

# Impurity Driven Metal-Insulator Transitions in Holography

### Yunseok Seo(Kookmin Univ.) February 20, 2023

Based on the arXiv:2302.07539 with Kyung Kiu Kim, Sang-Jin Sin, Keun-Young Kim and Yongjun Ahn

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#### Metal

- Freely moving electrons
- Described by Drude model: conventional metal
- Bad metal :  $\rho \sim T$
- Resistivity increases as temperature increased



#### ■ Insulator

- No freely moving electrons
- Strong electron-electron interaction: Mott
- Strong electron-disorder interaction: Anderson
- Resistivity decreases as temperature increased



### ■ Metal-Insulator Transition(MIT)

- The MIT is one of the oldest, not yet understood well in condensed matter physics
- MIT can capture properties of quantum critical point
- It is very hart to describe different excitations in one model



1112.6166: Dovrosavljevic



### Insulating mechanism

- Interaction induced insulator: Mott insulator
- Impurity(or disorder) induced insulator: Anderson insulator



Byczuk et al, IJMPB(2010)



Holography(gauge/gravity duality)



- Boundary system ↔ Bulk gravity
- Strongly interacting electron ↔ background geometry
- Temperature ↔ Hawking temperature of black hole
- Conserved charge  $\leftrightarrow$  U(1) gauge field
- Momentum relaxation  $\leftrightarrow$  linear axion field
- Operator  $\mathcal{O}_{\Delta} \leftrightarrow \text{field } \phi_m$

0 ...



Holography(gauge/gravity duality)



- Boundary system  $\leftrightarrow$  Bulk gravity
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0 ...
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#### ■ MIT in holography

- Helical lattice(Donos, Hartnoll: Nature, 2013)
- Scalar potential(Refford, Horowitz: PRD, 2014)
- In massive gravity(Baggioli: PRL, 2015)

0 ...



$$S_{tot} = S_0 + S_{int} + S_{bd},$$
  
$$S_0 = \int d^4x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{2} \left( \partial \chi \right)^2 - \frac{1}{4} F^2 - \frac{1}{2} \left( \partial \phi \right)^2 - \frac{1}{2} m^2 \phi^2 \right)$$





$$\begin{split} S_{tot} &= S_0 + S_{int} + S_{bd}, \\ S_0 &= \int d^4x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{2} \left( \partial \chi \right)^2 - \frac{1}{4} F^2 - \frac{1}{2} \left( \partial \phi \right)^2 - \frac{1}{2} m^2 \phi^2 \right) \\ \end{split}$$
 Electron interactions



$$S_{tot} = S_0 + S_{int} + S_{bd},$$

$$S_0 = \int d^4x \sqrt{-g} \left( R - 2\Lambda + \frac{1}{2} (\partial \chi)^2 - \frac{1}{4} F^2 - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} m^2 \phi^2 \right)$$
Momentum relaxation
$$E.O.M: \nabla^2 \chi = 0$$
Solution:  $(\chi_1, \chi_2) = (\beta x, \beta y)$ 
Ward identities:
$$\nabla^{\nu} < T_{\mu\nu} > = < \phi_l > \nabla_{\mu} \chi_l^{(0)} + F_{\mu\nu}^{(0)} < J^{\nu} >$$



$$\begin{split} S_{tot} &= S_0 + S_{int} + S_{bd}, \\ S_0 &= \int d^4 x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{2} \left( \partial \chi \right)^2 \left( -\frac{1}{4} F^2 \right) \frac{1}{2} \left( \partial \phi \right)^2 - \frac{1}{2} m^2 \phi^2 \right) \end{split}$$
  
Finite density  
E.O.M:  $\nabla_{\mu} F^{\mu\nu} = 0$   
Conserved charge:  $\sqrt{-g} F^{rt} = \mathcal{Q}$   
Chemical potential:  
 $A_t \sim \mu + \frac{\mathcal{Q}}{r} + \cdots$ 



■ Action in 3+1 dim.



$$S_{tot} = S_0 + S_{int} + S_{bd},$$
  

$$S_0 = \int d^4x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{2} (\partial \chi)^2 - \frac{1}{4} F^2 - \left( \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} m^2 \phi^2 \right) \right)$$
  
Order parameter

Asymptotic expansion: 
$$\phi \sim \frac{C_1}{r^{\Delta_+}} + \frac{C_2}{r^{\Delta_-}} + \cdots$$
  
Pure  $AdS_{d+1}$ :  $\Delta_{\pm} = \frac{3}{2} \pm \sqrt{m^2 + \frac{d^2}{4}}$   
Breitenlohner-Freedman (BF) bound:  $m_{BF}^2 \ge \frac{d^2}{4}$ 

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$$\begin{split} S_{tot} &= S_0 + S_{int} + S_{bd}, \\ S_0 &= \int d^4 x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{2} \left( \partial \chi \right)^2 - \frac{1}{4} F^2 - \frac{1}{2} \left( \partial \phi \right)^2 - \frac{1}{2} m^2 \phi^2 \right) \\ S_{int} &= -\int \sqrt{-g} \frac{\gamma_2}{4} \phi^2 F^2. \end{split} \qquad \gamma_2 = -0.2 < 0 \end{split}$$



$$\begin{split} S_{tot} &= S_0 + S_{int} + S_{bd}, \\ S_0 &= \int d^4 x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{2} \left( \partial \chi \right)^2 - \frac{1}{4} F^2 - \frac{1}{2} \left( \partial \phi \right)^2 - \frac{1}{2} m^2 \phi^2 \right) \\ S_{int} &= - \int \sqrt{-g} \frac{\gamma_2}{4} \phi^2 F^2. \end{split}$$



#### ■ Action in 3+1 dim.

$$\begin{split} S_{tot} &= S_0 + S_{int} + S_{bd}, \\ S_0 &= \int d^4 x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{2} \left( \partial \chi \right)^2 - \frac{1}{4} F^2 - \frac{1}{2} \left( \partial \phi \right)^2 - \frac{1}{2} m^2 \phi^2 \right) \\ S_{int} &= - \int \sqrt{-g} \frac{\gamma_2}{4} \phi^2 F^2. \end{split}$$

#### ■ Equations of motion

$$R_{MN} - \frac{1}{2}g_{MN}\mathcal{L} - \frac{1}{2}\partial_M\phi\partial_N\phi - \frac{1}{2}\partial_M\chi\partial_N\chi - \frac{1}{2}\left(1 + \gamma_2\phi^2\right)F_{MP}F_M^P = 0$$
  

$$\nabla^2\phi - \left(m^2 + \frac{1}{2}\gamma_2F^2\right)\phi = 0$$
  

$$\nabla_M\left(1 + \gamma_2\phi^2\right)F^{MN} = 0,$$

Ansaz

$$ds^{2} = -U(r)e^{2W(r)-2W(\infty)}dt^{2} + \frac{dr^{2}}{U(r)} + r^{2}(dx^{2} + dy^{2})$$
  
$$\chi^{I} = (\beta x, \beta y), \quad \phi = \phi(r), \quad A = A_{t}(r)dt.$$



#### Thermodynamic variables

Temperature : 
$$T = \frac{U'(r_h)}{4\pi}$$
  
Entropy density :  $s = 4\pi r_h^2$   
Charge density :  $Q = (1 + \gamma_2 \phi^2) F^{rt}$   
Chemical poential :  $\mu = A_t(\infty)$ 

Boundary behavior of scalar field

$$\phi(r)|_{r \to \infty} \sim \phi_{\infty} + \frac{\langle \mathcal{O} \rangle}{r} + \cdots$$







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Boundary behavior of scalar field

$$\phi(r)|_{r \to \infty} \sim \phi_{\infty} + \frac{\langle \mathcal{O} \rangle}{r} + \cdots$$

Spontaneous symmetry breaking



#### ■ Equations of motion

$$R_{MN} - \frac{1}{2}g_{MN}\mathcal{L} - \frac{1}{2}\partial_M\phi\partial_N\phi - \frac{1}{2}\partial_M\chi\partial_N\chi - \frac{1}{2}\left(1 + \gamma_2\phi^2\right)F_{MP}F_M^P = 0$$
  

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$$ds^{2} = -U(r)e^{2W(r)-2W(\infty)}dt^{2} + \frac{dr^{2}}{U(r)} + r^{2}(dx^{2} + dy^{2})$$
  
$$\chi^{I} = (\beta x, \beta y), \quad \phi = \phi(r), \quad A = A_{t}(r)dt.$$

#### ■ Conserved charge

$$\mathcal{Q} \equiv \frac{\sqrt{-g}}{L} (1 + \gamma_2 \phi^2) F^{tr}$$



# Conserved charge

■ Independent equations

$$w' - \frac{1}{4}r\varphi'^{2} = 0$$
  
$$\varphi'' + \left(\frac{1}{r} - \frac{2\beta^{2}r^{2} + \frac{Q^{2}}{\gamma_{2}\varphi^{2}+1} - 2(\varphi^{2}+6)r^{4}}{4Ur^{3}}\right)\varphi' + \frac{\gamma_{2}Q^{2}\varphi}{Ur^{4}(\gamma_{2}\varphi^{2}+1)^{2}} + \frac{2\varphi}{U} = 0$$
  
$$U' + \frac{1}{4}Ur\varphi'^{2} + \frac{\beta^{2} + 2U - (\varphi^{2}+6)r^{2}}{2r} + \frac{Q^{2}}{4r^{3}(\gamma_{2}\varphi^{2}+1)} = 0,$$

$$U'(1) = \frac{6 + \varphi(1)^2 - \beta^2}{2} - \frac{Q^2}{4(1 + \gamma_2 \varphi(1)^2)} \qquad \varphi'(1) = \frac{4\varphi(1)(\gamma_2 Q^2 + 2(\gamma_2 \varphi(1)^2 + 1)^2)}{(\gamma_2 \varphi(1)^2 + 1)(Q^2 + 2(\beta^2 - \varphi(1)^2 - 6)(\gamma_2 \varphi(1)^2 + 1))}.$$

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# **Background Geometry**

#### Equations of motion

$$R_{MN} - \frac{1}{2}g_{MN}\mathcal{L} - \frac{1}{2}\partial_M\phi\partial_N\phi - \frac{1}{2}\partial_M\chi\partial_N\chi - \frac{1}{2}\left(1 + \gamma_2\phi^2\right)F_{MP}F_M^P = 0$$
  

$$\nabla^2\phi - \left(m^2 + \frac{1}{2}\gamma_2F^2\right)\phi = 0$$
  

$$\nabla_M\left(1 + \gamma_2\phi^2\right)F^{MN} = 0,$$

$$ds^{2} = -U(r)e^{2W(r)-2W(\infty)}dt^{2} + \frac{dr^{2}}{U(r)} + r^{2}(dx^{2} + dy^{2})$$
  
$$\chi^{I} = (\beta x, \beta y), \quad \phi = \phi(r), \quad A = A_{t}(r)dt.$$

$$\mathcal{Q} \equiv \frac{\sqrt{-g}}{L} (1 + \gamma_2 \phi^2) F^{tr}$$

#### ■ Background Solution

- Without scalar condensation( $\phi = 0$ ): RN-AdS black hole
- With a scalar condensation ( $\phi \neq 0$ ): Hairy black hole



Source free condition



#### Background Solution

- Without scalar condensation( $\phi = 0$ ): RN-AdS black hole
- With a scalar condensation ( $\phi \neq 0$ ): Hairy black hole



Source free condition

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- Impurity effect on the scalar condensation
  - Impurity enhances scalar condensation through gravity(electron interaction)



$$S_{tot} = S_0 + S_{int} + S_{bd},$$

$$S_0 = \int d^4 x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{2} (\partial \chi)^2 - \frac{1}{4} F^2 - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} m^2 \phi^2 \right)$$

$$S_{int} = -\int \sqrt{-g} \frac{\gamma_2}{4} \phi^2 F^2.$$

$$R_{MN} - \frac{1}{2} \partial_M \phi \partial_N \phi - \frac{1}{2} \partial_M \chi \partial_N \chi - \frac{1}{2} (1 + \gamma_2 \phi^2) F_{MP} F_M^P = 0$$

$$\nabla^2 \phi - \left( m^2 + \frac{1}{2} \gamma_2 F^2 \right) \phi = 0$$

$$\nabla_M (1 + \gamma_2 \phi^2) F^{MN} = 0,$$











 $\blacksquare \gamma_2$  dependence of Quantum critical point







### KMU WYOOJ WEIGETU

#### ■ Quantum phase transition



$$\begin{split} S_{tot} &= S_0 + S_{int} + S_{bd}, \\ S_0 &= \int d^4 x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{2} \left( \partial \chi \right)^2 - \frac{1}{4} F^2 - \frac{1}{2} \left( \partial \phi \right)^2 - \frac{1}{2} m^2 \phi^2 \right) \\ S_{int} &= -\int \sqrt{-g} \frac{\gamma_2}{4} \phi^2 F^2. \end{split}$$

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■ Fluctuation around background solution

$$\delta G_{ti} = -t U(r)\zeta_i + \delta g_{ti}(r)$$
  

$$\delta G_{ri} = r^2 \delta g_{ri}$$
  

$$\delta A_i = t(-E_i + \zeta_i a(r)) + \delta a_i(r)$$
  

$$\delta A_i(r) \sim -E_i t + \frac{J^i}{r} + \cdots$$

■ Regularity condition on the horizon:

$$\delta g_{ti}(r) \sim \delta g_{ti}^0 + \cdots, \qquad \delta h_{ti}(r) \sim \frac{1}{r^2 U(r)} \delta g_{ti}^0 + \cdots,$$
  
$$\delta A_i(r) \sim -\frac{E_i}{4\pi T} \log(r - r_h) + \cdots, \qquad \delta \chi^{\mathcal{I}} \sim \delta \chi_0^{\mathcal{I}} + \cdots.$$



■ Fluctuation around background solution

$$\delta G_{ti} = -t U(r)\zeta_i + \delta g_{ti}(r)$$
  

$$\delta G_{ri} = r^2 \delta g_{ri}$$
  

$$\delta A_i = t(-E_i + \zeta_i a(r)) + \delta a_i(r)$$
  

$$\delta A_i(r) \sim -E_i t + \frac{J^i}{r} + \cdots$$

■ Boundary current

$$\mathcal{J}^{i} = \sqrt{-g}(1+\gamma_{2}\phi^{2})F^{ir}$$
  
=  $-U(r)(1+\gamma_{2}\phi(r)^{2})\delta a_{i}'(r) - a_{t}'(r)\delta g_{ti}(r) \longrightarrow \sim (1+\gamma_{2}\phi(\infty))J^{i}$ 

■ Fluctuation equation + Regularity condition on the horizon: DC conductivity

$$\sigma_{DC} = \left(1 + \gamma_2 \phi_h^2\right) + \frac{Q^2}{r_h^2 \beta^2}$$

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Two contributions to DC conductivity

$$\sigma_{DC} = \left(1 + \gamma_2 \phi_h^2\right) + \frac{Q^2}{r_h^2 \beta^2}$$
$$= \sigma_{ccs} + \sigma_{diss}$$

2015: Blake, Donos



Two contributions to DC conductivity





2015: Blake, Donos

Two contributions to DC conductivity





■ DC conductivity without charge carrier(Q = 0)

o Temperature dependence of DC conductivity





 $\sigma_{DC} = \left(1 + \gamma_2 \phi_h^2\right) + \frac{Q^2}{r_h^2 \beta^2}$  $= \sigma_{ccs} + \sigma_{diss}$ 



■ DC conductivity without charge carrier(Q = 0)

o Temperature dependence of DC conductivity





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![](_page_36_Picture_7.jpeg)

■ DC conductivity with charge carrier( $Q \neq 0$ )

$$\sigma_{DC} = (1 + \gamma_2 \varphi_h^2) + \frac{e^{W(\infty)} Q^2}{r_h^2 \beta^2} \quad \text{for hairy BH} \\ \sigma_{DC} = 1 + \frac{Q^2}{r_h^2 \beta^2} \quad \text{for RN AdS BH.}$$

![](_page_37_Figure_3.jpeg)

![](_page_37_Picture_6.jpeg)

■ DC conductivity with charge carrier( $Q \neq 0$ )

$$\sigma_{DC} = (1 + \gamma_2 \varphi_h^2) + \frac{e^{W(\infty)} Q^2}{r_h^2 \beta^2} \quad \text{for hairy BH} \\ \sigma_{DC} = 1 + \frac{Q^2}{r_h^2 \beta^2} \quad \text{for RN AdS BH.}$$

![](_page_38_Figure_3.jpeg)

![](_page_38_Picture_5.jpeg)

![](_page_38_Picture_6.jpeg)

■ DC conductivity with charge carrier( $Q \neq 0$ )

$$\sigma_{DC} = (1 + \gamma_2 \varphi_h^2) + \frac{e^{W(\infty)} Q^2}{r_h^2 \beta^2} \quad \text{for hairy BH} \\ \sigma_{DC} = 1 + \frac{Q^2}{r_h^2 \beta^2} \quad \text{for RN AdS BH.}$$

![](_page_39_Figure_3.jpeg)

![](_page_39_Picture_6.jpeg)

### ■ Phase diagram

![](_page_40_Figure_2.jpeg)

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![](_page_40_Picture_5.jpeg)

![](_page_41_Figure_2.jpeg)

![](_page_41_Figure_3.jpeg)

![](_page_41_Picture_5.jpeg)

### ■ Phase diagram

![](_page_42_Figure_2.jpeg)

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![](_page_42_Picture_5.jpeg)

![](_page_43_Figure_2.jpeg)

$$\sigma_{DC} = \left(1 + \gamma_2 \phi_h^2\right) + \frac{Q^2}{r_h^2 \beta^2}$$
$$= \sigma_{ccs} + \sigma_{diss}$$

- o Drude like behavior
- o Resistivity is increasing to T
- o Metallic phase

![](_page_43_Picture_10.jpeg)

![](_page_44_Figure_2.jpeg)

$$\sigma_{DC} = \left(1 + \gamma_2 \phi_h^2\right) + \frac{Q^2}{r_h^2 \beta^2}$$
$$= \sigma_{ccs} + \sigma_{diss}$$

- o  $\sigma_{ccs}$  suppression dominant
- o Resistivity is decreasing to T
- Impurity induced insulating phase
- o 'Anderson insulator'

![](_page_44_Picture_10.jpeg)

![](_page_45_Figure_2.jpeg)

$$\sigma_{DC} = \left(1 + \gamma_2 \phi_h^2\right) + \frac{Q^2}{r_h^2 \beta^2}$$
$$= \sigma_{ccs} + \sigma_{diss}$$

- No black hole solution
- o Hawking-Page transition(geometric transition)
- o Solitonic(or singular) solution
- o Insulating phase
- o 'Mott insulator'

![](_page_45_Picture_11.jpeg)

### ■ Phase diagram

![](_page_46_Figure_2.jpeg)

• Change of scaling dimension of order parameter in IR region

![](_page_46_Picture_6.jpeg)

# **Discussion and Future direction**

![](_page_47_Picture_1.jpeg)

- We construct a gravity system with scalar-gauge field interaction
  - We find a phase transition between RN AdS black hole and a hairy black hole
  - We find quantum phase transition at zero temperature from density effect in IR
  - Impurity enhances scalar condensation: "Order parameter enhanced by disorder"
  - Scalar condensation leads to the insulating phase
  - The insulating phase comes from the localization of electron-hole pair creation
  - We realize 'Anderson insulator'-metal transition in holography
- Physical meaning of the order parameter : AFM order?
- Calculation of AC conductivity: gap or pseudo-gap creating mechanism
- Couple to the complex scalar field: Superconducting dorm?

![](_page_48_Picture_0.jpeg)

# Thank you !!