The Role of Symmetry in the Universal Relations of Holography

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Based on works with:
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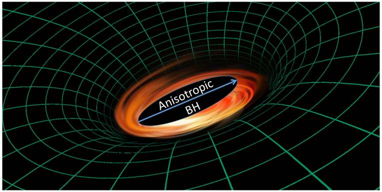
Online Talk given for : 4th International Conference on Holography, String Theory and Discrete Approach, 5 August, 2020

Outline

Motivation I.

- Strongly anisotropic systems have significantly richer structure compared to isotropic ones.
  - → Transport coefficients, Diffusion, Brownian Heavy Quark Motion...
  - → **Characteristic Example:** Shear viscosity η over entropy density s: takes parametrically low values wrt degree of anisotropy $\frac{\eta}{s} < \frac{1}{4\pi}$. (Rebhan,Steineder 2011; D.G. 2012; Jain, Samanta Trivedy 2015;... D.G., Gursoy, Pedraza, 2017;...)
  - → Universalities are very rare! Not surprising!



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(Anisotropy, Temperature)

  - → Anisotropic Universalities?

Motivation II.

- Existence of strongly coupled anisotropic systems.
  - $\rightarrow$ Quark - Gluon Plasma .
  - $\rightarrow$ Anisotropic Materials: Multilayer nanostructures...
    eg: (Pardo, Pickett 2009; Kobayashi, Suzumura, Piechon, Montambaux 2011; Fang, Fu 2015...)
- Strong (Magnetic) Fields in strongly coupled theories.
  - $\rightarrow$ New interesting phenomena in presence on such fields, i.e. inverse magnetic catalysis.
    eg: (Bali, Bruckmann, Endrodi, Fodor, Katz, Krieg et al. 2011)

Reminding-Example Slide: A Fixed Anisotropic Point

- The anisotropic hyperscaling violation metric in IR:

$$ds^2_{d+2} = u^{-\frac{2\theta}{d}}\left(u^{2z}\left(-f(u)dt^2 + dy_i^2\right) + u^2 dx_i^2 + \frac{du^2}{f(u)u^2}\right)$$

exhibits a critical exponent z and a hyperscaling violation exponent θ.

$$t \to \lambda^z t, \qquad y \to \lambda^z y, \qquad x \to \lambda x, \qquad u \to \frac{u}{\lambda}, \qquad ds \to \lambda^{\frac{\theta}{d}} ds\ .$$

- Thermodynamically it behaves as receiving contributions from a theory in $k-\theta$ dimensions with scaling z and from a $d-k$ dimensions conformal theory.

$$S \sim T^{\frac{k-\theta}{z}+d-k}\ , \quad \left(S_{hysca,iso} \sim T^{\frac{d-\theta}{z}}, \quad S_{Lif,iso} \sim T^{\frac{d}{z}}\right)$$

- Effective Space dimensionality for the dual theory!

How is Anisotropy introduced? A Pictorial Representation:

- For the Lifshitz-like IIB Supergravity solutions

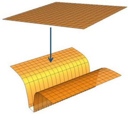
$$ds^2 = u^{2z}(dx_0^2 + dx_i^2) + u^2 dx_3^2 + \frac{du^2}{u^2} + ds_{S^5}^2 .$$

Introduction of additional branes: *(Azeyanagi, Li, Takayanagi, 2009)*

Spacetime

AdS →

	x_0	x_1	x_2	x_3	u	S^5
D3	x	x	x	x		
D7	x	x	x			x



- Which equivalently leads to the following AdS/CFT deformation.

$N = 4\; sYM$ ⟷ $AdS_5 \; x \; S^5 ,\; \varphi ,\; F_5$

$\tau = \frac{\vartheta}{4\pi} + \frac{4\,\pi\, i}{g_{ym}^2} = \chi + i e^{-\phi}$

$S = S_{N=4} + \delta S$
$\delta S = \int \theta(x_3)\, Tr\, F \wedge F$ ⟷ $\overline{AdS_5} \; x \; S^5 ,\; \varphi ,\; \chi(x_3) ,\; F_5$

- $dC_8 \sim \star d\chi$ with the non-zero component $C_{x_0 x_1 x_2 S^5}$.

A Theory with Phase Transitions in One Page:

- How the Field Theory looks like?
  - ✓ 4d $SU(N)$ Strongly coupled anisotropic gauge theory.
  - ✓ Its dynamics are affected by a scalar operator $\mathcal{O}_\Delta$.
  - ✓ Anisotropy is introduced by another operator $\tilde{\mathcal{O}} \sim \theta(x_3) TrF \wedge F$ with a space dependent coupling.

- The gravity dual theory is an Einstein-Axion-Dilaton theory in 5 dimensions with a non-trivial potential.
  - ✓ A "backreacting" scalar field depending on spatial directions, the axion; and a non-trivial dilaton.
  - ✓ Solutions are non-trivial RG flows:
    Conformal fixed point in the UV $\Rightarrow$ Anisotropic (Hyperscaling Lifshitz-like) in IR.

- The vacuum state confines color and there exists a phase transition at finite T_c above which a deconfined plasma state arises.

(D.G., Gursoy, Pedraza, 2017)

An Anisotropic Theory

The generalized Einstein-Axion-Dilaton action with a potential for the dilaton and an arbitrary coupling between the axion and the dilaton:

$$S = \frac{1}{2\kappa^2}\int d^5x\,\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi,\gamma,\sigma,\Delta)) - \frac{1}{2}Z(\phi,\gamma)(\partial\chi)^2\right].$$

- For $\sigma = 0, \gamma = 1, m(\Delta) = 0$ the action and the solution of eoms, are reduced of IIB supergravity.

(Mateos, Trancanelli, 2011)

- $V(\phi)$: Asymptotically AdS for small dilaton in the UV.

((Gubser, Nellore), Pufu, Rocha 2008a,b;Gursoy, Kiritsis, Nitti, 2007;...)

- Anisotropy: $\frac{\partial\chi}{\partial x_3} = \alpha$.
α: Uniform D7-brane charge density per unit length $\sim$ strength of Anisotropy.

A Solution : The RG Flow

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}(u)\mathcal{B}(u)\,dt^2 + dx_1^2 + dx_2^2 + \mathcal{H}(u)dx_3^2 + \frac{du^2}{\mathcal{F}(u)}\right),$$

$$\chi = \alpha x_3\,, \qquad \phi = \phi(u)\,, \qquad \mathcal{F}(u_h) = 0\,.$$

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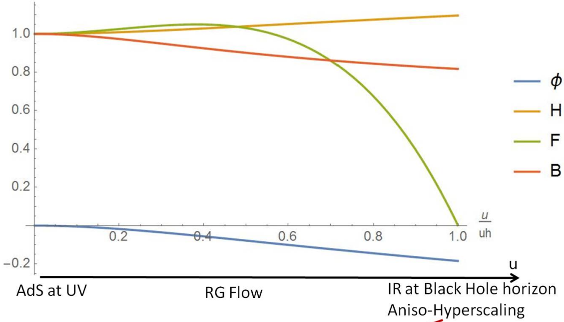

$$ds^2 = u^{-\frac{2\theta}{3}}\left(-u^{2z}\left(f(u)dt^2 + dx_{1,2}^2\right) + \tilde{\alpha}\,u^2 dx_3^2 + \frac{du^2}{f(u)u^2}\right)$$

Are the theories physical and stable?

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✓ Energy Conditions Analysis: $T_{\mu\nu}N^\mu N^\nu \geq 0\ , \quad N^\mu N_\mu = 0\ .$

$$R_1^1 - R_0^0 \geq 0\ , \qquad R_3^3 - R_0^0 \geq 0\ , \qquad R_u^u - R_0^0 \geq 0\ .$$

AND

✓ Local Thermodynamical Stability Analysis

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YES!

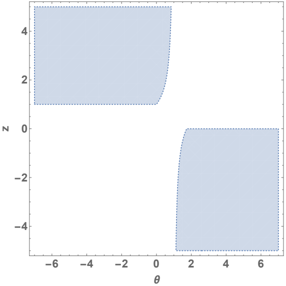
Three conditions that constrain (z, θ) and as a result (γ, σ).

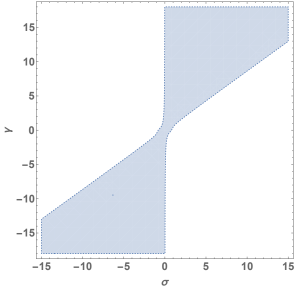
$$(z-1)(1-\theta+3z) \geq 0 \ , \qquad z = \frac{2+4\gamma^2-3\sigma^2}{2\gamma(2\gamma-3\sigma)} \ ,$$

$$\theta^2 - 3 + 3z(1-\theta) \geq 0 \ , \qquad \theta = \frac{3\sigma}{2\gamma} \ ,$$

$$1-\theta+2z \geq 0 \ .$$

The blue region is the acceptable for the theory parameters.





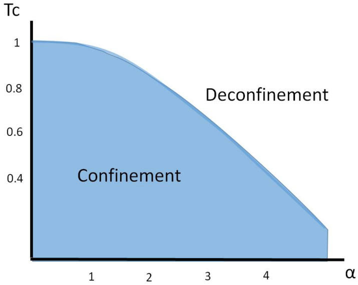
Phase Diagram

Properties of Heavy Quark Observables depend strongly on the Anisotropy.

(D.G. ; Chernicoff, Fernandez, Mateos, Trancanelli; Rebhan, Steineder 2012...)

Confinement/Deconfinement Phase Transitions?

- The Critical Temperature of the theories vs the anisotropy:



- Anisotropic strongly coupled systems have lower critical temperature.
- New Universal phenomenon: **Inverse Anisotropic Catalysis**.

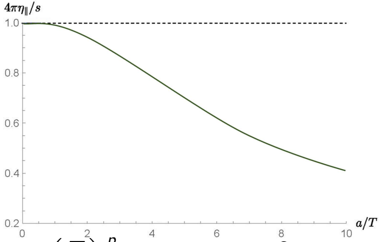
(DG, Gursoy, Pedraza 2018; related Aref'eva, Rannu 2018)

η/s in Theories with Broken Symmetry

Consider a finite T theory in the deconfined phase:

$$ds^2 = g_{tt}(u)dt^2 + g_{11}(u)(dx_1^2 + dx_2^2) + g_{33}(u)dx_3^2 + g_{uu}(u)du^2$$

- The anisotropic "shear viscosity" takes parametrical low values:



- The Ratio:

$$4\pi\frac{\eta_\parallel}{s} = \frac{g_{11}}{g_{33}}\bigg|_{u=u_h} \sim \left(\frac{T}{\alpha}\right)^p , \qquad p = 2 - \frac{2}{z} \sim [0,\infty) , \quad \alpha \gg T.$$

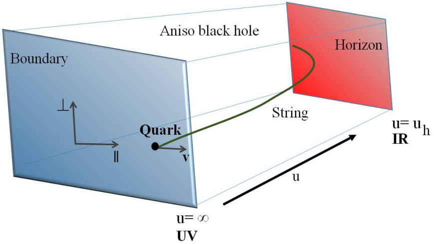
(Jain, Samanta Trivedy 2015; D.G., Gursoy, Pedraza, 2017)

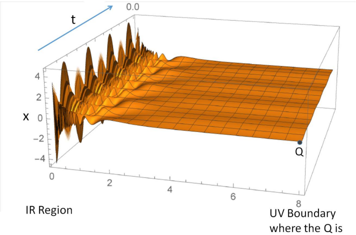
- New Universalities?

$$4\pi\frac{\eta_\parallel}{s}\frac{\sigma_\perp}{\sigma_\parallel} \geq 1$$

(Rebhan,Steineder 2011;Inkof, Gouteraux, Kiselev, Kuppers, Link, Narozhny, Schmalian 2018, 2019)

Langevin Dynamics and Brownian Motion





$$\frac{\kappa_{\parallel}}{\kappa_{\perp}} = \left.\frac{\left(g_{00}\, g_{\parallel\parallel}\right)'}{g_{\perp\perp}\, g_{\parallel\parallel}\left(\frac{g_{00}}{g_{\parallel\parallel}}\right)'}\right|_{u=u_{wh}}, \qquad \left\langle p^2_{\parallel,\perp}\right\rangle \sim \kappa_{\parallel,\perp}\,\mathcal{T}$$

A Universal Inequality for Isotropic Theory:

$\kappa_{\parallel} \geq \kappa_{\perp}$ for any isotropic strongly coupled plasma!

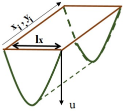
Can be inverted in the anisotropic theories: $\kappa_{\parallel} \geq < \kappa_{\perp}$.

(Gursoy, Kiritsis, Mazzanti, Nitti, 2010; D.G, Soltanpanahi, 2013a,b; D.G. 2018)

Anisotropic Monotonic Functions (c-function candidates)

- A proposed c-function wrt to entanglement entropy S
  ((aniso) Chu, D.G., 2019; (nrcft) Cremonini, Dong 2014; Myers, Singh 2012; (iso 2d+) Ryu, Takayanagi 2006; (2d) Casini, Huerta 2006)

$$c_x := \beta_x \frac{l_x^{d_x - 1}}{H_x^{d_1 - 1} H_y^{d_2}} \frac{\partial S_x}{\partial \ln l_x} \,, \qquad d_x := d_1 + d_2 \frac{n_2}{n_1}$$



H is the infrared regulator, d_1 (x_i), d_2 (y_i) are the spatial dimensions and n_1, n_2 are defined at the fixed point:

$$[t] = L^{n_t}, \quad [x_i] = L^{n_1}, \quad [y_j] = L^{n_2} \ .$$

- A relativistic "c-theorem" is guaranteed as long as the NEC: $T_u^u - T_0^0 \geq 0$ is satisfied ($u \to \infty \sim$UV):

$$\frac{dc}{du} \propto \int_0^l dx A'^{-2} \left(T_u^u - T_0^0\right) \geq 0 \qquad \Rightarrow \qquad c_{UV} \geq c_{IR}$$

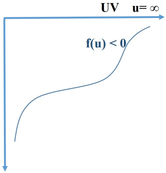
How about the Anisotropic Theories?

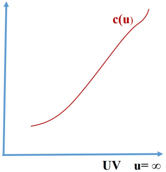
- Not a one-to-one correspondence between NEC and c-function monotonicity, but not surprising!
  (Chu, D.G. 2019; Aref'eva, Patrushev, Slepov ; Hoyos, Jokela, Penín, Ramallo 2020)
- The NEC can be written as

$$NEC := f_i'(u) > 0$$

  where $f_i(u) := F_i[g_{jj}(u)]$ are functionals of metric elements.
- If the metric boundary satisfies: e.g. $f_{i\ UV,\ u=\infty} \leq 0$ it guarantees the right monotonicity for only one of the c-functions along the RG flow

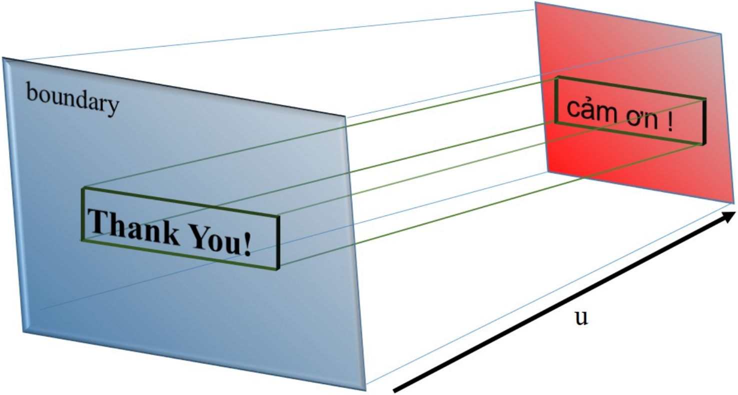
$$\frac{dc_x}{du} \sim -\int f_i(u) \qquad \Rightarrow \qquad c_{UV} \geq c_{IR}$$





Conclusions

- ✓ Observation: In strongly coupled theories many phenomena are more sensitive to the presence of the anisotropy than the source that triggers it.
- ✓ Several Universal Isotropic relations are anisotropically violated. Look for new Universalities!
- ✓ The phase transitions occur at lower critical Temperature as the anisotropy is increased = Inverse Anisotropic Catalysis!
- ✓ Holographic monotonic functions and conditions of monotonicity for (anisotropic) RG flows.
- c-functions as probes of phase transitions? (Baggioli, DG 2020)
- Are there any other observables that form functions, such that to have monotonic behavior along the (anisotropic) RG flow? (Chu, Derendinger, DG in progress)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Thank You!
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