JT Gravity from non-Abelian T-duality

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7th International Conference on Holography and String Theory

Duy Tan University, Da Nang

22-26 August 2024

Objective

The main goal of the presentation is describing a connection between

$$\text{JT gravity} \quad \xleftrightarrow{PSM} \quad \text{non-Abelian T-duality}$$

- 2d Dilaton-gravity model characterised by linear potential.
- 2d Principal Chiral Model with 3d background.
- The link is represented by Poisson sigma models (PSM).

The work is initially an attempt to better understand non-Abelian T-duality in superspace and this connection provides an interpretation of the observed results.

Outline

1. Introduction
2. T-duality of PCMs and the case of $\mathrm{OSp}(1|2)$
3. JT gravity from Poisson sigma models
4. Poisson sigma models from T-duality
5. Conclusions

Introduction

In physics the term *Duality* usually refers to the possibility of describing a certain physical system from two different perspectives, which are said to be dual to each other.

Target Space Duality arises in the context of sigma models and has the characteristic of relating different background geometries.

For this reason it has played an important role in the string theory framework, where sigma models are used to describe the propagation of strings in curved backgrounds.

Introduction: T-Duality in a Nutshell

Consider a 2d sigma model on a background with group of isometries G. We refer, with the name *T-Duality*, to the following series of steps

- Gauge a subgroup $\mathrm{K} \subseteq \mathrm{G}$ of the isometry group.
- Modify the action by adding a term which enforces the flatness of the gauge fields by means of Lagrange multipliers.
- Integrate out the gauge fields.
- Exploit the K-gauge freedom to remove the extra degrees of freedom.

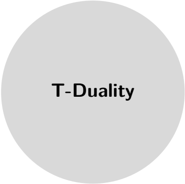
Another sigma model is obtained, in which the Lagrange multipliers play the role of dual coordinates. This is referred to as *T-Dual model*.

Introduction: Abelian vs Non-Abelian

T-Duality generally reduces the isometries of the model, since only the isometries commuting with the gauged ones survive dualisation **[Plauschinn 2013, Bugden 2018]**.

- Backgrounds with a single isometry or multiple commuting isometries preserve G, hence the dualisation can be repeated in the reverse direction. T-Duality is an exact symmetry of string theory **[Rocek, Verlinde 1991; Giveon, Rocek 1991]**.
- Backgrounds with a non-Abelian set of isometries generically break G to some smaller group, hence the procedure cannot be repeated in the reverse direction. T-Duality is not a symmetry of string theory **[Giveon, Rocek 1993]**.

The Landscape of T-Dualities



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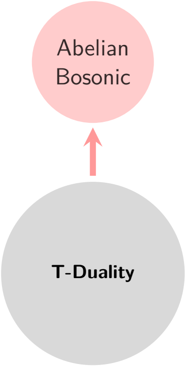

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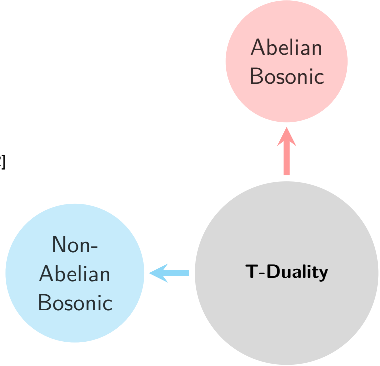

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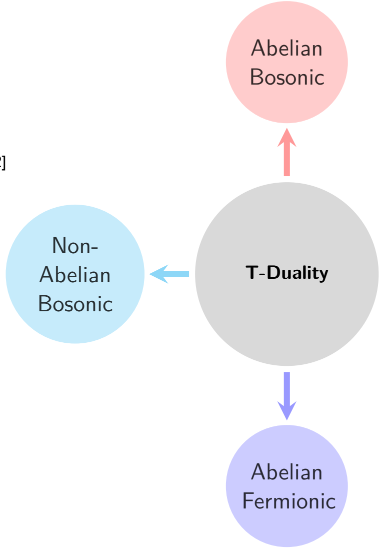

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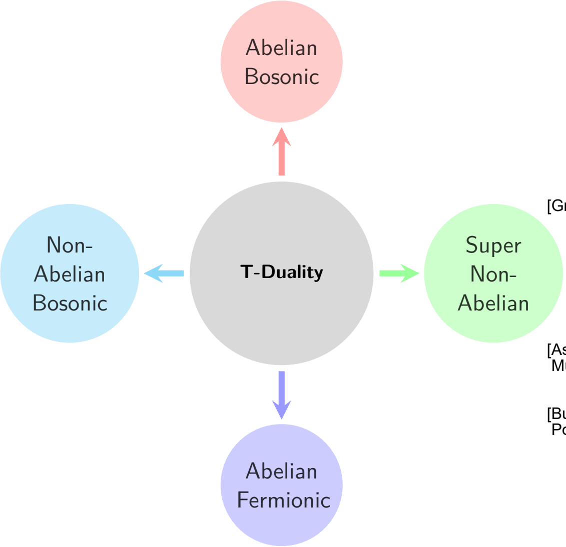

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[Grassi, Mezzalira 2011]

[Borsato, Wulff 2017]

[Borsato, Wulff 2018]

[Astrakhantsev, Bakhmatov, Musaev 2021,2022]

[Butter, Hassler, Pope, Zhang 2023]

[Berkovits, Maldacena 2008] [Beisert, Ricci, Tseytlin, Wolf 2008]

Introduction: Applications of T-duality

Among the applications and discoveries related to the Abelian setting

- self-duality of $AdS_5 \times S^5$ **[Berkovits, Maldacena 2008; Beisert, Ricci, Tseytlin, Wolf 2008]** and various other backgrounds under combined dualisation of Bosonic and Fermionic isometries **[Abbott, Murugan, Penati, Pittelli, Sorokin, Sundin, Tarrant, Wolf, Wulff 2015]**.

Bosonic non-Abelian duality has also played an important role

- Successfully used as a solution generating technique in supergravity **[Sfetsos, Thompson 2010; Lozano, Colgain, Sfetsos, Thompson 2011]**.
- Novel examples in the context of holography **[Lozano, Núñez 2016]**.
- Connection with integrable deformations of sigma models **[Sfetsos 2013;Osten, Van Tongeren 2016; Hoare, Tseytlin 2016; Borsato, Wulff 2016/17/18]**.

SNATD of PCMs: Initial Model and Gauging

PCMs are two-dimensional sigma models on a Lie group G defined as

$$S_{PCM} = -\frac{1}{2}\int_\Sigma \langle j, \star j\rangle = \int_\Sigma \mathrm{d}\tau\mathrm{d}\sigma\,\sqrt{-h}h^{ij}\,\partial_j Z^B \partial_i Z^A\, G_{AB}\ ,$$

with $j = \mathrm{g}^{-1}\mathrm{d}\mathrm{g}$ and isometry $\mathrm{G}_L \times \mathrm{G}_R$ due to invariance $\mathrm{g} \to \mathrm{g}_L^{-1}\mathrm{g}\mathrm{g}_R$.

- The G_L subgroup is naturally gauged by minimal coupling

$$j \quad \to \quad j_\omega := \mathrm{g}^{-1}(\mathrm{d}+\omega)\mathrm{g} \qquad \text{with} \qquad j,\omega \in \Omega^1(\Sigma,\mathfrak{g}).$$

- Flatness of ω is enforced adding Lagrange multipliers

$$S_\omega := -\frac{1}{2}\int_\Sigma \langle j_\omega, \star j_\omega\rangle - \int_\Sigma \langle \Lambda, F_\omega\rangle.$$

SNATD of PCMs: T-dual Model

Varying the action with respect to ω one finds

$$\star j_\omega + \mathrm{d}X - \mathrm{ad}_X(j_\omega) = 0 \qquad \text{with} \qquad X = \mathrm{g}^{-1}\Lambda\mathrm{g}\ ,$$

which can be solved for j_ω as

$$j_\omega = -\tfrac{1}{2}\frac{1}{1-\mathrm{ad}_X}(1+\star)\mathrm{d}X + \tfrac{1}{2}\frac{1}{1+\mathrm{ad}_X}(1-\star)\mathrm{d}X\ .$$

Substituting back and gauge fixing $\mathrm{g} = \mathbb{1}$, the dual action reads

$$\tilde{S} = -\tfrac{1}{2}\int_\Sigma \langle \mathrm{d}\Lambda, \frac{1}{1-\mathrm{ad}_\Lambda}(1+\star)\mathrm{d}\Lambda\rangle = \int_\Sigma \tilde{G} + \tilde{B}\ ,$$

with the identifications

$$\tilde{G} + \tilde{B} \simeq \frac{1}{1-\mathrm{ad}_\Lambda} \qquad \tilde{G} \simeq \frac{1}{1-\mathrm{ad}_\Lambda^2} \qquad \tilde{B} \simeq \frac{\mathrm{ad}_\Lambda}{1-\mathrm{ad}_\Lambda^2}\ .$$

SNATD of PCMs: The Case of $\mathrm{OSp}(1|2)$

Besides being a simple model, $\mathrm{OSp}(1|2)$ has interesting features.

- It represents an appropriate supergravity background interpreted as the $\mathcal{N} = 1$ supersymmetric version of AdS_3, see e.g. **[Buchbinder, Hutomo, Kuzenko, Tartaglino-Mazzucchelli 2017]**.
- The PCM on $\mathrm{SL}(2, \mathbb{R})$ has been dualised in **[Alvarez, Alvarez-Gaume, Barbon, Lozano 1993]**. The dual model containts non-trivial B_2 field and a metric corresponding to the spacetime of a black hole. It is natural to wonder if the theory could be embedded in a 3d supergravity.
- The T-dual model to $\mathrm{OSp}(1|2)$ breaks the requirements needed for a 3d supergravity interpretation **[DB, Penati, Sorokin, Wolf 2021]**.

SNATD of PCMs: The Case of $\mathrm{OSp}(1|2)$

The last property is related to a breaking pattern observed in T-dualisation of compact spaces **[Alvarez, Alvarez-Gaume, Barbon, Lozano 1993]** and, more recently, non-compact ones **[Ramirez 2021; Lozano, Núñez, Ramirez 2021]**

$$\mathrm{AdS}_3 \quad \xrightarrow[\text{T-duality}]{} \quad \mathrm{AdS}_2 \times \text{line} \; .$$

Performing an appropriate change of coordinates, one can indeed detect a similar behaviour in the case of $\mathrm{OSp}(1|2)$ **[DB, Penati, Ramirez 2024]**

$$\mathrm{sAdS}_3 \quad \xrightarrow[\text{T-duality}]{} \quad \mathrm{sAdS}_2 \times \text{superline} \; .$$

SNATD of PCMs: The Case of $\mathrm{OSp}(1|2)$

The T-dual model is described by 3 bosonic and 2 fermionic multipliers

$$\Lambda^A = \{x^0, x^1, x^2, \theta^1, \theta^2\} \ .$$

- Parametrising the bosonic ones in terms of the AdS_2 coordinates $\{t, u\}$ and an extra radial coordinate r, the purely bosonic sector of the metric takes the form

$$\mathrm{d}\tilde{s}^2|_{\mathrm{bos}} = \Omega(r)\mathrm{d}s^2_{\mathrm{AdS}_2} + \mathrm{d}r^2 \qquad \text{with} \qquad \mathrm{d}s^2_{\mathrm{AdS}_2} = \frac{1}{u^2}(-\mathrm{d}t^2 + \mathrm{d}u^2) \ .$$

- Making sense of the fermionic sector requires then an appropriate definition of what sAdS_2 is.

SNATD of PCMs: The Case of $\mathrm{OSp}(1|2)$

This can be constructed as the semi-symmetric coset space $\mathrm{OSp}(1|2)/\mathrm{SO}(1,1)$, for which an action was given in **[Verlinde 2004]**

$$S_{\mathsf{sAdS}_2} = -\int_\Sigma \mathrm{d}^2 z \frac{(\bar\partial Z + i\Theta\bar\partial\Theta)(\partial\bar Z + i\bar\Theta\partial\bar\Theta)}{(Z-\bar Z - i\Theta\bar\Theta)^2} = \int_\Sigma G_{\mathsf{sAdS}_2} + B_{\mathsf{sAdS}_2} \ .$$

The semi-symmetric structure ensures compatibility with the supergravity interpretation and one can read off the desired fields

$$\begin{aligned}\mathrm{d}s^2_{\mathsf{sAdS}_2} = &\tfrac{1}{8}(1-\tfrac{2i}{u}\tilde\theta^1\tilde\theta^2)\mathrm{d}s^2_{\mathsf{AdS}_2} - \tfrac{1}{2u^2}\tilde\theta^1\tilde\theta^2\mathrm{d}\tilde\theta^1\mathrm{d}\tilde\theta^2 + \\ &- \tfrac{i}{4u^2}\mathrm{d}u(\mathrm{d}\tilde\theta^1\tilde\theta^2 + \mathrm{d}\tilde\theta^2\tilde\theta^1) + \tfrac{i}{4u^2}\mathrm{d}t(\mathrm{d}\tilde\theta^1\tilde\theta^1 + \mathrm{d}\tilde\theta^2\tilde\theta^2)\end{aligned}$$

$$\begin{aligned}B_{\mathsf{sAdS}_2} = &\tfrac{1}{4}(1-\tfrac{2i}{u}\tilde\theta^1\tilde\theta^2)\mathsf{Vol}_{\mathsf{AdS}_2} - \tfrac{1}{4u^2}\tilde\theta^1\tilde\theta^2(\mathrm{d}\tilde\theta^1\wedge\mathrm{d}\tilde\theta^1 - \mathrm{d}\tilde\theta^2\wedge\mathrm{d}\tilde\theta^2) + \\ &- \tfrac{i}{4u^2}\mathrm{d}u\wedge(\mathrm{d}\tilde\theta^1\tilde\theta^1 + \mathrm{d}\tilde\theta^2\tilde\theta^2) + \tfrac{i}{4u^2}\mathrm{d}t\wedge(\mathrm{d}\tilde\theta^1\tilde\theta^2 + \mathrm{d}\tilde\theta^2\tilde\theta^1)\end{aligned}$$

SNATD of PCMs: The Case of $\mathrm{OSp}(1|2)$

The above expressions can be recast as the ones appearing in the T-dual model upon changing coordinates as

$$\tilde{\theta}^1 = t\theta^1 + \theta^2 \qquad\qquad \tilde{\theta}^2 = u\theta^1 \ .$$

This allows to rewrite the dual model as

$$\mathrm{d}\tilde{s}^2 = G_1\,\mathrm{d}s^2_{\mathsf{sAdS}_2} + G_2\,\mathrm{d}r^2 + + G_3\,\mathrm{d}(\theta^1\theta^2)\mathrm{d}r + G_4\,\mathrm{d}\theta^1\mathrm{d}\theta^2 \ .$$

- The functions G_i have the structure $G_i = g_i(r) + g_i^\theta(r)\,\theta^1\theta^2$.
- The last three terms in the metric form the line element of a $(1|2)$-dimensional supermanifold, the superline.
- A similar decomposition takes place for the B-field.
- The sugra interpretation seems to be broken down to a 2d subspace.

JT gravity from PSM: First-Order Form of JT

We consider now the JT gravity action **[Teitelboim 1993; Jackiw 1995]**

$$S = \frac{1}{4}\int_\Sigma \mathrm{d}^2x\sqrt{-g}\,\Phi(R+2)$$

this can be rewritten in first order form by introducing Lagrange multipliers X^i, with $i = 0, 1$, frame fields ${e_\mu}^i$ and a torsion-free connection ω:

$$S = \frac{1}{2}\int_\Sigma \Phi\mathrm{d}\omega - \frac{1}{2}\Phi\epsilon^{ij}e_i \wedge e_j + X^i(\mathrm{d}e_i - {\epsilon_i}^j\omega \wedge e_j)$$

- Integrating out the multipliers X^i we impose torsion-free condition.
- One can the use the 2d relations to go back to the JT action

$$\omega^{ij} = \epsilon^{ij}\omega \qquad e_0 \wedge e_1 = \mathrm{d}^2x\sqrt{-g} \qquad 2\mathrm{d}\omega = \mathrm{d}^2x\sqrt{-g}R\ .$$

JT gravity from PSM: Identification with PSM

The first order form of the JT action can then be identified with a Poisson sigma model **[Ikeda, Izawa 1993; Ikeda 1994; Shaller, Strobl 1994]**

$$S_{PSM} = \int_\Sigma \mathrm{d}X^a \wedge A_a + \tfrac{1}{2} P^{ab} A_b \wedge A_a \qquad \text{with} \qquad a = 0, 1, 2\ ,$$

upon identifying, for $i = 0, 1$,

$$A_i = e_i \qquad A_2 = \omega \qquad X^2 = \Phi\ .$$

$P^{ab} = \{X^a, X^b\}$ represents the Poisson structure of the underlying background and choosing components

$$P^{01} = \tfrac{1}{2}\Lambda\Phi \qquad P^{i2} = X^a {\epsilon_a}^i$$

reproduces the first order form of JT gravity.

PSM from NATD: First-Order Form of 2d Sigma Models

We start by noting that a generic 2d sigma model

$$S = \frac{1}{2}\int_\Sigma \mathrm{d}Z^b \wedge (1+\star)\mathrm{d}Z^a (G+B)_{ab} \ ,$$

can be written in first-order form by introducing 1-forms A_a

$$S = \int_\Sigma A_a \wedge \mathrm{d}Z^a + \frac{1}{2}A_a \wedge \star A_b\, g^{ba} + \frac{1}{2}A_a \wedge A_b\, P^{ba} \ ,$$

with the identifications

$$g^{ba} = [(G+B)^{-1}]^{(ba)} \qquad \text{and} \qquad P^{ba} = [(G+B)^{-1}]^{[ba]} \ .$$

PSM from NATD: Dualisation with a Coupling

Then consider again the general dualisation of PCMs, this time including an arbitrary coupling λ in front of the action

$$S = -\tfrac{\lambda}{2}\int_\Sigma \langle j, \star j\rangle \quad \longleftrightarrow \quad \tilde{S} = -\tfrac{1}{2\lambda}\int_\Sigma \langle \mathrm{d}\Lambda, \frac{1}{1-\lambda^{-1}\mathrm{ad}_\Lambda}(1+\star)\mathrm{d}\Lambda\rangle \ .$$

Now we can exploit the identification

$$\tilde{G}+\tilde{B} \simeq \frac{1}{1-\lambda^{-1}\mathrm{ad}_\Lambda} \quad \longleftrightarrow \quad (\tilde{G}+\tilde{B})^{-1} \simeq 1-\lambda^{-1}\mathrm{ad}_\Lambda \equiv g - \tfrac{1}{\lambda}P$$

with

$$g \equiv 1 \qquad \text{and} \qquad P \equiv \mathrm{ad}_\Lambda \ ,$$

to write down the first order form of the T-dual action.

PSM from NATD: First-Order Form of T-dual and $\lambda \to 0$

This takes the form

$$\tilde{S} = -\int_\Sigma \langle A, \mathrm{d}\Lambda\rangle + \frac{\lambda}{2}\langle A, \star A\rangle - \frac{1}{2}\langle A, P(A)\rangle$$

and one can finally take the limit $\lambda \to 0$, hence obtaining

$$\tilde{S} = \int_\Sigma \mathrm{d}\Lambda^a \wedge A_a + \frac{1}{2} P^{ab} A_b \wedge A_a \ ,$$

with

$$P^{ab} = \delta^{ad} \Lambda^c f_{cd}{}^b \ .$$

Identifying the 1-forms A with those of JT and the multipliers with

$$\Lambda^i \equiv X^i \qquad \Lambda^2 \equiv \Phi \qquad \text{for} \qquad i = 0, 1$$

we precisely recover the Poisson sigma models leading to JT gravity

Conclusions: Summary and Comments

In the attempt of better understanding non-Abelian T-duality in superspace we have found that:

- Dualising $sAdS_3$ background one obtains $sAdS_2$.
- There exists a connection between JT gravity and NATD via PSM.
- This requires considering a limiting behaviour, previously considered in **[Baulieu, Losev, Nekrasov 2002]**.
- The procedure can be extended to the superspace setting, allowing to recover super JT gravity **[Chamseddine 1991; Izquierdo 1998]** .
- The cases we considered can be regarded as special cases of a more general scheme relating NATD of PCMs and PSM.

Conclusions: Outlook

Some questions which remain open are:

- Do other limiting values of λ play any role?
- Can one find a way of connecting NATD with JT gravity, or possibly other 2d dilaton-gravity theories, for any λ?
- What happens to 10d supergravity backgrounds in this limit?
- Can we extend the procedure to other classes of sigma models?
- Does this procedure have any analogue in the holographically dual field theory?

Thank You for the Attention!