Fluxes of Scalar Field in Scattering to Rotating AdS Black Holes

Based on:
B. Gwak, **JCAP 10 (2021) 012** [arXiv:2105.07226 [gr-qc]] and B. Gwak, **JCAP 10 (2022) 077** [arXiv:2207.13822 [gr-qc]].
H. Han and B. Gwak, **JHEP 08 (2023) 102**, [arXiv:2306.10288 [gr-qc]].
J. Ko and B. Gwak, **JHEP 03 (2024) 072**, [arXiv:2312.17014 [gr-qc]].

Bogeun Gwak
Department of Physics, Dongguk University, Seoul Campus, Republic of Korea

2024.08.23. THE 7TH INTERNATIONAL CONFERENCE ON HOLOGRAPHY AND STRING THEORY IN DA NANG

Black Holes in GR

- In general relativity, the spacetime geometry is associated with energy and momentum.
- Einstein's equation:

Energy-momentum tensor

Einstein tensor $$G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$$

Cosmological constant **metric**

- The first and simplest black hole: Schwarzschild black hole (1916)

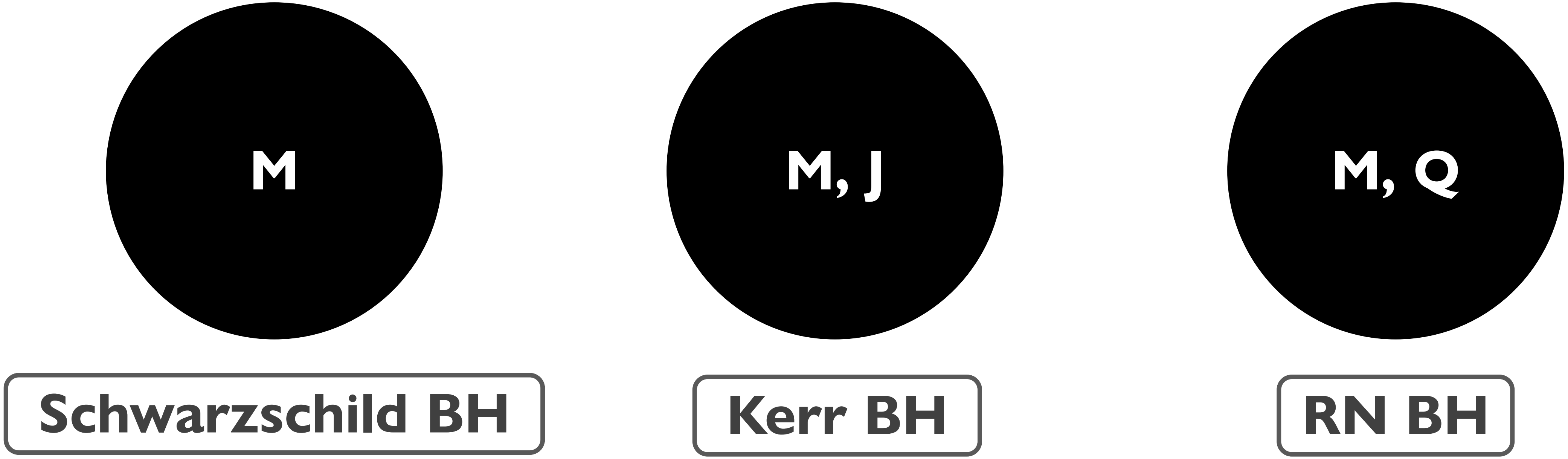
$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

- Kerr black hole and Reissner-Nordström (RN) black hole

Rotation **Electric charge**

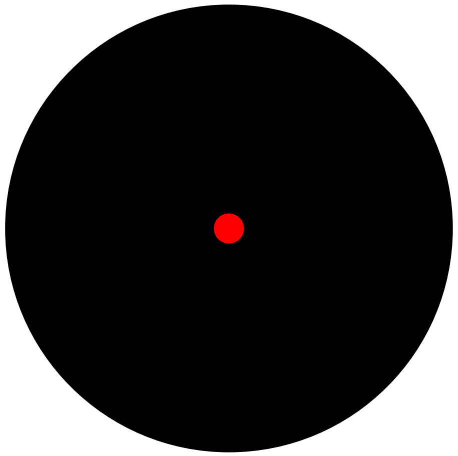
Black Holes with Observer

- There is a surface where no matter can escape from the black hole.
- No light from inside: **BLACK HOLE** to the observer.
- The surface is called **EVENT HORIZON.**
- The **CURVATURE SINGULARITY** is at their center.

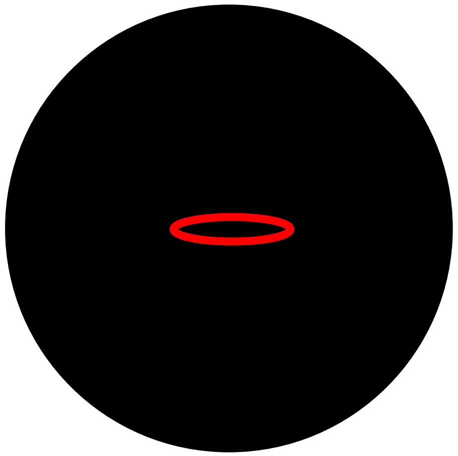


Black Holes with Observer

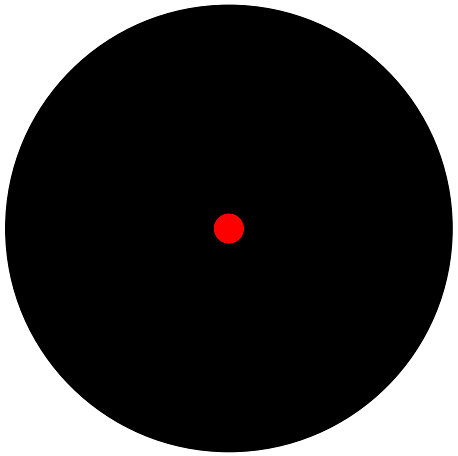
- There is a surface where no matter can escape from the black hole.
- No light from inside: **BLACK HOLE** to the observer.
- The surface is called **EVENT HORIZON.**
- The **CURVATURE SINGULARITY** is at their center.



Schwarzschild BH



Kerr BH



RN BH

Geodesics in Spacetime

- Their spacetimes are distinguishable from metric:

Schwarzschild BH

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

Kerr BH

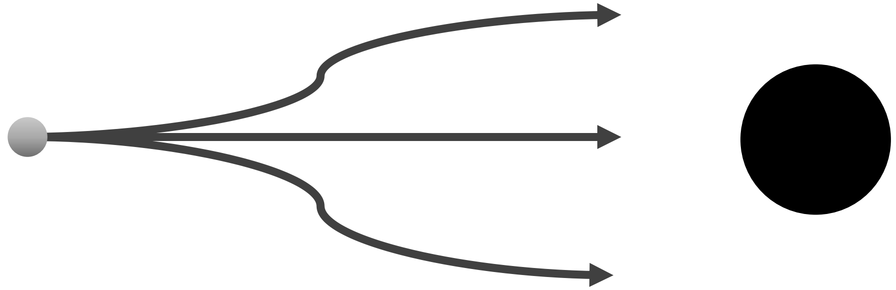
$$ds^2 = -\frac{\Delta_r}{\rho^2}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\rho^2}{\Delta^r}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left(adt - (r^2 + a^2)d\phi\right)^2$$

RN BH

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

- These differences can be observed in geodesics of a test particle.

Motions of a particle



Wave Scattering to Black Holes

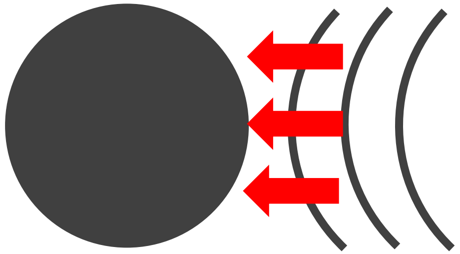
- Instead of a particle, considering a wave provides interesting phenomena different from cases of a particle.
- According to its spin, scalar, Dirac, gauge, and gravitational fields.
- Wave given by frequency, amplitude, and phase.

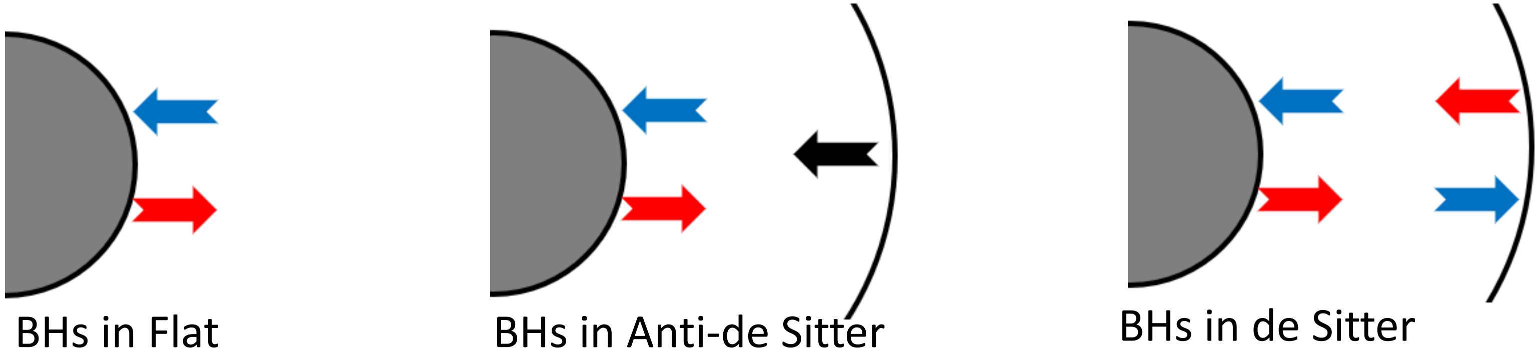
$$\Psi = \mathcal{I} e^{-i\omega(t+r)} + \mathcal{O} e^{-i\omega(t-r)}$$

- Observing the scattering of waves by a black hole.
- Penrose process by a particle corresponding to **SUPERRADIATION**.
- In scattering, the wave carries energy and momentum into BHs.
- The changes in a black hole: $(M_B, J_B, Q_B) \rightarrow (M_B + dM_B, J_B + dJ_B, Q_B + dQ_B)$

Superradiation

- Superradiation corresponds to Penrose process on a particle.
- External field can extract energy from a BH.
- By this interaction, BH energy is reduced (not increase).
- In flat ($\Lambda = 0$), extracted energy is dissipated to the infinity.
- However, AdS ($\Lambda < 0$) and dS ($\Lambda > 0$) provide different boundary conditions, so the behaviors are also different.
- AdS: amplification
  - BH bomb
- dS: complicated
  - cosmological horizon





Motivation

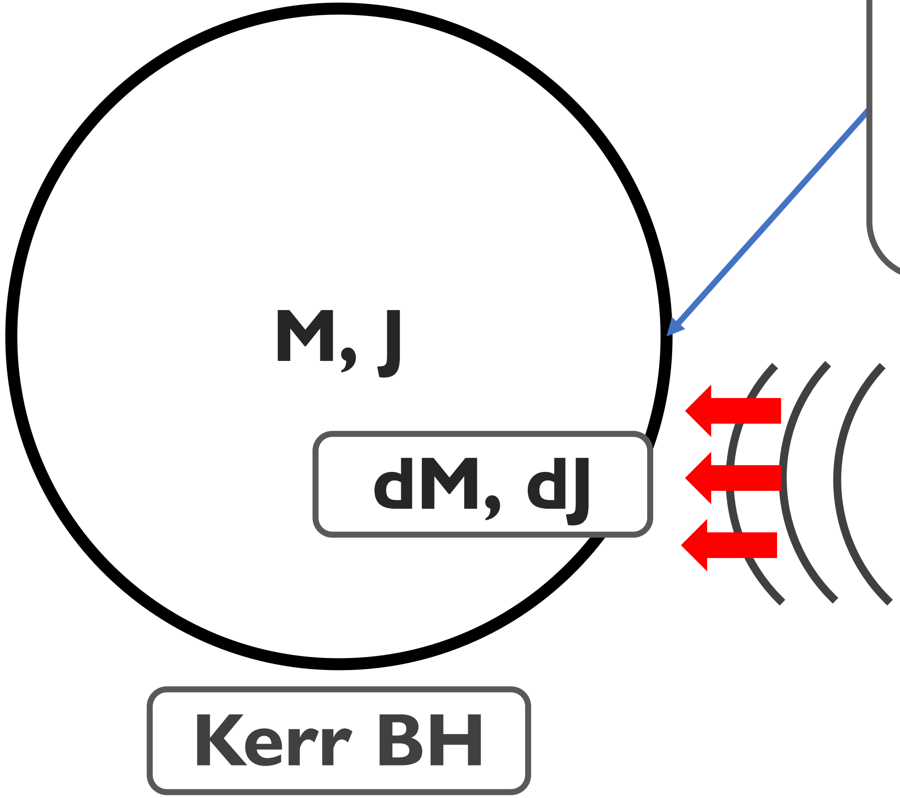
- Our general motivation: describing reactions of black hole when it is interacted with matters.
  - Focusing to BHs
  - BHs from gravity theories
    - GR and modified gravity theory
    - Considering interaction with matters
  - Matters from external particles or fields and their collisions to BH.
    - Scattering or geodesics
    - Features in spacetime geometries.
  - Reactions
    - Perturbation from matters : from scattering
    - BH structure stability under the interactions : event horizon, inner horizon, modification.
    - Explanation on changes in BH states

Black Hole and Scattering of Scalar Field

- Kerr-Newman-(Anti-)de Sitter Black Holes (KN(A)dS BHs)
  - Four-dimensional spacetime
  - Rotating and electrically charged black hole
  - Including negative, zero, and positive cosmological constant
- Scalar Field
  - Zero spin field
  - Electrically charged scalar field coupled with KN(A)dS BHs
- Scattering
  - External scalar field scattered by KN(A)dS BHs
  - Carried conserved charges from scalar field to KN(A)dS BHs
  - Changes in BHs during an infinitesimally short time interval
    - Changes in mass, angular momentum, and electric charge
    - The final state from a given initial state.

Simple View on Our Work

- Maybe, the previous slide is too long to explain shortly.
- Let see more simple structures on our work.



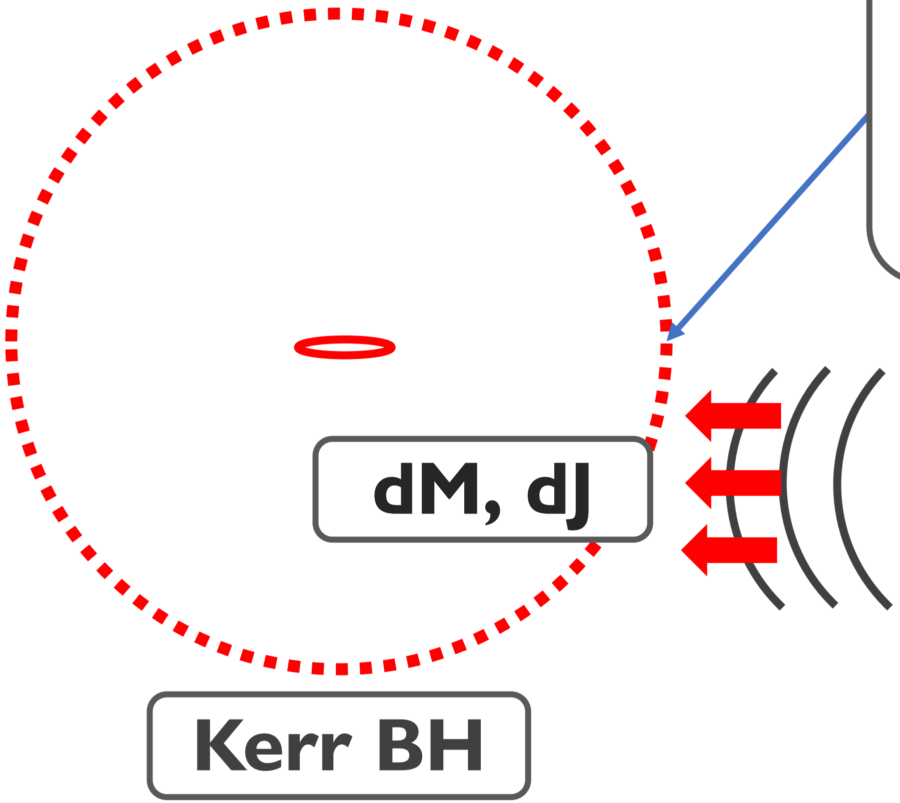
$$r_h = M + \sqrt{M^2 - \frac{J^2}{M^2}}$$

- If $(M + dM)^2 > \frac{(J+dJ)^2}{(M+dM)^2}$, it is still BH.
- If $(M + dM)^2 < \frac{(J+dJ)^2}{(M+dM)^2}$,

Simple View on Our Work

- Maybe, the previous slide is too long to explain shortly.
- Let see more simple structures on our work.

$$r_h = M + \sqrt{M^2 - \frac{J^2}{M^2}}$$

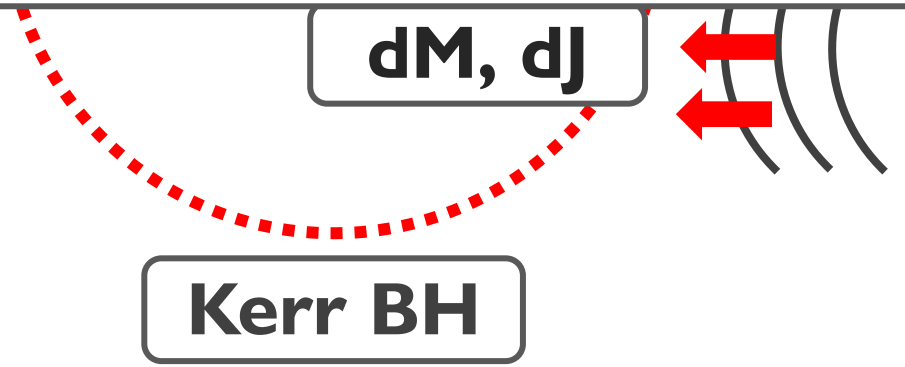


- If $(M + dM)^2 < \frac{(J+dJ)^2}{(M+dM)^2}$, it becomes a naked singularity.

Simple View on Our Work

- Maybe, the previous slide is too long to explain shortly.
- Let see more simple structures on our work.

Is it possible in considering a scalar field? (dM, dJ) constrained
Stability on the event horizon
The changes in BH by this scalar field
Can we see this singularity by this observer?:
Weak cosmic censorship conjecture



becomes a naked singularity

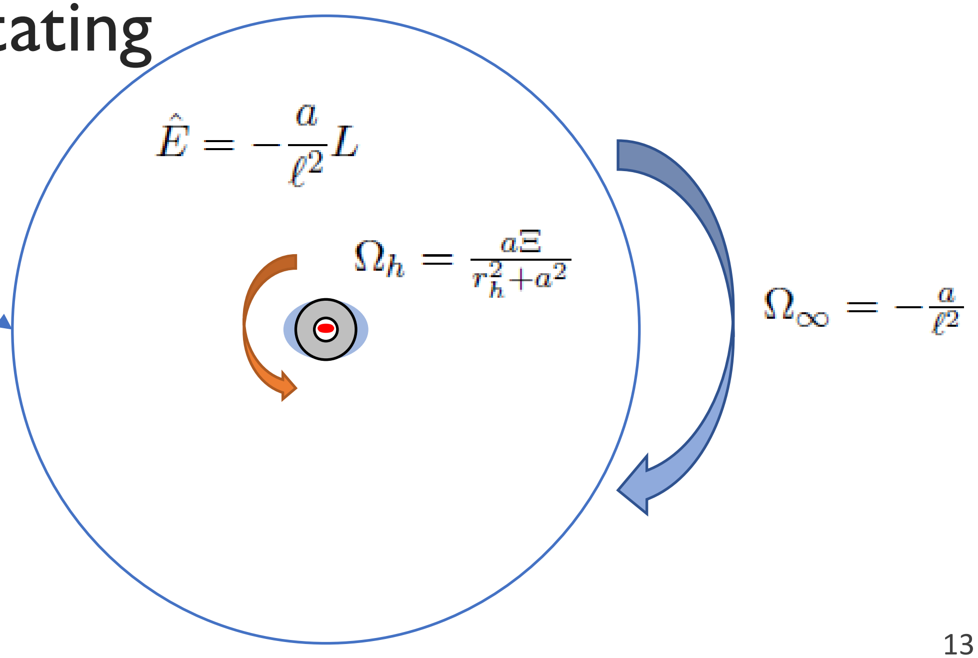
Metric: Kerr-Newman-(A)dS Black Holes

- KN(A)dS BHs in Boyer-Lindquist coordinates:

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right)^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta \sin^2\theta}{\rho^2}\left(a\,dt - \frac{r^2+a^2}{\Xi}d\phi\right)^2,$$

$$\Delta_r = (r^2+a^2)\left(1-\frac{1}{3}\Lambda r^2\right) - 2Mr + Q^2,\ \Delta_\theta = 1+\frac{1}{3}\Lambda a^2\cos^2\theta,\ \rho^2 = r^2 + a^2\cos^2\theta,\ \Xi = 1+\frac{1}{3}\Lambda a^2,$$

- In $r \gg 1$, the metric is not static: still rotating
- Asymptotic observer is rotating.
- Energy is <u>not well defined</u>.
- It needs correction.



Coordinate Transformations

- It is resolved by coordinate transformations:

Hawking, Hunter, Taylor (1999)

$$t \to T, \quad \phi \to \Phi + \frac{1}{3} a \Lambda T$$

- Then, the metric becomes asymptotically static:

$$ds^2 = -\frac{\Delta_r}{\rho^2 \Xi^2}\left(\Delta_\theta dT - a\sin^2\theta d\Phi\right)^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta \sin^2\theta}{\rho^2 \Xi^2}\left(a\left(1 - \frac{1}{3}\Lambda r^2\right)dT - (r^2 + a^2)d\Phi\right)^2.$$

BG (2021)

- The mass, angular momentum, and electric charge:

$$M_{\rm B} = \frac{M}{\Xi^2}, \quad J_{\rm B} = \frac{Ma}{\Xi^2}, \quad Q_{\rm B} = \frac{Q}{\Xi}$$

- Hawking temperature, entropy, angular velocity, and electric potential:

$$T_{\rm h} = \frac{r_{\rm h}\left(1 - \frac{\Lambda a^2}{3} - \frac{a^2 + Q^2}{r_{\rm h}^2} - \Lambda r_{\rm h}^2\right)}{4\pi (r_{\rm h}^2 + a^2)}, \quad S_{\rm h} = \frac{1}{4}A_{\rm h} = \frac{\pi (r_{\rm h}^2 + a^2)}{\Xi}, \quad \Omega_{\rm h} = \frac{a\left(1 - \frac{1}{3}\Lambda r_{\rm h}^2\right)}{r_{\rm h}^2 + a^2}, \quad \Phi_{\rm h} = \frac{r_{\rm h} Q}{r_{\rm h}^2 + a^2}$$

Scalar Field Equation

- Charged scalar field with a covariant derivative:

$$S_\Psi = -\frac{1}{2}\int d^4x\sqrt{-g}\left(\mathcal{D}_\mu\Psi\mathcal{D}^{*\mu}\Psi^* + \mu^2\Psi\Psi^*\right), \quad \mathcal{D}_\mu = \partial_\mu - iqA_\mu$$

- Field equations:

$$\frac{1}{\sqrt{-g}}\mathcal{D}_\mu\left(\sqrt{-g}g^{\mu\nu}\mathcal{D}_\nu\Psi\right) - \mu^2\Psi = 0, \quad \frac{1}{\sqrt{-g}}\mathcal{D}^*_\mu\left(\sqrt{-g}g^{\mu\nu}\mathcal{D}^*_\nu\Psi^*\right) - \mu^2\Psi^* = 0.$$

- Translation symmetries on (T, Φ).
- The solution will be a form:

$$\Psi(T, r, \theta, \Phi) = e^{-i\omega T}e^{im\Phi}R(r)\Theta(\theta)$$

- The scalar field equation is <u>separable</u> into radial and theta equations.

Radial Equation

- The separated radial equation:

$$\frac{1}{R(r)}\partial_r\left(\Delta_r\partial_r R(r)\right)+\frac{1}{\Delta_r}\left(\omega(r^2+a^2)-am\left(1-\frac{1}{3}\Lambda r^2\right)-qQr\right)^2-\mu^2r^2-\mathcal{K}=0.$$

- For all range, we need numerical calculation.
- However, focusing on carried conserved quantities: outer horizon
- Solving the radial equation at the outer horizon.
- With tortoise coordinate, the radial solution at the outer horizon:

$$\mathcal{R}(r^*)=\mathcal{T}e^{-i(\omega-m\Omega_{\rm h}-q\Phi_{\rm h})r^*},$$

- $\mathcal{T}$ is the transmission amplitude.
- The radial solution is only ingoing as a boundary condition.

θ-Directional Equation

- θ-directional equation is the generalized scalar hyper-spheroidal equation with a separate variable:

$$\frac{1}{\sin\theta\,\Theta(\theta)}\partial_\theta\left(\sin\theta\Delta_\theta\partial_\theta\Theta(\theta)\right) - \frac{1}{\Delta_\theta}\left(a\omega\sin\theta - m\Delta_\theta\csc\theta\right)^2 - a^2\mu^2\cos^2\theta + \mathcal{K} = 0,$$

- For $a = 0$ or $\Lambda = 0$, this is just spheroidal harmonics: $\mathcal{K} = \ell(\ell+1)$
- With s-spin field with $\Lambda = 0$, spin-weighted spheroidal harmonics
  Berti, Cardoso, Casals (2006)
- Generalized scalar hyper-spheroidal equation with higher dimension
  Cho, Cornell, Doukas, Naylor (2009)
- This is a known numerical solution and satisfies:

$$\int \Theta(\theta)\Theta^*(\theta)d\Omega_2 = 1.$$

- The details of $\Theta(\theta)$ is not important. It will be integrated into unity.

Carried Conserved Charges

- The scalar field solution at the outer horizon:

$$\Psi(T, r^*, \theta, \Phi) = \frac{\mathcal{T}}{\sqrt{r_{\rm h}^2 + a^2}} e^{-i\omega T} e^{-i(\omega - m\Omega_{\rm h} - q\Phi_{\rm h})r^*} \Theta(\theta) e^{im\Phi}$$

- The fluxes of the scalar field flowing into the horizon

$$dM_{\rm B} = \frac{4\pi|\mathcal{T}|^2}{\Xi}\omega(\omega - m\Omega_{\rm h} - q\Phi_{\rm h})dT,$$
$$dJ_{\rm B} = \frac{4\pi|\mathcal{T}|^2}{\Xi}m(\omega - m\Omega_{\rm h} - q\Phi_{\rm h})dT,$$
$$dQ_{\rm B} = \frac{4\pi|\mathcal{T}|^2}{\Xi}q(\omega - m\Omega_{\rm h} - q\Phi_{\rm h})dT.$$

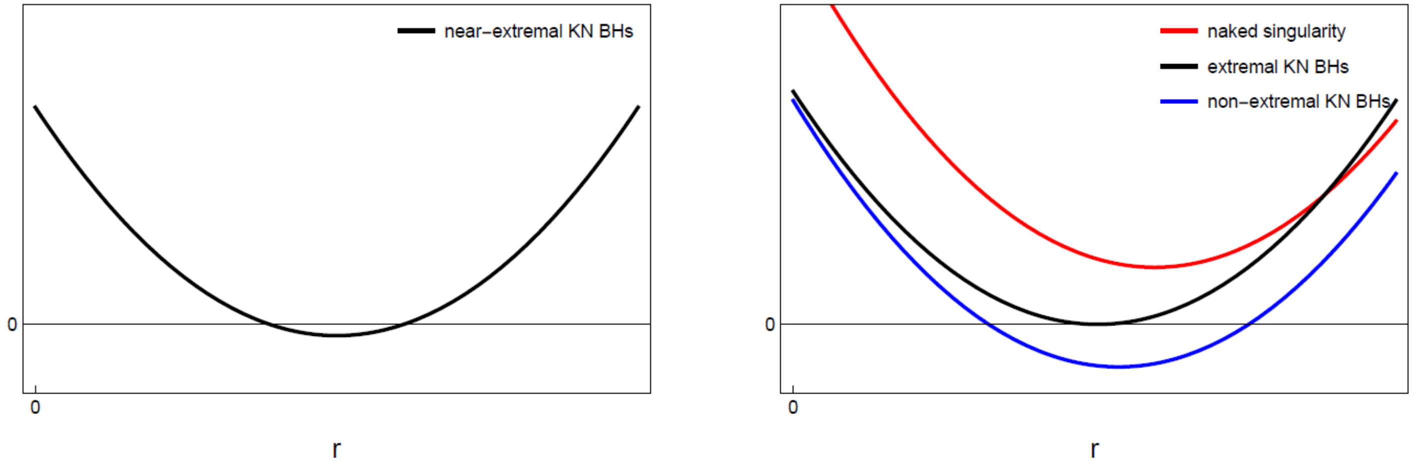
- The conserved charges should be preserved in the spacetime.

$$E, L, q \rightarrow M_{\rm B}, J_{\rm B}, Q_{\rm B}$$

- During a time interval dT, the changes in BH from fluxes.

Near-Extremal KN(A)dS Black Holes

- Under infinitesimal changes, BH into <u>BH</u> in the final state.
- Near-extremal (and extremal) one can be different: <u>naked singularity</u>
- By the changes: $(M_B, J_B, Q_B) \rightarrow (M_B + dM_B, J_B + dJ_B, Q_B + dQ_B)$
- Δ_r: determining locations of horizons: Minimum value is **IMPORTANT**!



(a) Δ_r with $(M_{\rm B}, J_{\rm B}, Q_{\rm B})$.

(b) Δ_r with $(M_{\rm B} + dM_{\rm B}, J_{\rm B} + dJ_{\rm B}, Q_{\rm B} + dQ_{\rm B})$.

- By over-spinning, near-extremal BH can be a naked singularity or not.

Change in Minimum Value

- Measuring minimum value in initial and final states

Minimum value	Near-extremal	Minimum location	Minimum condition

$$\Delta_{\rm min} \equiv \Delta_r\big|_{r=r_{\rm min}}, \quad -\Delta_{\rm min} \ll 1, \quad \frac{\partial \Delta_r}{\partial r}\bigg|_{r=r_{\rm min}} = \frac{\partial \Delta_{\rm min}}{\partial r_{\rm min}} = 0, \quad \frac{\partial^2 \Delta_r}{\partial r^2}\bigg|_{r=r_{\rm min}} = \frac{\partial^2 \Delta_{\rm min}}{\partial r_{\rm min}^2} > 0,$$

- The change in the minimum value is:

$$\begin{aligned} d\Delta_{\rm min} &\equiv \Delta_{\rm min}(M_{\rm B} + dM_{\rm B}, J_{\rm B} + dJ_{\rm B}, Q_{\rm B} + dQ_{\rm B}, r_{\rm min} + dr_{\rm min}) - \Delta_{\rm min}(M_{\rm B}, J_{\rm B}, Q_{\rm B}, r_{\rm min}) \\ &= \frac{\partial \Delta_{\rm min}}{\partial M_{\rm B}} dM_B + \frac{\partial \Delta_{\rm min}}{\partial J_{\rm B}} dJ_{\rm B} + \frac{\partial \Delta_{\rm min}}{\partial Q_{\rm B}} dQ_{\rm B} + \frac{\partial \Delta_{\rm min}}{\partial r_{\rm min}} dr_{\rm min}. \end{aligned}$$

- Applying near-extremal condition:

$$r_{\rm h} - r_{\rm min} = \epsilon \ll 1, \quad \Delta_{\rm min} = -\epsilon^2.$$

- Then, all values can be rewritten in terms of ϵ.

Black Hole in a Final State

- Under changes in conserved charges, the minimum value becomes:

$$\Delta_{\text{min}} + d\Delta_{\text{min}} = -\frac{8\pi|\mathcal{T}|^2(r_{\text{h}}^2 + a^2)(\omega - m\Omega_{\text{h}} - q\Phi_{\text{h}})^2}{r_{\text{h}}} dT + \mathcal{O}(\epsilon).$$

- The minimum value is always negative: **BLACK HOLE**
  - Cosmological constant Λ determines the boundary geometry and condition.
  - The asymptotic boundary is associated with the amplitude.
  - The square of the amplitude is always positive.
  - Regardless Λ, the conclusion is the same**.**
- It is impossible to overspin KN(A)dS BHs beyond extremal condition.
- The singularity cannot be observed by outside static detector.
- It is exactly what the weak cosmic censorship conjecture expects.

Our Recent Directions

- Fluxes of Scalar Field in Scattering to Rotating AdS Black Holes
  - Scattering problems
    - Quasinormal modes and superradiation instability
      - Scalar QNMs in C-metric and accelerating KNAdS black holes
        - BG, **EPJP** 138 (2023) 7, 582
        - J. B. Amado and BG, **JHEP** (2024) 02
  - Perturbation on black holes
    - Adding a mass fluctuation to black holes
      - Mass fluctuation in higher-dimensional black holes with large D limit.
        - H. Han and BG, **JHEP** 08 (2023) 102
  - Thermodynamics
    - Universal thermodynamic relation called Goon-Penco relation motivatied from WGC
      - Testing GP relation in black holes under near-extremal and near-Nariai limits.
      - J. Ko and BG, **JHEP** 03 (2024) 072

Summary

- We considered scalar field scattering which carries conserved quantities into a black hole.
- Owing to the conserved quantities, the black hole changes its mass, angular momentum, and electric charge.
- Even if these change the black hole, the black hole is still an outer horizon.

Thank You!