A path-integral toward non-perturbative effects in Hawking radiation

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2020. 8. 7.

Based on

Chen, Sasaki and DY, 1806.03766

Chen, Sasaki and DY, 2005.07011

Three roads to Hawking radiation

1. Bogoliubov transformation

Commun. math. Phys. 43, 199—220 (1975)

Particle Creation by Black Holes

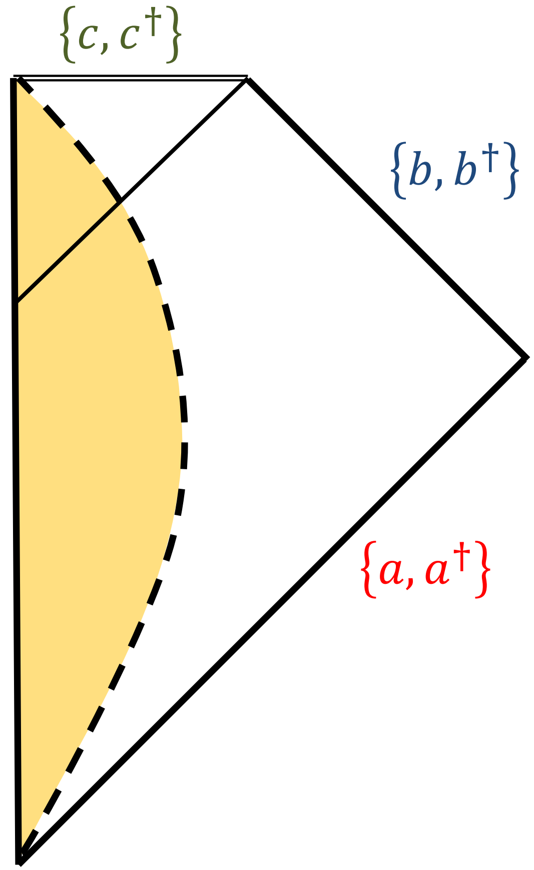
S. W. Hawking

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, England

Received April 12, 1975

Abstract. In the classical theory black holes can only absorb and not emit particles. However it is shown that quantum mechanical effects cause black holes to create and emit particles as if they were hot bodies with temperature $\frac{\hbar\kappa}{2\pi k} \approx 10^{-6}\left(\frac{M_\odot}{M}\right)$ °K where κ is the surface gravity of the black hole. This thermal emission leads to a slow decrease in the mass of the black hole and to its eventual disappearance: any primordial black hole of mass less than about 10^{15} g would have evaporated by now. Although these quantum effects violate the classical law that the area of the event horizon of a black hole cannot decrease, there remains a Generalized Second Law: $S+\frac{1}{4}A$ never decreases where S is the entropy of matter outside black holes and A is the sum of the surface areas of the event horizons. This shows that gravitational collapse converts the baryons and leptons in the collapsing body into entropy. It is tempting to speculate that this might be the reason why the Universe contains so much entropy per baryon.

1. Bogoliubov transformation



Unitary transformation between operators:

from

$$\boldsymbol{\phi}(\boldsymbol{x}) = \sum_i [\boldsymbol{a}_i \boldsymbol{f}_i(\boldsymbol{x}) + \boldsymbol{a}_i^\dagger \boldsymbol{f}_i^*(\boldsymbol{x})]$$

to

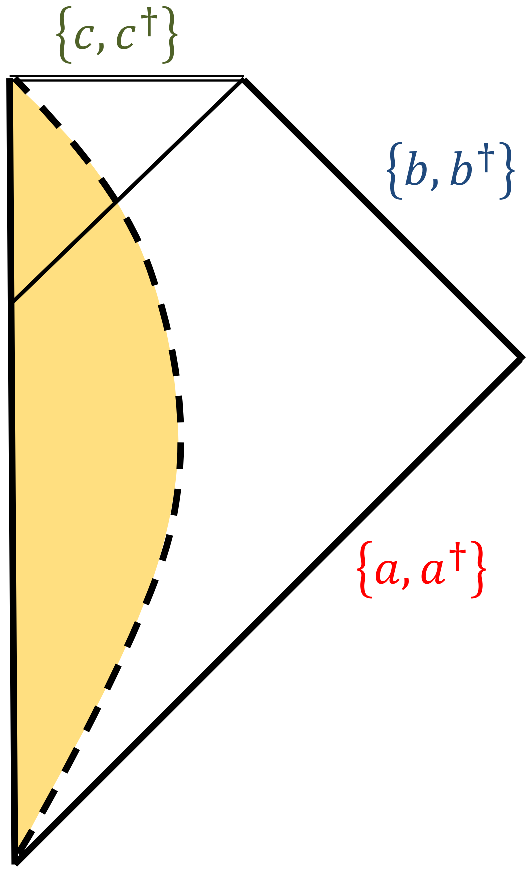
$$\boldsymbol{\phi}(\boldsymbol{x}) = \sum_i [\boldsymbol{b}_i \boldsymbol{p}_i(\boldsymbol{x}) + \boldsymbol{b}_i^\dagger \boldsymbol{p}_i^*(\boldsymbol{x}) + \boldsymbol{c}_i \boldsymbol{q}_i(\boldsymbol{x}) + \boldsymbol{c}_i^\dagger \boldsymbol{q}_i^*(\boldsymbol{x})]$$

so to speak,

$$\boldsymbol{p}_i = \sum_j [\boldsymbol{\alpha}_{ij} \boldsymbol{f}_j + \boldsymbol{\beta}_{ij} \boldsymbol{f}_j^*]$$

$$\boldsymbol{q}_i = \sum_j [\boldsymbol{\gamma}_{ij} \boldsymbol{f}_j + \boldsymbol{\eta}_{ij} \boldsymbol{f}_j^*]$$

1. Bogoliubov transformation



Even though we start from the vacuum

$$\boldsymbol{a_\omega}|\mathbf{0}\rangle = \mathbf{0}$$

by using

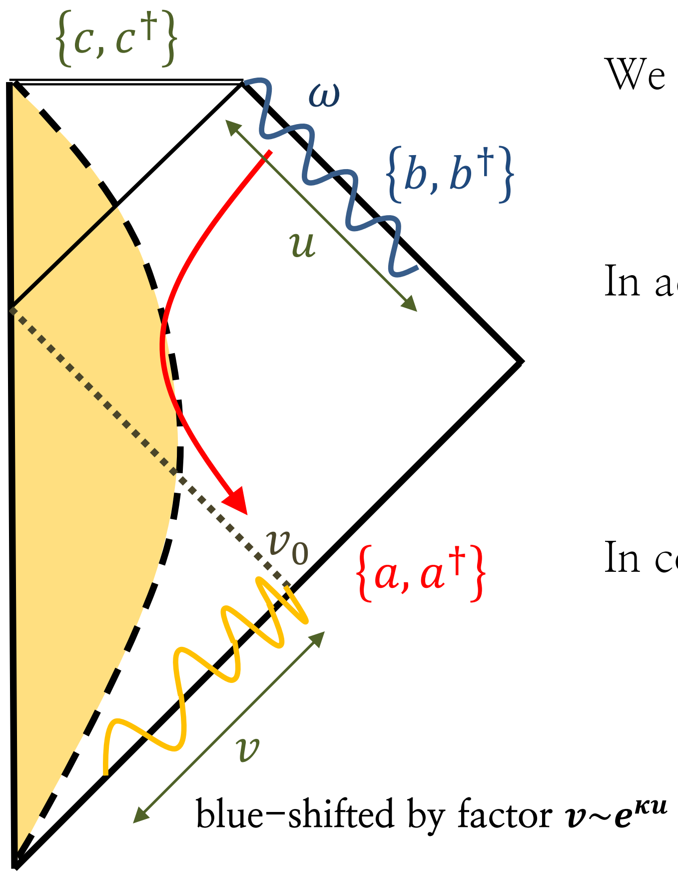
$$\boldsymbol{b_\omega} = \sum_{\omega'} [\boldsymbol{\alpha}^*_{\omega\omega'} \boldsymbol{a_{\omega'}} - \boldsymbol{\beta}^*_{\omega\omega'} \boldsymbol{a}^\dagger_{\omega'}]$$

we obtain

$$\langle \boldsymbol{n_\omega} \rangle = \langle \boldsymbol{b}^\dagger_\omega \boldsymbol{b_\omega} \rangle = \sum_{\omega'} |\boldsymbol{\beta}_{\omega\omega'}|^2$$

This may be non−zero.

1. Bogoliubov transformation



We obtain the relation

$$|\alpha_{\omega\omega'}|^2 = e^{2\pi\omega/\kappa}|\beta_{\omega\omega'}|^2$$

In addition, there is a normalization condition

$$\sum_{\omega'}(|\alpha_{\omega\omega'}|^2 - |\beta_{\omega\omega'}|^2) = 1$$

In conclusion,

$$\langle n_\omega \rangle \propto \frac{1}{e^{2\pi\omega/\kappa} - 1}$$

2. Renormalized energy–momentum tensor

2. Renormalized energy momentum tensor

We want to solve the equation

$$\boldsymbol{G_{\mu\nu} = 8\pi\langle T_{\mu\nu}\rangle}$$

where the energy−momentum tensor has ambiguities.

Moreover, the expectation values are divergent in general:

$$\langle\boldsymbol{\phi(x)\phi(x)}\rangle$$

2. Renormalized energy momentum tensor

Birrell and Davies, “Quantum fields in curved space”, 1982

In order to resolve these problems, renormalization techniques were developed.

Step 1. Obtain finite two−point correlation function, e.g., by the **point splitting method**:

$$\lim_{x \to x'} \langle \phi(x)\phi(x') \rangle = \frac{c}{x - x'} + \text{(finite terms)}$$

Step 2. Using the two−point correlation function, we obtain the energy−momentum tensor.

$$\langle T_\mu^\nu \rangle = \langle (\tfrac{2}{3}\phi_{;\mu}\phi_{;}{}^{\nu} - \tfrac{1}{6} g_\mu^\nu \phi_{;\alpha}\phi_{;}{}^{\alpha} - \tfrac{1}{3}\phi\phi_{;\mu}{}^{\nu}) \rangle$$

$$G(x,x') = i \langle \phi(x)\phi(x') \rangle$$

$$\langle T_\mu^\nu \rangle_{\mathrm{REN}} = \lim_{x' \to x} \{ -i [\tfrac{1}{3}(G_{;\mu\alpha'} g^{\alpha'\nu} + G_{;}{}^{\nu}{}_{\alpha'} g^{\alpha'}{}_{\mu}) - \tfrac{1}{6} G_{;\alpha\beta'} g^{\alpha\beta'} g_\mu^\nu - \tfrac{1}{6}(G_{;\mu}{}^{\nu} + G_{;\alpha'\beta'} g^{\alpha'}{}_{\mu} g^{\beta'\nu})] - \langle T_\mu^\nu \rangle_{\mathrm{subtract}} \}$$

Howard and Candelas, 1984

2. Renormalized energy momentum tensor

Davies, Fulling and Unruh, 1976

For two-dimensional cases, we can obtain the simpler form:

$$\langle \boldsymbol{T_{\mu\nu}} \rangle = \frac{\boldsymbol{P}}{\boldsymbol{\alpha^2}} \begin{pmatrix} (\boldsymbol{\alpha\alpha_{,uu} - 2\alpha_{,u}^2}) & -(\boldsymbol{\alpha\alpha_{,uv} - \alpha_{,u}\alpha_{,v}}) \\ -(\boldsymbol{\alpha\alpha_{,uv} - \alpha_{,u}\alpha_{,v}}) & (\boldsymbol{\alpha\alpha_{,vv} - 2\alpha_{,v}^2}) \end{pmatrix}$$

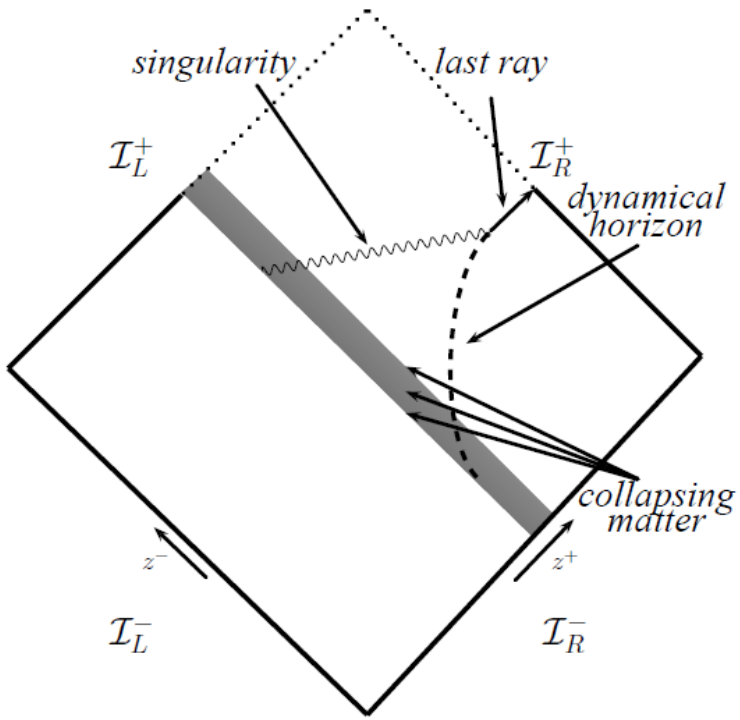
where $\boldsymbol{ds^2 = -\alpha^2 du dv}$.

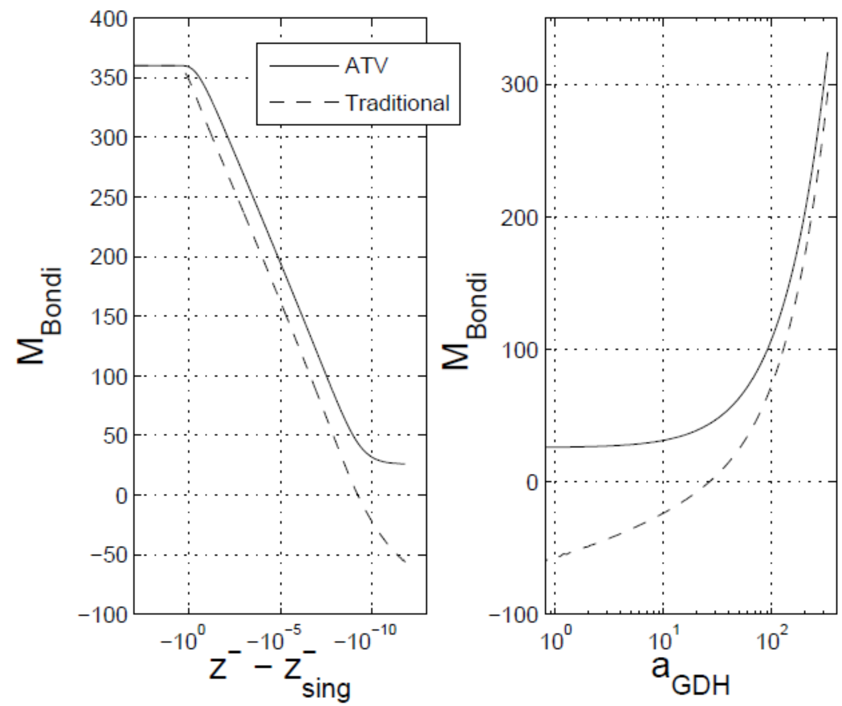
For dilaton black holes, there can be a back-reaction even for 2D: CGHS model.

Callan, Giddings, Harvey and Strominger, 1991

2. Renormalized energy momentum tensor

Ashtekar, Pretorius and Ramazanoglu, 2011





3. Tunneling picture

PHYSICAL REVIEW D VOLUME 13, NUMBER 8 15 APRIL 1976

Path-integral derivation of black-hole radiance*

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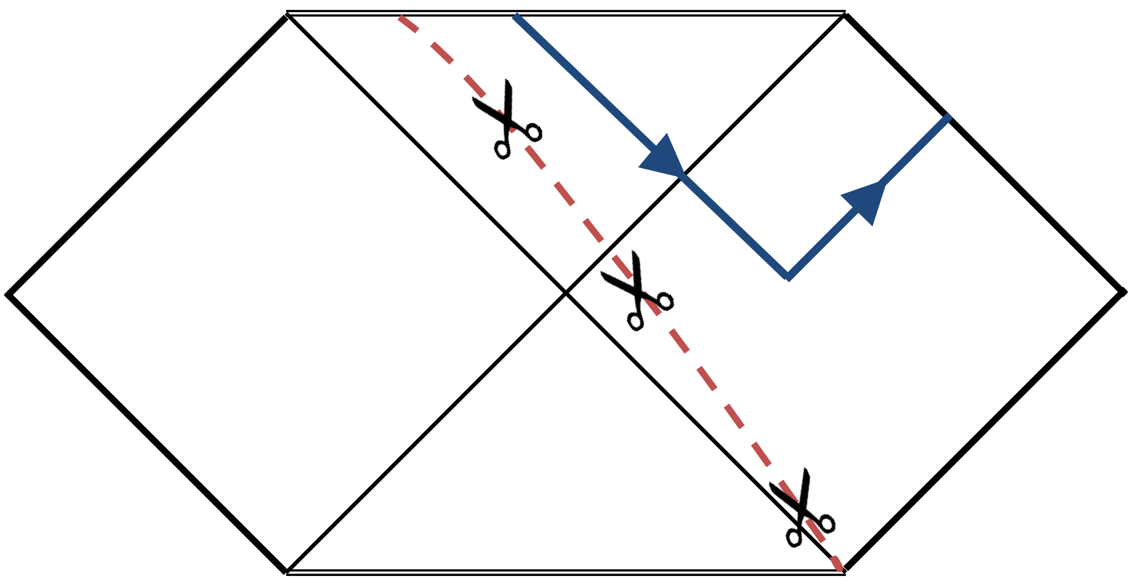
(Received 17 November 1975)

The Feynman path-integral method is applied to the quantum mechanics of a scalar particle moving in the background geometry of a Schwarzschild black hole. The amplitude for the black hole to emit a scalar particle in a particular mode is expressed as a sum over paths connecting the future singularity and infinity. By analytic continuation in the complexified Schwarzschild space this amplitude is related to that for a particle to propagate from the past singularity to infinity and hence by time reversal to the amplitude for the black hole to absorb a particle in the same mode. The form of the connection between the emission and absorption probabilities shows that a Schwarzschild black hole will emit scalar particles with a thermal spectrum characterized by a temperature which is related to its mass, M, by $T = \hbar c^3/8\pi GMk$. Thereby a conceptually simple derivation of black-hole radiance is obtained. The extension of this result to other spin fields and other black-hole geometries is discussed.

3. Tunneling picture

Hartle and Hawking, 1976

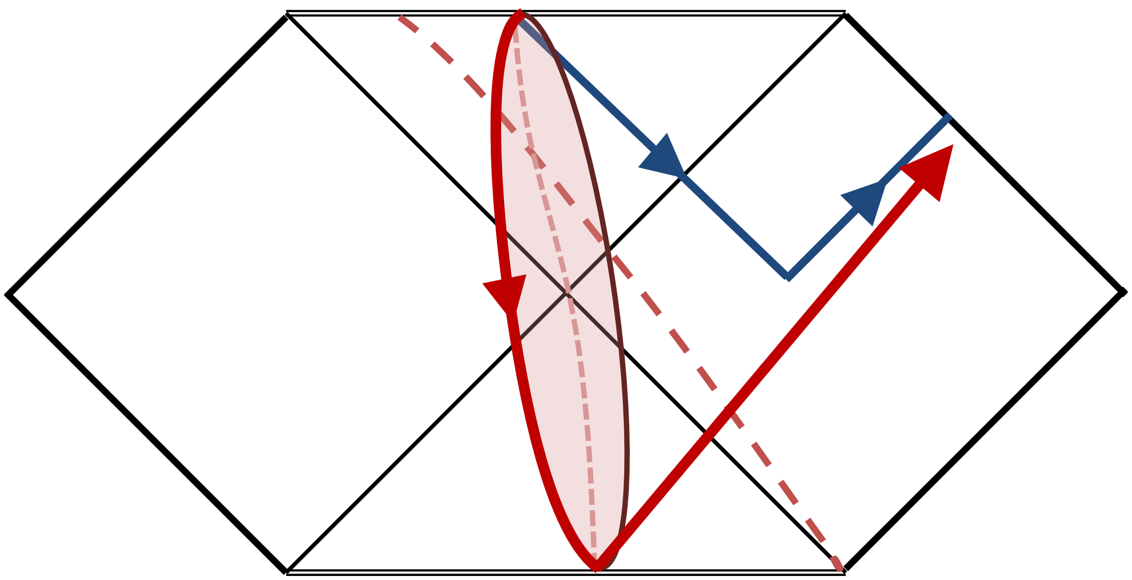
Hartle and Hawking considered particle tunneling from inside to outside the horizon.



3. Tunneling picture

Hartle and Hawking, 1976

Using the analytic continuation, one can calculate the emission rate.

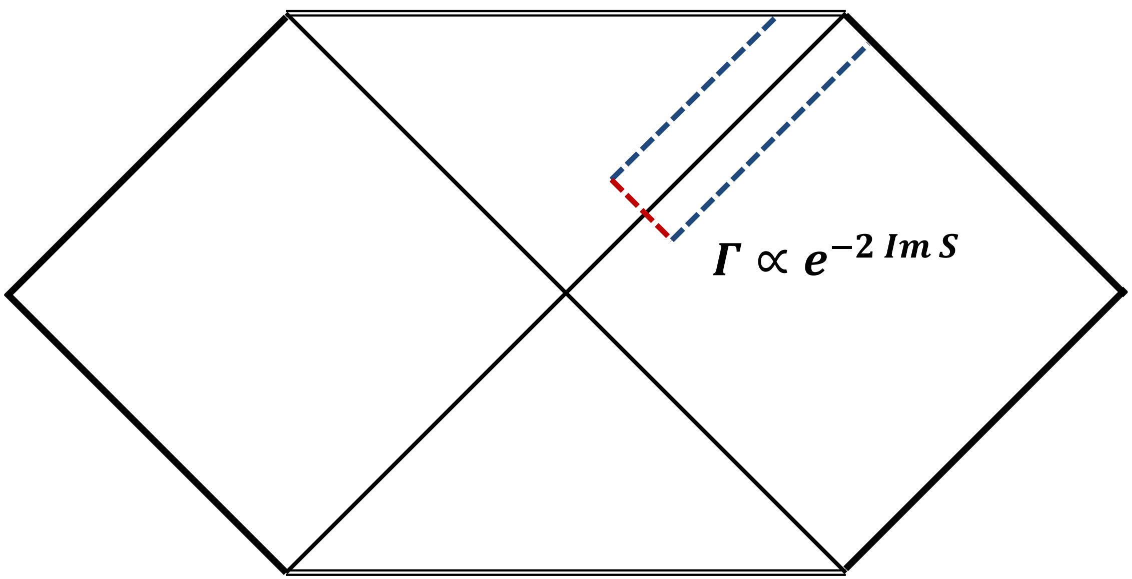


(probability to emit a particle with energy E) = $e^{-\frac{2\pi E}{\kappa}}$ × (probability to absorb a particle with Energy E)

3. Tunneling picture

Parikh and Wilczek, 2000

One can also calculate a tunneling between two null geodesics.



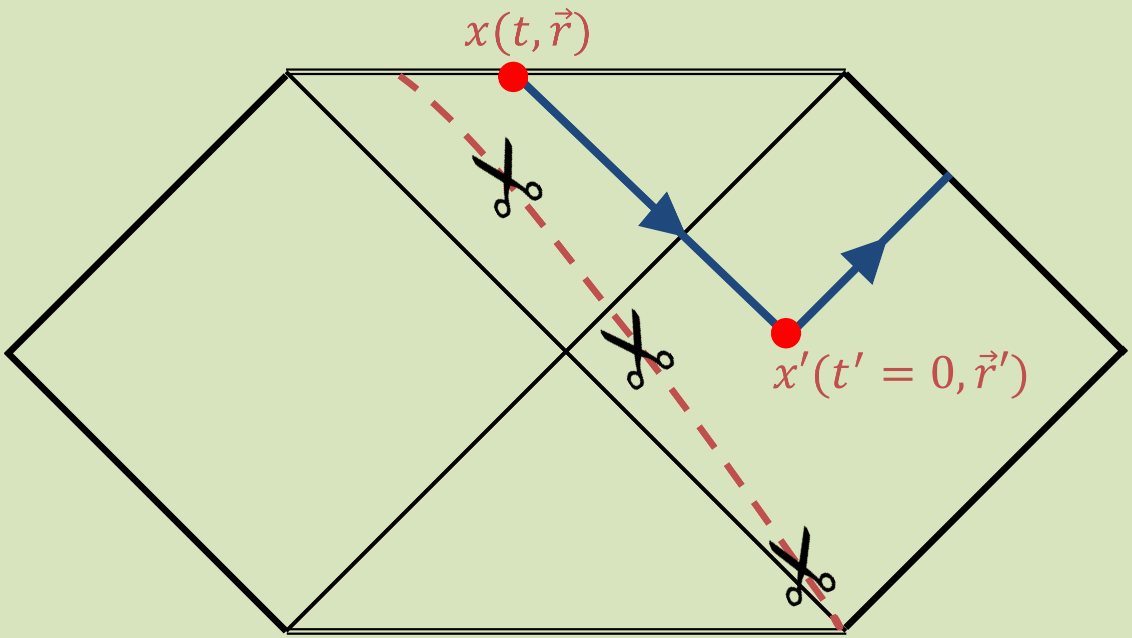
$$Im\, S = Im \int_{r_{in}}^{r_{out}} p_r dr = Im \int_{M}^{M-\omega} \int_{r_{in}}^{r_{out}} \frac{dr}{\dot{r}} dH = 4\pi\omega\left(M - \frac{\omega}{2}\right)$$

$$(H = M - \omega')$$

Why do we have to Wick-rotate

inside the horizon only?

Let's revisit the Hartle-Hawking's path integral.

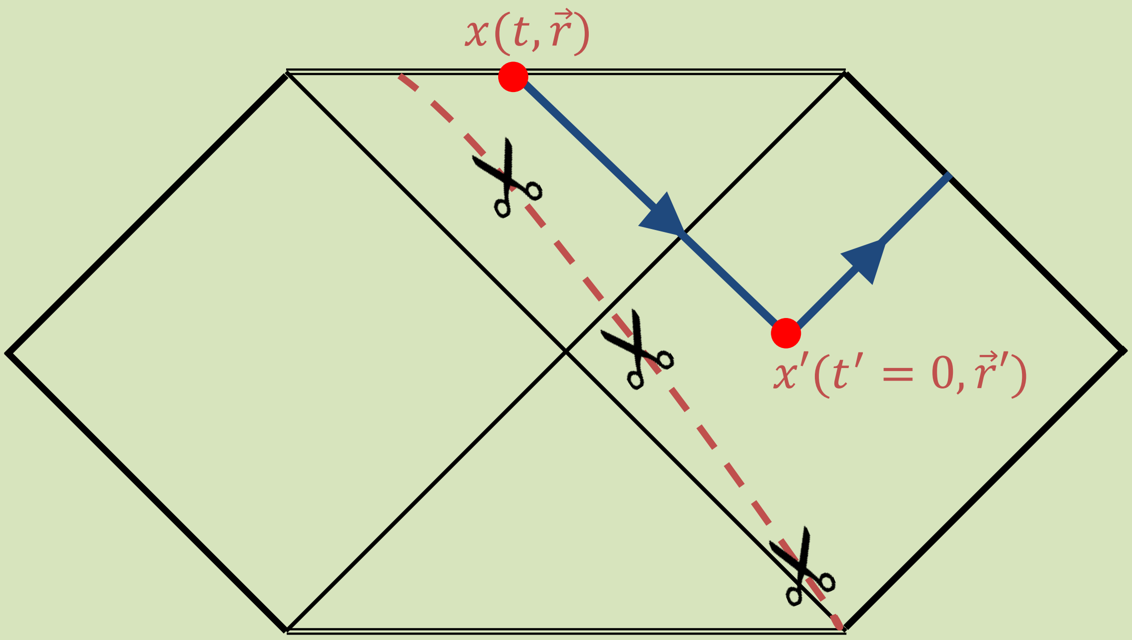


We calculate the transition amplitude

$$S(\vec{r}',\vec{r}\,) = \int_{-\infty}^{+\infty} dt\, e^{-i\omega t}\, K(0,\vec{r}';t,\vec{r}),$$

where $K(0,\vec{r}';t,\vec{r})$ is the propagator with a regulator:

$$K(0,\vec{r}';t,\vec{r}) = -\frac{i}{4\pi^2}\frac{1}{s(x,x')\pm i\epsilon}.$$

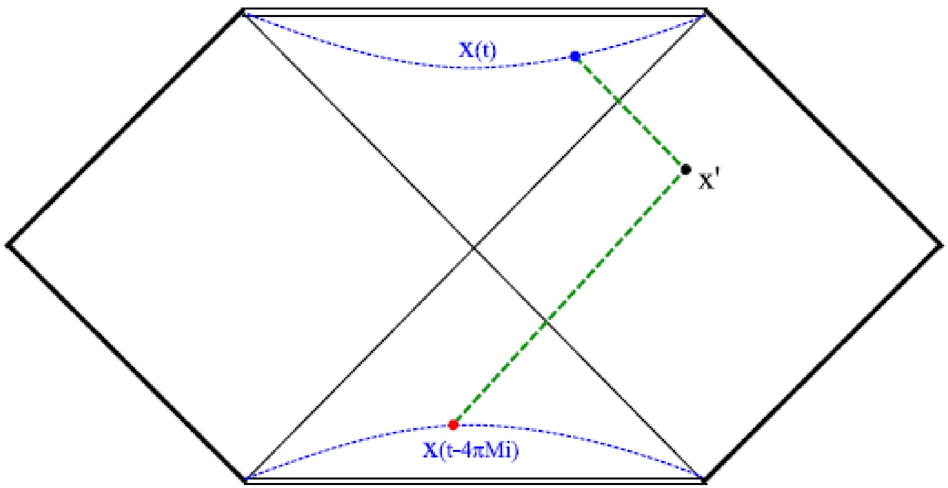


Since the propagator satisfies the relation

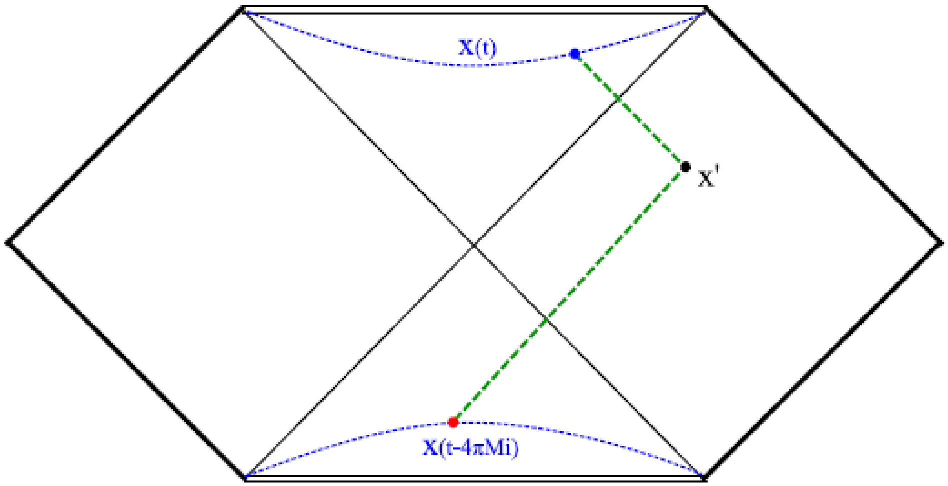
$$K(0,\vec{r}';t,\vec{r}) = K(t,\vec{r};0,\vec{r}') = K(-t,\vec{r}';0,\vec{r})$$

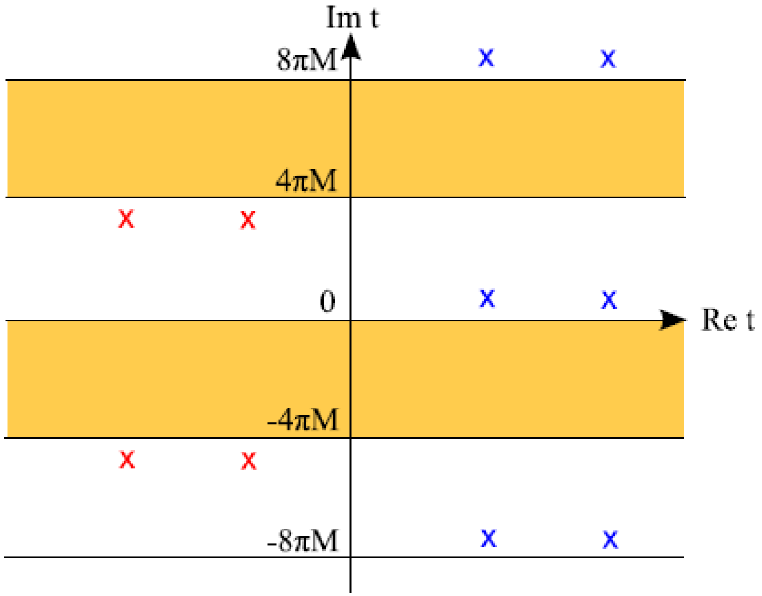
we may have a freedom to choose one of them.

Using $K(t,\vec{r}\,;0,\vec{r}\,')$

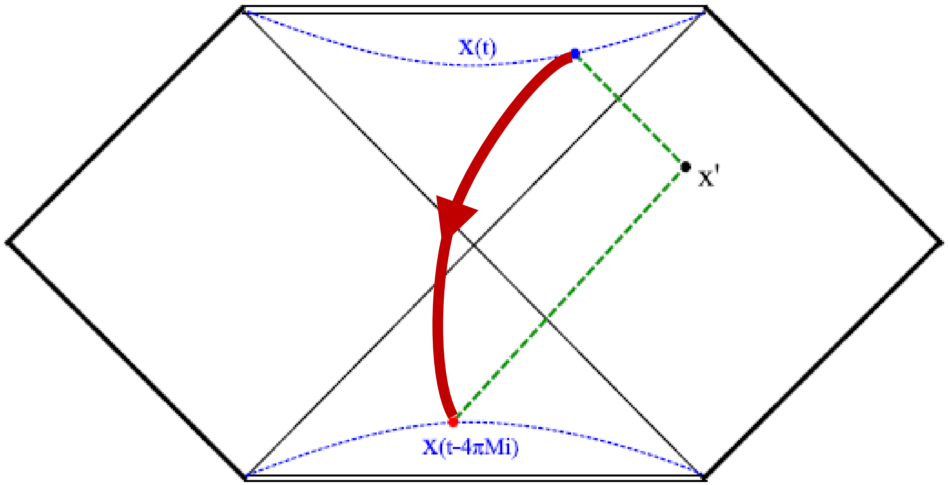


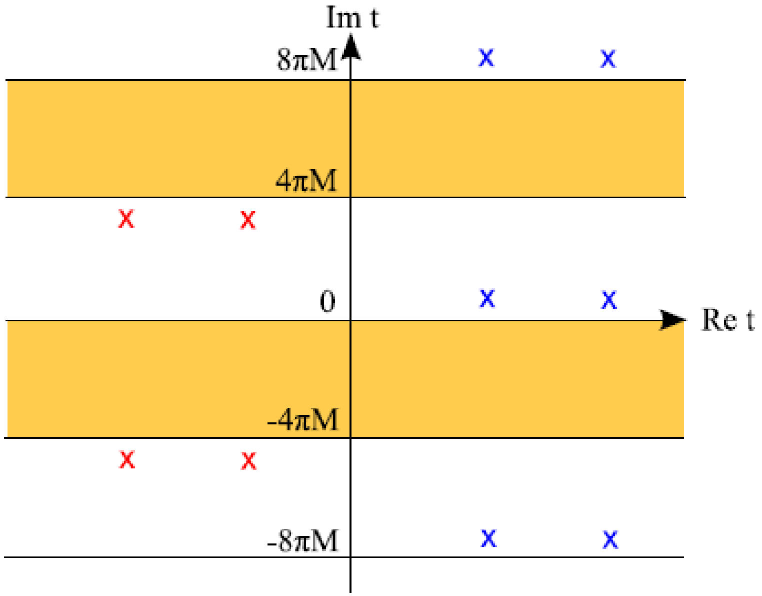
Using $K(t,\vec{r}\,;0,\vec{r}\,')$



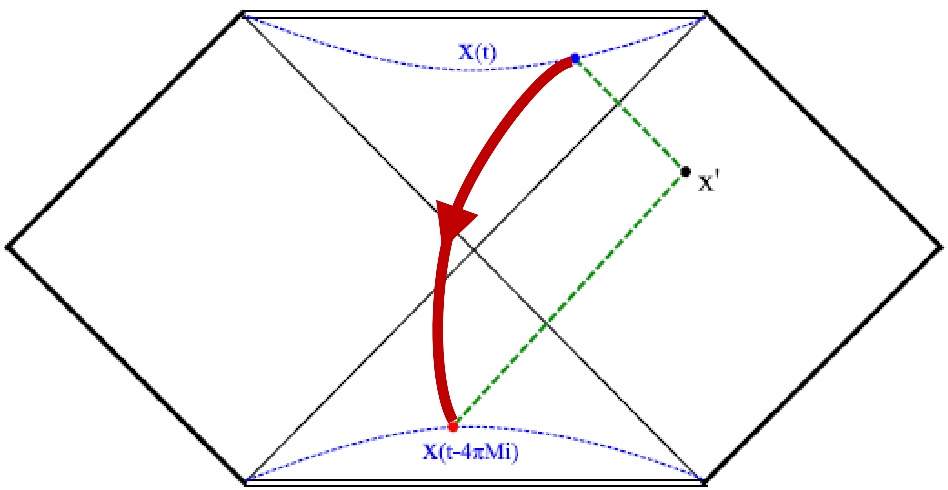


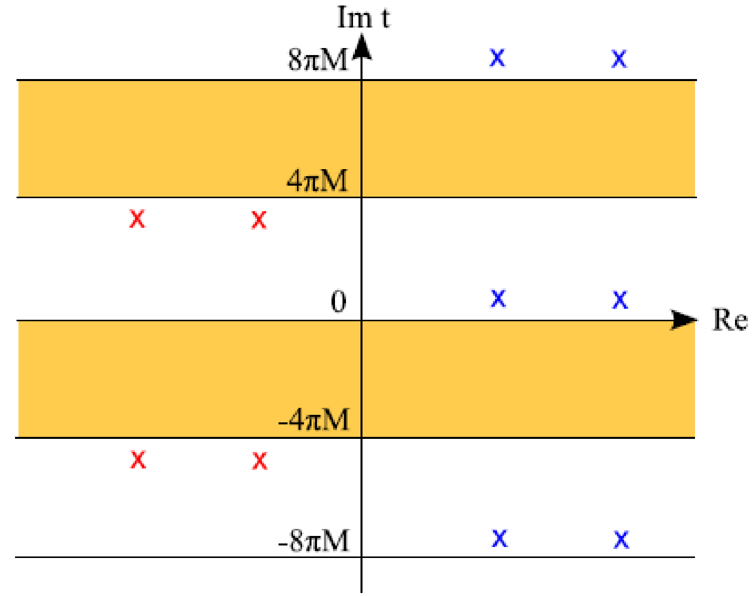
Using $K(t, \vec{r}\,; 0, \vec{r}\,')$



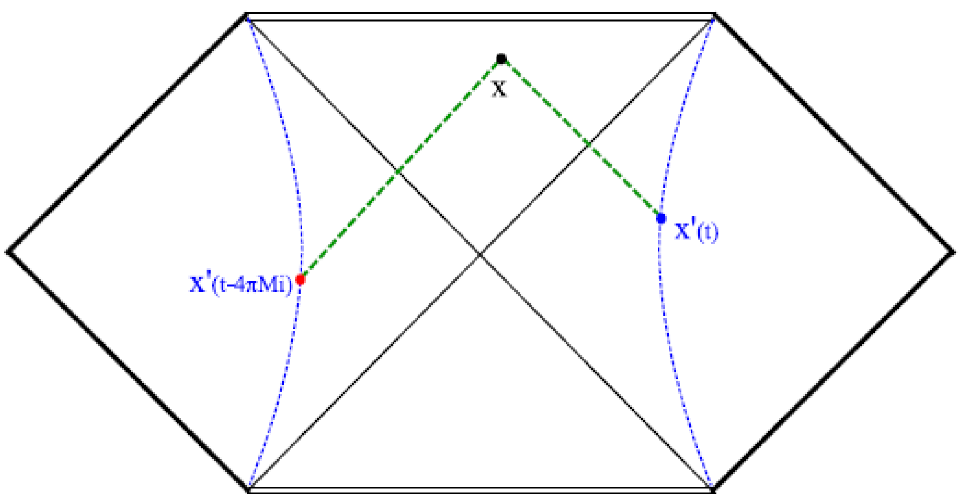


Using $K(t, \vec{r}\,; 0, \vec{r}\,')$

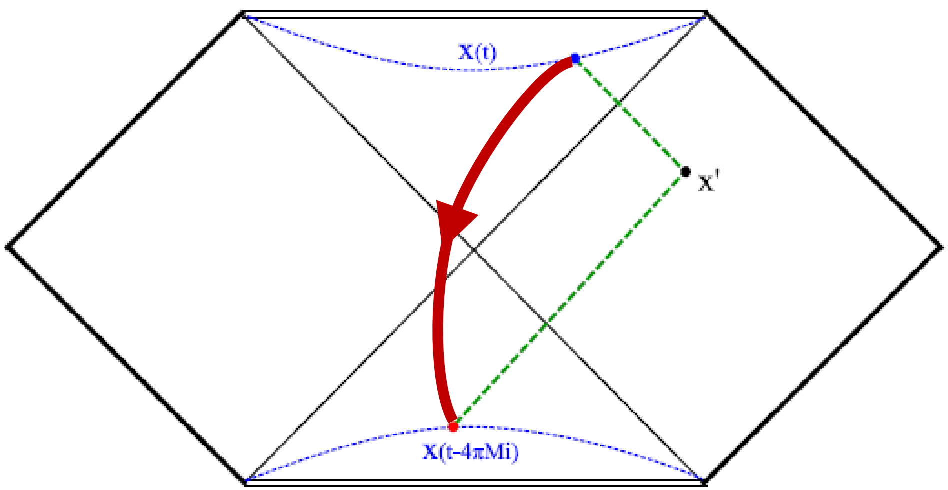


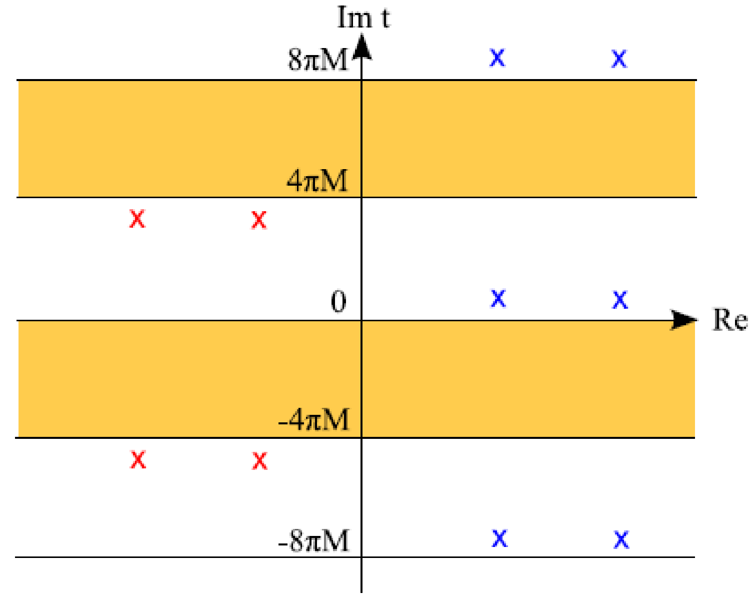


Using $K(-t, \vec{r}\,'\,; 0, \vec{r})$

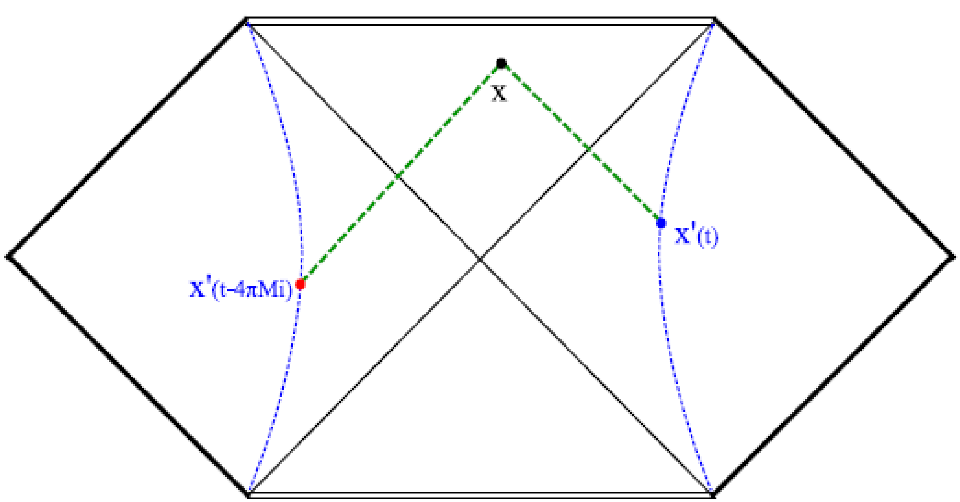


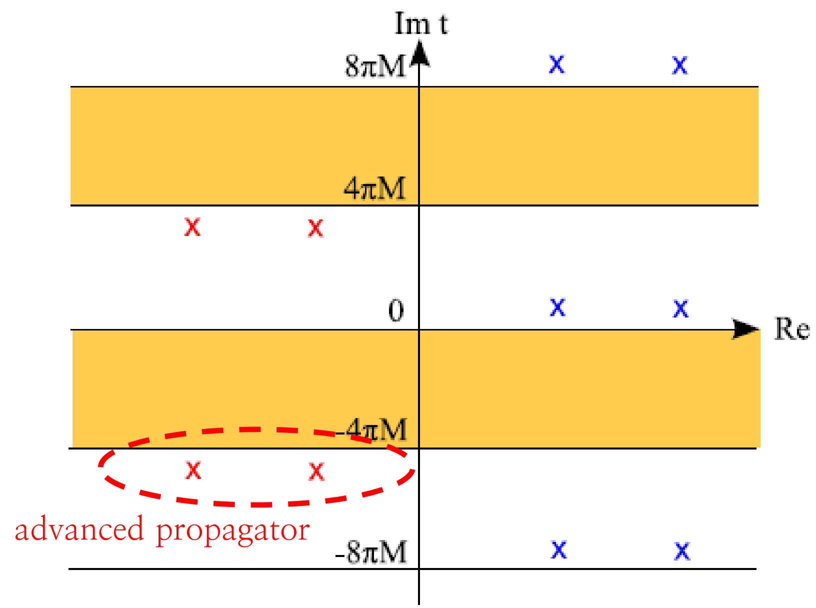
Using $K(t,\vec{r}\,;0,\vec{r}\,')$



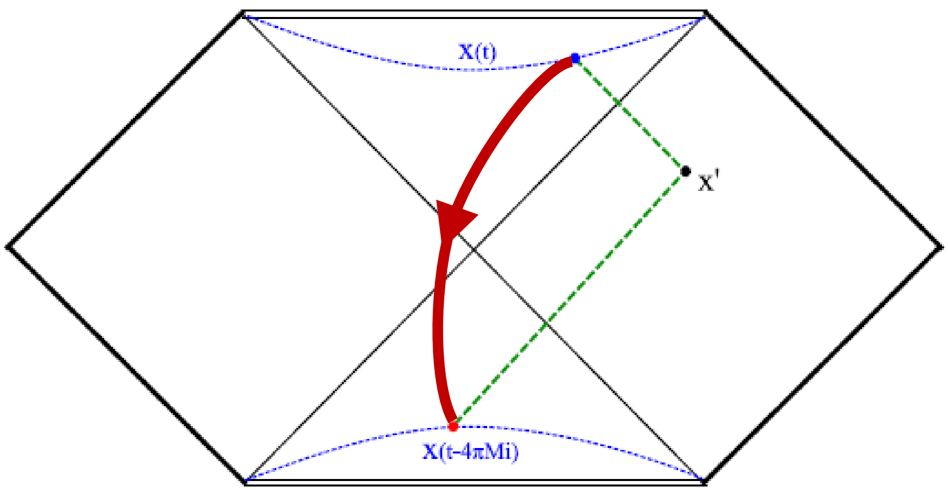


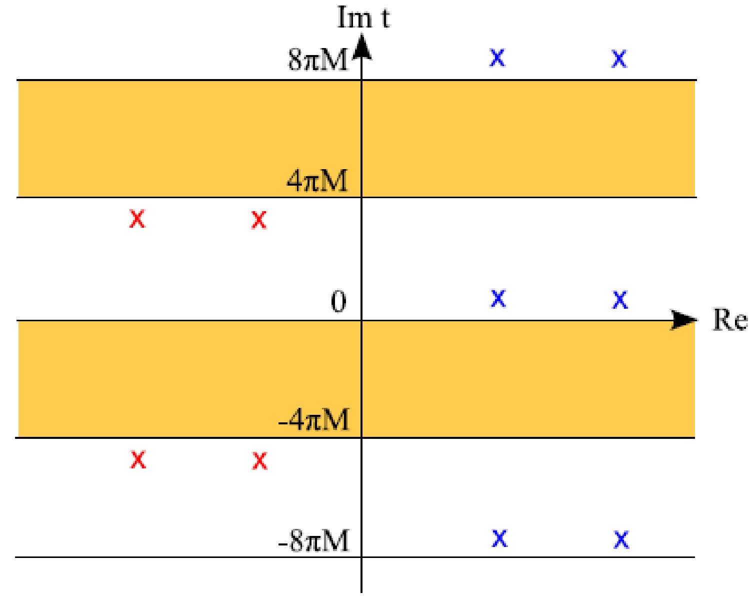
Using $K(-t,\vec{r}\,';0,\vec{r})$



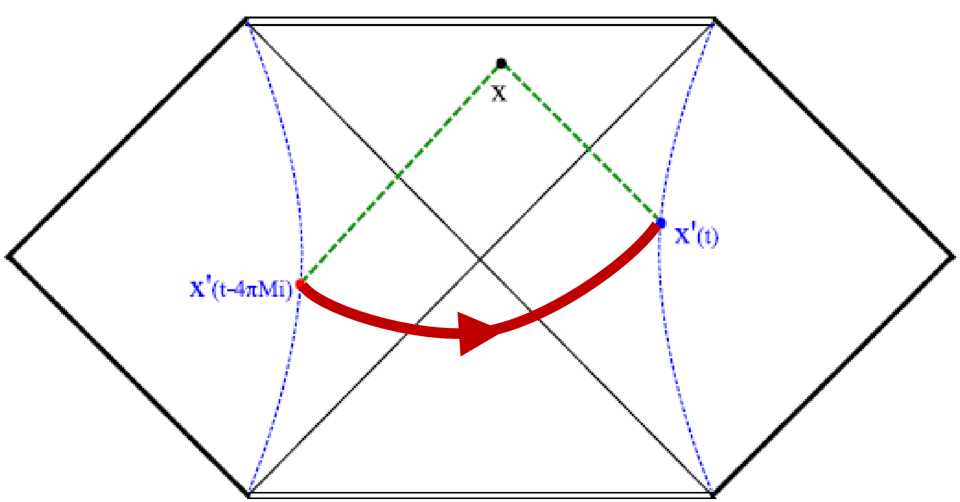


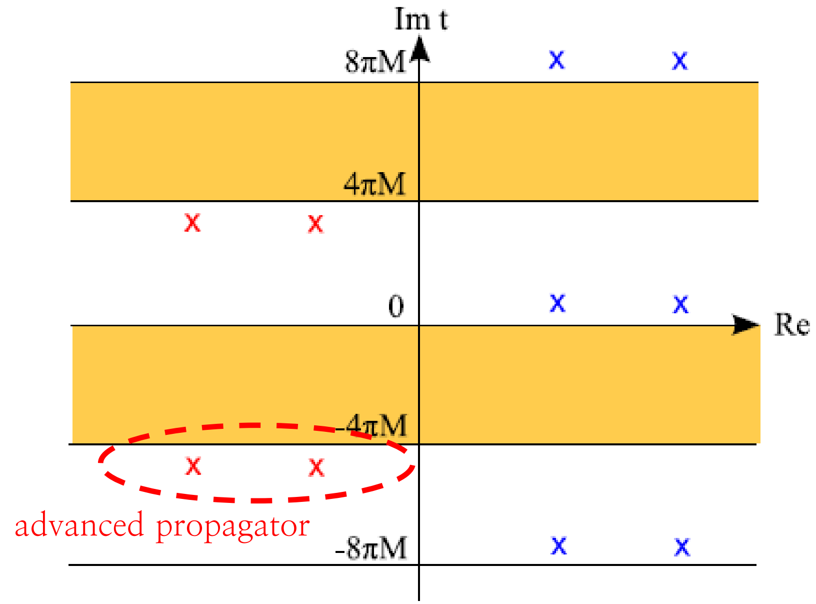
Using $K(t, \vec{r}\,; 0, \vec{r}\,')$

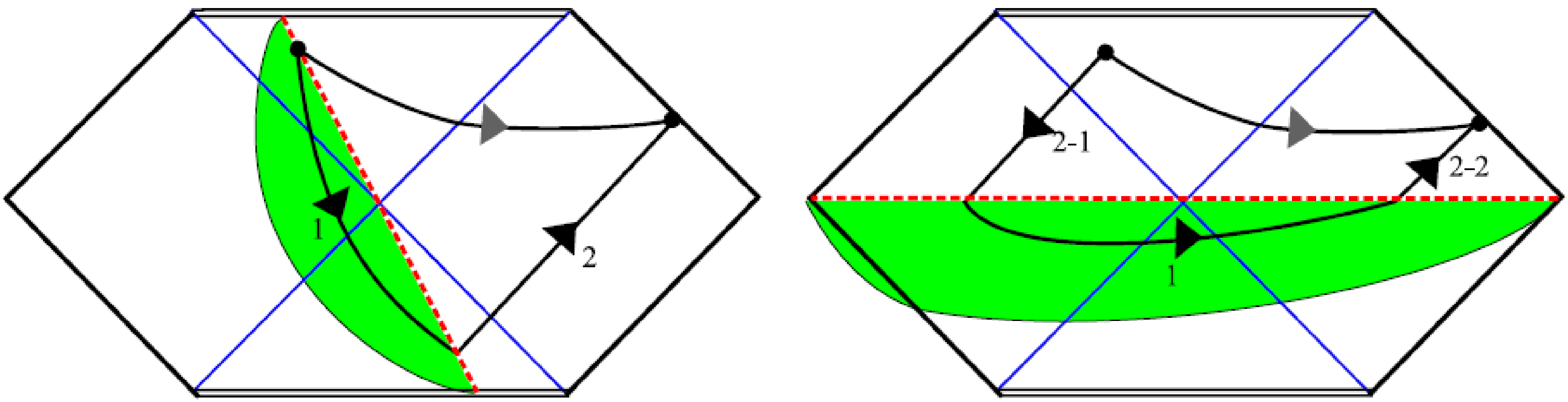




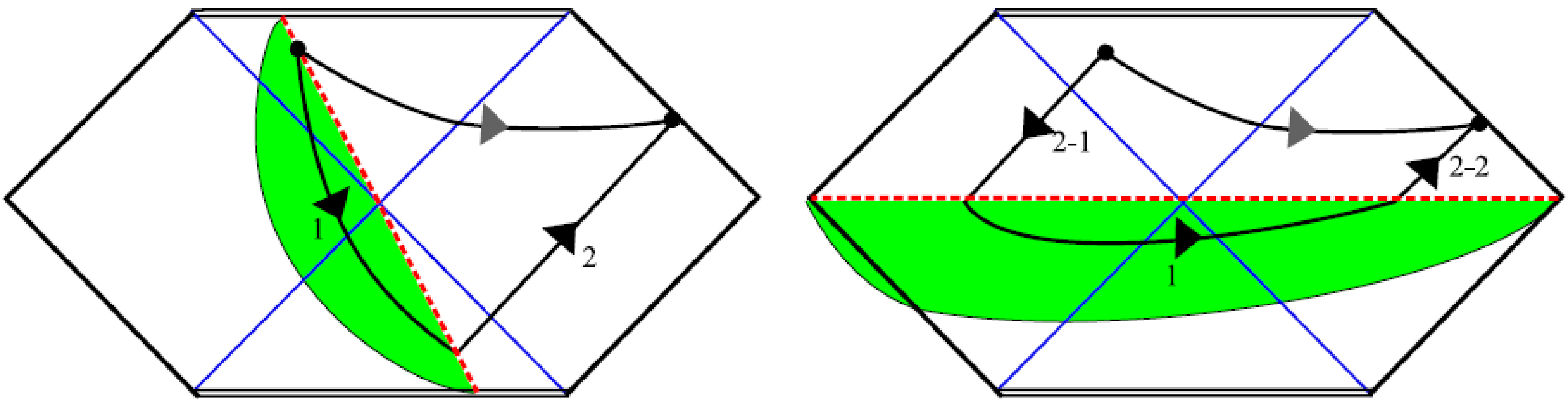
Using $K(-t, \vec{r}\,'; 0, \vec{r})$







Therefore, these two pictures are equivalent.

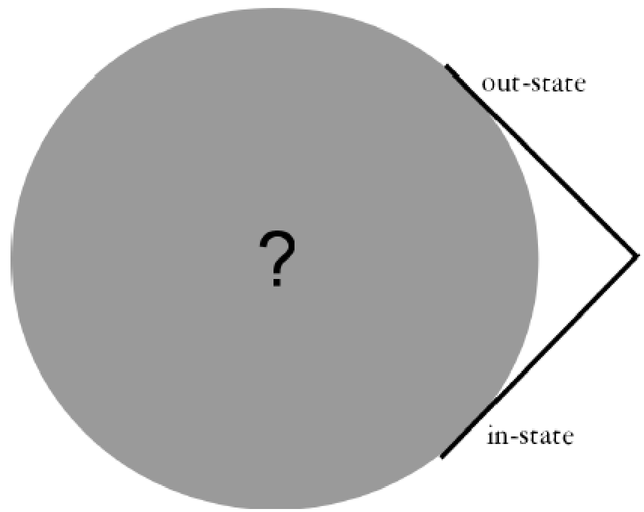


Then, why not generalize to many particles, e.g., instantons?

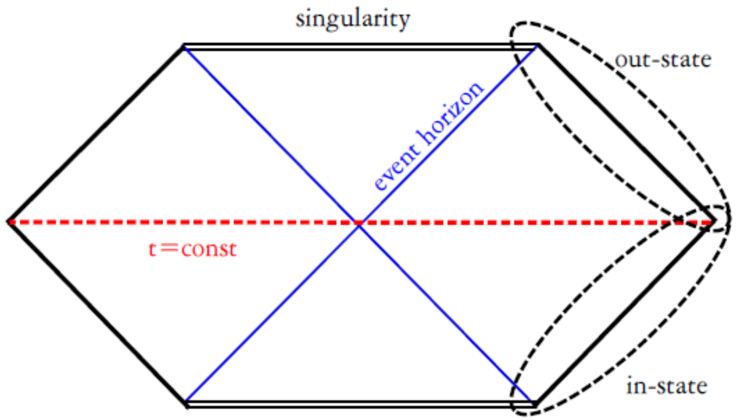
The Fourth Way:

Hawking radiation as instantons

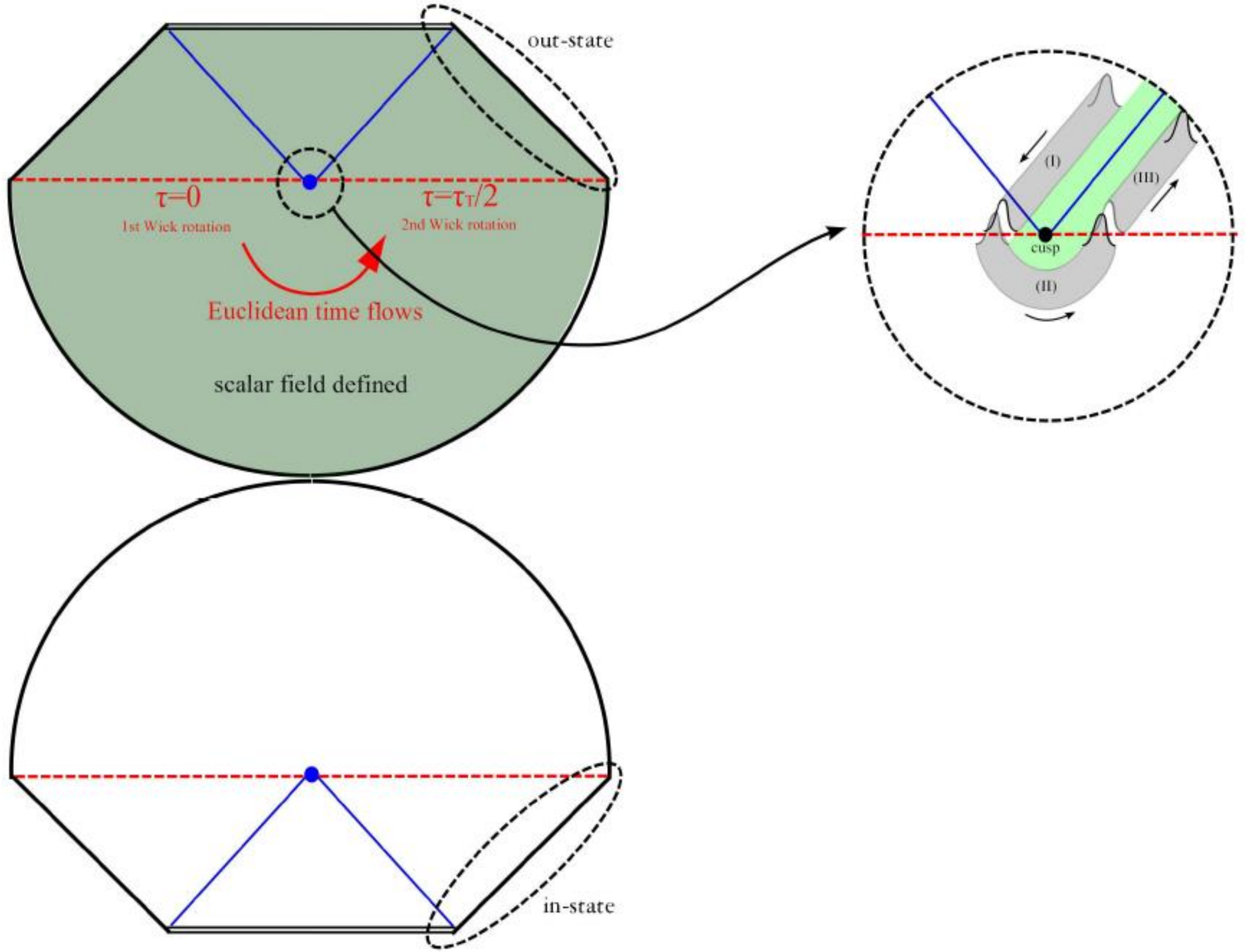
$$\langle f|i\rangle = \int_{i\to f^j} DgD\phi\, e^{-S_E}$$



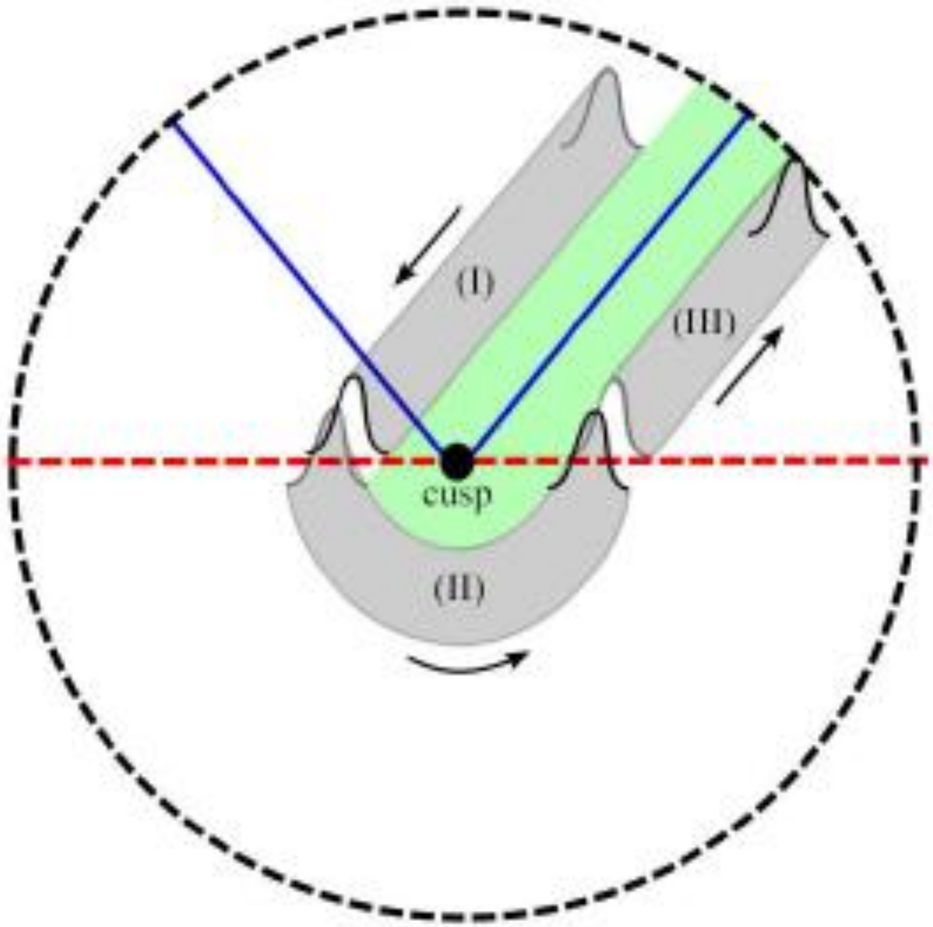
$$\langle f|i\rangle \cong \sum_{i\to f^j} e^{-S_E^{\text{on-shell}}}$$



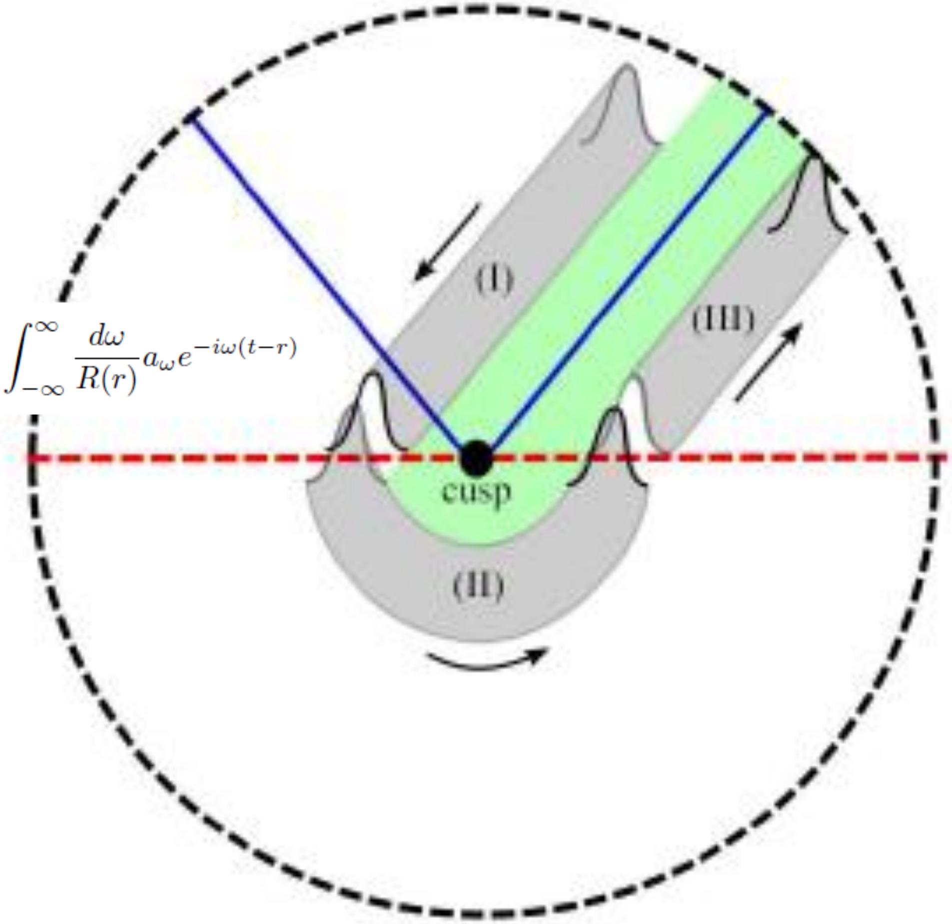
$$\langle f|i\rangle \cong \sum_{i\to f^j} e^{-S_E^{\text{on-shell}}}$$



$$\langle f|i\rangle \cong \sum_{i\to f^j} e^{-S_E^{\text{on-shell}}}$$

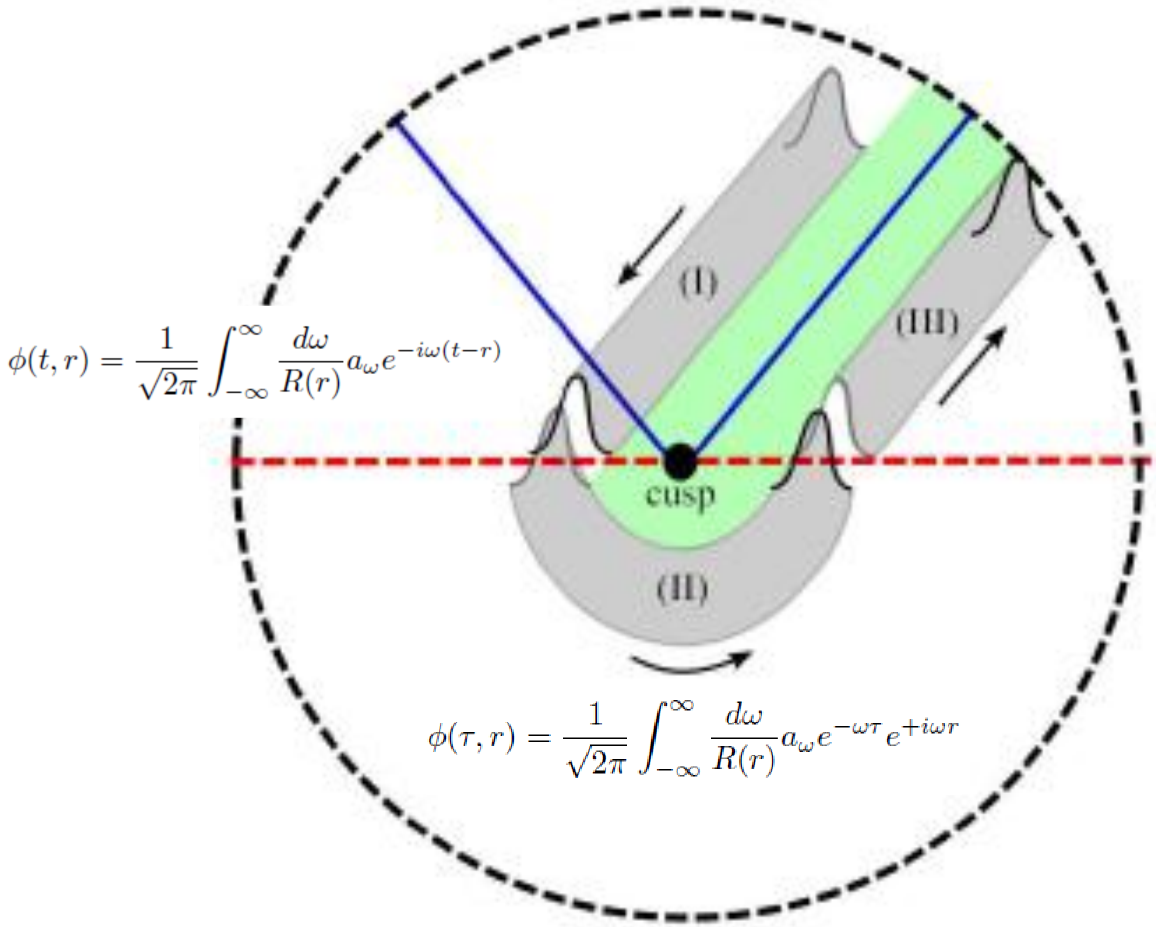


$$\langle f|i\rangle \cong \sum_{i\to f^j} e^{-S_E^{\text{on-shell}}}$$

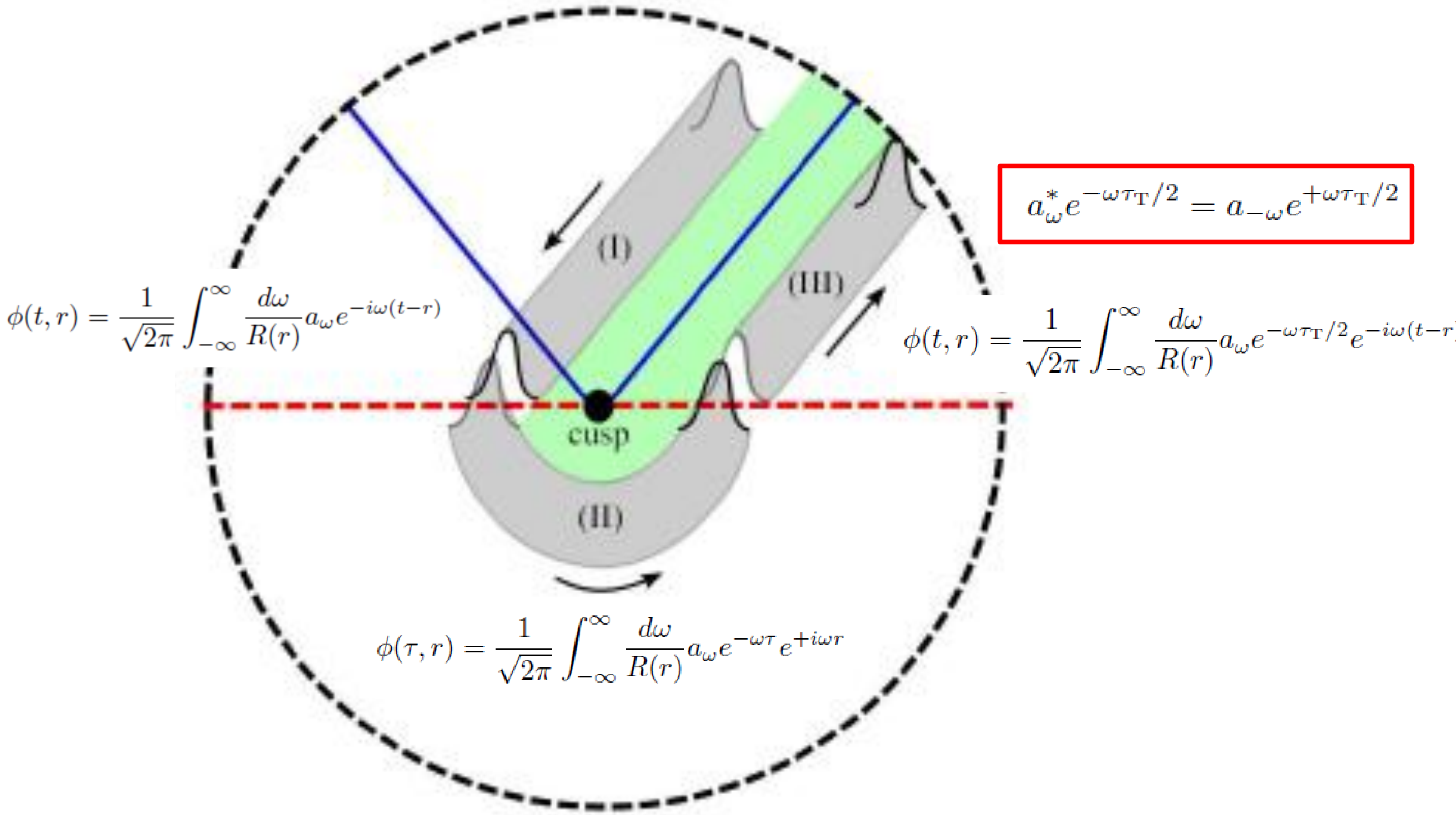


$$\phi(t,r) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{d\omega}{R(r)} a_\omega e^{-i\omega(t-r)}$$

$$\langle f|i\rangle \cong \sum_{i\to f^j} e^{-S_E^{\text{on-shell}}}$$



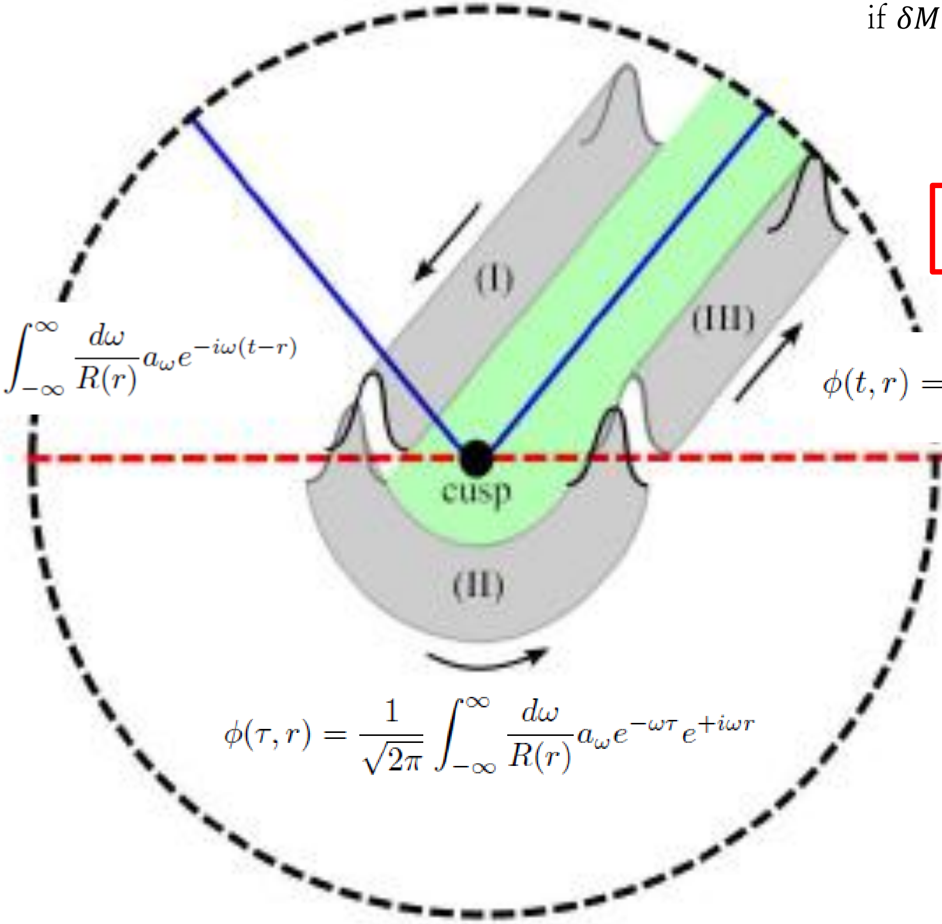
$$\langle f|i\rangle \cong \sum_{i\to f^j} e^{-S_E^{\text{on-shell}}}$$



$$\langle \boldsymbol{f} | \boldsymbol{i} \rangle \cong \sum_{\boldsymbol{i} \to \boldsymbol{f}^j} \boldsymbol{e}^{-S_E^{\text{on-shell}}}$$

$$\boxed{H = \delta M \propto \int_0^\infty d\omega \omega |A_\omega|^2 e^{-\omega \tau_{\mathrm{T}}}}$$

One can construct instantons for
an arbitrary amount of energy
if $\delta M \leq M$.



$$\boxed{a_\omega^* e^{-\omega \tau_{\mathrm{T}}/2} = a_{-\omega} e^{+\omega \tau_{\mathrm{T}}/2}}$$

$$\phi(t, r) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{d\omega}{R(r)} a_\omega e^{-i\omega(t-r)}$$

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$$\phi(\tau, r) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{d\omega}{R(r)} a_\omega e^{-\omega \tau} e^{+i\omega r}$$

For a free scalar field, the Euclidean action only depends on the boundary terms (at infinity and at horizon).

$$S_E = -\int_{\partial\mathcal{M}} \frac{K - K_0}{8\pi} \sqrt{+h} d^3x + (\text{contribution at horizon})$$

The boundary at infinity should be subtracted by the background term. The contribution at horizon is the areal entropy differences (Gregory, Moss and Withers, 2014).

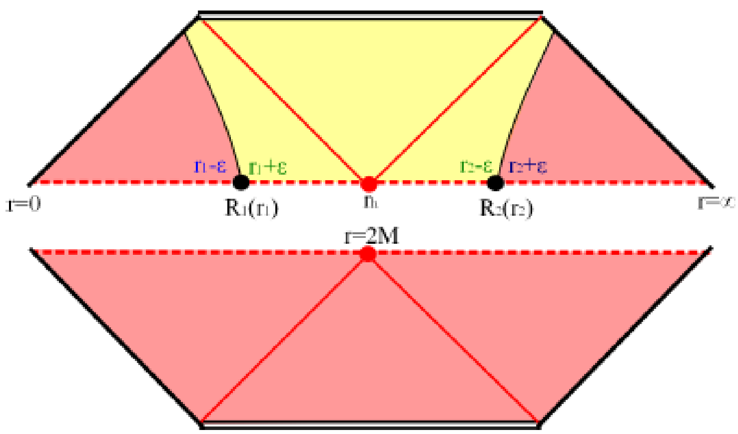
$$2B = \frac{\mathcal{A}_i - \mathcal{A}_f}{4}$$

This result is repeated by using the Hamiltonian approach of Fischler−Morgan−Polchinski

(Chen, Hu and DY, 2015)

$$i\Sigma = -\int_0^\infty dr \left[R\sqrt{L^2 f(R) - R'^2} - RR' \cos^{-1}\left(\frac{R'}{L\sqrt{f(R)}}\right)\right]$$

$$P \cong \left|\frac{\Psi_f}{\Psi_i}\right|^2 = e^{2i(\Sigma_f - \Sigma_i)}$$



$$\int_0^{r_1-\epsilon} dr(\dots) + \int_{r_1-\epsilon}^{r_1+\epsilon} dr(\dots) + \int_{r_1+\epsilon}^{r_2-\epsilon} dr(\dots) + \int_{r_2-\epsilon}^{r_2+\epsilon} dr(\dots) + \int_{r_2+\epsilon}^{\infty} dr(\dots)$$

$$\boxed{-\log P \cong 2B = \frac{\mathcal{A}_i - \mathcal{A}_f}{4}}$$

Therefore, finally we can recover Hawking's result.

$$\Gamma \propto e^{-2B} \simeq e^{-8\pi M \delta M} \quad \text{if} \;\; \delta M \ll M$$

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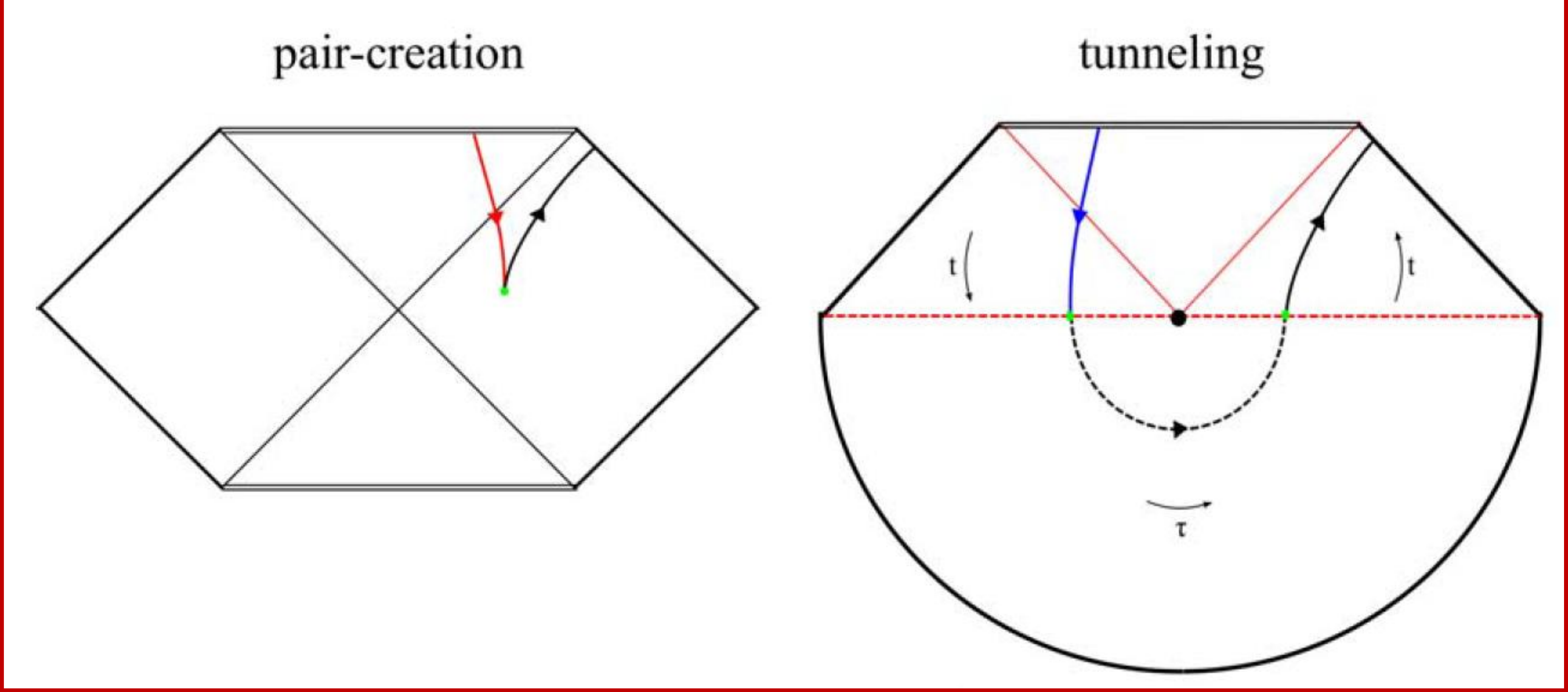
$$\Gamma \propto e^{-2B} \simeq e^{-8\pi M \delta M} \quad \text{if} \;\; \delta M \ll M$$

We further observe that there exist plenty of instantons with $\delta M / M \leq 1$.

Dual interpretation

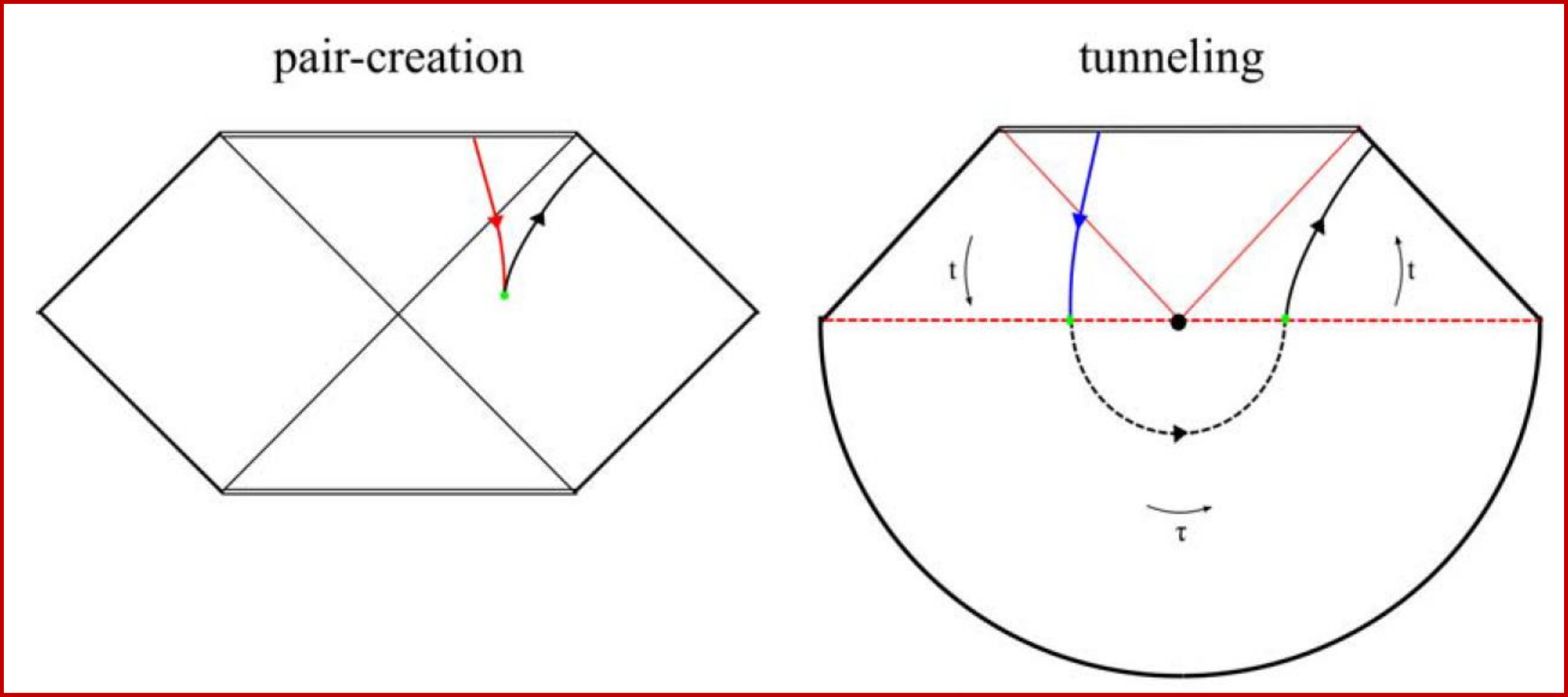
pair−creation vs. instanton tunneling



Can this be extended

not only perturbative regime (Hawking radiation)

but also non–perturbative processes?



We can study thin-shell tunneling
as an example of the non-perturbative process.

$$\Psi\left[h_{ab}^{\mathrm{out}},\phi^{\mathrm{out}};h_{ab}^{\mathrm{in}},\phi^{\mathrm{in}}\right]=\int\mathcal{D}g_{\mu\nu}\mathcal{D}\phi\ e^{-S_{\mathrm{E}}[g_{\mu\nu},\phi]}\simeq\sum_{\mathrm{on-shell}}e^{-S_{\mathrm{E}}^{\mathrm{on-shell}}}$$

$$S_{\mathrm{E}}=-\int dx^4\sqrt{g}\left[\frac{1}{16\pi}\mathcal{R}-\frac{1}{2}\left(\nabla\phi\right)^2\right]+\int_{\partial\mathcal{M}}\frac{\mathcal{K}-\mathcal{K}_o}{8\pi}\sqrt{h}dx^3$$

$$\Psi\left[h_{ab}^{\text{out}},\phi^{\text{out}};h_{ab}^{\text{in}},\phi^{\text{in}}\right]=\int\mathcal{D}g_{\mu\nu}\mathcal{D}\phi\ e^{-S_{\text{E}}[g_{\mu\nu},\phi]}\simeq\sum_{\text{on-shell}}e^{-S_{\text{E}}^{\text{on-shell}}}$$

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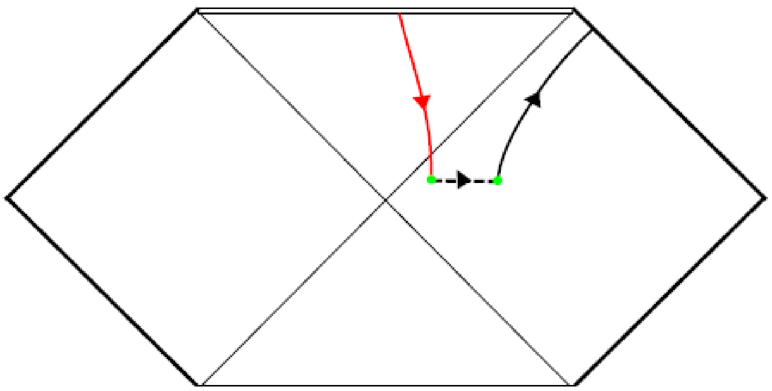
junction condition of the shell

$$ds_{\pm}^2=-f_{\pm}(R)dT^2+\frac{1}{f_{\pm}(R)}dR^2+R^2d\Omega^2$$

$$ds_{\text{shell}}^2=-dt^2+r^2(t)d\Omega^2$$

$$\epsilon_-\sqrt{\dot{r}^2+f_-(r)}-\epsilon_+\sqrt{\dot{r}^2+f_+(r)}=4\pi r\sigma$$

pair-creation

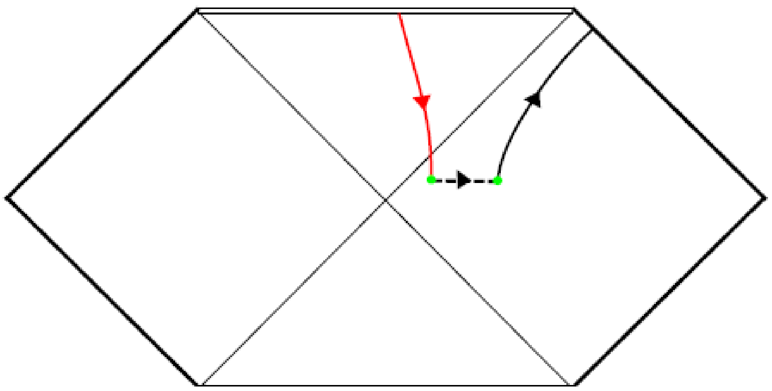


We interpret a **negative−tension shell** falls in and the entropy of the black hole decreases.

The Euclidean action difference becomes

$$\Delta S_E = \frac{\Delta F}{T} = \frac{\Delta(E - ST)}{T} = -\Delta S = -\frac{\Delta A}{4}$$

where the energy is conserved.



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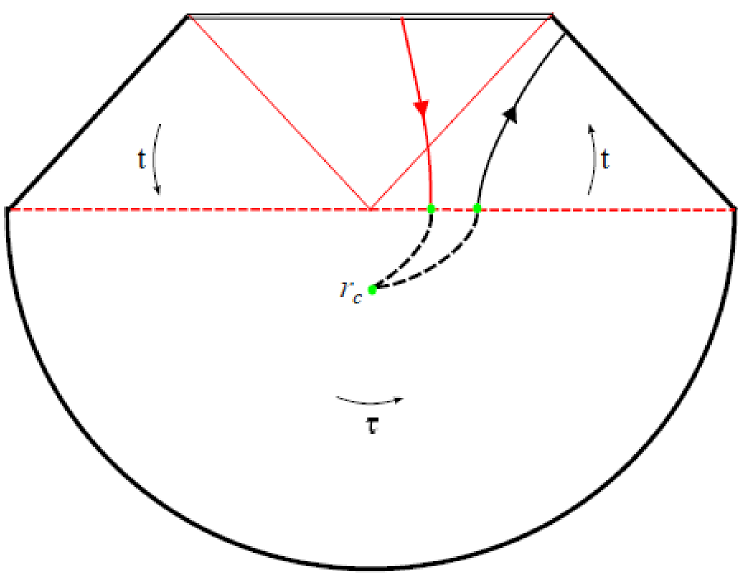
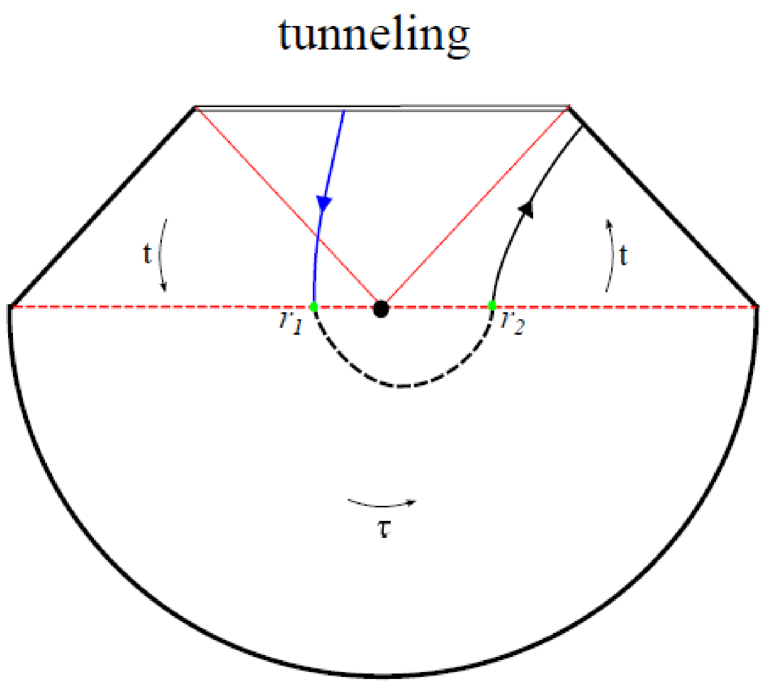


In addition, there exist contributions from the shell−dynamics, which has the following form.

$$\text{(shell integration)} = 2\int_{r_1}^{r_2} dr r \left| \cos^{-1}\left(\frac{f_+ + f_- - 16\pi^2\sigma^2 r^2}{2\sqrt{f_+ f_-}} \right) \right|$$



On the other hand, in the **instanton** picture, **instanton** lives on the Euclidean manifold.

The contribution comes from the cusp of the Euclidean manifold

$$\Delta S_E = -\frac{\Delta A}{4}$$

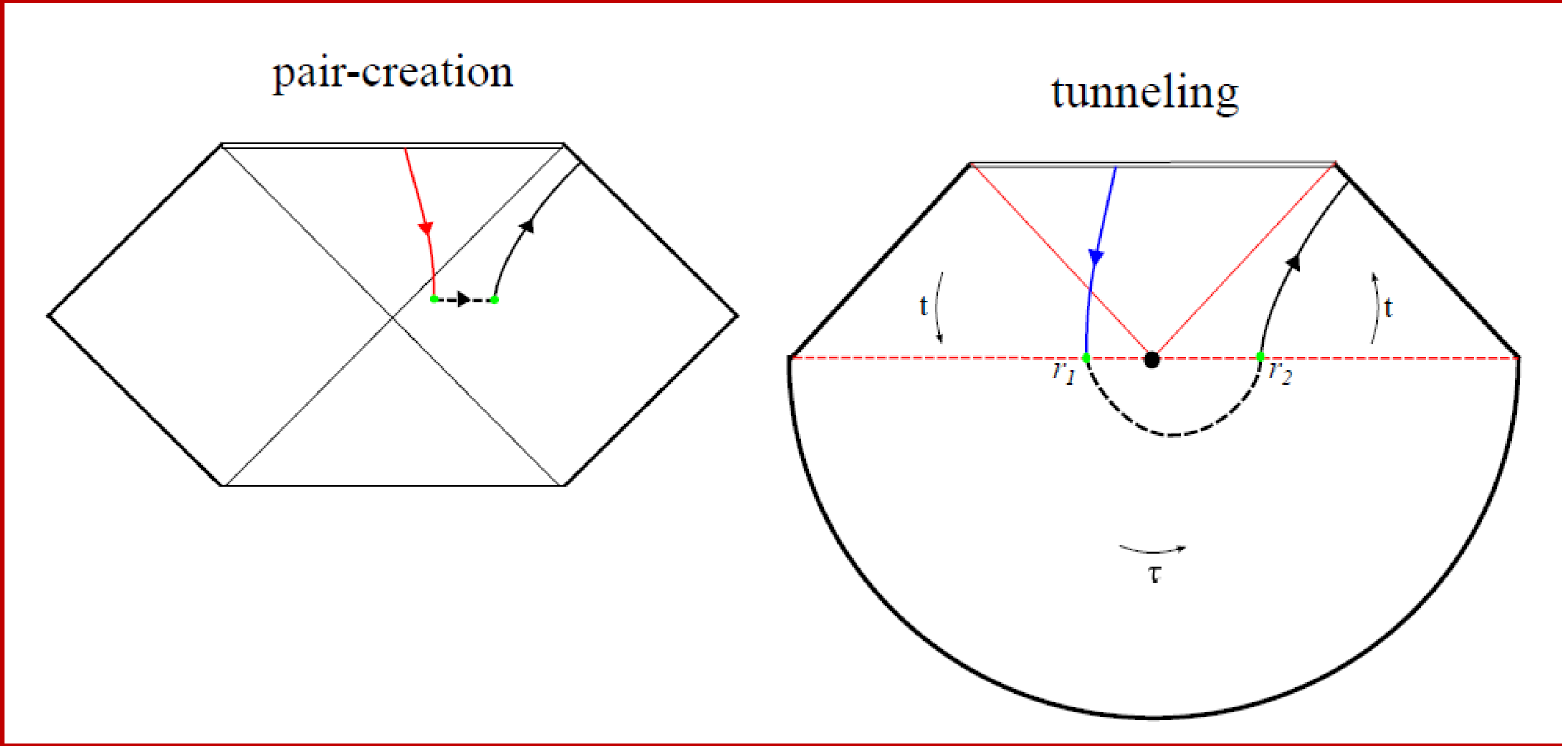
after a proper regularization.

Due to the symmetry, they have the same shell−integration term.

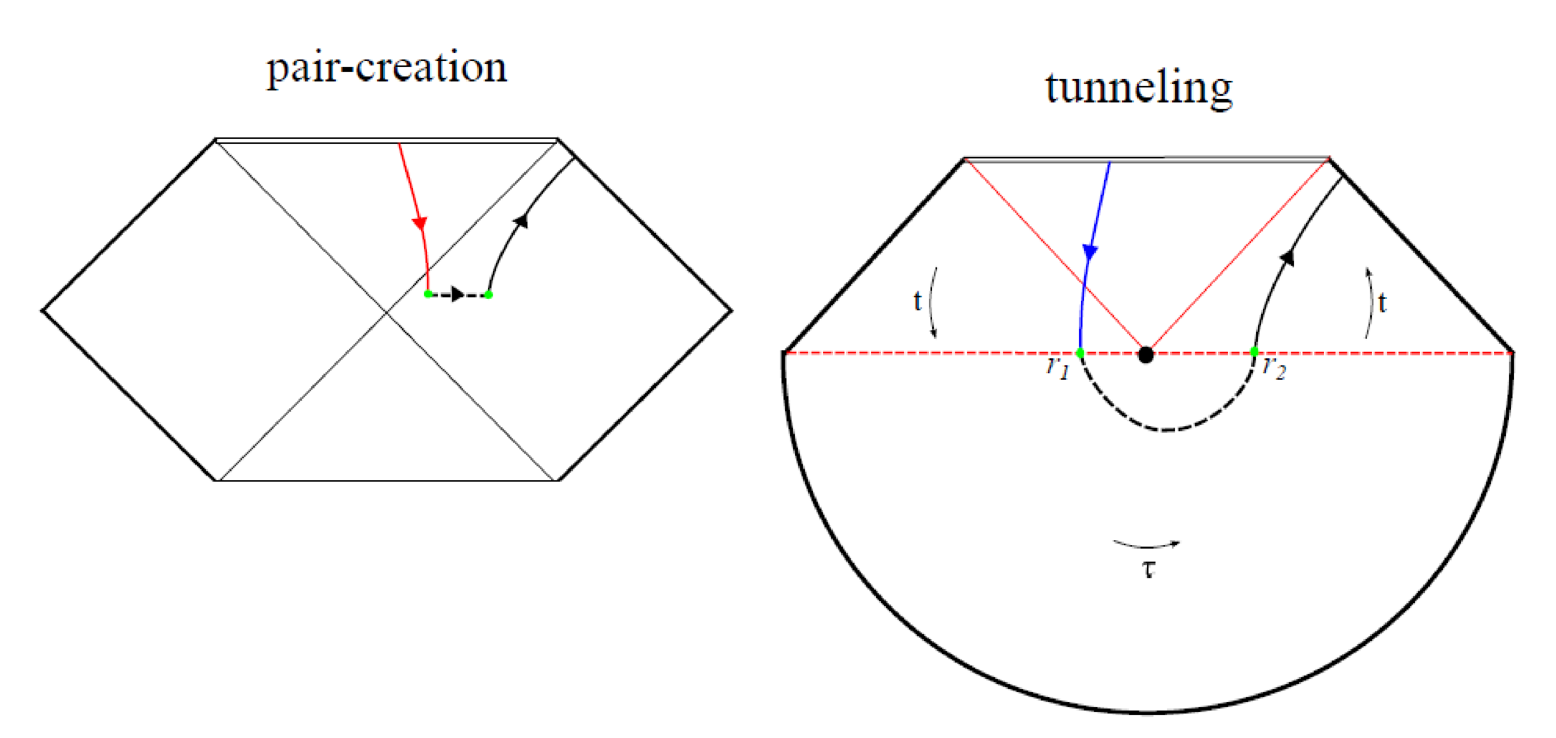
$$(\text{shell integration}) = 2\int_{r_1}^{r_2} dr\, r \left|\cos^{-1}\left(\frac{f_+ + f_- - 16\pi^2\sigma^2 r^2}{2\sqrt{f_+ f_-}}\right)\right|$$

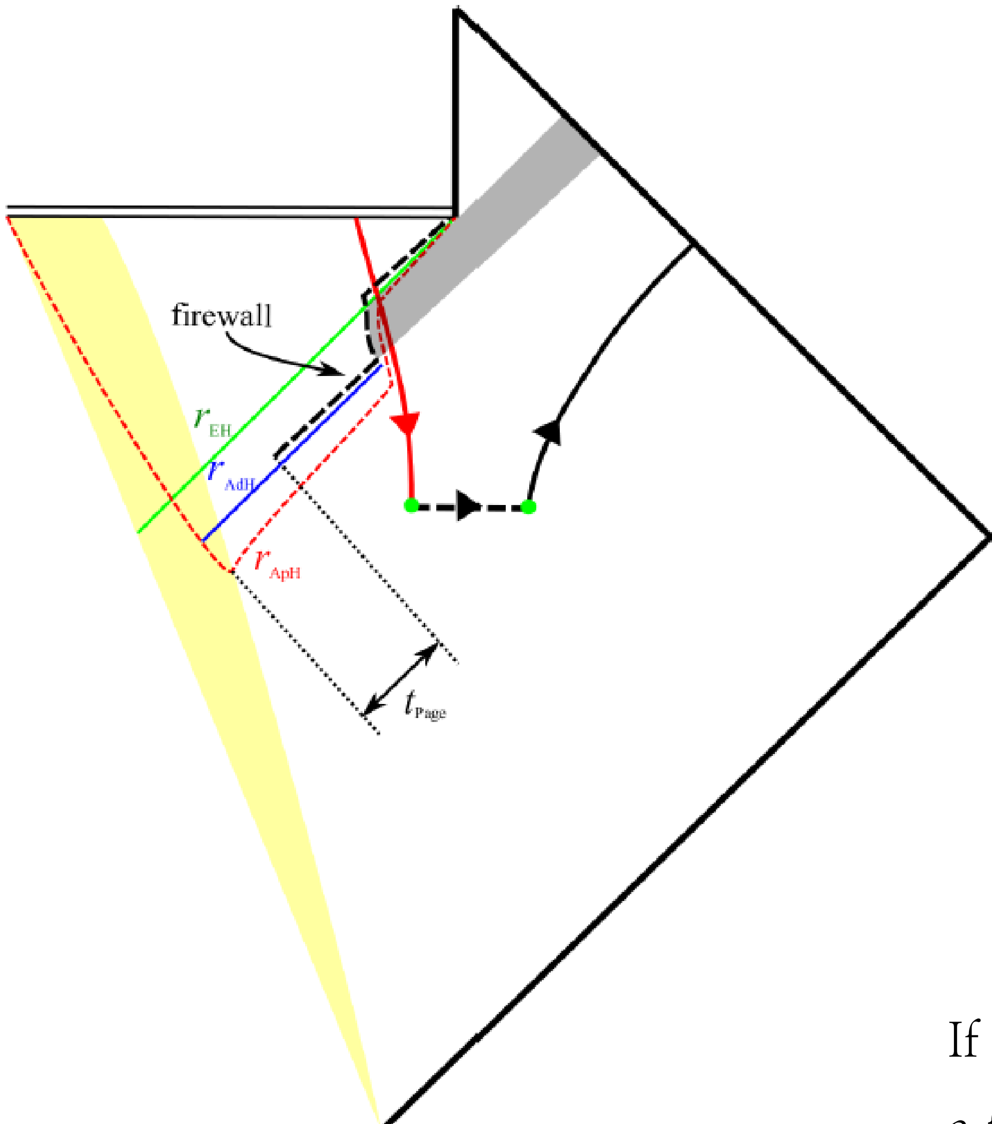
Dual interpretation

pair–creation vs. instanton tunneling

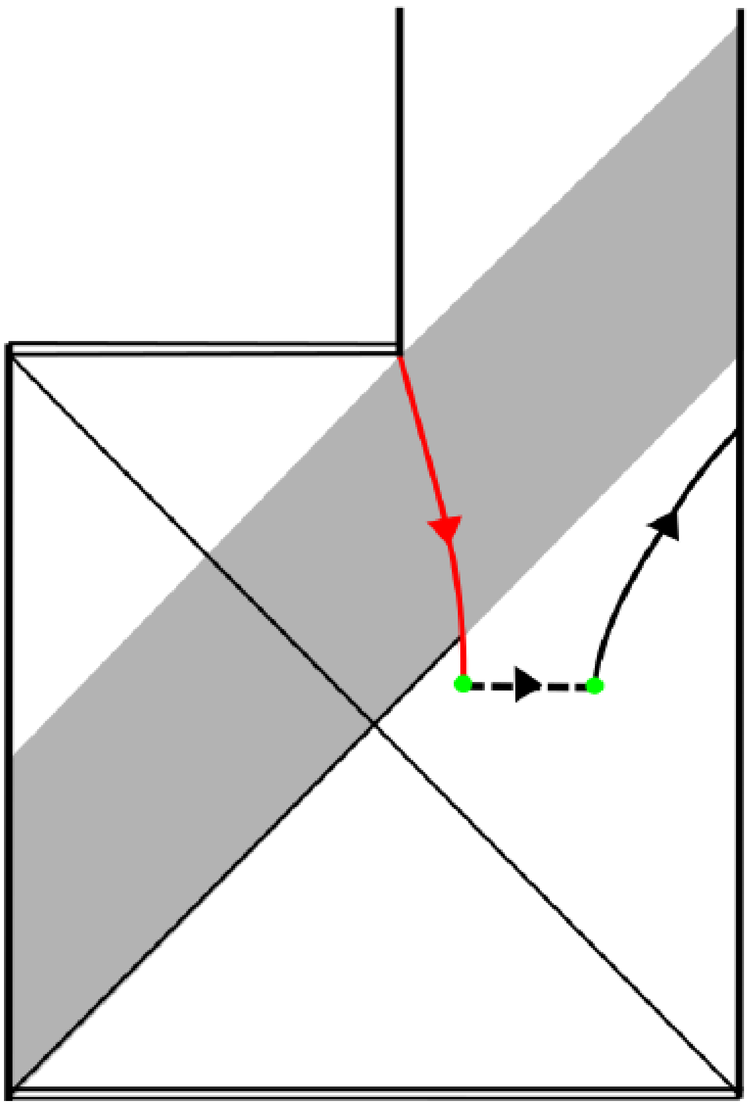


The second picture is rather mathematically complete, while the first picture provides easier conceptual interpretations.





If there exists a non-perturbative process,
a **firewall** can be naked (if exists).
(Chen, Ong, Page, Sasaki and DY, 2016)



If there exists a non−perturbative process,
an **Einstein−Rosen bridge** can be traversable
which is inconsistent to the ER=EPR philosophy.
(Chen, Wu and DY, 2017)

Thank you very much