New Developments in Non-relativistic String Theory

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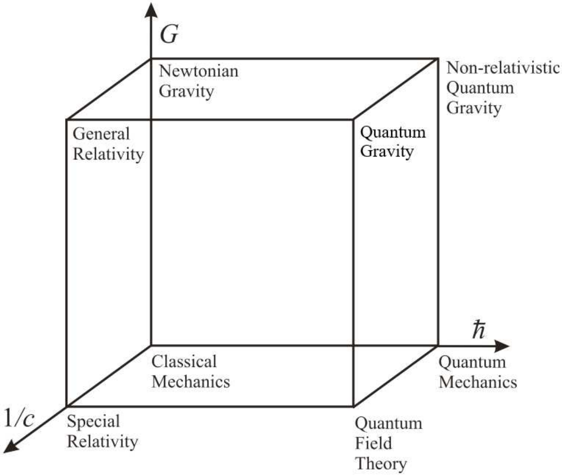
4th International Conference on
Holography, String Theory and Discrete Approach

Hanoi, Vietnam, August 8 2020

Does Quantum Gravity require Relativity ?



Some References

NR gravity/string theory without taking NR limit

T. Harmark, J. Hartong and N. A. Obers (2017); Kluson (2018); T. Harmark, J. Hartong, L. Menculini, N. A. Obers and Z. Yan (2018); Kluson (2019); Roychowdhury; T. Harmark, J. Hartong, L. Menculini, N. A. Obers and G. Oling (2019); A.D. Gallegos, U. Gürsoy and N. Zinnato (2019)

NR strings with NR worldsheet

C. Batlle, J. Gomis and D. Not (2017); C. Batlle, J. Gomis, L. Mezincescu and P. K. Townsend (2017); T. Harmark, J. Hartong and N. A. Obers (2017); T. Harmark, J. Hartong, L. Menculini, N. A. Obers and Z. Yan (2018)

Outline

Newton-Cartan Gravity

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Summary and Open Issues

From Galilei to Bargmann

Andringa, Panda, de Roo + E.B. (2011)

the zero commutator

$$[G_a, P_b] = 0$$

implies that a massive particle with non-zero spatial momentum P_b cannot by any boost transformation G_a be brought to a rest frame $\Rightarrow$

$$[G_a, P_b] = \delta_{ab} M \qquad \rightarrow \qquad \text{extra gauge field } M_\mu$$

Newton-Cartan Geometry

The independent NC fields $\{\tau_\mu, E_\mu{}^a, M_\mu\}$ transform as follows:

$$\begin{aligned} \delta\tau_\mu &= 0\,, \\ \delta E_\mu{}^a &= \lambda^a{}_b\, E_\mu{}^b + \lambda^a\tau_\mu\,, \\ \delta M_\mu &= \partial_\mu\sigma + \lambda_a\, E_\mu{}^a \end{aligned}$$

The spin-connection fields $\Omega_\mu{}^{ab}$ and $\Omega_\mu{}^a$ are functions of τ_μ , $E_\mu{}^a$ and M_μ

The Zero Torsion Constraint

$$\tau_{\mu\nu} \equiv \partial_{[\mu}\tau_{\nu]} = 0 \quad \rightarrow \quad \tau_\mu = \partial_\mu \rho \quad \text{with} \quad \tau_\mu \quad \text{clock function}$$

$$\Delta T = \int_{\mathcal{C}} \mathrm{d}x^\mu \tau_\mu = \int_{\mathcal{C}} \mathrm{d}\rho \ \text{ is path-independent} \quad \rightarrow \quad \text{absolute time}$$

Metrics

general relativity has one non-degenerate metric that is invariant under Lorentz transformations

NC gravity has two degenerate metrics

$$\tau_{\mu\nu} = \tau_\mu \tau_\nu \qquad \text{and} \qquad h^{\mu\nu} = E^\mu{}_a E^\nu{}_b \delta^{ab}$$

that are invariant under the Bargmann transformations

the symmetric tensor

$$H_{\mu\nu} \equiv E_\mu{}^a E_\nu{}^b \delta_{ab} + 2M_{(\mu}\tau_{\nu)}$$

is invariant under Galilean transformations but not under central charge transformations!

NR Limits

The NR limit of a particle coupled to general relativity is finite provided that the leading divergence is cancelled by coupling the particle to a (zero flux) gauge field $\hat{M}_\mu$ via a WZ term

$$\dot{x}^\mu \hat{M}_\mu \qquad \text{with} \qquad \hat{M}_\mu = \omega\tau_\mu + M_\mu \qquad \text{and} \qquad \partial_{[\mu}\tau_{\nu]} = 0$$

where $\omega \to \infty$ in the NR limit

Gomis, Ooguri (2001), Klebanov, Maldacena (2001), Danielsson, Guijosa, Kruczenski (2000)

- this limit is brane-dependent!

$$\mathcal{L} = m\frac{\dot{X}^\mu \dot{X}^\nu}{\tau_\rho \dot{X}^\rho}\Big[E_\mu^a E_\nu^b \delta_{ab} + 2M_{(\mu}\tau_{\nu)}\Big]$$

$$= e^{-1}\Big[\dot{X}^\mu \dot{X}^\nu H_{\mu\nu} + \lambda\big(\dot{X}^\mu \tau_\mu - me\big)\Big] \quad \leftrightarrow \quad e = \frac{1}{m}\dot{X}^\mu \tau_\mu$$

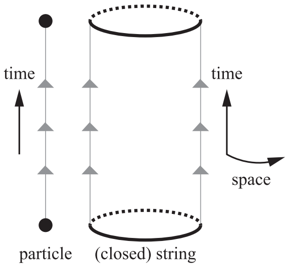
Outline

String Newton-Cartan Geometry

The string should be coupled to a (zero flux) 2-form gauge field $\hat{B}_{\mu\nu}$ with

$$\hat{B}_{\mu\nu} = \omega^2 \epsilon_{AB} \tau_\mu{}^A \tau_\nu{}^B + B_{\mu\nu} \quad \rightarrow \quad D_{[\mu}(\omega)\tau_{\nu]}{}^A = 0$$

defining a string NC geometry with $\tau_\mu \rightarrow \tau_\mu{}^A$ with $A = 0, 1$



String Newton-Cartan Gravity

$$D \text{ flat indices} \rightarrow \begin{cases} 2 \ \text{longitudinal indices } A \\ D-2 \ \text{transverse indices } a \end{cases}$$

The independent string NC fields are

$$\{\tau_\mu{}^A, E_\mu{}^a, M_\mu{}^A\}$$

longitudinal metric: $\tau_{\mu\nu} \equiv \tau_\mu{}^A \tau_\nu{}^B \eta_{AB}$

transverse 'metric': $H_{\mu\nu} \equiv E_\mu{}^a E_\nu{}^b \delta_{ab} + \left(\tau_\mu{}^A M_\nu{}^B + \tau_\nu{}^A M_\mu{}^B\right)\eta_{AB}$

The Nonrelativistic String

J. Gomis, Z. Yan + E.B. (2018); J. Gomis, J. Rosseel, C. Şimşek, Z. Yan + E.B. (2019)

$$S_{\rm NR\ string} = -\frac{T}{2}\int d^2\sigma\,\Big[\sqrt{-h}\,h^{\alpha\beta}\,\partial_\alpha x^\mu \partial_\beta x^\nu H_{\mu\nu} + \epsilon^{\alpha\beta}\partial_\alpha x^\mu \partial_\beta x^\nu B_{\mu\nu}\Big]$$

with world-sheet metric $h_{\alpha\beta} \sim \tau_{\alpha\beta} \equiv \partial_\alpha x^\mu \partial_\beta x^\nu \tau_{\mu\nu}$

and constraints $D_{[\mu}(\omega)\tau_{\nu]}{}^A = 0$: conventional and geometric

This is the generalization of flat spacetime to a string NC background

Gomis, Ooguri (2001); Danielsson, Guijosa, Kruczenski (2000)

Equations of Motion

One can calculate the E.O.M. of the SNC background fields either by taking the NR limit of the (matter coupled) Einstein E.O.M. or by calculation the β-functions

Gomis, Oh, Yan (2019), Yan, Yu (2019); see also Gallegos, Gürsoy and Zinnatos (2019)

There is a surprise: there are less E.O.M. than background fields due to an emergent Stückelberg symmetry:

$$S_{\mathrm{NR\ particle}} = -\frac{m}{2}\int d\tau \left\{\frac{\dot{x}^\mu \dot{x}^\nu}{\tau_\rho \dot{x}^\rho} H_{\mu\nu} - B_\mu \dot{x}^\mu\right\}$$

is invariant under

$$H_{\mu\nu} \to H_{\mu\nu} + \frac{1}{2}\left(\tau_\mu C_\nu + \tau_\nu C_\mu\right)\,, \qquad B_\mu \to B_\mu + C_\mu$$

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Summary and Open Issues

Non-relativistic T-duality

The T-duality rules (a.k.a. Buscher's rules) depend on whether the T-duality direction is transverse or longitudinal to the string

1. The transverse T-dual of a non-relativistic string gives the usual $R \leftrightarrow \ell_s^2/R$ duality like in the relativistic case

2. The spatial longitudinal T-dual of a nonrelativistic string gives a string moving in a general relativity (plus Kalb-Ramond plus dilaton) background with

- a null Killing vector
- constraints that are the T-dual of the geometric constraints

The Null σ Model

The Null σ Model is defined by

$$S_{\mathrm{N\sigma M}} = \frac{T}{2}\int d^2\sigma\left[\sqrt{-h}\,h^{\alpha\beta}\partial_\alpha x^\mu\partial_\beta x^\nu G_{\mu\nu}(x) + \epsilon^{\alpha\beta}\partial_\alpha x^\mu\partial_\beta x^\nu B_{\mu\nu}(x)\right]$$

with

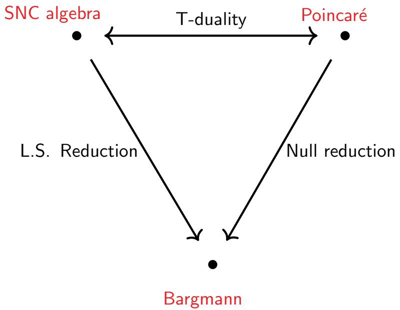
1. $G_{vv} = 0$: null isometry
2. $\partial_{[i}G_{j]v} = \partial_{[i}B_{j]v} = 0$: geometric constraints

$\mu = (v, i)$

Embedding the Bargmann Algebra

10D

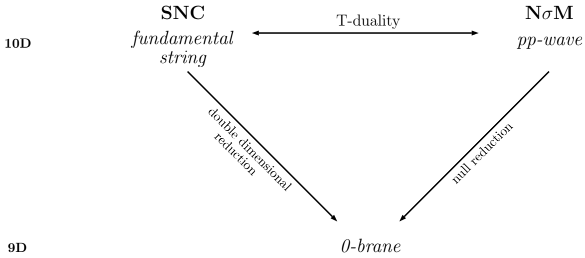
9D



The 9D Bargmann algebra has not only a null-embedding in a 10D Poincaré algebra but also a spatial embedding in a 10D SNC algebra

The Fundamental String Solution

J. Lahnsteiner, L. Romano, J. Rosseel, C. Şimşek, T. ter Veldhuis + E.B., work in progress



- the fundamental string solution:

$$\tau_\mu{}^A = \delta_\mu{}^A\,, \qquad E_\mu{}^{A'} = \delta_\mu{}^{A'}\,, \qquad e^\phi = g_s$$

$$\epsilon^{AB}\mathcal{B}_{AB} \equiv \epsilon^{AB}B_{AB} - M_A{}^A = K(z_{(8)})\,, \qquad B_{A'B'} = 0$$

Outline

Summary

- String NC Geometry is to NR string theory what Riemannian geometry is to relativistic string theory

  - String Action : new geometry with emerging Stückelberg symmetry

  - T-duality : NσM Model

  - Background Field E.O.M. : fundamental string solution

Open Issues

- Going beyond the fundamental: NS-NS branes and D-branes

  K. Grosvenor, J. Lahnsteiner, L. Romano, J. Rosseel, C. Şimşek + E.B., work in progress

- Supersymmetry

- Nonrelativistic holography ?

  Gomis, Gomis, Kamimura (2005), Gopakumar, Bagchi (2009)

- Double Field Theory ?

  S. M. Ko, C. Melby-Thompson, R. Meyer and J.-H. Park (2015); C. Blair (2019, 2020)

- NR Quantum Gravity ?

Take Home Message

nonrelativistic string theory can be studied on its own!