CONSTRAINTS ON LOW ENERGY EFFECTIVE THEORIES FROM WEAK COSMIC CENSORSHIP

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MOTIVATION

- Due to quantum correction, the Einstein-Maxwell theory will turn into a higher derivative theory (HDT):

$$I = \int d^4x \sqrt{-g}(\frac{1}{2\kappa}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \Delta L)$$

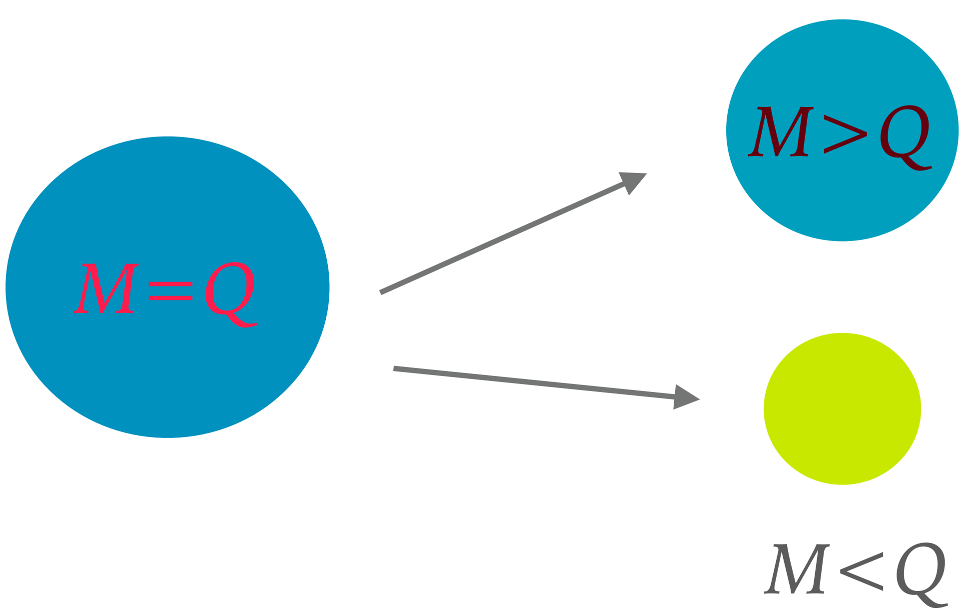
$$\begin{aligned}\Delta L = \quad & c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \\ & + c_4 R F_{\mu\nu}F^{\mu\nu} + c_5 R_{\mu\nu}F^{\mu\rho}F^{\nu}{}_{\rho} + c_6 R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + \\ & + c_7 F_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} + c_8 F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu} \,.\end{aligned}$$

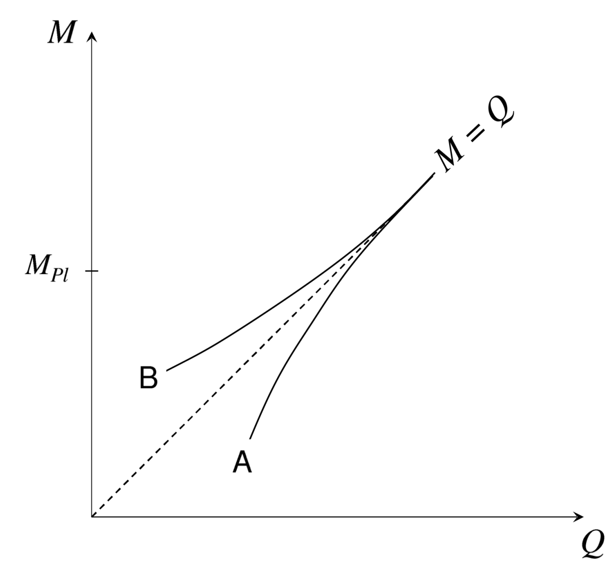
- Could we have some non-perturbative principle to constrain the Wilson coefficients for the higher derivative terms?

N.B. $c_1, c_2, c_3, \kappa^{-1}c_4, \kappa^{-1}c_5, \kappa^{-1}c_6, \kappa^{-2}c_7, \kappa^{-2}c_8 \ll 1$, *with* $\kappa = 8\pi G_N$

MOTIVATION

- One such example is the Weak Gravity Conjecture (WGC) by requiring the gauge force must be stronger than gravity.
- WGC is motivated by distinguishing the swampland in string landscape. This requires the number of light charged particles are thermodynamically finite [Arkani-Hamed et al, '06].
- Since extremely BH behaves like elementary particles, it requires the quantum effect to change the extremal condition from M=Q to M<Q so that most of the extremal BHs can decay. $(\kappa = 2)$





EXTREMAL RN BLACK HOLE

- In Einstein-Maxwell, the metric of an RN BH:
  $ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$,
  $e^{\nu^{(0)}} = e^{-\lambda^{(0)}} = 1 - \dfrac{\kappa m}{r} + \dfrac{\kappa q^2}{2r^2}$, $\quad m = M/4\pi, q = Q/4\pi$, and the extremality bound is $m \geq \sqrt{\dfrac{2}{\kappa}}|q|$.

In HDT,

$$
\begin{aligned}
e^{\nu} = 1 - \frac{\kappa m}{r} + \frac{\kappa q^2}{2r^2} &+ c_2\left(\frac{\kappa^3 m q^2}{r^5} - \frac{\kappa^3 q^4}{5r^6} - \frac{2\kappa^2 q^2}{r^4}\right) \\
&+ c_3\left(\frac{4\kappa^3 m q^2}{r^5} - \frac{4\kappa^3 q^4}{5r^6} - \frac{8\kappa^2 q^2}{r^4}\right) \\
&+ c_4\left(-\frac{6\kappa^2 m q^2}{r^5} + \frac{4\kappa^2 q^4}{r^6} + \frac{4\kappa q^2}{r^4}\right) \\
&+ c_5\left(\frac{4\kappa^2 q^4}{5r^6} - \frac{\kappa^2 m q^2}{r^5}\right) \\
&+ c_6\left(\frac{\kappa^2 m q^2}{r^5} - \frac{\kappa^2 q^4}{5r^6} - \frac{2\kappa q^2}{r^4}\right) \\
&+ c_7\left(-\frac{4\kappa q^4}{5r^6}\right) + c_8\left(-\frac{2\kappa q^4}{5r^6}\right) + \mathcal{O}(c_i^2)\,.
\end{aligned}
$$

- [Kats et al, '06]

Then, the extremity bound becomes

$$m \geq \sqrt{\frac{2}{\kappa}}|q|\left(1 - \frac{4}{5q^2}c_0\right)$$

$$c_0 \equiv c_2 + 4c_3 + \frac{c_5}{\kappa} + \frac{c_6}{\kappa} + \frac{4c_7}{\kappa^2} + \frac{2c_8}{\kappa^2}$$

MOTIVATION

- When applying to WGC for the extremal RN BH of the above HDTs, one arrive [Kats et al, '06]

$$c_0 \equiv c_2 + 4c_3 + \frac{c_5}{\kappa} + \frac{c_6}{\kappa} + \frac{4c_7}{\kappa^2} + \frac{2c_8}{\kappa^2} > 0$$

- This non-perturbative constraint can be used to compare with some perturbative results [Cheung, '14 & '18]. For example, the one-loop of Einstein-Maxwell yields only nonzero positive c_2 [Deser, '74]. But for minimally coupled scalar and spinor, one has [Bastianelli, '08 & '12]

$$\mathcal{L}_{\text{spinor}} \propto 5RF^2 - 26R_{\mu\nu}F^{\mu\rho}F^{\nu}{}_{\rho} + 2R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

$$\mathcal{L}_{\text{scalar}} \propto -\frac{5}{2}RF^2 - 2R_{\mu\nu}F^{\mu\rho}F^{\nu}{}_{\rho} - 2R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

c_4 c_5 c_6

MOTIVATION

- In this talk, we will invoke another non-perturbative principle to constrain the HDTs. It requires no violation of weak cosmic censorship conjecture (WCCC).
- WCCC states that a gravitational curvature singularity should be hidden inside a black hole horizon [Penrose, '69].
- In Einstein-Maxwell theory, a Kerr-Newman black hole will not have naked curvature singularity if its mass M, charge Q and angular momentum J=aM satisfying ($\kappa = 2$)

$$M^2 \geq a^2 + Q^2.$$

- The equality hold for extremal BH.

MOTIVATION

- However, in HDTs the extremality bound for RN BH becomes

$$m \geq \sqrt{\frac{2}{\kappa}}|q|\left(1-\frac{4}{5q^2}c_0\right)$$

- If one follow Wald's gedanken experiment of destroying an extremal BH by throwing the charged matter, this turns the original BH into a one-parameter family solutions with $m(\tau) = m + \tau\delta m$ and $q(\tau) = q + \tau\delta q$. Up to the first order of $\tau \ll 1$, the extremality bound turns into a discriminant condition for WCCC:

$$\delta m - \sqrt{\frac{2}{\kappa}}\left(1+\frac{4c_0}{5q^2}\right)\delta q \geq 0\,.$$

WALD'S GEDENKEN EXPERIMENT

- Can we overspin or overcharge to destroy a black hole by throwing matter of large spin or charge into it?
- Consider throwing a charged particle of mass m and charge e into an extremal Reissner-Nordstrom (RN) black hole with M=Q ($\kappa = 2$). The energy of the particle is given by

$$E = -(mu_\mu + eA_\mu)\xi^\mu \geq e\Phi_H = e \qquad \text{with } \Phi_H = (-A_\mu \xi^\mu)|_H = 1 \text{ for extermal RN black hole.}$$

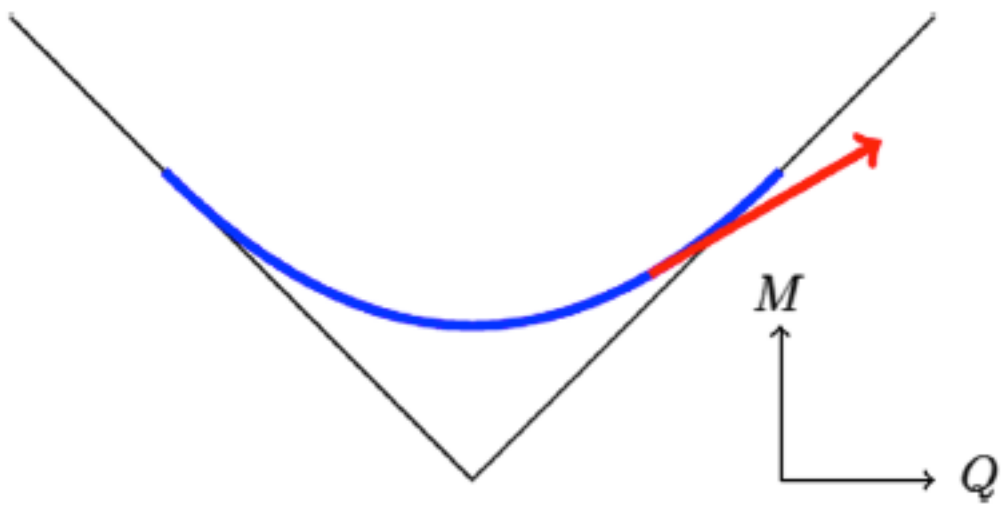
Thus, M+E>=Q+e, impossible to overcharge.

- However, this simple argument cannot be generalized to generic matter and does not work beyond Einstein-Maxwell theory. Moreover, it also fails for near-extremal BH.

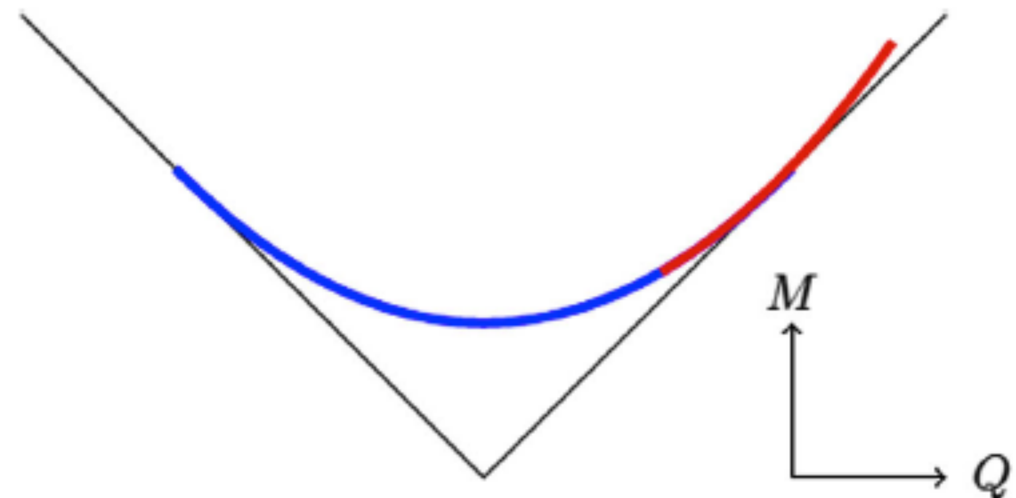
HUBENY'S ARGUMENT

- Hubeny (1999) argued that it is possible to overcharge a near-extremal black hole. Parametrizing the near-extremality by $\varepsilon = \sqrt{1 - Q^2/M^2}$.
- The EM potential now is $\Phi_{\mathrm{H}} = Q/r_+ \simeq 1 - \varepsilon$, and the energy of the charged particle $E > (1 - \varepsilon)\, e$. Thus, we have $M + E - (Q + e) \simeq -\varepsilon e + \varepsilon^2 M/2$
- It seems that we can overcharge to destroy a black hole if $e > \epsilon M/2$. However, this is not the whole story since the e^2 effect is involved for the argument without also including it in estimating E.

- In 2017, Sorce & Wald gave a general proof of WCCC based on the variational identities, which is the generalization of BH's first law when considering the falling-in of the generic matters up to 2nd order variation.



Hubeny's argument (1999)

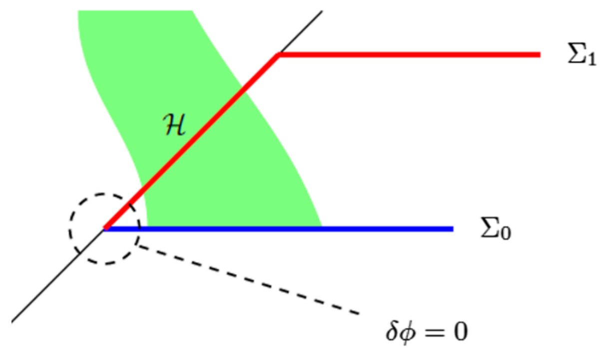


Including 2nd order effect by Sorce & Wald (2017)

- In this talk, we will focus only on the WCCC for extremal RN BHs in HDTs.

DESTROY A EXTREMAL BH: SORCE & WALD

- The charged matter falls through the event horizon of an extremal BH in the finite time interval. The perturbed initial data is chosen to vanish near the horizon.



- Since the BH is extremal, the process causes no gravitational & EM radiation, the black hole mechanics is manifested as a variational identity $\delta\mathcal{M} - \Phi_{\rm H}\int_{\mathcal{H}}\epsilon_{abcd}\,\delta j^a = -\int_{\mathcal{H}}\epsilon_{ebcd}\,\xi^a\delta T^e{}_a\,,$ where $\Phi_{\rm H} := -(\xi^a A_a)\,|_{\mathcal{H}}$
- and $\delta\mathcal{M}$ and $\delta\mathcal{Q} \equiv \int_{\mathcal{H}}\epsilon_{abcd}\,\delta j^a$ are the changes of mass and charge of BH caused by in-falling matter which obeys NEC:
- Using $\epsilon_{ebcd} = -4n_{[e}\tilde{\epsilon}_{bcd]}$ and NEC, the variation Id turns into

$$\delta\mathcal{M} - \Phi_{\rm H}\delta\mathcal{Q} \geq 0.$$

WALD'S DERIVATION OF VARIATION ID

- Start with Lagrangian 4-form $\mathbf{L} = L(\phi)\epsilon$ with $\phi = (g_{ab}, A_a)$, its variation yields $\delta\mathbf{L} = \mathbf{E}(\phi)\delta\phi + d\mathbf{\Theta}(\phi, \delta\phi)$, where $\mathbf{E}(\phi) = 0$ is EoM, and $\mathbf{\Theta}(\phi, \delta\phi)$ is the symplectic 3-form.
- For a vector ξ^a, define the conserved Noether current $\mathbf{J}_\xi = \mathbf{\Theta}(\phi, \mathscr{L}_\xi\phi) - i_\xi\mathbf{L}$. Since $d\mathbf{J}_\xi = 0$, $\mathbf{J}_\xi = d\mathbf{Q}_\xi + \xi_d\mathbf{C}^d$ with the 3-form constraint $\mathbf{C}^d = 0$ whenever $\mathbf{E}(\phi) = 0$.
- If $\mathscr{L}_\xi\phi = 0$ (a timeline Killing symmetry), one can show $\delta\mathbf{J}_\xi = di_\xi\mathbf{\Theta}(\phi, \delta\phi)$. Together with $\delta\mathbf{J}_\xi = d\delta\mathbf{Q}_\xi + \xi^a\delta\mathbf{C}_a$, one arrives variational Id
- $$\delta\mathscr{M} := \int_{\partial\Sigma=\infty} [\delta\mathbf{Q}_\xi - i_\xi\mathbf{\Theta}(\phi, \delta\phi)] = -\int_H \epsilon_{ebcd}\xi^a(\delta T^e{}_a + A_a\delta j^e)$$
- Used: $(\delta\mathbf{C}_a)_{bcd} := \epsilon_{ebcd}(\delta T^e{}_a + A_a\delta j^e)$

FORMAL RESULTS FOR HDTS

- Define some notations: $E^{abcd} \equiv \dfrac{\delta L}{\delta R_{abcd}}, E_F^{ab} \equiv \dfrac{\delta L}{\delta F_{ab}}$
- Symplectic 3-form: $\mathbf{\Theta}_{b_2b_3b_4} = \epsilon_{ab_2b_3b_4}(2E^{abcd}\nabla_d\delta g_{bc} - 2\delta g_{bc}\nabla_d E^{abcd} + 2E_F^{ab}\delta A_b)$
- Noether charge: $(\mathbf{Q}_\xi)_{b_3b_4} = \epsilon_{abb_3b_4}\left((-2\nabla_d E^{abcd} + E_F^{ab}A^c)\,\xi_c - E^{abcd}\nabla_{[c}\,\xi_{d]}\right)$
- Constraints: $(\mathbf{C}^d)_{b_2b_3b_4} = \epsilon_{eb_2b_3b_4}\left(2E^{pqre}R_{pqr}{}^d + 4\nabla_f\nabla_h E^{efdh} + 2E_F^{eh}F_h^d - 2A^d\nabla_h E_F^{eh} - g^{ed}L\right)$
- E.g., $\mathbf{L} = \mathbf{L}_0 + \sum_i c_i \mathbf{L}_i$
- $\delta \mathbf{L}_4 = \delta g_{ab}(E_4^g)^{ab}\epsilon + \delta A_a(E_4^A)^a\epsilon + \mathbf{d}\mathbf{\Theta}_4,$
- $(E_4^g)^{ab} = \left(-R^{ab} + \frac{1}{2}g^{ab}R - g^{ab}\nabla^2 + \nabla^{(a}\nabla^{b)}\right)F^2 - 2RF^{ac}F_b{}^c,$
  $(E_4^A)^a = 4\nabla_b\left(RF^{ab}\right)\ .$
- $(Q_\xi^4)_{ab} = \epsilon_{abcd}\left(F^2\nabla^d\xi^c - 2\xi^c\nabla^d F^2 + 2RF^{cd}A_e\xi^e\right)$
- $C_{bcda}^4 = -2\epsilon_{ebcd}(E_4^g)^e{}_a - \epsilon_{ebcd}(E_4^A)^e A_a$ $\quad\longleftrightarrow\quad$ $\mathbf{E}\delta\phi = -\epsilon\left(\frac{1}{2}T^{ab}\delta g_{ab} + j^a\delta A_a\right)$, $C_{bcda} = \epsilon_{ebcd}\left(T^e{}_a + j^e A_a\right)$

DESTROY AN EXTREMAL KERR-NEWMAN BH IN EINSTEIN-MAXWELL?

- We now see the case of Maxwell-Einstein (set $\kappa = 2$):
- Extremality bound: $M^2 \geq (a^2 + Q^2)$ with $a := J/M$.
- Its variation gives the condition for WCCC:
  $$M\delta M \geq \frac{J}{M^3}(M\delta J - J\delta M) + Q\delta Q = \frac{a}{M}(\delta J - a\delta M) + Q\delta Q.$$
- On the other hand, Variational Id & NEC yields
  $\delta M - \Omega_{\rm H}\delta J - \Phi_{\rm H}\delta Q \geq 0$
- For extremal BH, $\Omega_{\rm H} = \dfrac{a}{M^2 + a^2}$, $\Phi_{\rm H} = \dfrac{MQ}{M^2 + a^2}$, then the above inequality is coincident with the condition for WCCC.

DESTROY AN EXTREMAL RN BH IN HIGHER DERIVATIVE THEORIES?

- First, note that the horizon of the extremal BH in HDT is shifted: $r_{\rm H} = \frac{m\kappa}{2} + \frac{4}{5m}\left(c_2 + 4c_3 + \frac{10c_4}{\kappa} + \frac{c_5}{\kappa} + \frac{c_6}{\kappa} - \frac{16c_7}{\kappa^2} - \frac{8c_8}{\kappa^2}\right)$
- The gauge potential is changed, too:
$A_t = -\frac{q}{r} + \frac{2q^3}{5r^5}\left(c_5\kappa + 6c_6\kappa - \frac{5c_6\kappa m r}{q^2} + 8c_7 + 4c_8\right)$
- Combine both we can evaluate the chemical potential at horizon: $\Phi_{\rm H} := -(\xi^a A_a)|_H = \sqrt{\frac{2}{\kappa}}\left(1 + \frac{4c_0'}{5q^2}\right)$ where
$c_0' = -\frac{10c_4}{\kappa} - \frac{2c_5}{\kappa} - \frac{2c_6}{\kappa} + \frac{4c_7}{\kappa^2} + \frac{2c_8}{\kappa^2}.$
- recall our notations: $m \equiv M/4\pi$, $q \equiv Q/4\pi$ and $\kappa = 8\pi G_N$

DESTROY AN EXTREMAL RN BH IN HIGHER DERIVATIVE THEORIES?

- Recall the variation inequality: $\delta\mathcal{M} - \Phi_{\rm H}\delta\mathcal{Q} \geq 0$
- Using the formal results we can evaluate $\delta\mathcal{M}$ and $\delta\mathcal{Q}$ for the extremal charged BH in HDT.
- It is easy to see that the higher derivative corrections fall off quickly and will not change the ADM mass, i.e, $\delta\mathcal{M} = \delta M$.
- Similar, we find that $\delta\mathcal{Q} \equiv \int_{\mathcal{H}} \epsilon_{abcd}\, \delta j^a = \delta Q + \mathcal{O}(c_i^2).$
- Combine the above we arrive: $\delta m - \sqrt{\frac{2}{\kappa}}\left(1 + \frac{4c_0'}{5q^2}\right)\delta q \geq 0$
- Compare with extremality bound: $\delta m - \sqrt{\frac{2}{\kappa}}\left(1 + \frac{4c_0}{5q^2}\right)\delta q \geq 0$, we obtain the condition for WCCC: $c_0' \geq c_0$, or
$c_2 + 4c_3 + \frac{10c_4}{\kappa} + \frac{3c_5}{\kappa} + \frac{3c_6}{\kappa} \leq 0.$

KEY RESULT

- Our key result: condition for WCCC to hold in HDT —
  $c_2 + 4c_3 + \frac{10c_4}{\kappa} + \frac{3c_5}{\kappa} + \frac{3c_6}{\kappa} \leq 0$
- Note that c_7 and c_8 do not appear in the above, this means that there is no constraint on the box diagram of QED.
- The 1-loop result of Einstein-Maxwell violates the WCCC.
- However, the 1-loop EFT for the minimally coupled scalar and spinor violate the WGC: $c_2 + 4c_3 + \frac{c_5}{\kappa} + \frac{c_6}{\kappa} + \frac{4c_7}{\kappa^2} + \frac{2c_8}{\kappa^2} \geq 0$, but do not violate WCCC.

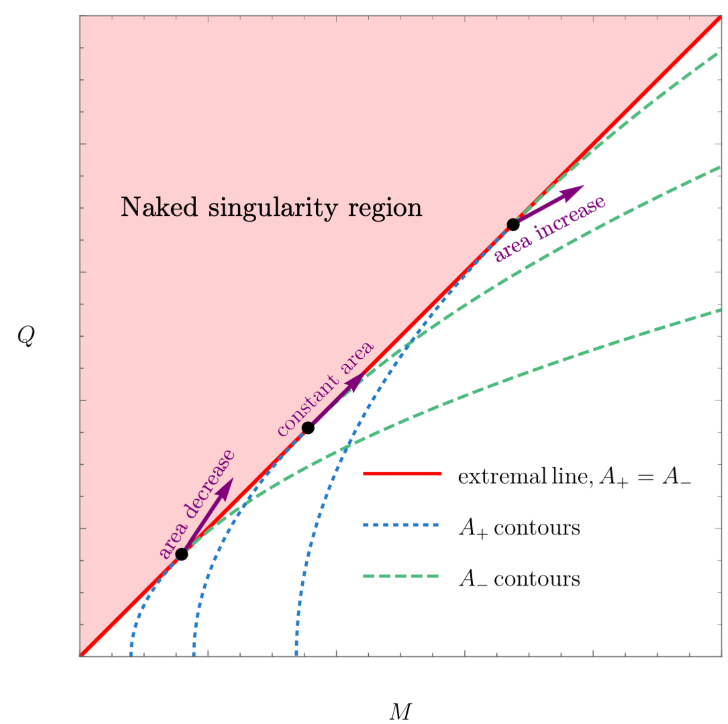
$$\mathscr{L}_{\text{spinor}} \propto 5RF^2 - 26R_{\mu\nu}F^{\mu\rho}F^{\nu}{}_{\rho} + 2R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

$$\mathscr{L}_{\text{scalar}} \propto -\frac{5}{2}RF^2 - 2R_{\mu\nu}F^{\mu\rho}F^{\nu}{}_{\rho} - 2R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

DISCUSSIONS & CONCLUSIONS

- Unlike the WGC bound, our WCCC bound are not invariant under field redefinitions: $g_{\mu\nu} \longrightarrow g_{\mu\nu} + \delta g_{\mu\nu}$ with $\delta g_{\mu\nu} = r_1 R_{\mu\nu} + r_2 g_{\mu\nu} R + r_3 \kappa F_{\mu\rho} F^{\rho}_{\nu} + r_4 \kappa g_{\mu\nu} F^2$. However, this may be turned into a requirement to fix how the matter couples to gravity and Maxwell.

- Moreover, it is easy to see that the extremality contour is coincident with the constant-area contour:



Suppose $F(M, Q, A) = 0$ so that the tangent vector satisfies

$$\partial_M F \Delta M + \partial_M F \Delta Q + \partial_A F \Delta A = 0$$

Since the 3rd term vanishes for either constant-area ($\Delta A = 0$) or extremality ($\partial_A F = 0$ for degenerate horizons), so

$$\left(dQ/dM\right)_A = \left(dQ/dM\right)_{\text{ext}} = -\partial_M F/\partial_Q F.$$

DISCUSSIONS & CONCLUSIONS

- However, in HDT the constant-area is not the same as constant-entropy, it remains to see how WCCC based on variational Id can be related to the generalized second law of BH mechanics.
- In summary: based on the WCCC condition derived from the formalism of Sorce & Wald, we have constrained the Wilson coefficients of HDT, and may serve as a new principle to distinguish the part of swampland in the string landscape.
- One may try to derive the WCCC condition for near-extremal BH in HDT, which may give more subtle constraints due to the GW radiation and self-forces.
- It is also interesting to compare with other constraints from WGC or general principle of QFT & quantum gravity.

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Nature abhors a naked singularity!

-Stephen Hawking '91