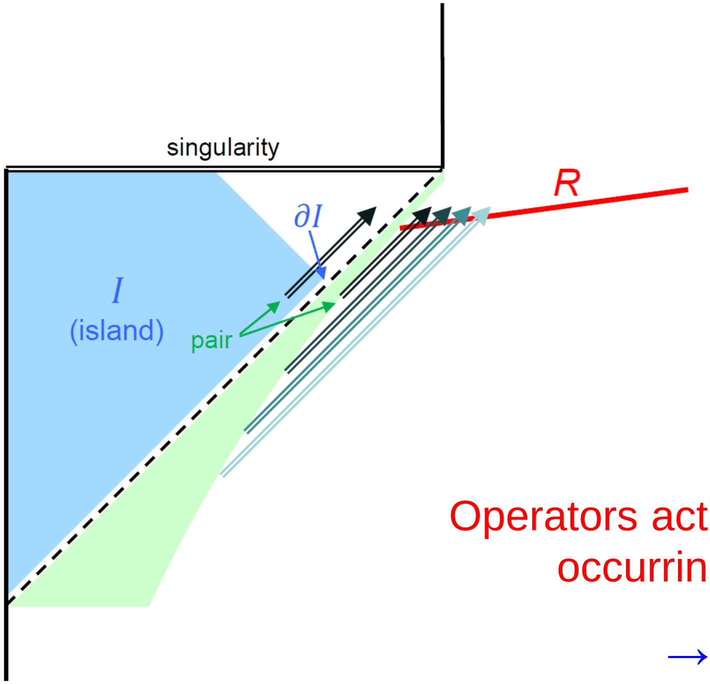

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Global spacetime

(embracing the **interior**)

Replica wormholes

(nonperturbative effects of gravity)

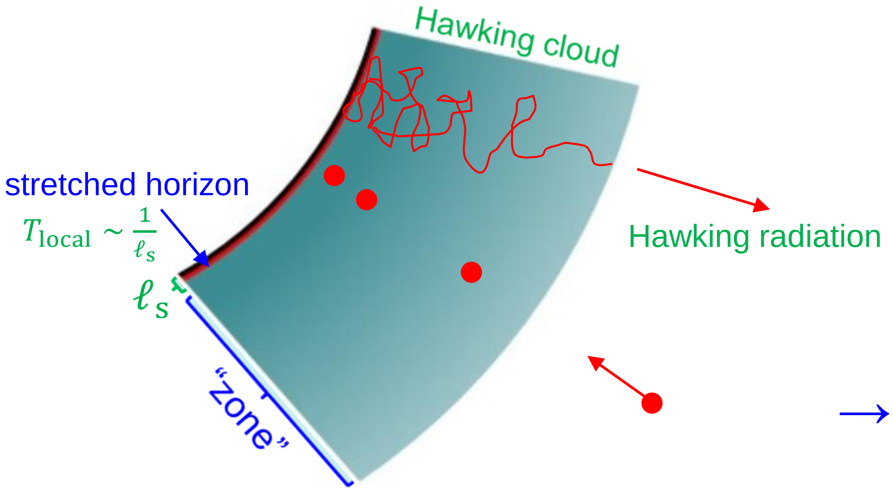
Page curve

(signifying **unitarity**)

Picture based on Holography

— unitary gauge construction —

Start with a “distant” (holographic) description



The d.o.f.s outside the horizon comprise the **entire** system.

→ The evolution is unitary.

→ How does the “interior” emerge?

Papadodimas, Raju (’12–’15); Verlinde, Verlinde (’12–’13);
Y.N., Sanches, Varela, Weinberg (’12–’15); …
Y.N. (’19, 20)

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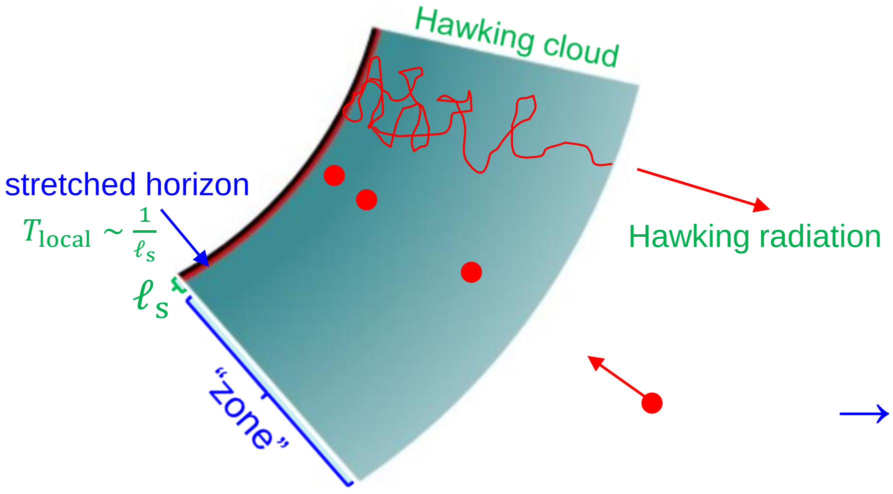

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Y.N. (’19, 20)

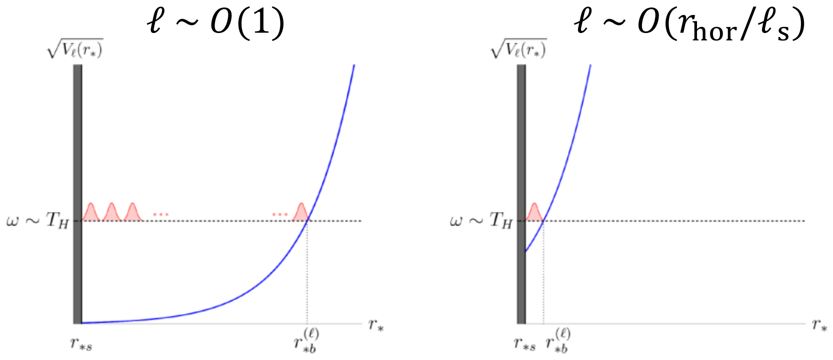
Key features

Y.N. (’19, 20), Murdia, Y.N., Ritchie (’22)

— defining characteristics of BHs

(I) Huge redshift

Spacetime does not exist below $\ell_{\text{proper}} \sim \ell_s$ $\longrightarrow$ $S_{\text{BH}} \sim \text{area}$, $\Delta E \sim e^{-S_{\text{BH}}}$



Relevant modes:

$$\begin{cases} \text{BH} & \begin{cases} \text{horizon} \\ \text{zone} \end{cases} \\ \text{far} & \end{cases}$$

horizon } soft (cloud)
zone ⊃ hard (objects)

(II) Dynamics at the stretched horizon

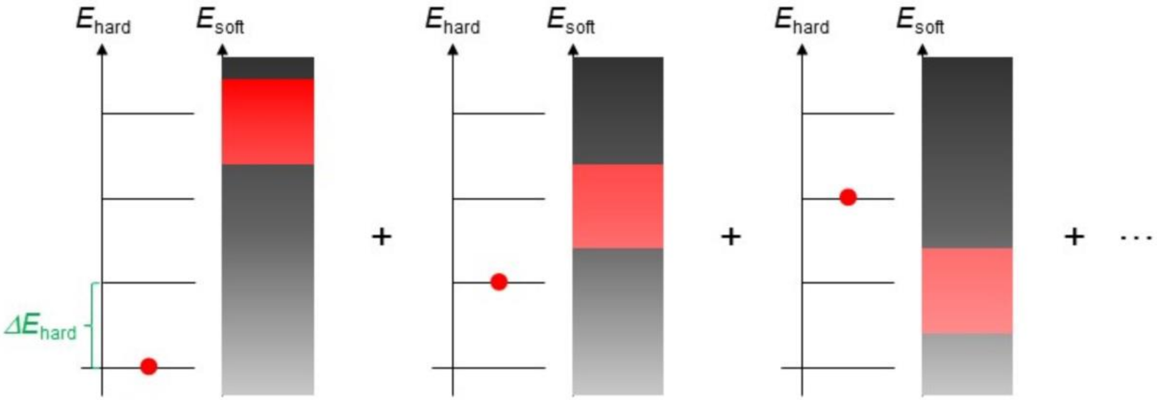
$T_{\text{local}} \sim M_{\text{string}}$

… string dynamics

- quantum chaos Maldacena, Shenker, Stanford ('15)
- fast scrambling Hayden, Preskil ('07); Sekino, Susskind ('08)
- universal Banks, Seiberg ('10); …; Harlow, Ooguri ('18)

(*e.g.* no global symmetry)

→ "ultimate" thermalization in the zone

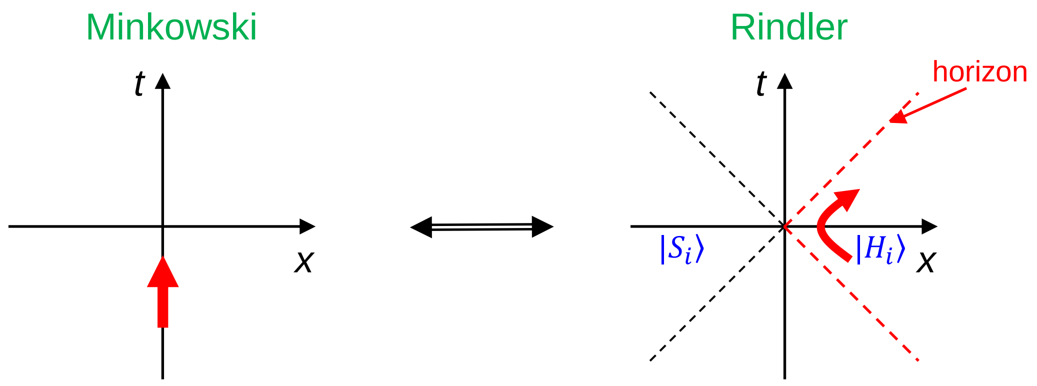


$$\sum_{i=1}^{e^{S_{\text{bh}}(M)}} c_i^{(0)} |0\rangle|\Psi_i^{(0)}\rangle + \sum_{i=1}^{e^{S_{\text{bh}}(M-\Delta E)}} c_i^{(1)} |1\rangle|\Psi_i^{(1)}\rangle + \sum_{i=1}^{e^{S_{\text{bh}}(M-2\Delta E)}} c_i^{(2)} |2\rangle|\Psi_i^{(2)}\rangle + \cdots$$

$$\xrightarrow[c_i^{(0)} \sim c_i^{(1)} \sim c_i^{(2)} \sim \cdots]{} \propto |0\rangle\langle 0| + e^{-\frac{\Delta E}{T_{\text{H}}}} |1\rangle\langle 1| + e^{-\frac{2\Delta E}{T_{\text{H}}}} |2\rangle\langle 2| + \cdots$$

… **universal** across all low energy species

Emergence of the interior: Basic picture



$$|0\rangle$$

$$\propto \sum_i e^{-\frac{E_i}{2T}} |R_i\rangle |L_i\rangle$$

… thermal state Fulling ('73); Davies ('75); Unruh ('76)

Near empty
Interior spacetime

(An object thrown "sees" interior spacetime)

frame change

$$\Longleftrightarrow$$

Hard mode states

$$|\Psi_{\rm BH}\rangle \propto \sum_i e^{-\frac{E_i}{2T_{\rm H}}} |H_i\rangle \overbrace{|S_i\rangle}$$

… play the role of the mirror partners

Coarse-grained soft mode states
(representing their collective excitations)

… **universally** thermal

At late times, the BH is entangled with radiation

Hard mode states

$$|\Psi_{\rm BH}\rangle \propto \sum_i e^{-\frac{E_i}{2T_{\rm H}}} |H_i\rangle \overbrace{|(S+R)_i\rangle}$$

… play the role of the mirror partners

Coarse-grained soft mode **and** radiation states
(representing collective excitations of these modes)

… Interior d.o.f.s involve early Hawking radiation.

More details and math behind them

- Black hole **vacuum** state at time *t*

$$|\Psi_{A,0}(M)\rangle = \sum_n \sum_{i_n=1}^{e^{S_{\rm bh}(M-E_n)}} \sum_{a=1}^{e^{S_{\rm rad}}} c^A_{n i_n a} \underbrace{|\{n_\alpha\}\rangle}_{\text{Hard mode}} \overbrace{|\psi^{(n)}_{i_n}\rangle}^{\text{Soft mode … the density of states: } e^{S_{\rm BH}(E_{\rm soft})}} \underbrace{|\phi_a\rangle}_{\text{Far mode (radiation)}}$$

index specifying microstate: $A = 1, \cdots, e^{S_{\rm tot}}$

$$\left(\langle\{m_\alpha\}|\{n_\alpha\}\rangle = \delta_{mn}, \quad \langle\psi^{(m)}_{i_m}|\psi^{(n)}_{j_n}\rangle = \delta_{mn}\delta_{i_m j_n}, \quad \langle\phi_a|\phi_b\rangle = \delta_{ab} \right)$$

$$e^{S_{\rm tot}} \equiv \sum_n e^{S_{\rm bh}(M-E_n)} e^{S_{\rm rad}} = z\, e^{S_{\rm bh}(M)+S_{\rm rad}} \quad \left(z \equiv \sum_n e^{-\frac{E_n}{T_{\rm H}}} \right)$$

Complete thermalization by (redshifted) string dynamics

$$\langle c^A_{n i_n a}\rangle = 0, \quad \sqrt{\langle |c^A_{n i_n a}|^2\rangle} = \frac{1}{e^{\frac{1}{2}S_{\rm tot}}} \quad \Longrightarrow \quad {\rm Tr}_{\rm soft}|\Psi_{A,0}(M)\rangle\langle\Psi_{A,0}(M)| = \frac{1}{z}\sum_n e^{-\frac{E_n}{T_{\rm H}}} |\{n_\alpha\}\rangle\langle\{n_\alpha\}| \otimes \rho_\phi$$

… thermal density matrix for the hard modes

Excitations in the zone:

$$b_\gamma = \sum_n \sqrt{n_\gamma}\, |\{n_\alpha - \delta_{\alpha\gamma}\}\rangle\langle\{n_\alpha\}|$$

$$b^\dagger_\gamma = \sum_n \sqrt{n_\gamma + 1}\, |\{n_\alpha + \delta_{\alpha\gamma}\}\rangle\langle\{n_\alpha\}|$$

… standard annihilation and creation operators

What about the interior?

More details and math behind them

- Black hole **vacuum** state at time *t*

$$|\Psi_{A,0}(M)\rangle = \sum_n \sum_{i_n=1}^{e^{S_{\rm bh}(M-E_n)}} \sum_{a=1}^{e^{S_{\rm rad}}} c^A_{n i_n a} |\{n_\alpha\}\rangle |\psi^{(n)}_{i_n}\rangle |\phi_a\rangle$$

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… thermal density matrix for the hard modes

Mirror microstates **for each *A***

cf. Papadodimas, Raju ('12–'15); Verlinde, Verlinde ('12–'13); Y.N., Sanches, Varela, Weinberg ('12–'15)

$$\|\{n_\alpha\}_A\rangle\!\rangle = \alpha^A_n \sum_{i_n=1}^{e^{S_{\rm bh}(M-E_n)}} \sum_{a=1}^{e^{S_{\rm rad}}} c^A_{n i_n a} |\psi^{(n)}_{i_n}\rangle |\phi_a\rangle \quad \Longrightarrow \quad |\Psi_{A,0}(M)\rangle = \frac{1}{\sqrt{z}} \sum_n e^{-\frac{E_n}{2T_{\rm H}}} |\{n_\alpha\}\rangle \|\{n_\alpha\}_A\rangle\!\rangle$$

… thermofield double form

normalization: $\alpha^A_n = \dfrac{1}{\sqrt{\sum_{i_n=1}^{e^{S_{\rm bh}(M-E_n)}} \sum_{a=1}^{e^{S_{\rm rad}}} c^{A*}_{n i_n a} c^A_{n i_n a}}}$

$$= \sqrt{z}\, e^{\frac{E_n}{2T_{\rm H}}} \left(1 - \frac{1}{2}\varepsilon^{AA}_n \right)$$

exponentially small correction

Mirror microstates **for each *A***

$$\|\{n_\alpha\}_A\rangle\!\rangle = \alpha_n^A \sum_{i_n=1}^{e^{S_{\mathrm{bh}}(M-E_n)}} \sum_{a=1}^{e^{S_{\mathrm{rad}}}} c^A_{n i_n a} |\psi^{(n)}_{i_n}\rangle |\phi_a\rangle \quad \Longrightarrow \quad |\Psi_{A,0}(M)\rangle = \frac{1}{\sqrt{z}} \sum_n e^{-\frac{E_n}{2T_{\mathrm{H}}}} |\{n_\alpha\}\rangle \|\{n_\alpha\}_A\rangle\!\rangle$$

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… **thermofield double form**

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$$= \sqrt{z}\, e^{\frac{E_n}{2T_{\rm H}}} \left(1 - \frac{1}{2}\varepsilon_n^{AA}\right)$$

exponentially small correction

Mirror operators **for each microstate** (representing collective excitations of soft and far modes)

$$\tilde{b}_\gamma^A = \sum_n \sqrt{n_\gamma}\, \|\{n_\alpha - \delta_{\alpha\gamma}\}_A\rangle\!\rangle \langle\!\langle \{n_\alpha\}_A\|$$

$$\tilde{b}_\gamma^{A\dagger} = \sum_n \sqrt{n_\gamma + 1}\, \|\{n_\alpha + \delta_{\alpha\gamma}\}_A\rangle\!\rangle \langle\!\langle \{n_\alpha\}_A\|$$

… satisfy the commutation relation of ann./cre. operators
up to exponentially suppressed corrections of $\sim e^{-S}$.

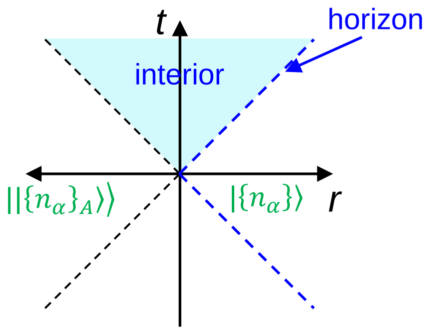
Infalling mode operators for each microstate

$$a_\xi^A = \sum_\gamma \left(\alpha_{\xi\gamma} b_\gamma + \beta_{\xi\gamma} b_\gamma^\dagger + \zeta_{\xi\gamma} \tilde{b}_\gamma^A + \eta_{\xi\gamma} \tilde{b}_\gamma^{A\dagger}\right)$$

$$a_\xi^{A\dagger} = \sum_\gamma \left(\beta^*_{\xi\gamma} b_\gamma + \alpha^*_{\xi\gamma} b_\gamma^\dagger + \eta^*_{\xi\gamma} \tilde{b}_\gamma^A + \zeta^*_{\xi\gamma} \tilde{b}_\gamma^{A\dagger}\right)$$

standard Bogoliubov coefficients

… describe interior spacetime
for **the hard modes** (objects).



Sufficient? → No

Hard modes (a falling object) may be entangled with soft modes.

$$|\Psi(t_*)\rangle = \sum_{A=1}^{S_{\rm tot}} \sum_{I} d_{AI}(t_*) |\Psi_{A,I}(M)\rangle$$

→ Which infalling micro-operators should we use?

Global promotion

$$\mathcal{M} = \left\{ \sum_{A=1}^{e^{S_{\rm tot}}} a_A |\Psi_{A,0}(M)\rangle \,\middle|\, a_A \in \mathbb{C}, \sum_{A=1}^{e^{S_{\rm tot}}} |a_A|^2 = 1 \right\}$$

… space of vacuum microstates

$$\tilde{\mathcal{M}} = \left\{ \sum_{A'=1}^{e^{S_{\rm eff}}} a_{A'} |\Psi_{A',0}(M)\rangle \,\middle|\, a_{A'} \in \mathbb{C}, \sum_{A'=1}^{e^{S_{\rm eff}}} |a_{A'}|^2 = 1 \right\}$$

… subspace of $\mathcal{M}$ with $S_{\rm eff} < S_{\rm bh}(M) + S_{\rm rad}$

Globally promoted operators:

$$\tilde{\mathcal{B}}_\gamma = \sum_{A'=1}^{e^{S_{\rm eff}}} \tilde{b}_\gamma^{A'}, \quad \tilde{\mathcal{B}}_\gamma^\dagger = \sum_{A'=1}^{e^{S_{\rm eff}}} \tilde{b}_\gamma^{A'\dagger}$$

⟹

$$\mathcal{A}_\xi = \sum_\gamma \left(\alpha_{\xi\gamma} b_\gamma + \beta_{\xi\gamma} b_\gamma^\dagger + \zeta_{\xi\gamma} \tilde{\mathcal{B}}_\gamma + \eta_{\xi\gamma} \tilde{\mathcal{B}}_\gamma^\dagger \right)$$

$$\mathcal{A}_\xi^\dagger = \sum_\gamma \left(\beta_{\xi\gamma}^* b_\gamma + \alpha_{\xi\gamma}^* b_\gamma^\dagger + \eta_{\xi\gamma}^* \tilde{\mathcal{B}}_\gamma + \zeta_{\xi\gamma}^* \tilde{\mathcal{B}}_\gamma^\dagger \right)$$

… satisfy the correct algebra
for **any** vacuum state in $\tilde{\mathcal{M}}$.

(up to exponentially suppressed corrections)

Sufficient? → No

Hard modes (a falling object) may be entangled with soft modes.

$$|\Psi(t_*)\rangle = \sum_{A=1}^{S_{\rm tot}} \sum_{I} d_{AI}(t_*)|\Psi_{A,I}(M)\rangle \xrightarrow[\text{decomposition}]{\text{Schmidt}} |\Psi(t_*)\rangle = \sum_{I=1}^{\mathcal{K}} g_I |\Psi_{A(I),I}(M)\rangle$$

where $\mathcal{K} \le S_{\rm exc} < S_{\rm bh}(M) + S_{\rm rad}$

$\Downarrow$ One can always choose

$$\boxed{\tilde{\mathcal{M}} \supseteq \tilde{V}\big[|\Psi(t_*)\rangle\big]}$$

where $\tilde{V}\big[|\Psi(t_*)\rangle\big] = \mathrm{span}\big(\{|\Psi_{A(I),0}(M)\rangle\}\big)$

Global promotion

$$\mathcal{M} = \left\{ \sum_{A=1}^{e^{S_{\rm tot}}} a_A |\Psi_{A,0}(M)\rangle \,\middle|\, a_A \in \mathbb{C},\ \sum_{A=1}^{e^{S_{\rm tot}}} |a_A|^2 = 1 \right\}$$

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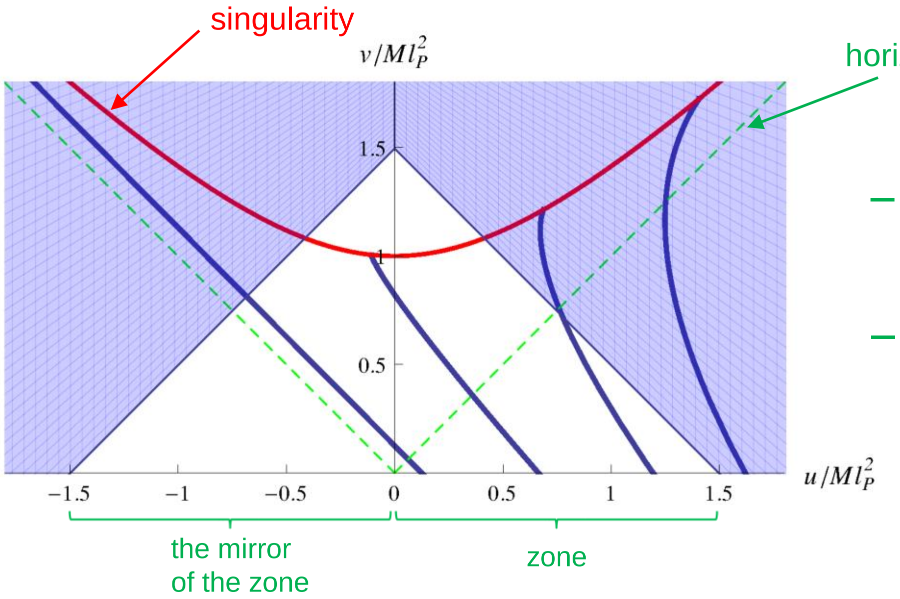
$$\Longrightarrow \boxed{\begin{aligned} \mathcal{A}_\xi &= \sum_\gamma \big(\alpha_{\xi\gamma} b_\gamma + \beta_{\xi\gamma} b_\gamma^\dagger + \zeta_{\xi\gamma} \tilde{\mathcal{B}}_\gamma + \eta_{\xi\gamma} \tilde{\mathcal{B}}_\gamma^\dagger\big) \\ \mathcal{A}_\xi^\dagger &= \sum_\gamma \big(\beta_{\xi\gamma}^* b_\gamma + \alpha_{\xi\gamma}^* b_\gamma^\dagger + \eta_{\xi\gamma}^* \tilde{\mathcal{B}}_\gamma + \zeta_{\xi\gamma}^* \tilde{\mathcal{B}}_\gamma^\dagger\big) \end{aligned}}$$

… satisfy the correct algebra
for **any** vacuum state in $\tilde{\mathcal{M}}$.

(up to exponentially suppressed corrections)

Effective theory of the interior

… erected at each time *t* (as measured in the asymptotic region)



– describes only a **limited** region of spacetime

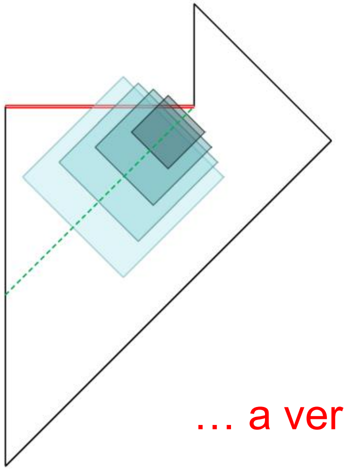
(causal region of the zone and its mirror at *t*)

– **intrinsically** semiclassical

(coarse-grained; the unique infalling vacuum)

→ no cloning problem !

Describing a large interior region
requires **multiple** effective theories
erected at different times.



… a version of complementarity

Susskind, Thorlacius Uglum ('93)

Comments

– Intrinsic ambiguity

Infalling mode operators are not strictly orthogonal to $\tilde{\mathcal{M}}$.

$\Longrightarrow$ ambiguity of the procedure of erecting the effective theory of order $\epsilon = \max\left\{ \frac{e^{S_{\rm eff}}}{e^{S_{\rm bh}(M)+S_{\rm rad}}}, \frac{1}{e^{\frac{1}{2}\{S_{\rm bh}(M)+S_{\rm rad}\}}} \right\}$

… manifestation of the fact that the theory has only a finite # of d.o.f.s.

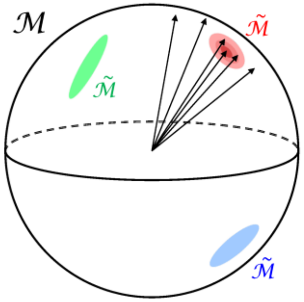
– State dependence

$\tilde{\mathcal{M}}$, of dimension $e^{S_{\rm eff}}$, can be taken large,

$S_{\rm eff} = c\,\{S_{\rm bh}(M) + S_{\rm rad}\}$ for any c (< 1)

unless c is exponentially close to 1.

$\Longrightarrow$ A single set of global operators cannot cover the entire $\mathcal{M}$.



… state dependence

Papadodimas, Raju ('13–'15)

– Young black holes

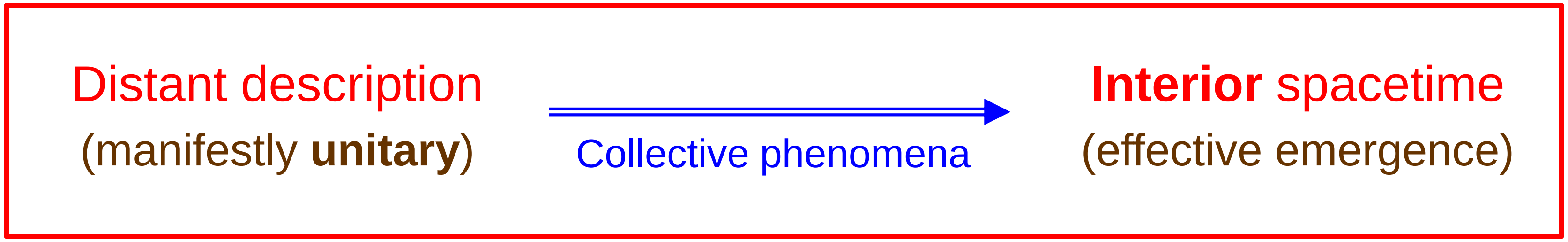
For a young BH, $S_{\rm bh} > S_{\rm rad}$, infalling operators can be taken to act only on soft modes, using the Petz map.

This option is **not** available for an old BH.
→ **must** involve early radiation.

$$\tilde{\mathcal{O}}^A[\mathcal{O}] = \sum_\kappa \sum_\lambda \mathcal{O}_{\kappa\lambda}\, \alpha^A_\kappa \alpha^{A*}_\lambda \sum_{i_\kappa, i'_\kappa=1}^{e^{S_{\rm bh}(M-E_\kappa)}} \sum_{j_\lambda, j'_\lambda=1}^{e^{S_{\rm bh}(M-E_\lambda)}} \sum_{a=1}^{e^{S_{\rm rad}}} X^{(\kappa,A)}_{i'_\kappa i_\kappa} c^A_{\kappa i_\kappa a} c^{A*}_{\lambda j_\lambda a} X^{(\lambda,A)}_{j_\lambda j'_\lambda} |\psi^{(\kappa)}_{i'_\kappa}\rangle\langle\psi^{(\lambda)}_{j'_\lambda}| \quad \left(X^{(\kappa,A)}_{i_\kappa j_\kappa} = e^{-\frac{1}{2}S_{\rm rad}} \sum_{a=1}^{e^{S_{\rm rad}}} \frac{c^A_{\kappa i_\kappa a} c^{A*}_{\kappa j_\kappa a}}{\left|\alpha^A_\kappa \sum_{k_\kappa=1}^{e^{S_{\rm bh}(M-E_\kappa)}} c^{A*}_{\kappa k_\kappa a} c^A_{\kappa k_\kappa a}\right|^2} \right)$$

Summary

— unitary gauge construction —



Distant description

(manifestly **unitary**)

Collective phenomena

Interior spacetime

(effective emergence)

— It is crucial for the string dynamics, $T_{\mathrm{loc}} \sim M_{\mathrm{string}}$,
to lead to "ultimate" (universal) thermalization.

… the defining characteristic for BHs in this description

(→ A similar construction does **not** work for the surface of regular material.)

— Emergence of the interior does not require
detailed knowledge about the UV physics.

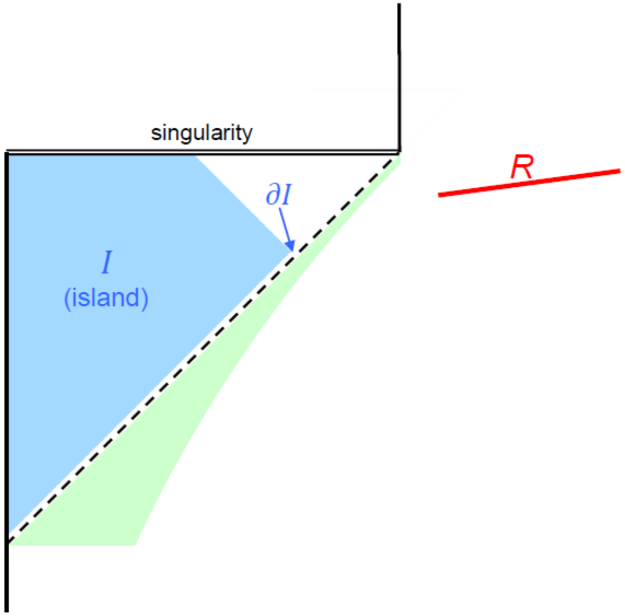
… Some basic features (quantum chaos, fast scrambling, universality) are sufficient.

Relation between the Two Pictures

— canonical vs path-integral views —

Reconstructing the interior of an old BH

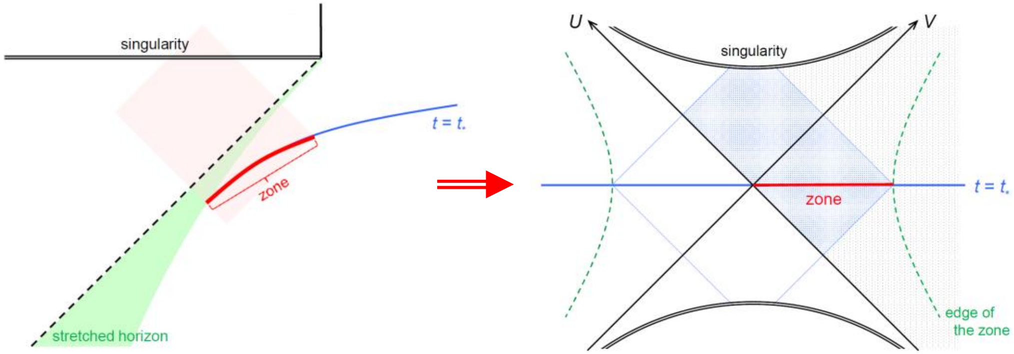
- Entanglement wedge reconstruction



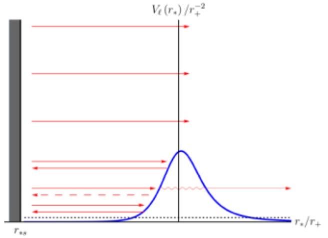
Operators acting **only** on radiation can represent the interior.

- Effective theory of the interior

$$|\Psi_{\rm BH}\rangle \propto \sum_i e^{-\frac{E_i}{2T_{\rm H}}} |Z_i\rangle\, |(H+R)_i\rangle$$



For an evaporating BH,
ingoing modes are not thermalized.

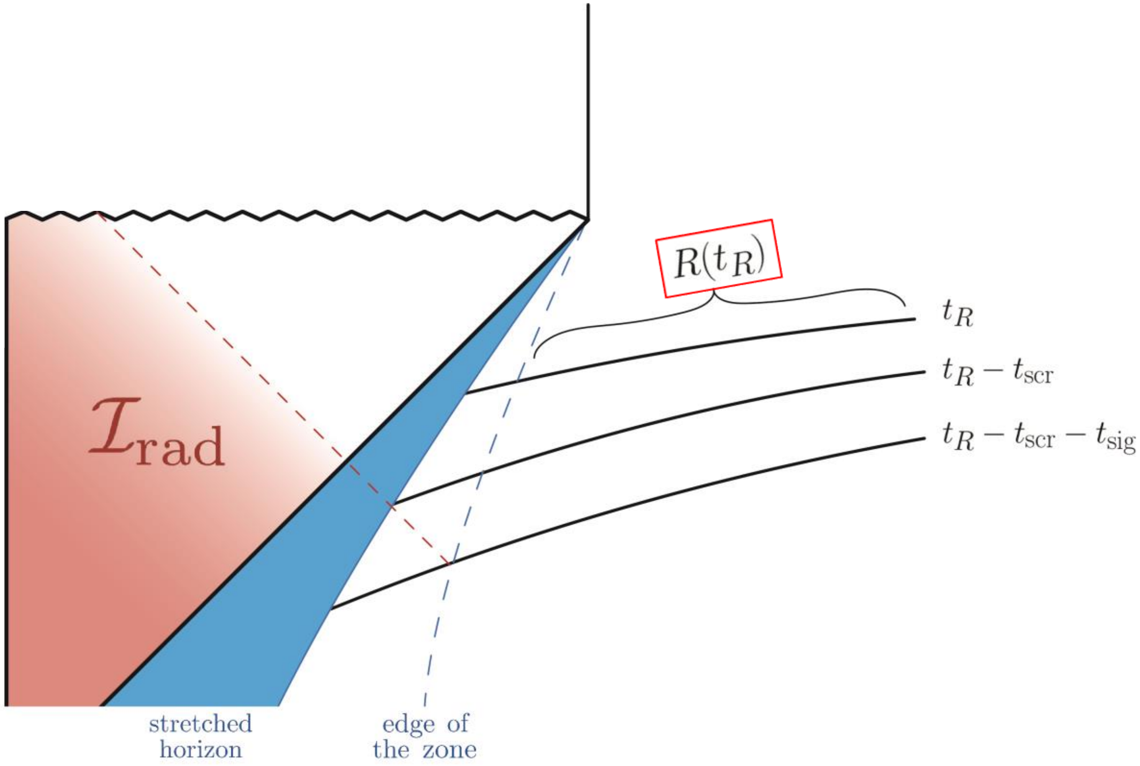


Operators representing the interior must involve **both** the horizon **and** radiation modes.

Spacetime vs State

Langhoff, Y.N. ('20); Murdia, Y.N., Ritchie ('22)

Entanglement wedge reconstruction assumes
we know the "boundary" time evolution.



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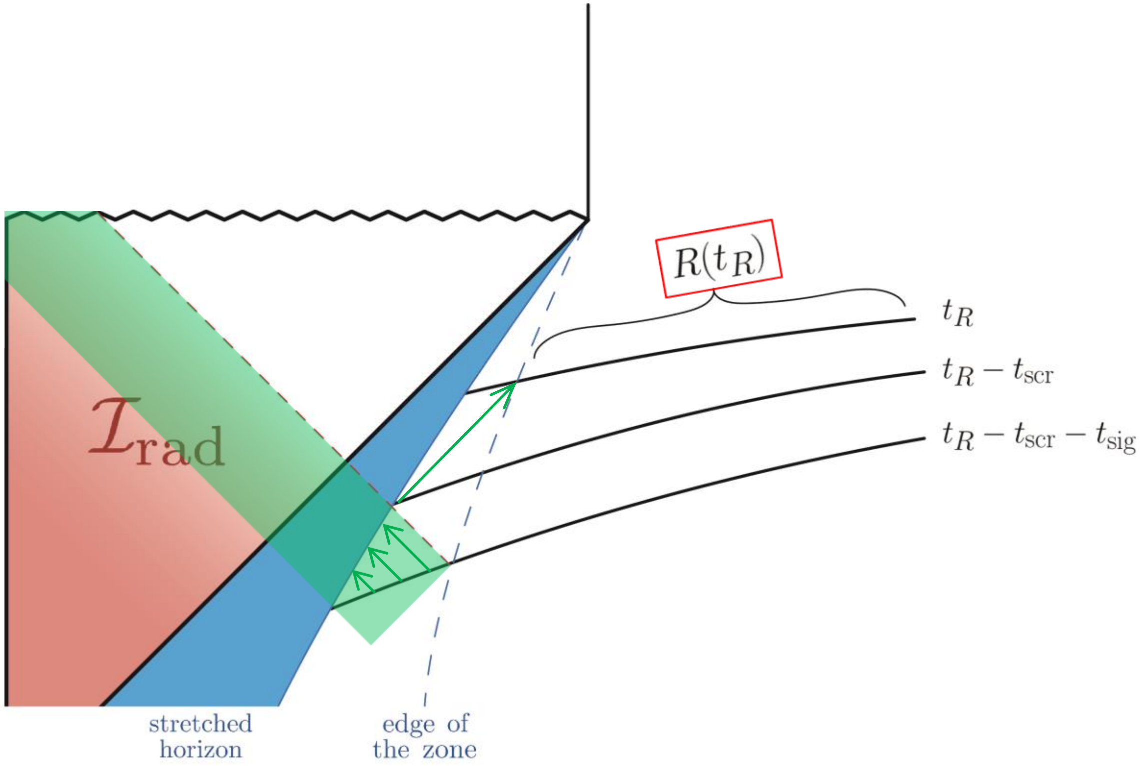


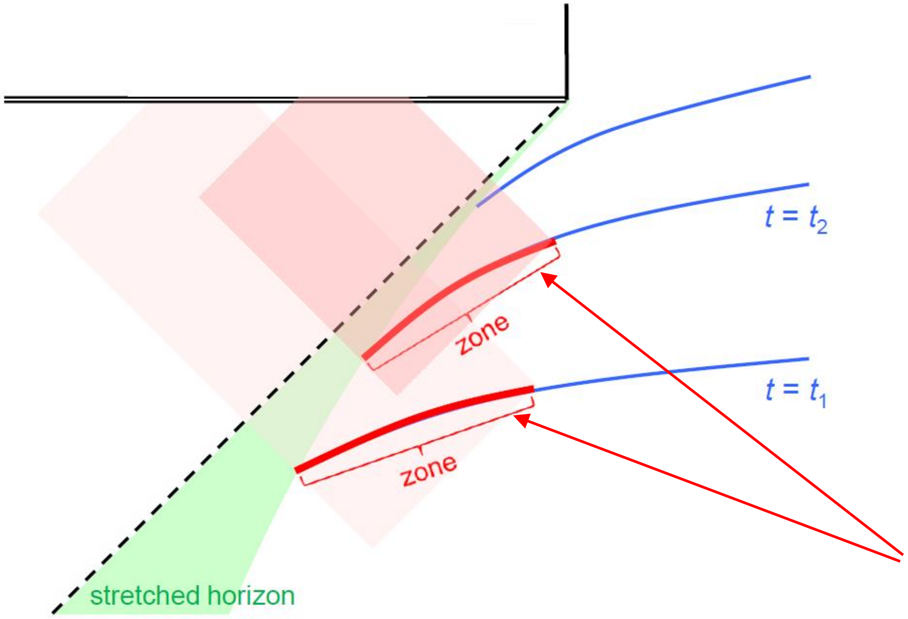
Effective theory construction uses
an **instantaneous state** to construct (the portion of) the interior.

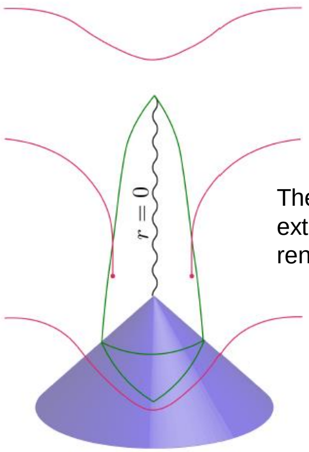
Spacetime vs State

Effective theories of the interior:

… representing the portion of the interior reconstructable from the state **at a given time** by simple operations

(state in the Schrödinger picture)





The boundary time can be extended into the bulk by a certain renormalization procedure.

need anchor (zone) modes to interpret the horizon/radiation modes as constituents of the interior modes

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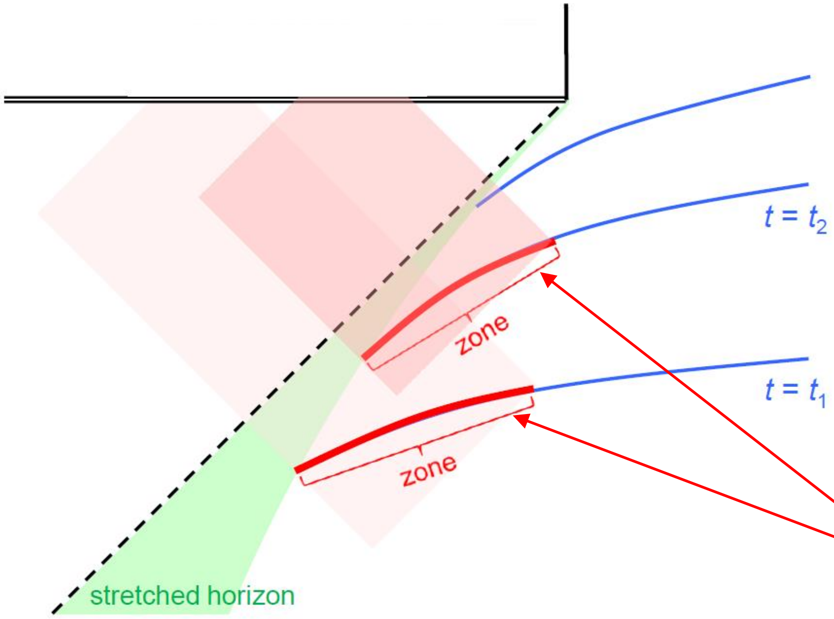
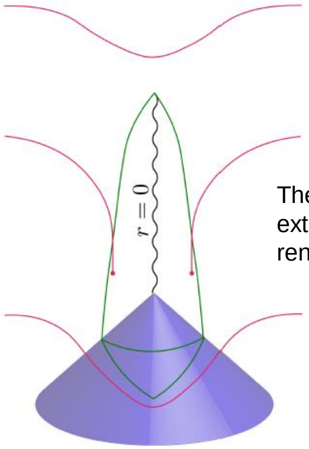




The boundary time can be extended into the bulk by a certain renormalization procedure.

need anchor (zone) modes
to interpret the horizon/radiation modes
as constituents of the interior modes

As time passes, the information in the horizon/radiation modes
become inaccessible at the semiclassical level
because of their complexity generated by the horizon dynamics.

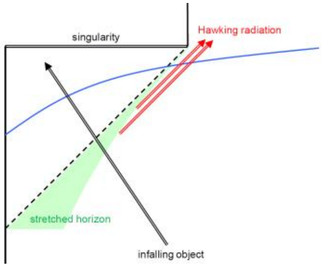
(Entanglement wedge reconstruction describes these “at once.”)

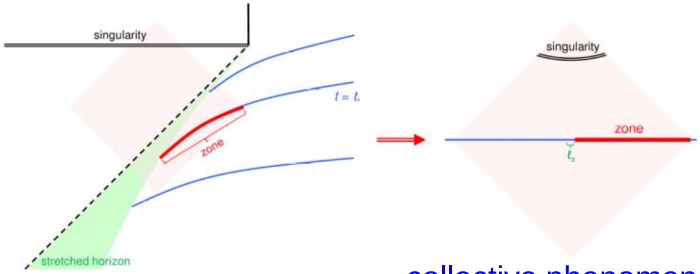
Global spacetime

Unitary / holographic

- Interior

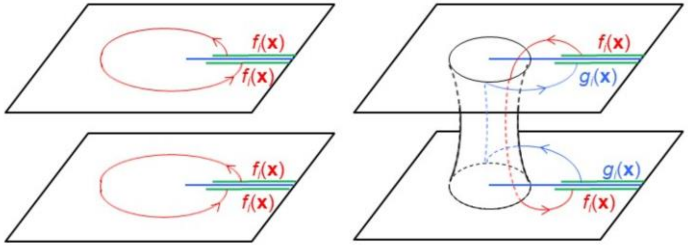
Evident





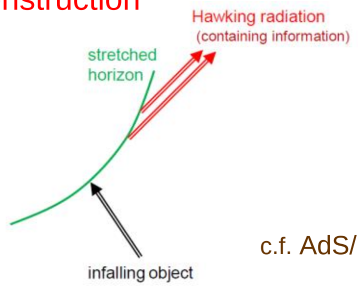
collective phenomenon

- Unitarity



replica wormholes

By construction



c.f. AdS/CFT

- Apparent violation of BH entropy

huge interior spatial volume at late times

semiclassically orthogonal states in fact have $\langle\Psi_1|\Psi_2\rangle \sim e^{-S_{\rm BH}/2}$

$\rightarrow$ $e^{S_{\rm BH}}$ states (+ null states)

Effective theory of the interior has a finite maximal volume.

Hilbert space of dimension $e^{S_{\rm BH}}$ can host $e^{e^{S_{\rm BH}}}$ approximately orthogonal states.

- Ensemble nature

Wormhole contributions

→ “statistical” results

$\langle\psi_I|\psi_J\rangle = 0,\ \left|\langle\psi_I|\psi_J\rangle\right|^2 \sim e^{-S_{\rm BH}/2} \neq 0$

Penington, Shenker, Stanford, Yang (’19)

← Wormholes calculate (incoherent) average

Averaging over horizon modes

$\int dU_{\rm soft} \langle\psi_{U(i)}|\psi_{U(j)}\rangle = 0,$

$\int dU_{\rm soft} \left|\langle\psi_{U(i)}|\psi_{U(j)}\rangle\right|^2 \sim e^{-S_{\rm BH}/2}$

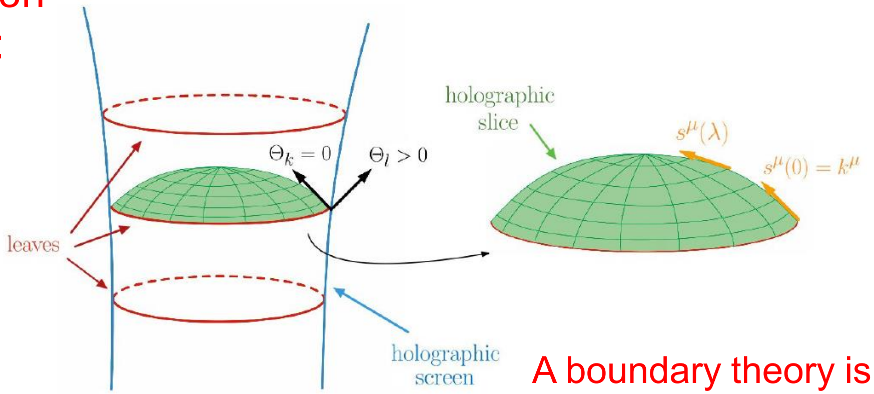
Langhoff, Y.N. (’20); Chakravarty (’20); Pollack, Rozali, Sully Wakeham (’20)

Beyond Black Holes

— de Sitter spacetime and others —

A similar construction for de Sitter spacetime

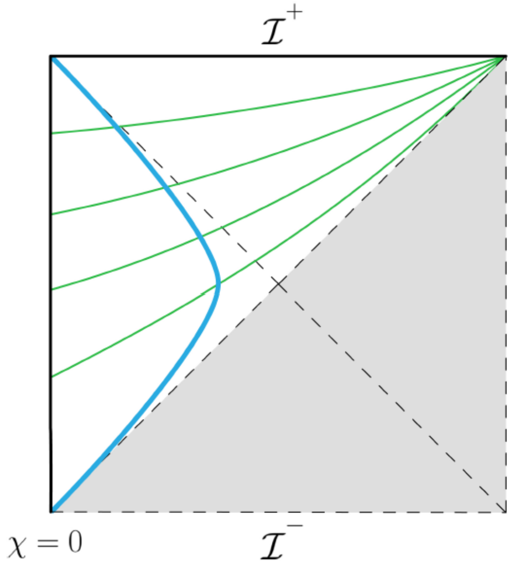
A generalization of AdS/CFT:



A boundary theory is located on the "holographic screen."

Y.N., Salzetta, Sanches, Weinberg ('16)

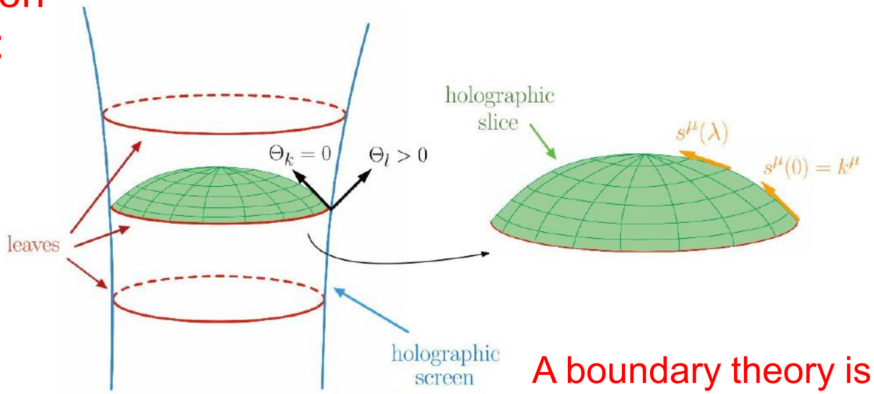
→ apply to regularized/"realistic" de Sitter spacetime



FRW de Sitter bubble universe

A similar construction for de Sitter spacetime

A generalization of AdS/CFT:



A boundary theory is located on the “holographic screen.”

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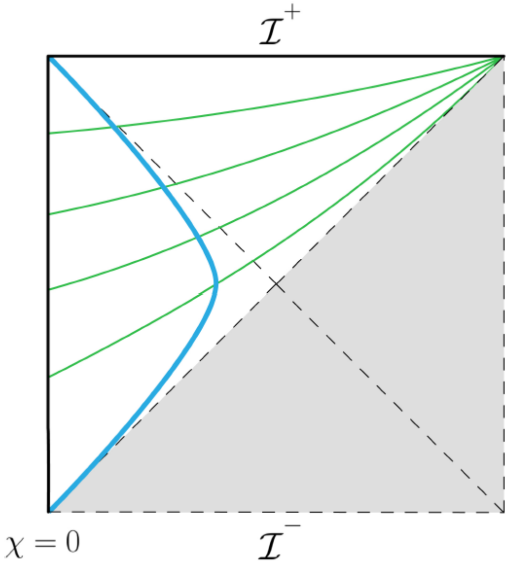
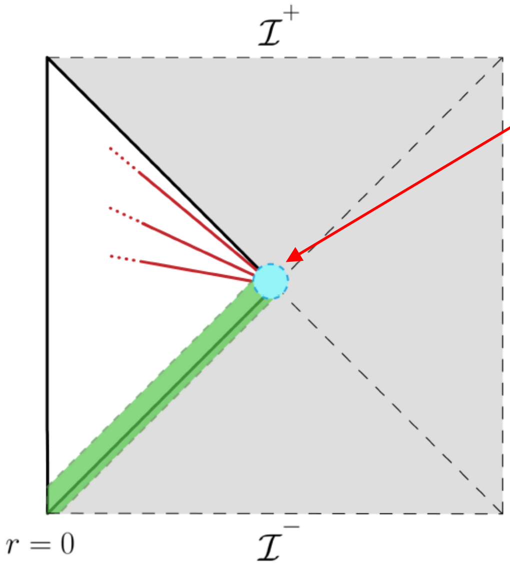
→ apply to regularized/“realistic” de Sitter spacetime



FRW de Sitter bubble universe

de Sitter holography



The holographic theory is on the **regularized/stretched horizon**.

… inside-out version of the black hole case

Y.N. (’11); Sanches, Weinberg (’16);
Y.N., Rath, Salzetta (’17); Y.N. (’19);
…
cf. Shaghoulian, Susskind (’21, ’22)

Reconstructing spacetime outside the dS horizon

States of modes
in the static patch

$$|\Psi_{\mathrm{dS}}\rangle \propto \sum_i e^{-\frac{E_i}{2T_{\mathrm{H}}}} \, |S_i\rangle \, \overbrace{|H_i\rangle}$$

… play the role of the modes in the other hemisphere

Coarse-grained horizon mode states

(representing their collective excitations)

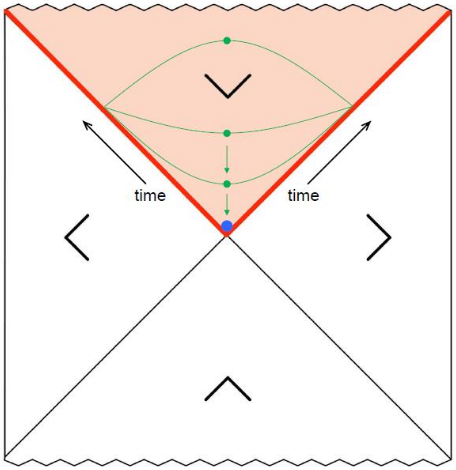
… emergence of spacetime beyond the horizon from horizon d.o.f.s

(applicable for Rindler regions in general)

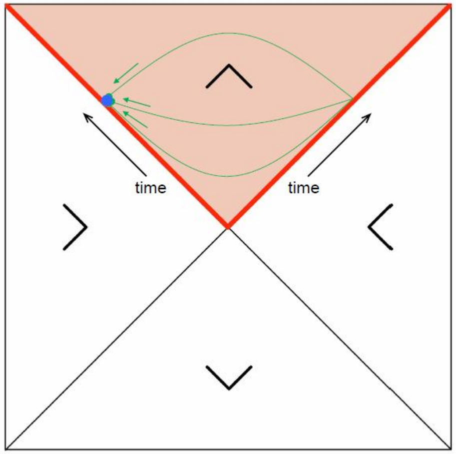
Relation to the Shaghoulian-Susskind proposal

… effective description extracting semiclassical d.o.f.s “outside” the horizon

Black hole:



dS space:



Summary

Black hole conundrum

⇓

Structure of quantum gravity

⊃ Quantm mechanics & General relativity, but in a subtle manner!

… only one of them being manifest

Global spacetime

- interior — evident
- unitarity — nonperturbative gravity

… path integral (GR friendly)

Unitary / holographic

- unitarity — by construction
- interior — collective phenomenon

… operator (QM friendly)

⟹ Lower energy physics without details of microscopic physics

And yet, we want to understand the microscopic theory of quantum gravity

… string theory, quantum information science, …