D-brane effective actions

and their all order α' corrections

Ehsan Hatefi

SNS, Pisa & Jilin University

Outline

- D-branes
- Motivations
- Effective action DBI+CS+WZ
- Exact Math Results
- Brane-Anti Brane
- Open questions & Conclusion

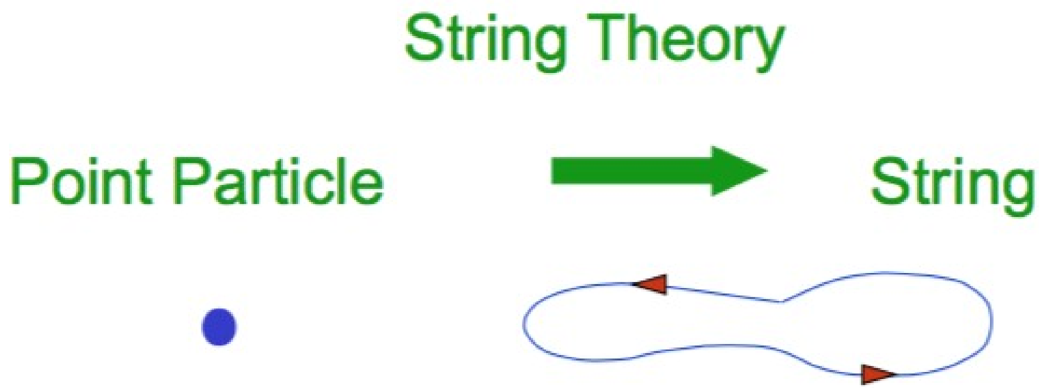
Gravity results from the curvature of spacetime, but there is a problem with it.

The nonlinear mathematics of Einstein's theory is not compatible with the QFT.

Modern physics is based on quantum physics for atoms, while Einstein's GR is used for stars, galaxies, and the whole Universe.

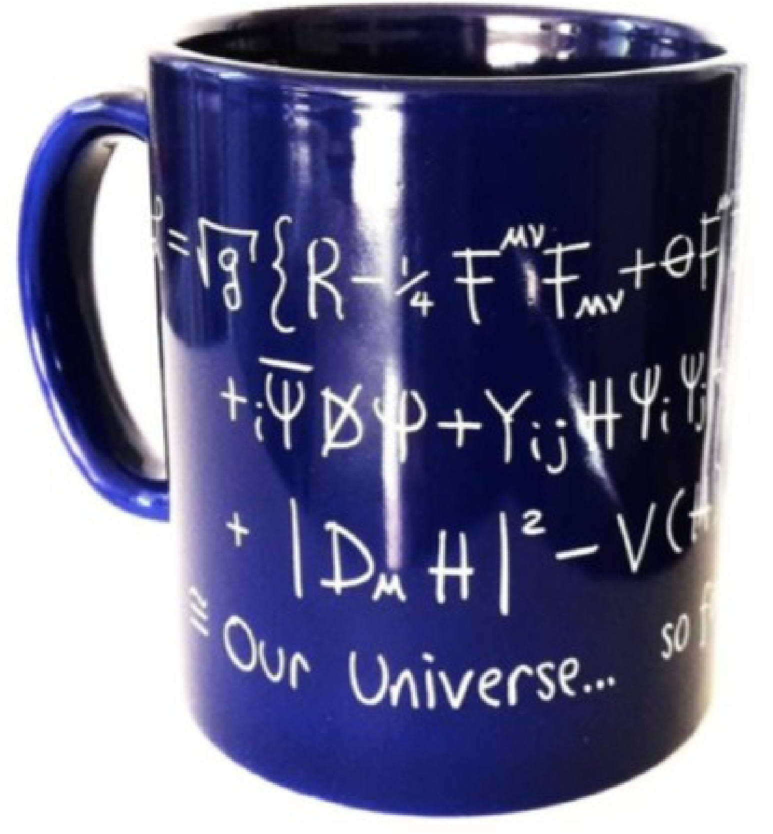
They all have to be described by the same theory.

One real idea about how to combine quantum mechanics with general relativity is **"String Theory"**



It sounds like an amazing idea, but it has incredibly far-reaching consequences.

Standard Model + Gravity



D-branes are fundamental non perturbative objects where the end points of open strings must be attached to them.

BPS Dp-branes in type II

IIA(IIB),includes BPS branes (stable) with even p (odd p) and the only difference with non BPS branes is the absence of Tachyon.

The charge of a Dp-brane is **Ramond-Ramond field**

BPS branes have orientation, while anti branes carry anti charge and their orientation is reverse of BPS branes.

For stable Dp-branes **(p is even in IIA , odd in IIB)** preserve half of supersymmetry.

Stability, Supersymmetry, conserved (RR) charge and having no tachyons are, all properties of these type II branes.

We talk about a universal conjecture that holds even brane-anti brane.

Using scattering of Strings, we point out how to look for effective actions for D-brane and anti D-brane

Some new Wess-Zumino couplings with their corrections will be presented.

These new actions/couplings are neither inside Myers‘ terms nor within **pull-back/Taylor** expansions.

Motivations

1) To obtain Universality for all-order alpha-prime corrections to BPS/non-BPS systems

2) It seems that, description of world volume dynamics of D-brane is still lacking at fundamental level.

3) Holographic QCD Models, Cosmology,…

4) Working out with Mathematical Structures behind Scattering amplitudes (world-sheet integrals)

The world-volume theory of a Dp-brane involves a massless U(1) vector, 9 − p real massless scalars, fermions.

At leading order, the low-energy action is DBI.

There are higher $\alpha' = l_s^2$ order corrections.

When derivatives of the Field strength are small on string scale, the action takes BI form

If we consider N coincident Dp-branes, the U(1) symmetry of a Dp-brane gets enhanced to non-abelian U(N).

The low energy action of Dp-branes consists of 2 parts.
Born-Infeld and Chern-Simons

The action for constructing **non-abelian Dp-branes** is

R. Myers , arXiv:9910053, JHEP

$$S_{BI} = -T_p \int d^{p+1}\sigma \, \mathrm{STr} \left(e^{-\phi} \sqrt{-\det \left(P \left[E_{ab} + E_{ai}(Q^{-1} - \delta)^{ij} E_{jb} \right] + \lambda F_{ab} \right) \det(Q^i{}_j)} \right),$$

$$E_{ab} = G_{ab} + B_{ab} \qquad , \qquad Q^i{}_j \equiv \delta^i{}_j + i\lambda \left[\Phi^i, \Phi^k \right] E_{kj}$$

To find interactions expected from DBI, we expand the action, set all background field to zero, that is, working on flat empty space background

The second part is the Wess-Zumino action, contains the coupling of the U(N) massless world volume vectors to RR field

Effective Field Theory on the World-Volume

The states in S-matrix are gauge, scalars and tachyons from DBI action and RR field from the WZ action

$$S_{WZ} = \mu_p \int \mathrm{STr}\left(P\left[e^{i\lambda\, i_\Phi i_\Phi}\left(\sum C^{(n)}\right)\right] e^{\lambda F}\right)$$

The scalar are transverse coordinate of the D-brane, These scalars appear in 3 different ways

1^{st} : explicit appearance in the exponential

$$i_\Phi i_\Phi C^{(n)} = \frac{1}{2(n-2)!}[\Phi^i, \Phi^j]\, C^{(n)}_{ji\mu_3\cdots\mu_n} dx^{\mu_3}\cdots dx^{\mu_n}$$

2nd , covariant derivatives pull-back.

$$P[E]_{ab} = E_{ab} + \lambda\, E_{ai}\, D_b\Phi^i + \lambda\, E_{ib}\, D_a\Phi^i + \lambda^2\, E_{ij} D_a\Phi^i D_b\Phi^j$$

3rd , the action includes derivatives of closed string fields through the Taylor expansion of these fields

$$G_{\mu\nu} = \exp\left[\lambda\Phi^i\,\partial_{x^i}\right] G^0_{\mu\nu}(\sigma^a, x^i)|_{x^i=0}$$

$$= \sum_{n=0}^{\infty} \frac{\lambda^n}{n!}\,\Phi^{i_1}\cdots\Phi^{i_n}\,(\partial_{x^{i_1}}\cdots\partial_{x^{i_n}})G^0_{\mu\nu}(\sigma^a, x^i)|_{x^i=0}$$

non trivial interactions, we need 3 open and a closed string.

To study these effective actions, we use the

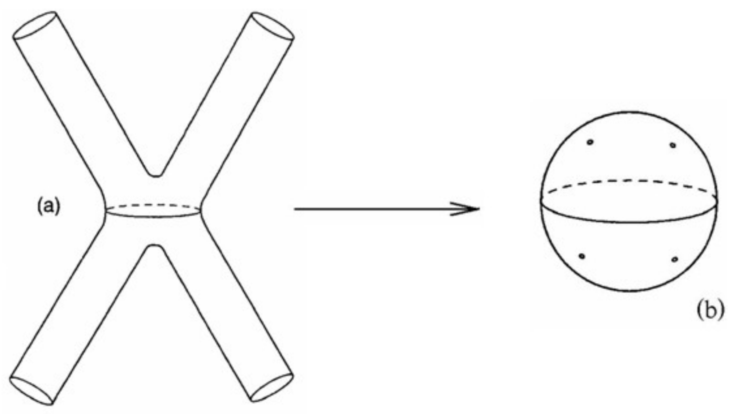
S-matrix method (Scattering amplitudes)

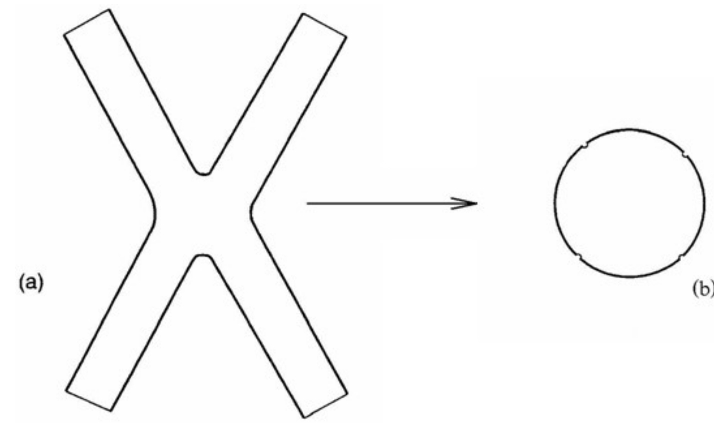
Using CFT, we evaluate the amplitudes to find

all closed string couplings to open strings on Dp-branes

Four Point amplitude

We calculate scattering amplitudes of strings by Conformal Field Theory (CFT) methods





From CFT, one evaluates the correlation functions of all fields

TTTT we consider a world-sheet with the topology of

a disk with vertex operator insertions on its boundary

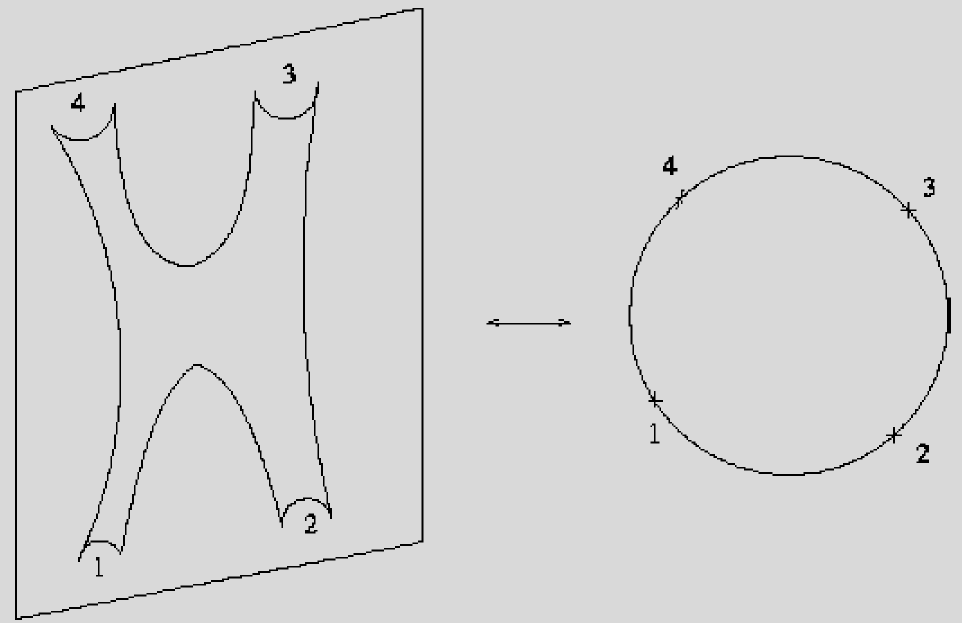


Fig. 1: World-sheet corresponding to the scattering of four open strings.

Vertex Operators

Vertex includes information about the properties of strings. Form of every vertex is calculated by using the conformal invariance of S-matrix.

$$V_{RR}^{(-1)}(z,\bar{z}) = \quad (P_- H\!\!\!/_{(n)} M_p)^{\alpha\beta} e^{-\phi(z)/2} S_\alpha(z) e^{ip\cdot X(z)} e^{-\phi(\bar{z})/2} S_\beta(\bar{z}) e^{ip\cdot D\cdot X(\bar{z})}$$

$$V_A^{(0)}(x) = \xi_a \bigg(\partial X^a(x) + 2ik\cdot\psi\psi^a(x) \bigg) e^{2ik.X(x)}$$

$$k^2 = 0, k.\xi = 0$$

R.Antonelli, I.Basile, EH , arXiv: 1903.07540, JCAP

$$\int d^2z|1-z|^a|z|^b(z-\bar{z})^c(z+\bar{z})^d = (2i)^c 2^d \pi \frac{J_1+J_2}{\Gamma(-\frac{a}{2})\Gamma(-\frac{b}{2})\Gamma(d+2+c+\frac{a+b}{2})}.$$

where

$$\begin{aligned} J_1 &= \frac{1}{2}\Gamma(d+\frac{b+c}{2})\Gamma(d+\frac{a+c}{2})\Gamma(-d-\frac{a+b+c}{2})\Gamma(\frac{1+c}{2}) \\ J_2 &= \Gamma(d+1+\frac{b+c}{2})\Gamma(1+\frac{a+c}{2})\Gamma(-1-\frac{a+b+c}{2})\Gamma(\frac{1+c}{2}). \end{aligned}$$

Universality in all order alpha-prime Corrections

[E.H, IY Park, NPB]

There exists a regularity in higher derivatives.

"prescription"

1st we find S-matrix of desired amplitudes.

2nd, using Mandelstam variables, we rewrite amplitudes so that all poles are seen.

3rd step is finding leading couplings from DBI action.

4th step is to express the symmetric trace in term of ordinary trace and apply higher derivative corrections on them.

$$D_{nm}(EFGH) = D_{b_1}\cdots D_{b_m} D_{a_1}\cdots D_{a_n} EFD^{a_1}\cdots D^{a_n} GD^{b_1}\cdots D^{b_m} H,$$
$$D'_{nm}(EFGH) = D_{b_1}\cdots D_{b_m} D_{a_1}\cdots D_{a_n} ED^{a_1}\cdots D^{a_n} FGD^{b_1}\cdots D^{b_m} H$$

The crucial step seems to extract symmetric trace in terms of

ordinary trace and apply higher derivative operators on them

1^{st} example

In [1003.0314,JHEP, 1203.5553,PRD, E.H,IY Park], it was shown that the string theory

result of (CAAA) is reproduced by

$$-T_p(2\pi\alpha')^4 STr\left(-\frac{1}{8}F_{bd}F^{df}F_{fh}F^{hb}+\frac{1}{32}(F_{ab}F^{ba})^2\right)$$

The closed form of all corrections to all orders is found.

Extension of these interaction vertices to higher derivative couplings

reproduce all infinite massless poles

$$(2\pi\alpha')^4 \frac{1}{8\pi^2} T_p \, (\alpha')^{n+m} \sum_{m,n=0}^{\infty} (L_5^{nm} + L_6^{nm} + L_7^{nm})$$

$$L_5^{nm} = -Tr\left(a_{n,m} D_{nm}[F_{bd}F^{df}F_{fh}F^{hb}] + b_{n,m} D'_{nm}[F_{bd}F_{fh}F^{df}F^{hb}] + h.c.\right)$$

$$L_6^{nm} = -Tr\left(a_{n,m} D_{nm}[F_{bd}F^{df}F_{hb}F^{fh}] + b_{n,m} D'_{nm}[F_{bd}F_{hb}F^{df}F^{fh}] + h.c.\right)$$

$$L_7^{nm} = \frac{1}{2} Tr\left(a_{n,m} D_{nm}[F_{ab}F^{ab}F_{cd}F^{cd}] + b_{n,m} D'_{nm}[F_{ab}F_{cd}F^{ab}F^{cd}] + h.c.\right)$$

These corrections are checked by explicit computations of the amplitude of **one RR and 3 gauge fields**

2^{nd} Ex. for the pure SYM vertices

The same is true for 2 scalar-2 gauge field couplings
[E.H,IY Park 1203.5553,PRD] (CAA\phi)

$$-\frac{T_p(2\pi\alpha')^4}{2}\mathrm{STr}\left(D_a\phi^i D^b\phi_i F^{ac}F_{bc} - \frac{1}{4}(D_a\phi^i D^a\phi_i F^{bc}F_{bc})\right)$$

3rd Ex : **2 tachyons, 2 scalar fields on** Brane-Anti-Brane
[E.H,1601.06667,JCAP].

However, for 4 scalars the coefficients change (dualities)
[E.H,IY Park, 1205.5079,NPB].

Non-BPS Branes

This action was proposed by S-Matrix method which is a generalization of DBI action for gauge fields and MSF's on WV of branes.

$$L = -V(T)\sqrt{-\det(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu} + 2\pi\alpha' \partial_\mu T \partial_\nu T)}$$

$$V(T) = 1 - \frac{T^2}{4} + O(T^4)$$

For non –BPS branes, this potential produces tachyon's mass on branes.

To have consistency with S-Matrix method we generalized tachyonic action so that it reproduces all desired couplings for brane-anti brane systems.

arXiv: 1707.06609, JHEP,EH

Field Theory of Brane-Anti brane

The effective action of a *brane-anti brane in IIA(IIB) theory is given by* extension of the DBI action and the WZ terms which include the tachyon fields.

$$S_{DBI} = -\int d^{p+1}\sigma Tr\left(V(\mathcal{T})\sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab} + 2\pi\alpha' D_a\mathcal{T} D_b\mathcal{T})}\right)$$

The trace in the above action should be completely symmetric between all matrices of

$$F_{ab}, D_aT, T$$

$$F_{ab} = \begin{pmatrix} F_{ab}^{(1)} & 0 \\ 0 & F_{ab}^{(2)} \end{pmatrix},\; D_a\mathcal{T} = \begin{pmatrix} 0 & D_aT \\ (D_aT)^* & 0 \end{pmatrix},\; \mathcal{T} = \begin{pmatrix} 0 & T \\ T^* & 0 \end{pmatrix}$$

Consistency with S-Matrix imposed

If one uses ordinary trace, instead, the above action reduces to the action proposed by A.Sen after making the kinetic term symmetric and performing the trace. This latter action is not consistent with S-matrix calculation.

The tachyon potential which is consistent with S-matrix element calculations has

$$V(|T|) = 1 + \pi\alpha' m^2 |T|^2 + \frac{1}{2}(\pi\alpha' m^2 |T|^2)^2 + \cdots$$

consistent with the tachyon potential of BSFT

The Lagrangian of the above action which has contribution to the S-matrix CTTA is

$$
\begin{aligned}
\mathcal{L}_{DBI} = & -T_p(2\pi\alpha')\left(m^2|T|^2 + DT\cdot(DT)^* - \frac{\pi\alpha'}{2}\left(F^{(1)}\cdot F^{(1)} + F^{(2)}\cdot F^{(2)}\right)\right) + T_p(\pi\alpha')^3 \\
& \times\Big(\frac{2}{3}DT\cdot(DT)^*\left(F^{(1)}\cdot F^{(1)} + F^{(1)}\cdot F^{(2)} + F^{(2)}\cdot F^{(2)}\right) \\
& +\frac{2m^2}{3}|\tau|^2\left(F^{(1)}\cdot F^{(1)} + F^{(1)}\cdot F^{(2)} + F^{(2)}\cdot F^{(2)}\right) \\
& -\frac{4}{3}\left((D^\mu T)^* D_\beta T + D^\mu T(D_\beta T)^*\right)\left(F^{(1)\mu\alpha}F^{(1)}_{\alpha\beta} + F^{(1)\mu\alpha}F^{(2)}_{\alpha\beta} + F^{(2)\mu\alpha}F^{(2)}_{\alpha\beta}\right)\Big)
\end{aligned}
$$

So just this Lagrangian could consistently produce CTTA amplitude.

Note that the term $F^1.F^2$ in the tachyon

DBI action is necessary for the above consistency

It can not be derived by field redefinition of fields nor by Sen’s action.

A proposal by A. Sen for brane anti brane effective action

$$S = -\int d^{p+1}\sigma V(T)\left(\sqrt{(}-detA^{1}) + \sqrt{(}-detA^{2})\right)$$

$$A^{n}_{\mu\nu} = \eta_{\mu\nu} + 2\pi\alpha' F^{n}_{\mu\nu} + \pi\alpha'\left(D_{\mu}T(D_{\nu}T)^{*} + D_{\nu}T(D_{\mu}T)^{*}\right)$$

Non-BPS branes

Field strength and **Covariant derivative of the Tachyons** are

$$F = \frac{1}{2} F_{ab} dx^a \wedge dx^b, \quad DT = (\partial_a T - i[A_a, T]) dx^a$$

Using **S-matrix method** the kinetic term of tachyon appears in the DBI action as

$$S_{DBI} \sim \int d^{p+1}\sigma \mathrm{STr} \Bigg(V(T^i T^i) \sqrt{1 + \frac{1}{2}[T^i, T^j][T^j, T^i])} \times \sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab} + 2\pi\alpha' D_a T^i (Q^{-1})^{ij} D_b T^j)} \Bigg)$$

$$V(T^i T^i) = e^{-\pi T^i T^i/2} = 1 - \pi T^i T^i/2 + \frac{1}{2}(-\pi T^i T^i/2)^2$$

On D-brane anti D-brane actions, their corrections to all orders in alpha-prime [1601.06667,EH,JCAP]

Discovering all higher derivative corrections to produce all scalar poles of $< V_C V_\phi V_T V_T >$

Explored the presence of new coupling

$$D\phi^1 . D\phi^2$$

The symmetric trace effective action has a non-zero coupling (like above) while this coupling does not exist in ordinary trace action.

The only consistent effective action for D-brane anti D-brane systems, based on direct S-matrix computations of
CATT, CTTTT

was appeared in [1707.06609, EH,JHEP] and in

R. Antonelli, I.Basile, EH , arXiv: 1903.07540, JCAP

We have shown that there were non-zero couplings between 2 field strengths, 2 covariant derivatives of scalars on brane-anti brane, we found all their higher derivative corrections.

Conclusions

We analyzed all the 3,4, 5 point functions of BPS, non-BPS, D-brane anti-D-brane system.

It is not clear how to produce all contact terms of string amplitude. Possibly pull back may need modification.

We found **universality** in all order higher derivative corrections of brane-anti-brane system.

Fermionic amplitudes and Universality in Type IIA,IIB?

The supersymmetric generalization of DBI is **still unknown.**

Thank you


THE MAGIC ROUNDABOUT
Type I
Heterotic
E8xE8
SO(32)
R
R
11D
SUGRA
Type IIA
Type IIB