Cosmological Islands

Chethan KRISHNAN

Indian Institute of Science, Bangalore

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Based on –

- arXiv:2007.06551 [CK]
- See also complementary work which we will not discuss – arXiv:2005.02993 [CK, Vaishnavi Patil, Jude Pereira]

The (Weak) Information Paradox

- There are many questions of increasing subtlety, which go by the moniker, “information paradox".
- Hawking’s original version can be viewed as a statement about loss of unitarity during evaporation of the black hole.
- This problem is basically solved if AdS/CFT is true.
- But where did Hawking’s semi-classical calculation go wrong is not clarified immediately by AdS/CFT.

- This is because an implicit part of Hawking's semi-classical calculation is the picture that the black hole is a bulk-localized system in flat space or AdS.
- And bulk-locality questions are not easy to ask in holography, because the true diff-invariant observables are integrals over the bulk and therefore have support only at the boundary.

- One can elevate Hawking's implicit assumption that black holes are localized objects, into an explicit assumption. In other words, we split the Hilbert space into two tensor factors and imagine the black hole as living in one.
- Then one can ask more generally, how the entanglement entropy of the sub-system should evolve, if degrees of freedom are steadily leaving the sub-system into the rest of the system. This will be a model for Hawking radiation. [Page, Mathur, AMPS]

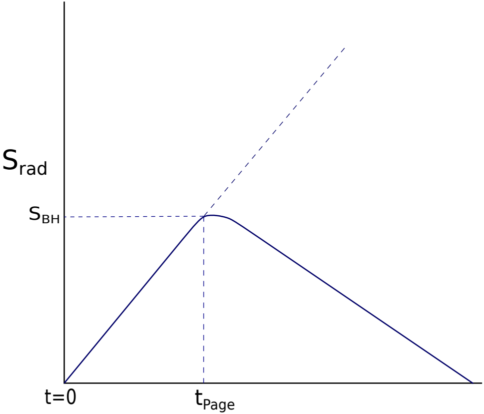

Figure: The shape of the Page curve.

- Operationally, the difference between the Hawking expectation and Page's curve happens because Hawking's version of the entropy calculation treats the entanglement entropy as entirely due to the entanglement entropy between ingoing and outgoing QFT modes in the BH background. This can only relentlessly increase (at least until the entire black hole is gone) as more and more modes are emitted (because the emission is always happening from a smooth, ie., vacuum-like, horizon). This entanglement entropy is sometimes denoted S_{bulk}.
- One can think of S_{bulk} as the entanglement entropy of the bulk quantum field modes that have left the black hole sub-system (eg. AdS, when AdS is coupled to a sink to extract radiation).

- Can we somehow modify this bulk prescription for entanglement entropy to get the right Page curve?
- The recent developments can be viewed as the osbervation that there is a natural way to do this. (But natural does not mean that a full understanding of why this prescription is correct, is known.)
- Motivated by Bekenstein's old idea that entropy of quantum fields needs the addition of black hole area to protect thermodynamics when there are black holes, one might consider the possibility that the correct entropy should contain not just S_{bulk} but also a bulk geometrical area piece.
- Formalization of this leads to the notion of a Quantum Extremal Surface. [Engelhardt-Wall]

In an AdS/CFT context, the idea is that the the entanglement entropy of a CFT subregion should be calculated by extremizing

$$S_{gen}[\chi] = \frac{A[\chi]}{4G} + S_{bulk}[\chi] \tag{1}$$

over bulk χ (and then taking the minimal one if there are multiple choices), with $\partial\chi$ coinciding with the boundary of the CFT subregion. The surface that extremizes the generalized entropy is the QES.

(In AdS/CFT, it is believed that the region within the QES can be reconstructed only based on the sub-region data of the boundary sub-region, and is sometimes called the Entanglement Wedge).

It turns out that the entanglement entropy of the boundary, computed using the QES prescription for an AdS/CFT black hole that is leaking to an extrenal sink, turns out to be dominated by two distinct surfaces before and after the Page time, and the “phase transition" between them can explain the overall shape of the Page curve. [Penington, Almheiri-Engelhardt-Marolf-Maxfield].

We will not discuss this particular Page curve in this talk. We will discuss –

- A related information problem for the eternal (as opposed to evaporating) black hole, and the associated entanglement wedge phase transition there. [Almheiri-Mahajan-Maldacena (AMM)].
- We will work with a “doubly holographic" set up. [Almheiri-Mahajan-Maldacena-Zhao (AMMZ), Almheiri-Mahajan-Santos (AMS)]

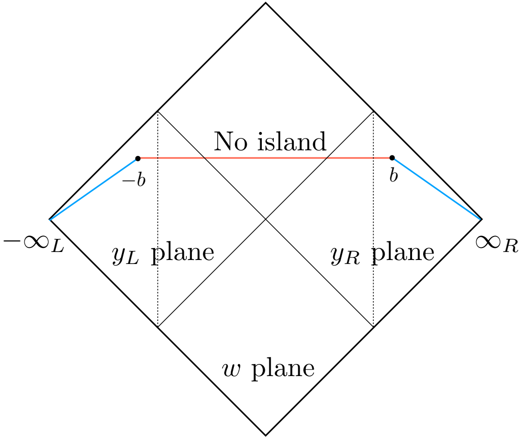

Figure: The Eternal Black Hole Entanglement entropy increases relentlessly, because of nice slices. (Figure from [AMM], idea from previous papers of [Mathur] and [Hartman-Maldacena]).

Double holography is another name for a braneworld – in other words, we are considering information paradox for a braneworld black hole.

We take the matter part of the brane AdS gravity+matter theory to be a strongly interacting theory with a large central charge. The claim is that we can compute QES using ordinary Ryu-Takayanagi surfaces in a doubly holographic bulk geometry. This is basically the bulk in which the braneworld lives [AMMZ, AMS].

If we can find a phase transition in the RT surface after a suitable Page time, so that there exists an alternate RT surface of fixed area at late times, then that will resolve the paradox. Such alternate surfaces are called islands. They are the key to the entanglement phase transitions that are key to the recent discussions on the information paradox.

Such islands have been found via direct calculation in JT-gravity on an AdS_2 braneworld [AMM], and also numerically via double holography in an AdS_4 braneworld [AMS].

In the doubly holographic setting, one views the black hole coupled to sink as a sub-critical brane hanging in the bulk, reaching an AdS boundary.

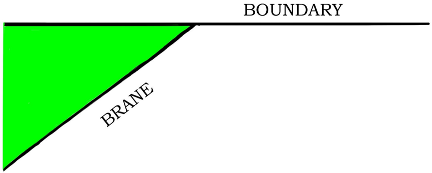

Figure: The Sub-critical Karch-Randall Braneworld. The brane has AdS gravity with a leaky boundary condition. The angle determine the brane cosmological constant. When it goes to zero, the brane Newton constant, brane cosmological constant and gravton mass on the brane all go to zero.

The geometry in [AMS] is such that there exists an RT surface anchored at the boundary, that reaches the brane (we will call this the second extremal surface). When the bulk has a planar AdS black hole horizon, there is also an RT surface that passes through the planar horizon which we call the first extremal surface. This is the surface that leads to the paradox at late times.

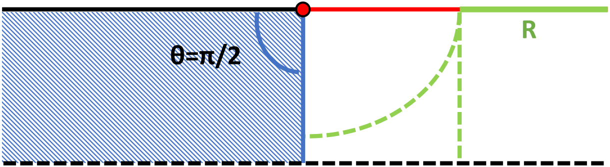

Figure: Picture taken from Karch-Geng, based on figure in AMS. The angle need not be $\pi/2$ in general.

The competition between these two extremal surfaces is what leads to a phase transition in the entropy curve of the eternal black hole at late times. In more detail –

the extremal surface that passes through the planar horizon starts out at $t = 0$, as the smaller of the two. So the entanglement entropy is determined by the first extremal surface at early times. But as time progresses, we expect this area to increase linearly with time – this is because nice slices inside horizons get stretched with time because the black hole interior is a time-dependent background [Mathur, Hartman-Maldacena]. Since the area of the second extremal surface stays fixed, this means that at late times, the second extremal surface is what determines the RT surface. This is the origin of islands. The island in this case happens to be right outside the brane horizon.

[Karch-Geng] have emphasized that the gravitons on the AdS braneworld of Karch-Randall are massive, and in the limit when the mass of the gravitons goes to zero, the brane goes to the boundary and there is no gravitational dynamics and no information paradox.

It is known that the massiveness of gravitons is a result of the transparent boundary conditions at the AdS boundary [Porrati].

Is this a feature of AdS or is it a feature of islands – that gravitons are massive? The latter would be problematic for islands as a general solution to the information paradox.

What we would like to see is if there is a version of the information paradox that can be phrased in the crictical braneworlds of the usual Randall-Sundrum II type, where the braneworld has no cosmological constant. This means that the brane is at some finite cut-off, and it does not reach the boundary. None of this angle brane funny business.

This would be a doubly holographic picture for black holes is non-AdS braneworlds.

In what follows, we will argue that the natural place to look for such a formulation is a in the context of a so-called AdS black funnel geometry attached to a critical Randall-Sundrum II braneworld.

Since black funnels may seem like a somewhat esoteric ingredient, let us give some context/story.

The black funnel is an AdS geometry where the horizon has a neck region that reaches the boundary.

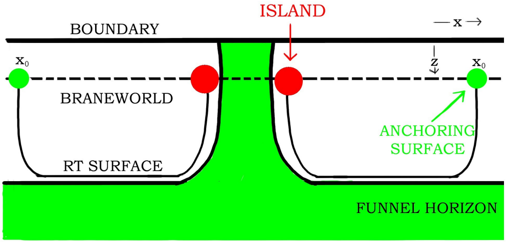

Figure: Black funnel with candidate islands and RT surface.

- AdS geometries where black holes on the asymptotic boundary are interacting with deconfined plasma are known, eg., [Hubeny et al., Santos-Way]. These are the AdS black funnels.
- Note that boundary black holes are fixed backgrounds with non-dynamical gravity, and so ultimately we are interested in finite cut-off braneworld versions of these solutions which need to be explicitly constructed. But they provide useful intuition. In the bulk, these solutions extend to two broad categories of black solutions - black funnels and black droplets.
- The black funnel solutions are believed to be duals of the strongly coupled Hartle-Hawking state, ie., they are dual to the deconfined phase of the boundary field theory in equilibrium with the boundary black hole.

- Analytic funnel solutions are known in AdS_3 and AdS_4 [Hubeny, et al.]. The latter are adaptations of the so-called AdS C-metrics [Plebanski-Demianski]. The analytic AdS_4 cases suffer from the feature that the boundary black hole lives in an asymptotically hyperbolic slicing and not in asymptotically flat space. We will use the analytic funnel in AdS_3 to do some explicit calculations later.
- Even though analytic solutions are not known, there do exist numerical black funnel solutions in AdS_5 that are dual to asymptotically flat boundary Schwarzschild black holes. We will take this as conclusive evidence that asymptotially flat black funnels exist. Numerical black funnels in AdS_4 that asymptote to three dimensional asymptotically flat black holes have also been constructed.

- So far of course, we have said nothing about braneworlds and black holes on them.
- In a very interesting paper [Figueras-Wiseman] have shown that braneworld black holes can be constructed from the associated AdS-CFT problem. More precisely, they argued that given an asymptotic boundary metric and the demand that the behavior of the solution matches with AdS at the Poincare horizon, one can construct corresponding braneworld geometries that are solutions of the induced braneworld gravity.
- Using this method, Schwarzschild black holes that are asymptotically flat (on the brane) have been constructed on RSII braneworlds [Figueras-Wiseman], starting from numerical AdS geometries [Figueras-Lucietti-Wiseman] that asymptote to Schwarzschild black holes at the boundary.

- These solutions are not quite what we want, because on the boundary they asymptote to the vacuum, and are more analogous to Unruh/Boulware states than Hartle-Hawking states. In the bulk this is tied to the fact that they asymptote to Poincare horizons and not planar black holes. To get corresponding black funnels, we need to have a horizon that asymptotes to the planar black hole horizon in the interior, in the boundary asymptotic directions.
- But nonetheless, from the above construction, it seems quite plausible that there should also exist a braneworld black hole that in the bulk asymptotes to a planar black hole. We will call this a braneworld black funnel, even though more accurately it is a black funnel attached to the brane world black hole.

- We will assume that demanding the planar black hole behavior in the interior and using the numerical black funnels of [Santos-Way] that are anchored to the conformal boundary, and following the philosophy and techniques of [Figueras-Wiseman] such a solution can be constructed.
- Assuming it exists, this task will at least be of as much difficulty as the construction of the numerical braneworld black hole in [Figueras-Wiseman].
- One key difference we expect is that we expect these black holes to live in braneworld cosmologies, because braneworlds in planar AdS black holes are FRW cosmologies [Kraus]. More about this, later.

The arguments above indicate that braneworld black funnels are closely related to the AdS/CFT black funnels, which have been constructed numerically in some interesting cases even in high enough dimensions [Santos-Way].

In what follows, we will present a doubly-holographic prescription motivated by [AMM, AMS] assuming that such braneworld black funnel constructions are possible.

It turns out that there exists a toy analytic black funnel in AdS_3 [Hubeny et al.]. So we will be able to check our guess for RT surfaces with islands.

Since it is the structure of the geometry that affects the existence of the extremal surface (and not directly the presence or absence of propagating gravitons), it may be reasonable to think that the calculation in the AdS_3 funnel that we will do may be relevant for physics even in higher dimensions. This is loosely analogous to how [AMS] generalized [AMM].

To begin with, let us note that the black funnel is an AdS geometry where the horizon has a neck region that reaches the boundary.

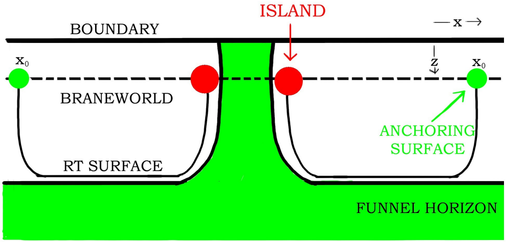

Figure: Black funnel with candidate islands and RT surface.

Clearly, the first kind of RT surfaces that are anchored to the asymptotic regions of the boundary and fall through the planar horizon, should still exist.

The question is, what replaces the second class of extremal surfaces. It cannot be the case that these extremal surfaces cut the neck of the funnel. Because if they do, by the argument we made above [Hartman-Maldacena], their areas will keep increasing quasi-linearly with time.

This would mean that no matter which extremal surface we picked, the late-time entropy is relentlessly increasing – there is no saturation of the entropy as one expects in physical systems, and we are left with a version of the information paradox.

Cutting the horizon neck also means that on the brane there are no islands, at least none that are immediately parallel to the sub-critical brane case.

The natural other possibility is that the island contribution comes from a region on the brane near (and outside) the location where the funnel horizon cuts the braneworld.

This would mean that there should exist bulk horizon straddling extremal surfaces that are anchored to the asymptotic region of the brane.

This would be consistent with the fact that on eternal black holes, the island can be outside the horizon, [AMM]. The relevant RT surface area has a chance of remaining constant as time evolves (caveats later).

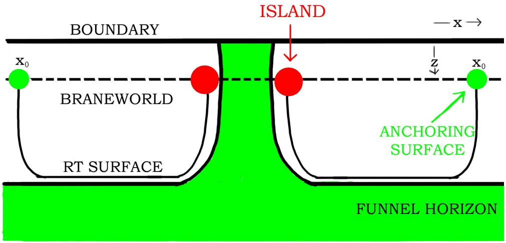

Figure: Black funnel with candidate islands and RT surface.

A key question we did not address is that of explicit construction of such extremal surfaces in higher dimensions. Our motivation for believing their existence is threefold.

- Firstly, the structure of these solutions on a given timeslice, in light of our discussions in this section, is strikingly parallel to the sub-critical case considered in [AMS]. Indeed, one of our motivations to consider black funnels was that they naturally geometrize the intuition of [AMS] in a critical braneworld context.
- Secondly, in the next section, in a three dimensional funnel, we will be able to explicitly demonstrate that bulk-horizon straddling RT surfaces that are anchored at the asymptotic regions of the boundary, do exist in the underlying AdS/CFT geometry.
- Thirdly, we will see that there is a very natural way to incorporate the cosmological nature of the braneworld into these discussions.

Now, we will specialize the discussion to the case of an AdS_3 black funnel presented in [Hubeny et al.]. This is a simple context where an analytic funnel metric is known. The result of our calculation will demonstrate the existence of the kind of extremal surface in this geometry that has a chance of leading to islands in higher dimensions.

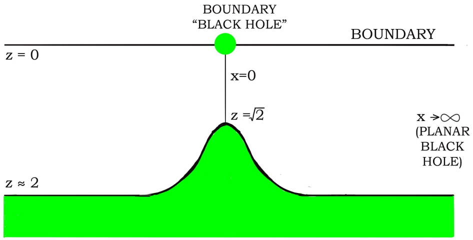

Figure: Spatial slice of the AdS_3 black funnel.

- One difference between the higher dimensional settings of the last section and the present one, is that the collapsed neck of the AdS_3 funnel (which has some similarities to the zero energy hyperbolic black hole in higher dimensions) extends a finite distance from the boundary.
- A related point is that if one wants to take our calculation as a statement about a 1+1 dimensional information paradox, one should embed a brane in the 2+1 dimensional AdS funnel geometry. This brane can be nearby and parallel to the $z = 0$ boundary in the boundary asymptotic directions ($x \to \infty$), but will have to cut the $z = \sqrt{2}$ end of the green horizon in the figure in order for it to be truly analogous to the higher dimensional discussions. It is conceivable that the behavior of such a brane at late times, especially near the bulk horizon, needs a more detailed analysis.

- Another comment is that a 1+1 dimensional brane needs to have a gravity action on it (as in the JT gravity examples in the sub-critical case), but in higher dimensions one can simply work with the induced gravity on the brane. These things are important for various purposes, but they are details for us, because our goal here will be to simply demonstrate the existence of boundary-anchored bulk RT surfaces that straddle the bulk horizon. It is the existence of the latter that we expect, will generalize to higher dimensions.

With these caveats emphasized, we turn to the AdS_3 black funnel. The metric is

$$ds^2 = \frac{1}{z^2}\left(-f(x,z)dt^2 + g(x,z)dx^2 + dz^2\right) \tag{2}$$

with

$$\begin{aligned} f(x,z) &= \tanh^2 x\left(1 - z^2\frac{\cosh^2 x + 1}{4\cosh^2 x}\right)^2, \\ g(x,z) &= \left(1 + z^2\frac{\cosh^2 x - 3}{4\cosh^2 x}\right)^2 \end{aligned} \tag{3}$$

where x is a radial coordinate and z is the holographic direction with boundary at $z = 0$.

The horizon is obtained by setting the metric function $f = 0$, and it has one branch coming from $x = 0$ corresponding to the $\tanh$ piece (this corresponds to the collapsed neck), and another that can solved explicitly for z in terms of x which yields the green horizon in the figure. Asymptotically as $x \to \infty$ the latter tends to the planar black hole horizon at $z \sim 2$. It deviates from the planar black hole near $x \sim 0$ and we get the bump in the figure which is peaked at $z = \sqrt{2}$.

Based on our higher dimensional prejudices discussed earlier, we expect that there should be an RT surface close to this bump that reaches out to the asymptotic region of the boundary $z = 0$ as $x \to \infty$.

We are to look for an extremal surface anchored on the brane between a fixed large $x = x_0$ and an $x = x_h \sim 0$, and pick the minimal one by varying over x_h.

Since we wish to avoid the complication of explicitly introducing the brane, we will instead look for extremal surfaces anchored to $x = 0$ and $x = x_0$, with $dz/dx|_{x=0} = 0$.

Our goal below will be to demonstrate that such extremal surfaces do exist and that they have properties that one might expect from islands.

The relevant integral that we have to extremize then is

$$I = \int_{x=0}^{x=x_0} \frac{dx}{z} \sqrt{g(x,z) + \left(\frac{dz}{dx}\right)^2} \tag{4}$$

with the boundary conditions that $dz/dx|_{x=0} = 0$ and $z(x = x_0) = 0$.

Note that I has dependence on both the independent variable x as well as z and dz/dx. So even though the variational equation of motion arising from this integral is an ODE, it is too complicated to be instructive (as far as I could see). So we will not exhibit it.

Solving it for fixed values of x_0 numerically is possible by using a shooting method.

What we will do is to integrate the Euler-Lagrange equation from $x = 0$ with $dz/dx|_{x=0} = 0$ for various values of $z(x = 0)$. From the plots of such curves, it will become clear that any value of x_0 can be arrived at if one chooses a suitable $z(x = 0)$, establishing the existence of the RT surface.

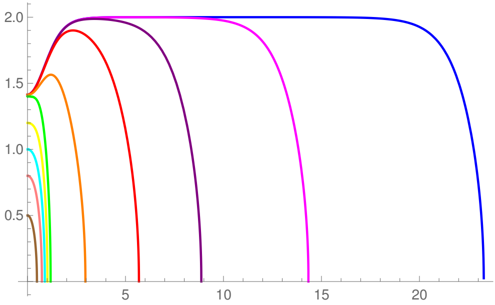

Figure: Plots of extremal surfaces with $x = 0$ and $dz/dx|_{x=0} = 0$. The vertical axis is z and the horizontal axis is x. The “island" extremal surfaces are the ones that straddle the horizon, note that the bump in the horizon as shown in Fig. 8 happens at $(x = 0, z = \sqrt{2})$.

The evidence for island like physics is clear from the plots. As the anchoring point at $x = 0$ gets closer to the horizon, the second anchoring point gets pushed out to larger and larger x_0. The extremal surfaces straddle the green horizon for longer and longer before diverging from it and hitting the boundary/brane. This is again consistent with the observations of [AMM] and is also intuitive.

The above discussion takes care of the “second" extremal surfaces that we defined earlier. Now we turn to the “first" kind of extremal surfaces, those that cut the planar black hole horizon. Here it turns out that it is beneficial to write the integral that captures the area in a notation where z is the independent variable:

$$I' = \int_{z=0}^{z=z_*} \frac{dz}{z} \sqrt{1 + g(x,z) \left(\frac{dx}{dz}\right)^2} \tag{5}$$

Here z_* is the location on the horizon where $dx/dz = 0$ (at least for large enough x_0, we can expect this to be true.), and $x(z=0) = x_0$. The usefulness of this form is that the Euler-Lagrange equation has the property that when $x' = 0$, it reduces to $x'' = 0$. In other words, if we are imagining starting the integration from the horizon where $dx/dz = 0$, we can see that the extremal surface is simply $x = x_0$. This is the first kind of extremal surface.

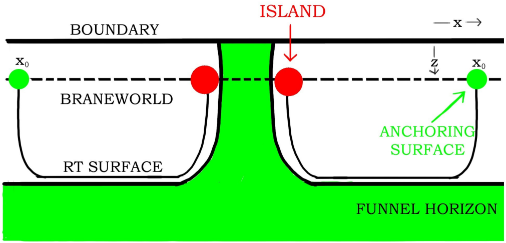

Figure: Black funnel with candidate islands and RT surface.

Qualitatively similar comments apply to general dimensions. We expect anchoring them as in the AdS3 case, to exhibit the presence of horizon-straddling extremal surfaces. Of course, in higher dimensions the construction will have to be numerical.

Interpretation: Cosmological Islands

- We showed the existence of these extremal surfaces on the constant time slices of the metric, by direct calculation.
- Furthermore, it is natural that for large enough x_0, the second kind of extremal surfaces have bigger area at some initial time than the first kind – the former scale extensively with x_0, the latter do not.
- Also as time elapses, the contribution from inside the horizon to the area of the first extremal surface can be expected to increases linearly with time.
- Because of all these reasons, one might hope that there is a phase transition at late times, and that the argument for avoiding the information paradox is just a re-run of that in [MMS].

- But there is one fly in the ointment – the brane is moving in the bulk. This means that the island argument here needs some adaptations from that in [MMS].
- We will have to find a way to keep track of only the black hole contribution to the entanglement entropy, and this is the purpose of this part of the talk.
- Remarkably, we find that there is a quite compelling way to do this, both from the bulk and the brane perspectives. Since the argument here contains some interesting new ingredients that did not play a role in [MMS], this can be viewed as additional evidence for the role of islands.
- Note also that this discussion goes a bit beyond the purely kinematical discussion of RT surfaces that we were concerned with in the rest of this talk.

- Even though the geometry for a full braneworld black funnel is not known, we know that it tends to a planar black hole far away from the funnel.
- A brane in a planar black hole has to accelerate, so that it does not fall through the horizon. On the worldvolume of the brane this results in an FRW cosmology, with the scale factor directly captured by the radial position of the brane [Kraus].

- Even though the braneworld attached to a black funnel is likely hard to (numerically) construct, a natural guess is that the horizon region of the brane also moves, and therefore experiences the FRW scale factor. (If it does not, it is easy to convince oneself that the arguments of MMS apply with minor changes here as well, and the information problem is solved already at this stage. But we believe the situation is more interesting.)
- Note eg., that the AdS/CFT black funnel has a horizon on the conformal boundary. This seems more natural to understand in terms of the braneworld funnel if the horizon were to *not* get stuck at a finite location in the AdS radial direction.

- Now a moving brane with a cosmology on it makes our problem different from that in AMM-AMS. So let us first observe that the eternal black hole information paradox in this braneworld system is still well-posed.
- The entire system is getting red-shifted and cooling down due to cosmological expansion, but the temperature of the black hole remains the same as that of radiation and therefore it remains in equilibrium.
- By picking the confinement scale of the matter and the initial size of the black hole suitably, we can ensure that we stay above the deconfinement temperature for order of a few Page times. The relevant Page time is given by $\sim \beta S_{BH}/c$ where c is the large central charge of the matter. Therefore the paradox can still be arranged to exist, by choosing parameters, if we pick the black hole to be small enough (but still macroscopic) with a Page time that is small enough.

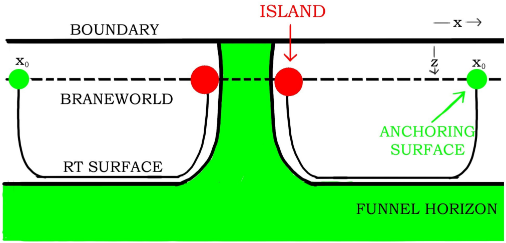

Figure: Black funnel with candidate islands and RT surface.

The matter on the brane is deconfined radiation and the scale factor goes as a power law $a(t) \sim t^{1/2}$ for a 4D braneworld (and temperature falls as $\sim 1/a$), even though the specific time dependence of the scale factor does not affect our discussions below.

Let us note that primordial-sized black holes have been argued to exist upto the Planck limit of 10^{-8} kg [Hawking]. But a black hole of mass 1 kg is sufficiently macroscopic, but will have a Hawking temperature of $\sim 10^{23}$ K, which is easily higher than the deconfinement temperature of real world QCD, which is about $\sim 10^{12}$ K. Note that the (large) central charge and the deconfinement scale of matter, we can treat as free inputs.

A second point worthy of note is that because of the FRW scale factor, there are two natural candidates for the entanglement entropy that one can compute on the braneworld. The first possibility is that we compute the entanglement entropy of a sphere located at fixed physical distance from the braneworld black hole. The other is that we look at a sphere that is at a fixed comoving radius. Together with this choice, we also have to make a corresponding choice for the UV cut-off in physical distance or comoving distance.

We will argue that since our goal is to keep track of the entanglement entropy due to the black hole and not due to the cosmological expansion, it is the comoving entanglement entropy that one should compute. Let us present a few reasons why we think this is an eminently natural choice.

- From the point of view of an entanglement entropy computation on the brane, note that if we fix a comoving length scale as the UV cut-off, we will never trace over the modes that become accessible due to cosmological red-shifting. This is precisely what we would like, since we wish to avoid counting the cosmological contribution to the entanglement entropy and only keep track of the black hole physics.

- Note that this statement can be understood also from the bulk/brane geometric point of view. The time-dependence is a direct result of embedding the brane in the bulk planar black hole geometry, and on the brane it gets reflected in the fact that the black hole is living in a cosmology. The key point is that even when there is no funnel or braneworld black hole, but only the bulk planar black hole, we get cosmology/time-dependence on the brane. What the comoving approach does is to regulate this contribution away.
- A practical reason for preferring comoving coordinates is that since the brane horizon is stretching, it will reach any fixed physical radius in finite time. It may perhaps be possible to choose one's scales so that this happens after the Page time, but it certainly does not look too appealing as a starting point.

- Another way to see that the natural choice is the comoving location, is to look at the bulk picture of the extremal surfaces. In the picture, the brane is moving “up" because it is cosmological, and fixed locations in boundary coordinates to anchor the extremal surfaces naturally translate to fixed comoving coordinates on the braneworld. On the contrary, if we were to anchor the extremal surfaces at fixed physical radius, as the brane evolves, they will have to move inward in the x-direction in the figure. This is a bulk manifestation of the expansion of the horizon.
- Note that the extremal surfaces familiar from AdS/CFT, naturally map on to comoving Ryu-Takayanagi surfaces from the brane point of view. This is evident from the structure of cosmological branes in general, as well as from our figures (note that the scale factor on the brane is a direct result of the brane moving “up").

A crucial observation is that if we are computing entanglement entropy of regions defined by their comoving coordinates, and with a fixed comoving cut-off, the scale factor drops out of the entanglement entropy and therefore its leading behavior is time-independent. This is *not* the case if we fix a physical cut-off instead. Note again that this is consistent with our previous expectation that the comoving calculation is insensitive to cosmological production of entanglement entropy.

Finally, let us also emphasize that since RT surfaces anchored on the brane are of manifestly finite area, the dual entanglement entropy defined on the brane should necessarily be understood with a cut-off.

Now we turn to our bulk computation of entanglement entropy using the RT surfaces and how they should be interpreted. An immediate observation is that because the branes are moving “up", the contribution coming from near the anchoring surfaces has time dependence. Our discussion in the last paragraph is a strong indication that this time dependence is a consequence of implicitly working with a *physical* UV cut-off.

ASIDE: One might hope that this will go away in the subtraction with respect to the trivial (“first") RT surface, if ones goal was to simply try to reproduce the methods of MMS. But it is clear that the island contribution also has the same time dependence and that will not go away in the subtraction. In any event, as we argued, this time dependence has a physical origin and hoping that subtraction would save us, is unsatisfactory. Note in particular that these are manifestly finite quantities, and therefore they should be explicitly understood with a cut-off from the brane perspective.

Further evidence for this comes by a direct calculation – we can estimate how the RT surface area depends on the (moving brane) location r. Here r is the Poincare radial coordinate at the anchoring surface on the brane. Note that the scale factor on the brane is determined by the time dependence of its radial position [Kraus], which means that we should eventually take $r \sim a(t)$. In $(d+1)$-dimensional AdS, the cut-off dependence of RT area is well-known and gives rise to the area law for the entanglement entropy on the boundary, and therefore should scale as r^{d-2}.

It can also be estimated by a direct calculation, noting that near the cut-off in AdS, the RT surface will have the metric $dr^2/r^2 + r^2(dX^2)$ where dX^2 denotes some r-independent, $(d-2)$-dimensional geometry. This leads to an area (which is the volume of the RT surface) that can be esimated as $\int \frac{dr}{r} r^{d-2} \sim r^{d-2}$.

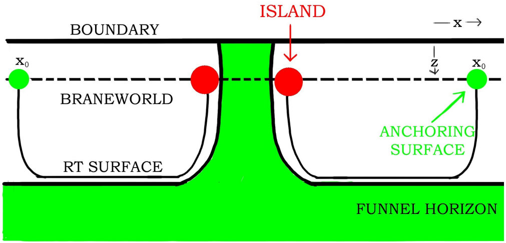

Figure: Black funnel with candidate islands and RT surface.

For a moving brane, with the brane position $r \sim a(t)$ this therefore gives precisely the scale factor dependence one expects due to a physical UV cut-off in the braneworld theory. And the remedy is clear - one has to change the cut-off to a comoving cut-off, and all one has to do to accomplish this at leading order is to delete the scale factor dependence arising at the anchoring surfaces in our RT calculation. Essentially, the object we are calculating is

$$S \sim \frac{A(t)}{\epsilon^{d-2}(t)} \sim \frac{a(t)^{d-2}}{\epsilon^{d-2}(t)} \tag{6}$$

where A is a suitable area on the brane.

There are two natural choices for the UV cut-off $\epsilon(t)$. The physical cutoff takes $\epsilon(t) = \epsilon_{phys}$, a constant length independent of time. The comoving cutoff instead fixes the coordinate grid length as the fundamental quantity, and sets $\epsilon(t) \sim a(t)\epsilon_{comov}$ where ϵ_{comov} is a constant cutoff in the comoving grid size. The key point is that to go between the two definitions of S one needs to scale by a factor of $a(t)^{d-2}$, and the key observation is that it is precisely this factor that shows up as the time-dependence of the RT surfaces at the anchoring surfaces on the brane.

Of course, since the full geometry of the braneworld black funnel is a complicated time-dependent geometry, evaluating these explicitly will require a computer. But by identifying the relevant object as the entanglement entropy with a comoving cut-off, we have understood how the superficially time-dependent RT area leads ultimately to a meaningful time-independent result. And it is at this stage, that we are free to blissfully apply the argument of MMS.

We note that both the first and second RT surface areas (when computed correctly with the comoving regulator) are time independent, and therefore there will indeed be a phase transition at late times.

This is a very strong suggestion that cosmological islands resolve a version of the information paradox for these eternal cosmological black holes. Because they are critical braneworlds, the gravitons in them are massless.

Outlook

Let us first summarize the context of this paper. When JT-gravity is coupled to a sink CFT in 2D, the emergence of the Page curve can be understood via semi-classical islands in a 3D “doubly-holographic" geometry. To this end, one views JT-gravity as living on a Karch-Randall braneworld. The higher dimensional analogues of this set up lead to black holes in AdS braneworlds, but with massive gravitons. It has been observed that in the massless limit of these massive gravitons, islands also vanish, raising questions about the validity of the prescription for real world gravity.

In this paper, we noted that by working with a critical Randall-Sundrum II set up instead of Karch-Randall, we can get braneworlds with vanishing cosmological constants and massless gravitons. Stringing together various known facts in the literature (some from numerical relativity), we argued that a version of the doubly-holographic Ryu-Takayanagi calculation should make sense in the critical RSII braneworld, in terms of a black funnel attached to the braneworld black hole. The geometry of this configuration makes it plausible that islands exist. We checked this by direct calculation in a (not fully realistic) 3-dimensional example, where the analytic funnel metric is explicitly known. This could resolve an eternal black hole version of the information paradox, in a suitable braneworld in this system. Along the way, we understood a thing or two about the role of islands in cosmology – in particular the usefulness of the comoving entanglement entropy on the brane.

Note that critical braneworlds are interesting for two reasons. Firstly, they provide a doubly holographic geometry for non-AdS black holes. Secondly, they give us massless gravitons. The arguments we present here are complementary to those in [CK-Patil-Pereira], and further strengthen the case that islands maybe of more general significance than 1+1 dimensions or AdS, in resolving the Page curve version of the information paradox.

Thank you!

One interpretation of the recent developments on the information paradox is that the entropy problem in Hawking's calculation can be solved already at the semi-classical level. On the other hand, a full understanding of the microstate of the system, will require more work. Doubly holographic (indeed any semi-classical) calculation assumes that there is some UV completion to which these calculations can be viewed as approximations to. The outstanding question, as always, is to understand this UV completion better.

To directly check these claims in higher dimensions, we will need to take three steps.

- Firstly, we need explicit black funnel metrics in higher dimensional AdS. The state of the art in this is the work of Santos and Way and the results are numerical.
- Secondly, we will need to construct explicit numerical RSII braneworld black holes at finite cut-off, induced from these AdS black funnels. This will loosely follow the structure of [Figueras-Wiseman], but note that the “boundary" condition in the deep bulk that should be used for constructing this solution is that of a planar black hole.

- This makes the braneworld time-dependent and potentially more complicated. This existence step is necessary, if we want to be fully confident that the claims that are being made by the RT surface calculations in the next step, are indeed saying something about the information problem.
- Thirdly, we should explicitly construct the RT surfaces. As every step requires numerics, it is clear that this is a problem tailor-made for numerical relativists.

Once one convinces oneself that the second extremal surfaces exists in black funnel backgrounds, that resolves (modulo a caveat we will expand on) the eternal black hole version of the information paradox, following the logic of [AMM,AMS]. It would of course be interesting to see if one can make such an argument in the context of an evaporating black hole as well. A doubly-holographic calculation of this type may be doable in a dynamical black droplet geometry [**?**, **?**].