

HOLOGRAPHY FLAT^{IN} SPACE

Hanoi: Conference on Holography,
String Theory & Discrete Approach
Aug 6, 2020

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Harvard

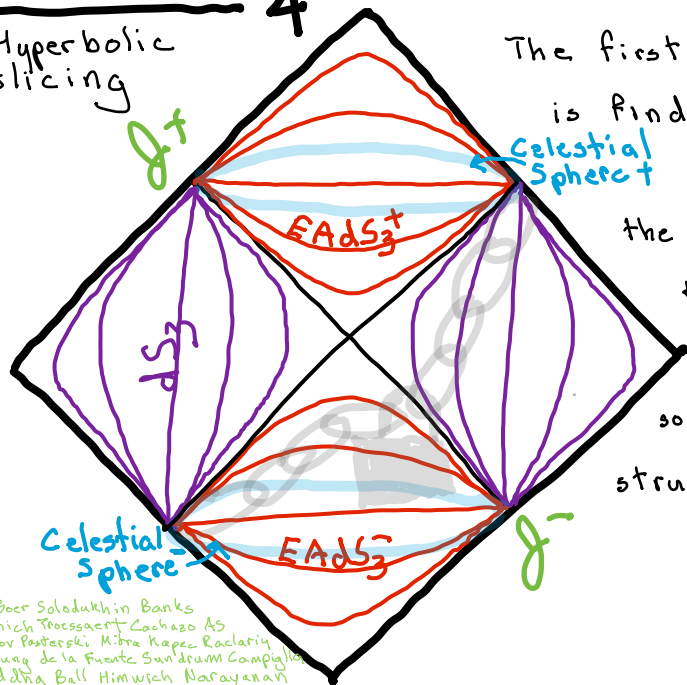
The black hole area-entropy law

$$S_{\text{BH}} = \frac{\text{Area (horizon)}}{4 G_N \hbar}$$

is the primeval holographic relationship: $N_{\text{dof}} \propto \partial V$
Understanding it for BHs with near-horizon AdS regions went hand-in-hand with understanding AdS/CFT holography. One might guess that understanding black holes in flat space - where the information paradox takes its most striking form - will go hand-in-hand with understanding holography for quantum gravity in asymptotically flat spacetimes.

Minkowski 4

Hyperbolic
slicing



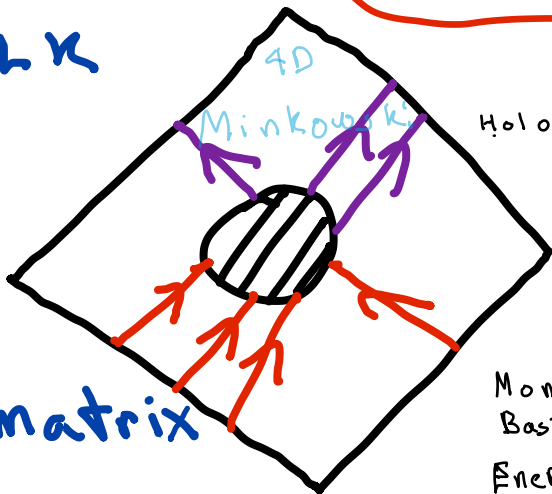
de Boer, Solodukhin, Banks,
Barnich, Troessaert, Cachazo, As
Lysor, Pasterski, Mitra, Nappi, Raclariu,
Cheung, de la Fuente, Sundrum, Campiglia,
hadadna, Ball, Himwich, Narayanan
...

The first step in finding a holographically dual pair is finding the symmetries both sides obey. Thanks to the subleading soft graviton ^{talks} ^{P. Mao, G. Compere} ~~thm~~, we now know the ∞ -dimensional 2D conformal symmetry is uplifted to the celestial sphere. But there are ∞ -ly many more from supertranslations, subsubleading soft gravitons, subleading soft gluons... a rich & beautiful structure to explore.

At crossroad of GR/QFT.

Basic Idea

BULK

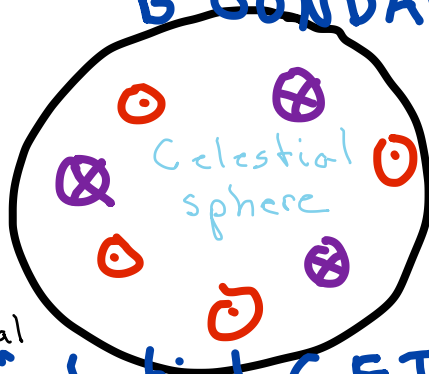


S-matrix

Holographic Duality

=

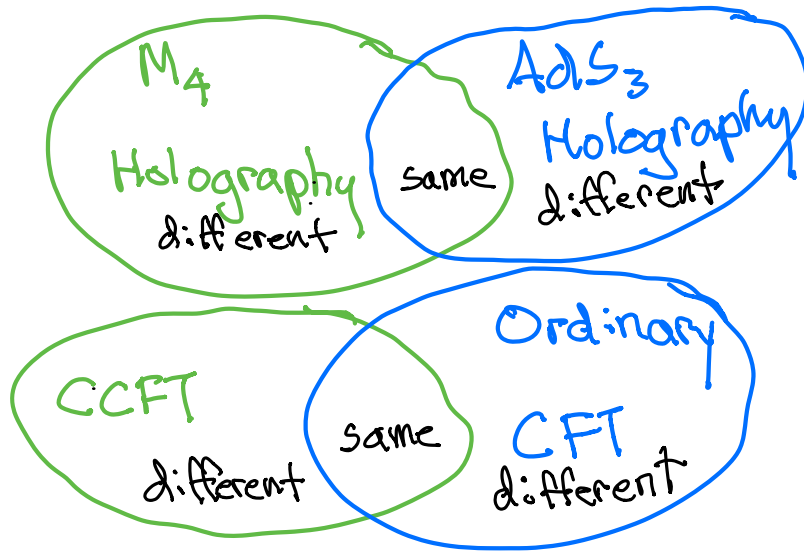
BOUNDARY



Momentum Basis $\Leftrightarrow$ Conformal Basis
Energy $\Leftrightarrow$ Boost weight

Celestial CFT correlator

$SL(2, \mathbb{C})$ Lorentz = global 2D conformal
Superrotations = local 2D conformal
Supertranslations $\mathcal{O}_\Delta(z, \bar{z}) \rightarrow \mathcal{O}_{\Delta+1}(z, \bar{z})$!
... more ...



SO WHAT ARE THE PROPERTIES OF A CCFT?



So far, mostly BOTTOM $\rightarrow$ UP. LHS defines RHS.
c.f. Brown-Henneaux.

TOP $\rightarrow$ DOWN approach from string theory
not ruled out.

String WS CFT $\xrightarrow{\Delta \rightarrow \infty}$ CCFT stieberger
Taylor

CHY WS CFT $\xrightarrow{?}$ CCFT

Intrinsic construction of a CCFT = Major Breakthrough!

Many string-like structures encountered.
Strings without strings?

CCFT

Two Virasoros
Current Algebras
Collinear Singularities

Continuous Complex
Highest Weight Reps
Single valued

$$S^\dagger S = 1$$

Spacetime crossing
 ∞ -dim supertranslation
& subleading soft
symmetry

New!

USUAL CFT

Two Virasoros
Current Algebras
OPE Singularities
Discrete Highest
Weight Reps

Locality



Diagram illustrating a crossing of two lines, labeled $\frac{2}{n}$, followed by an equals sign and a circle containing three question marks, suggesting a non-local or singular behavior.

No analog

Old!

A lot to understand

A hand-drawn illustration of a stick figure with a speech bubble saying "Wow what is this???" surrounded by various celestial bodies like stars, planets, and moons, with the text "COSMOS" written in large, stylized letters in the background.



For massless particles, the map between momentum-space and conformal basis amplitudes is a Mellin transform

$$A(\lambda_1, z_1, \dots, \lambda_n, z_n) \equiv \int d\omega_1 \dots d\omega_n \omega_1^{h_1 + \bar{h}_1 - 1} \dots \omega_n^{h_n + \bar{h}_n - 1} A(p_1, \dots, p_n) \\ = \langle \mathcal{O}_1(z_1) \dots \mathcal{O}_n(z_n) \rangle_{\text{CCFT}}$$

$$h = \frac{1}{2}(1 + s + i\lambda)$$

$$\bar{h} = \frac{1}{2}(1 - s + i\lambda)$$

$$z_k = \frac{p_k^1 + i p_k^2}{p_k^3 + p_k^0}$$

$\lambda \in \mathbb{R}$ (subtleties range, $\lambda=0$)
 || Donnay, Buhm, AS
 Pasternski & Shao

complete basis of states

Full CCFT operator content is known! $\Leftrightarrow$
 Spectrum of stable particles.

Conformal basis for massive particles involves bulk-to-boundary propagator on H_3 .

$A(\lambda_1, z_1, \dots, \lambda_n, z_n)$ has been computed in a number of examples, sometimes via AdS-Witten diagrams:

① Scalars: massive, massless, 1-loop

Pasterski Shao AS Huang Cordona Lavin

② All tree-level n -point YM

Pasterski Shao AS Cheung de la Fuente Sundrum Schreiber Volovich Zlotnik

Kinematic singularities @ 3 & 4 pt, 5+ are smooth.

MARGINALLY CONVERGENT

DEFINES

Good UV behavior
"Baked-in".

③ All n -point Einstein gravity
 ∞ !!!

④ String theory up to 4pt gauge + gravity

CONVERGENT

Stieberger Taylor

CONFORMALLY SOFT THEOREM

A conformally soft operator is one for which

$$(h, \bar{h}) = (n, 0) \quad (\text{or } (0, n))$$

It is not energetically soft! Nevertheless it has been recently shown in gauge theory

$$\lim_{\lambda_n \rightarrow 0} A(\lambda_1 z_1, \dots, \lambda_n z_n) = -\frac{i}{\lambda_n} \sum_k \frac{1}{z_n - z_k} A(\lambda_1 z_1, \dots, \lambda_k z_k, \dots, \lambda_{n-1} z_{n-1})$$

Fan Fofopoulos Taylor, Puhm
Pete Raclariu AS

using UV convergence & usual soft thm.

Expected in conformal picture, surprising in momentum space picture. Subleading soft graviton $\Rightarrow$

$(2, 0)$ celestial stress tensor T_{ww} is a further example

SUPERTRANSLATIONS are more subtle. For gravity:

$$(h, \bar{h}) = \left(\frac{3}{2} + \frac{i\lambda}{2}, -\frac{1}{2} + \frac{i\lambda}{2} \right)$$

for $\lambda \rightarrow 0$

$$(h, \bar{h}) = \left(\frac{3}{2}, -\frac{1}{2} \right)$$

The associated operator P_z obeys

$$P_z(z) \mathcal{O}_{h, \bar{h}}(w) \sim \frac{1}{z-w} \mathcal{O}_{h+\frac{1}{2}, \bar{h}+\frac{1}{2}} !$$

∞ constraints on amplitudes/OPEs
correlators. Even global translations
nontrivial. Illustration...

YM + + + OPE from asymptotic symmetries

M. Pate
A. Raclariu
A.S.
F. Yuan
Fan Fotopoulos
Stieberger Taylor

collinear singularity

$$\sigma_{h_1}^{+a}(z_1) \sigma_{h_2}^{+b}(z_2) \sim \frac{B(h_1, h_2)}{z_1 - z_2} f^{abc} \sigma_{h_1+h_2-1}^{+c}(z_2)$$

Global translation invariance requires

$$B(h_1, h_2) = B(h_1 + \frac{1}{2}, h_2) + B(h_1, h_2 + \frac{1}{2})$$

Subleading soft symmetry requires ($h_1 = 0$ pole)

$$(2h_1 - 3) B(h_1 - \frac{1}{2}, h_2) = (2h_2 - 3) B(h_1, h_2 - \frac{1}{2})$$

These two recursion relations, together with $h \rightarrow \infty$ falloff, imply the OPE coefficient

$$B(h_1, h_2) = B(2h_1 - 2, 2h_2 - 2) \quad B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

in agreement w/ direct Mellin transform of collinear singularities. Fan Fotopoulos Taylor

In fact, all the

The mixed helicity gluon OPEs are

$$O_{\Delta_1}^{+a,\epsilon}(z_1, \bar{z}_1) O_{\Delta_2}^{-b,\epsilon}(z_2, \bar{z}_2) \sim \frac{-i f^{ab}}{z_{12}} \epsilon B(\Delta_1 - 1, \Delta_2 + 1) O_{\Delta_1 + \Delta_2 - 1}^{-c,\epsilon}(z_2, \bar{z}_2) \\ + \frac{\kappa \bar{z}_{12}}{2 z_{12}} \delta^{ab} B(\Delta_1, \Delta_2 + 2) G_{\Delta_1 + \Delta_2}^{-,\epsilon}(z_2, \bar{z}_2), \\ O_{\Delta_1}^{+a,\epsilon}(z_1, \bar{z}_1) O_{\Delta_2}^{-b,-\epsilon}(z_2, \bar{z}_2) \sim \frac{-i f^{ab}}{z_{12}} \delta [-B(\Delta_2 + 1, 1 - \Delta_1 - \Delta_2) O_{\Delta_1 + \Delta_2 - 1}^{-c,\epsilon}(z_2, \bar{z}_2) \\ + B(\Delta_1 - 1, 1 - \Delta_1 - \Delta_2) O_{\Delta_1 + \Delta_2 - 1}^{-c,-\epsilon}(z_2, \bar{z}_2)] \\ + \frac{\kappa \bar{z}_{12}}{2 z_{12}} \delta^{ab} [B(\Delta_2 + 2, -1 - \Delta_1 - \Delta_2) G_{\Delta_1 + \Delta_2}^{-,\epsilon}(z_2, \bar{z}_2) \\ + B(\Delta_1, -1 - \Delta_1 - \Delta_2) G_{\Delta_1 + \Delta_2}^{-,-\epsilon}(z_2, \bar{z}_2)]. \quad (106)$$

The graviton OPEs are

$$G_{\Delta_1}^{+,\epsilon}(z_1, \bar{z}_1) G_{\Delta_2}^{\pm,\epsilon}(z_2, \bar{z}_2) \sim -\frac{\kappa \bar{z}_{12}}{2 z_{12}} B(\Delta_1 - 1, \Delta_2 + 1 \mp 1) G_{\Delta_1 + \Delta_2}^{\pm,\epsilon}(z_2, \bar{z}_2), \\ G_{\Delta_1}^{+,\epsilon}(z_1, \bar{z}_1) G_{\Delta_2}^{\pm,-\epsilon}(z_2, \bar{z}_2) \sim \frac{\kappa \bar{z}_{12}}{2 z_{12}} [B(\Delta_2 + 1 \mp 2, 1 \pm 2 - \Delta_1 - \Delta_2) G_{\Delta_1 + \Delta_2}^{\pm,\epsilon}(z_2, \bar{z}_2) \\ + B(\Delta_1 - 1, 1 \pm 2 - \Delta_1 - \Delta_2) G_{\Delta_1 + \Delta_2}^{\pm,-\epsilon}(z_2, \bar{z}_2)]. \quad (107)$$

The gluon-graviton OPEs are

$$G_{\Delta_1}^{+,\epsilon}(z_1, \bar{z}_1) O_{\Delta_2}^{\pm a,\epsilon}(z_2, \bar{z}_2) \sim -\frac{\kappa \bar{z}_{12}}{2 z_{12}} B(\Delta_1 - 1, \Delta_2 + 1 \mp 1) O_{\Delta_1 + \Delta_2}^{\pm a,\epsilon}(z_2, \bar{z}_2), \\ G_{\Delta_1}^{+,\epsilon}(z_1, \bar{z}_1) O_{\Delta_2}^{\pm a,-\epsilon}(z_2, \bar{z}_2) \sim -\frac{\kappa \bar{z}_{12}}{2 z_{12}} [B(\Delta_2 + 1 \mp 1, 1 \pm 1 - \Delta_1 - \Delta_2) O_{\Delta_1 + \Delta_2}^{\pm a,\epsilon}(z_2, \bar{z}_2) \\ - B(\Delta_1 - 1, 1 \pm 1 - \Delta_1 - \Delta_2) O_{\Delta_1 + \Delta_2}^{\pm a,-\epsilon}(z_2, \bar{z}_2)]. \quad (108)$$

From (8), we recall a factor of g_{YM} is absorbed in f_c^{ab} . The $\bar{z}_{12} \rightarrow 0$ celestial OPEs are obtained in a similar way by imposing the δ symmetry instead.

The presence of higher-dimension operators due to quantum, stringy or other corrections is expected to augment this list with the finite number of additions allowed by the general formula (13). A list of all possible corrections in theories with only gluons and gravitons is given in appendix B.

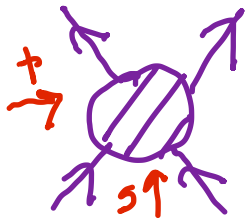
Acknowledgements

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Analyticity, Factorization & IR Safety on the Celestial Sphere

Arkani-Hamed, Pate, Raclariu & AS, to appear

Start w/ tree 4-pt:



$$A(s, t) \xrightarrow{s \rightarrow \infty} e^{-s B_M} \sim e^{-s} \quad \text{exponentially}$$

$$\underset{s \rightarrow 0}{=} \sum_P s^P a_P(z); z = \frac{t}{s} \quad \text{soft due to BMs}$$

= conformal cross ratio

2.4f, EFT hedron coefficients
Arkani-Hamed, Huang & Huang

$$\tilde{A}(\Delta, z) = \int ds s^{\Delta-3} A(s, z) \sim \sum_P \frac{a_P(z)}{\Delta - 4 + 2P} + \text{finite}$$

Celestial tree amplitudes are meromorphic in Δ with poles at even negative integers! Residues = coefficients of 2.4f/EFT hedron with rich positivity constraints.

Quantum Theory

- i) UV behavior still soft, divergences in $\tilde{A}(\Delta, z)$ all from IR - still meromorphic
- ii) IR divergences w/ logs in exclusive $A(s, t) \sim \frac{s^2}{t} \ln \frac{s}{\Lambda_{IR}} + \dots$ ↗ regulator required
- iii) logs controlled by soft theorem. For QED/photons:

$$A(p_i, Q) = e^{-\Gamma \sum_{i,j} Q_i Q_j \ln(p_i \cdot p_j)} A_{\text{hard}}; \quad \Gamma = \frac{e^2}{4\pi} \ln \frac{\Lambda_{UV}}{\Lambda_{IR}} \quad \text{cusp anomalous dimension}$$

- iv) Transforming to celestial sphere

$$\tilde{A}(z_i, \Delta_i) = \tilde{A}_{\text{soft}} + \tilde{A}_{\text{hard}}(z_i, \Delta_i - \Gamma Q_i^2); \quad \tilde{A}_{\text{soft}} = \prod_{i,j} |z_i - z_j|^{-\Gamma Q_i Q_j}$$

↙ shift in dimension

reveals exact current algebra factorization

- v) $\tilde{A}_{\text{hard}}$ are nothing but the IR safe scattering amplitudes w/ unique conformal primary Fadeev-Kulish dressings!!

SIM. FOR GRAVITY, WITH $Q_i \rightarrow P_i$, $\tilde{A}_{\text{soft}} \rightarrow \text{operator!}$

Finally we have an S-matrix!

Needed for holography, BHs

Arkani-Hamed, Pate, Raclariu AS; Himwich, Narayanan, Nisarga, Pate, AS

MORE
COOL STUFF!

A QUESTION

What is the most general form of the graviton 4pt amplitude consistent with

1. Conformal invariance
 2. Supertranslation invariance
 3. Causality
 4. Crossing symmetry
 5. Subleading soft symmetries
 6.
- 

Does it look like string scattering?

CONCLUSION

The reformulation of Minkowski scattering as correlators on the celestial sphere is leading to novel insights. There is much to understand.

Thank you
for listening!

