The BMS group in (A)dS

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4th International Conference on
Holography, String Theory and Discrete Approach in Hanoi, Vietnam,
August 5th 2020

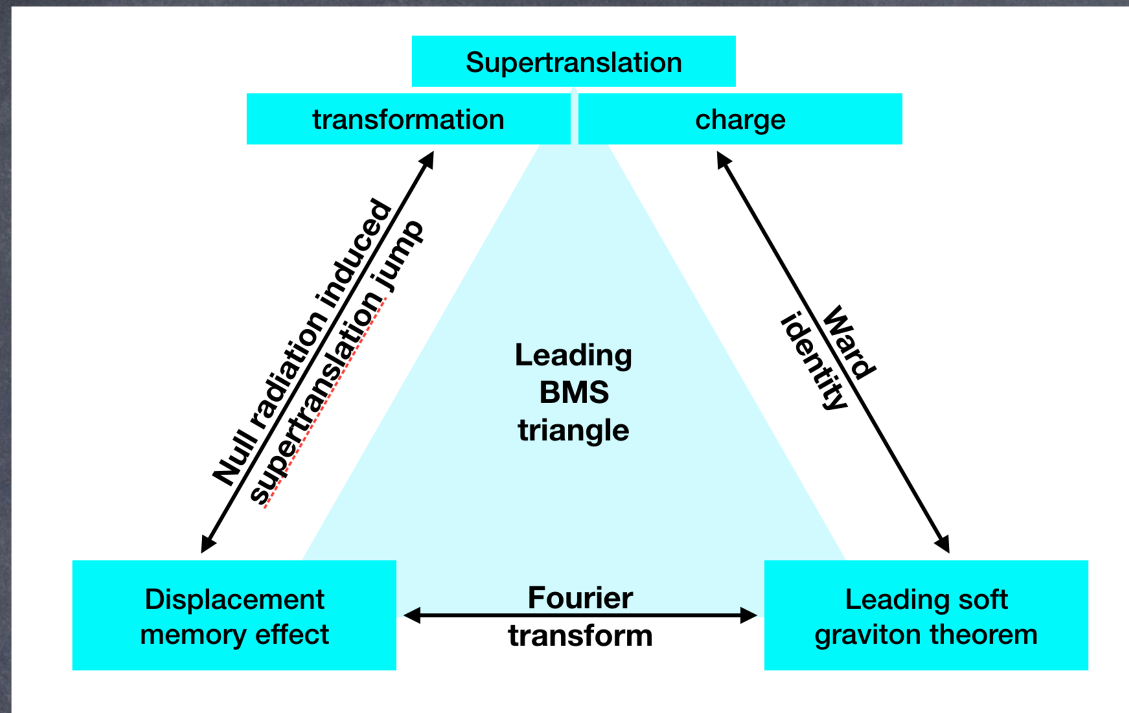
References

- G.C., Adrien Fiorucci and Romain Ruzziconi,
  The Λ-BMS_4 Charge Algebra
  2004.10769

- G.C., Adrien Fiorucci and Romain Ruzziconi,
  The Λ-BMS_4 group of dS_4 and new boundary conditions for AdS_4.
  1905.00971

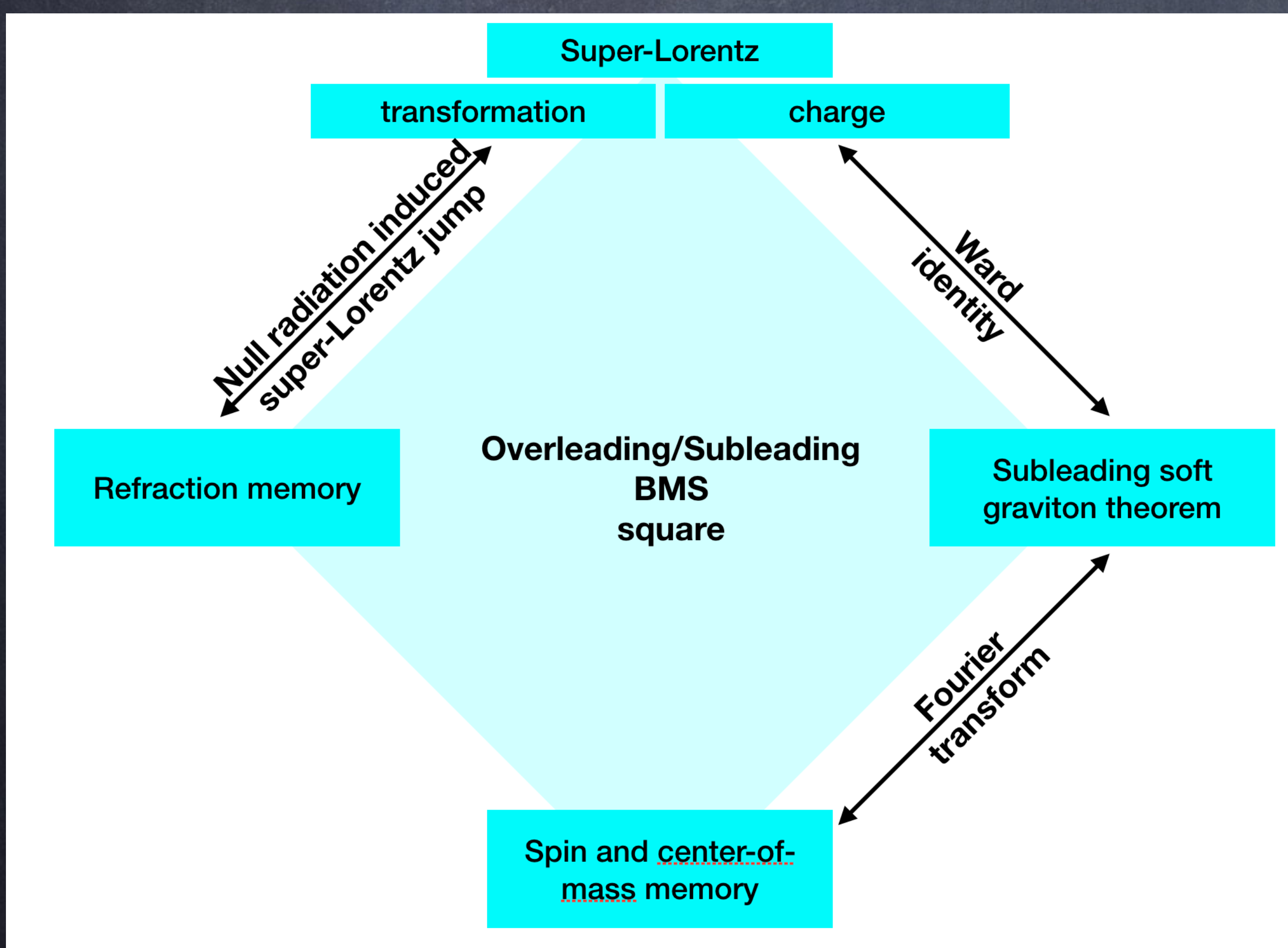
- G.C., Adrien Fiorucci and Romain Ruzziconi,
  Superboost transitions, refraction memory and super-Lorentz charge algebra
  1810.00377

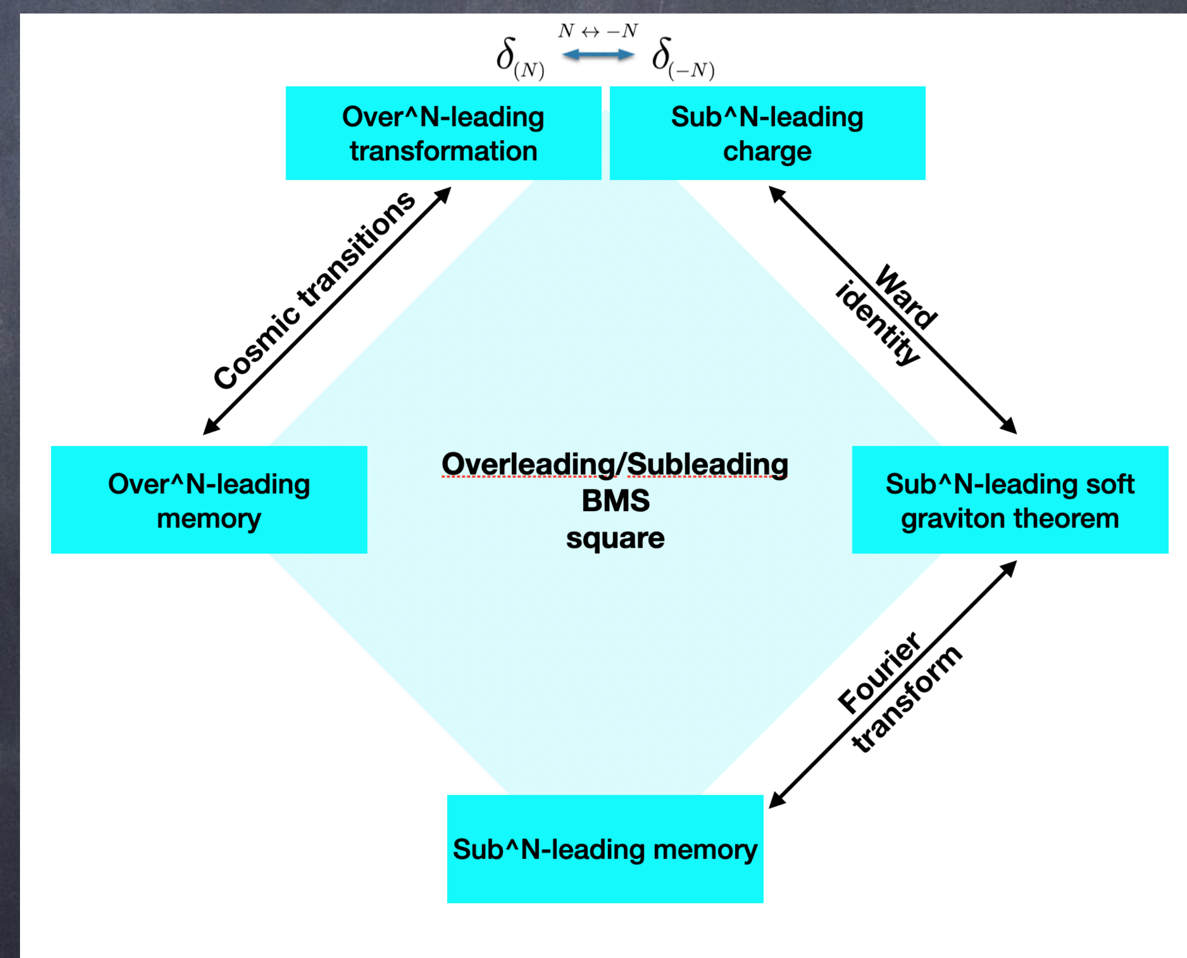
Leading, subleading, sub^N-leading



Lectures on the Infrared Structure of Gravity and Gauge Theory
Andrew Strominger

An Infinite Set of Ward Identities for Adiabatic Modes in Cosmology
Kurt Hinterbichler[a], Lam Hui[b] and Justin Khoury[c]





Superboost transitions, refraction memory and super-Lorentz charge algebra
Geoffrey Compère, Adrien Fiorucci, Romain Ruzziconi

Infinite towers of supertranslation and superrotation memories
Geoffrey Compère

Supertranslations

$$T(\theta,\phi)\partial_u + \frac{1}{2}\nabla^2 T\partial_r - \frac{1}{r}(\partial_\theta T\partial_\theta + \frac{1}{\sin^2\theta}\partial_\phi T\partial_\phi) + \dots$$

- Associated Noether charge: Bondi mass aspect $m(u,x^A)$
- 4 lowest harmonics: translations/ Momenta

[Bondi, Metzner, van der Burg, Sachs, 1962]

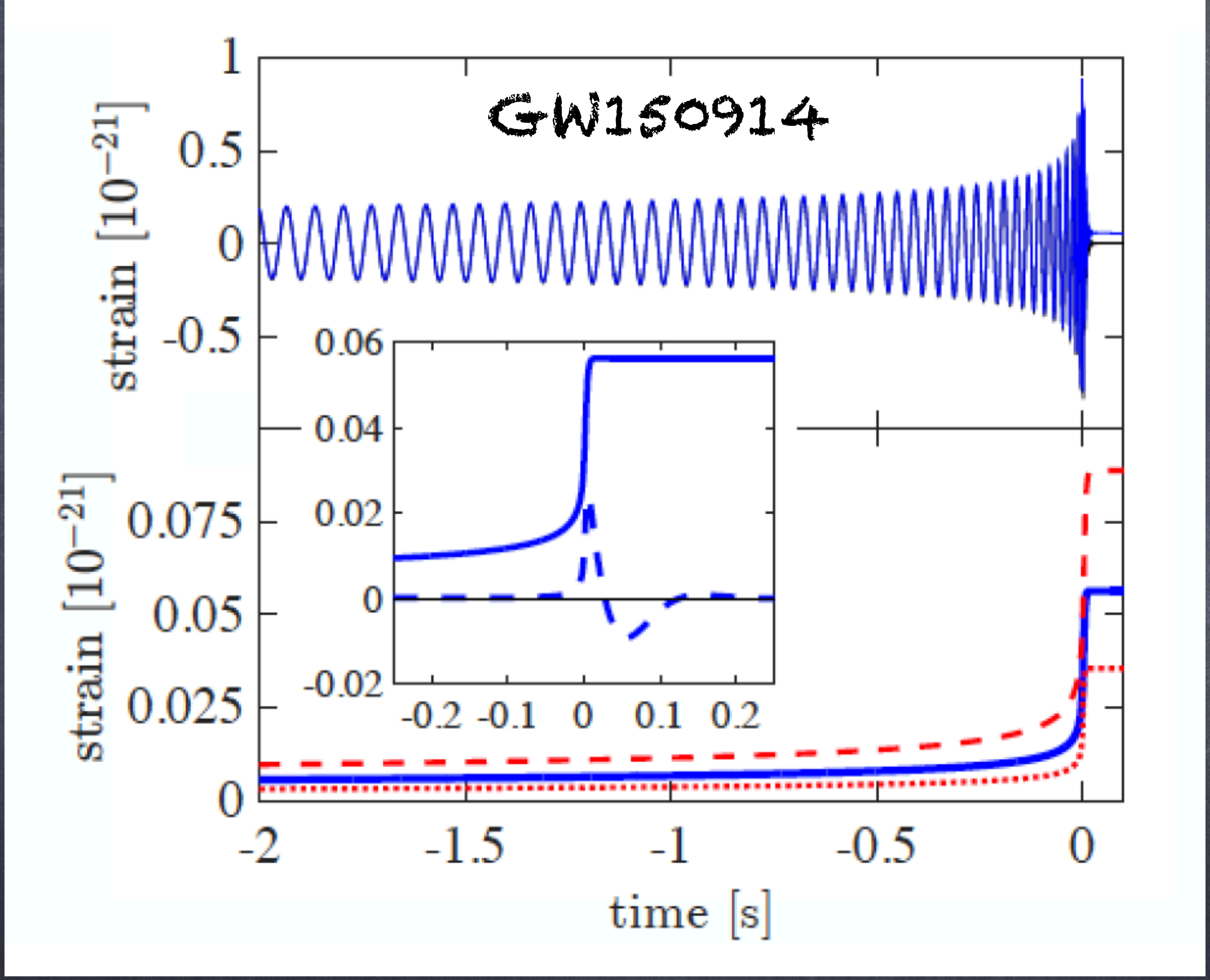
Super-Lorentz transformations

$$\frac{1}{2}uD_AR^A\partial_u + (-\frac{1}{2}(r+u)D_AR^A + \mathcal{O}(\frac{1}{r})\partial_r + (R^A - \frac{u}{2r}D^AD_BR^B + \mathcal{O}(\frac{1}{r^2}))\partial_A$$

- Associated Noether charge: Bondi angular momentum aspect $N_A(u,x^A)$
- 6 lowest harmonics: Lorentz/Angular Momenta and center-of-mass charges.
- Only Lorentz are asymptotic symmetries! The others are overleading transformations!

[Barnich, Troessaert] [Campiglia, Laddha] [G.C., Fiorucci, Ruzziconi]

Prospect of observing displacement memory at LIGO



[Lasky et al. 2016]

[Boersma, Nichols, Schmidt, 2020]

Question

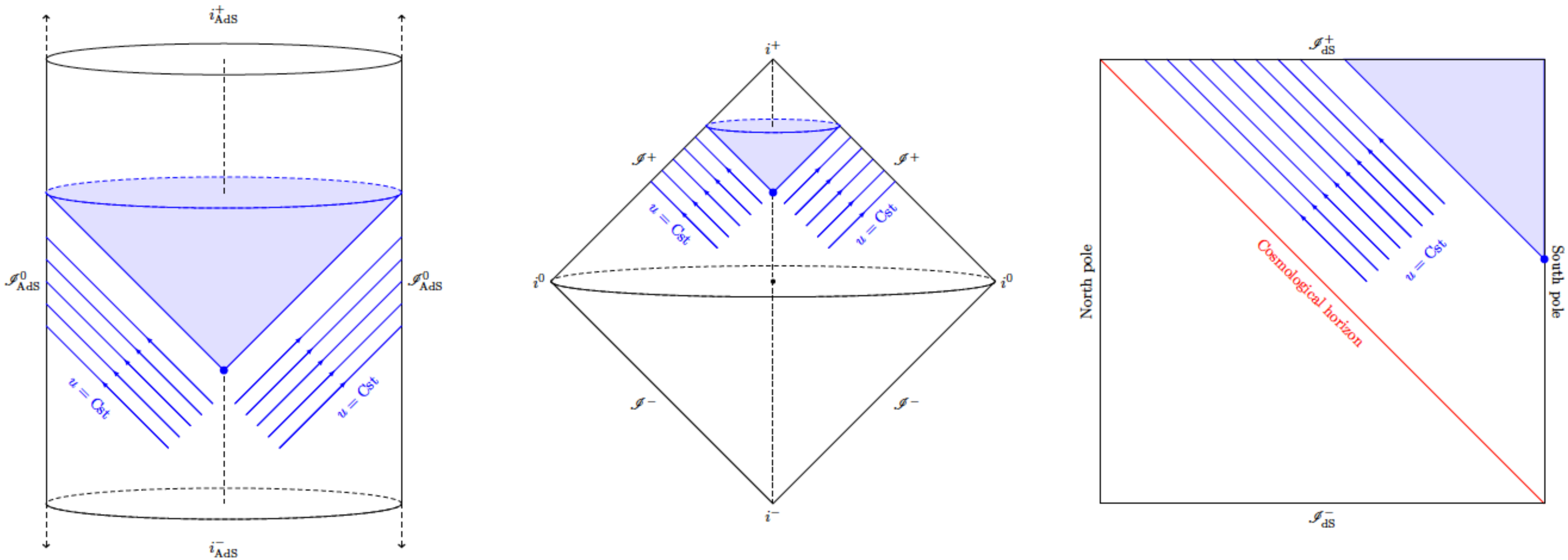
- Three-dimensional case :

Asymptotically AdS_3 : $\text{Diff}(S^1) \times \text{Diff}(S^1)$ [Brown-Henneaux '86]	$\xRightarrow[\Lambda \to 0]{}$	Asymptotically flat : $BMS_3 = \text{Diff}(S^1) \ltimes \text{Vect}(S^1)$ [Ashtekar-Bicak-Schmidt '96] [Barnich-Compère '07]

- Four-dimensional case :

Asymptotically AdS_4 : ? ? ?	$\xRightarrow[\Lambda \to 0]{}$	Asymptotically flat : $BMS_4 = \text{Diff}(S^2) \ltimes \mathcal{T}$ [Bondi-van der Burg-Metzner '62] [Sachs '62]

Universal BMS structure

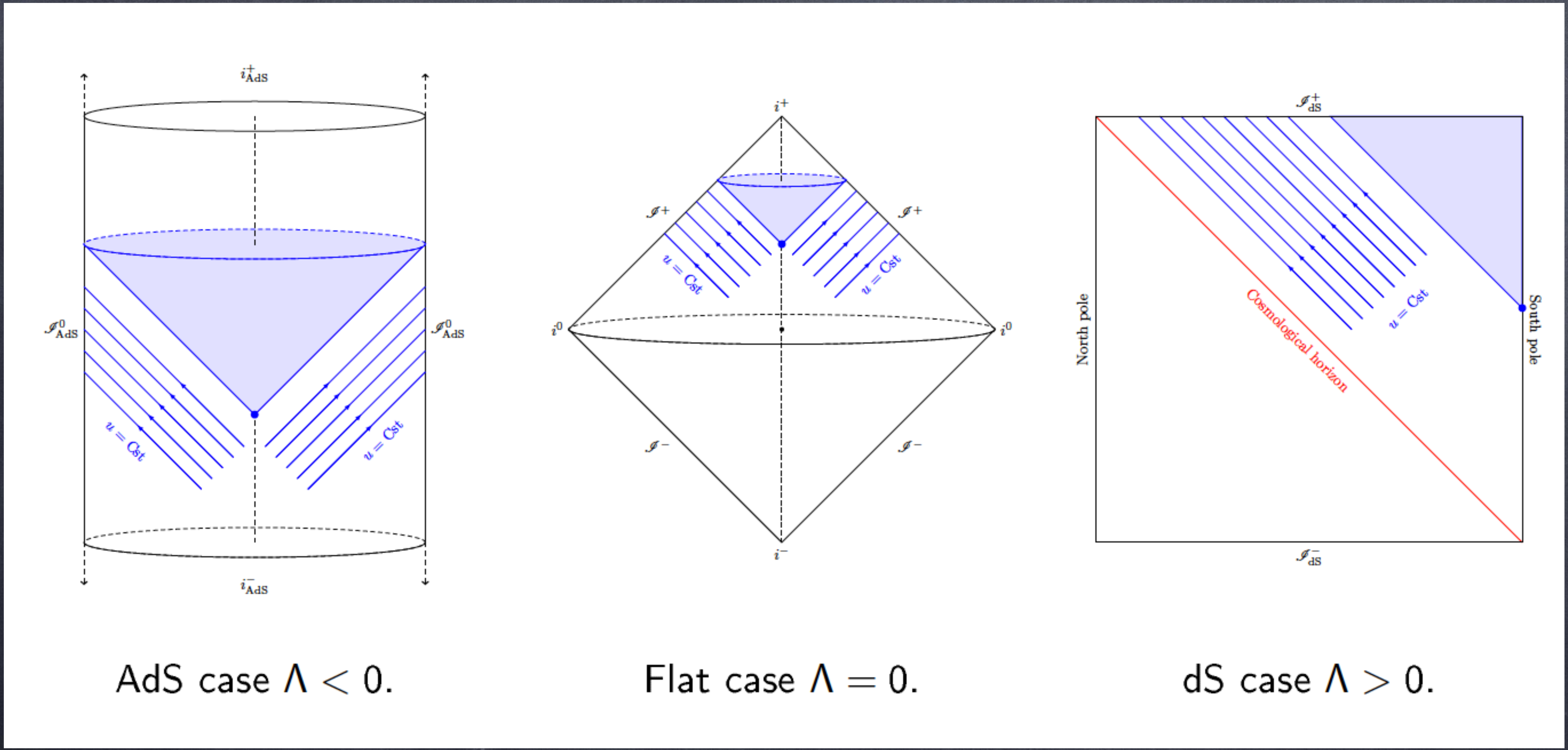


AdS case $\Lambda < 0$. Flat case $\Lambda = 0$. dS case $\Lambda > 0$.

A foliation and a measure

$$ds^2 = sign(\Lambda) du^2 + q_{AB} dx^A dx^B \qquad \sqrt{q}$$

We need a dictionary



Fefferman-Graham

$$g^{(0)}_{ab}, T^{ab}$$

Bondi

$$C_{AB}, M, N_A,$$
$$D_{AB}, E_{AB}, \dots$$

Starobinsky

$$g^{(0)}_{ab}, T^{ab}$$

Definitions

Starobinsky/
Fefferman-Graham
(SFG) gauge

$$(\rho, x^a)$$

$$g_{\rho a} = 0,$$

$$g_{\rho\rho} = -\frac{3}{\Lambda}\frac{1}{\rho^2}$$

Bondi gauge

$$(u, r, x^A)$$

$$g_{rr} = 0, \quad g_{rA} = 0$$

$$\partial_r \left(\frac{\det(g_{AB})}{r^4}\right) = 0$$

The dictionary between Bondi and Starobinsky/Fefferman-Graham gauge has been worked out

- One can solve the large radius expansion of Einstein's equations in both gauges
- We found a diffeomorphism between the two gauges when $\Lambda \neq 0$
- We found the (2-covariant) map between the free fields in each gauge

See also [Poole, Skenderis, Taylor, 2018]

Solution space (AL(A)dS$_4$)

SFG gauge

$$ds^2 = -\frac{3}{\Lambda}\frac{d\rho^2}{\rho^2} + \gamma_{ab}(\rho, x^c)dx^a dx^b.$$

$$\gamma_{ab} = \frac{1}{\rho^2} g^{(0)}_{ab} + \frac{1}{\rho} g^{(1)}_{ab} + g^{(2)}_{ab} + \rho\, g^{(3)}_{ab} + \mathcal{O}(\rho^2)$$

[FG theorem]

$$T_{ab} = \frac{\sqrt{3|\Lambda|}}{16\pi G} g^{(3)}_{ab}$$

Bondi gauge

$$ds^2 = e^{2\beta}\frac{V}{r}du^2 - 2e^{2\beta}du dr + g_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

$$g_{AB} = r^2 q_{AB} + r\, C_{AB} + D_{AB} + \frac{1}{r} E_{AB} + \frac{1}{r^2} F_{AB} + \mathcal{O}(r^{-3})$$

[Blanchet-Damour]

$$l = l^A_A = \frac{1}{2} q^{AB}\partial_u q_{AB} = \partial_u \ln\sqrt{q}.$$

$$U^A = U^A_0(u, x^B) + \overset{(1)}{U^A}(u, x^B)\frac{1}{r} + \overset{(2)}{U^A}(u, x^B)\frac{1}{r^2} + \overset{(3)}{U^A}(u, x^B)\frac{1}{r^3} + \overset{(L3)}{U^A}(u, x^B)\frac{\ln r}{r^3} + o(r^{-3})$$

$$\beta(u, r, x^A) = \beta_0(u, x^A) + \frac{1}{r^2}\Big[-\frac{1}{32}C^{AB}C_{AB}\Big] + \frac{1}{r^3}\Big[-\frac{1}{12}C^{AB}\mathcal{D}_{AB}\Big] + \frac{1}{r^4}\Big[-\frac{3}{32}C^{AB}\mathcal{E}_{AB} - \frac{1}{16}\mathcal{D}^{AB}\mathcal{D}_{AB} + \frac{1}{128}(C^{AB}C_{AB})^2\Big] + \mathcal{O}(r^{-5}).$$

$$\frac{V}{r} = \frac{\Lambda}{3}e^{2\beta_0}r^2 - r(l + D_A U^A_0) - e^{2\beta_0}\Big[\frac{1}{2}\Big(R[q] + \frac{\Lambda}{8}C_{AB}C^{AB}\Big) + 2D_A\partial^A\beta_0 + 4\partial_A\beta_0\partial^A\beta_0\Big] - \frac{2M}{r} + o(r^{-1})$$

Solution space: free fields $\Lambda \neq 0$

SFG gauge | **Bondi gauge**

$$g^{(0)}_{ab} dx^a dx^b \;\neq\; \frac{\Lambda}{3} e^{4\beta_0} du^2 + q_{AB}(dx^A - U_0^A du)(dx^B - U_0^B du)$$

$$T^{ab}$$

$$T_{ab} = \frac{\sqrt{3|\Lambda|}}{16\pi G} \begin{bmatrix} -\frac{4}{3} M^{(\Lambda)} & -\frac{2}{3} N_B^{(\Lambda)} \\ -\frac{2}{3} N_A^{(\Lambda)} & J_{AB} + \frac{2}{\Lambda} M^{(\Lambda)} q_{AB} \end{bmatrix}$$

$$M^{(\Lambda)} = M + \frac{1}{16}(\partial_u + l)(C_{CD}C^{CD}),$$

$$N_A^{(\Lambda)} = N_A - \frac{3}{2\Lambda} D^B(N_{AB} - \frac{1}{2} l C_{AB}) - \frac{3}{4}\partial_A(\frac{1}{\Lambda} R[q] - \frac{3}{8} C_{CD}C^{CD}),$$

$$\begin{aligned} J_{AB} = & -\mathcal{E}_{AB} - \frac{3}{\Lambda^2}\Big[\partial_u(N_{AB} - \frac{1}{2} l C_{AB}) - \frac{\Lambda}{2} q_{AB} C^{CD}(N_{CD} - \frac{1}{2} l C_{CD})\Big] \\ & + \frac{3}{\Lambda^2}(D_A D_B l - \frac{1}{2} q_{AB} D_C D^C l) \\ & - \frac{1}{\Lambda}(D_{(A} D^C C_{B)C} - \frac{1}{2} q_{AB} D^C D^D C_{CD}) \\ & + C_{AB}\Big[\frac{5}{16} C_{CD}C^{CD} + \frac{1}{2\Lambda} R[q]\Big]. \end{aligned}$$

Constraints

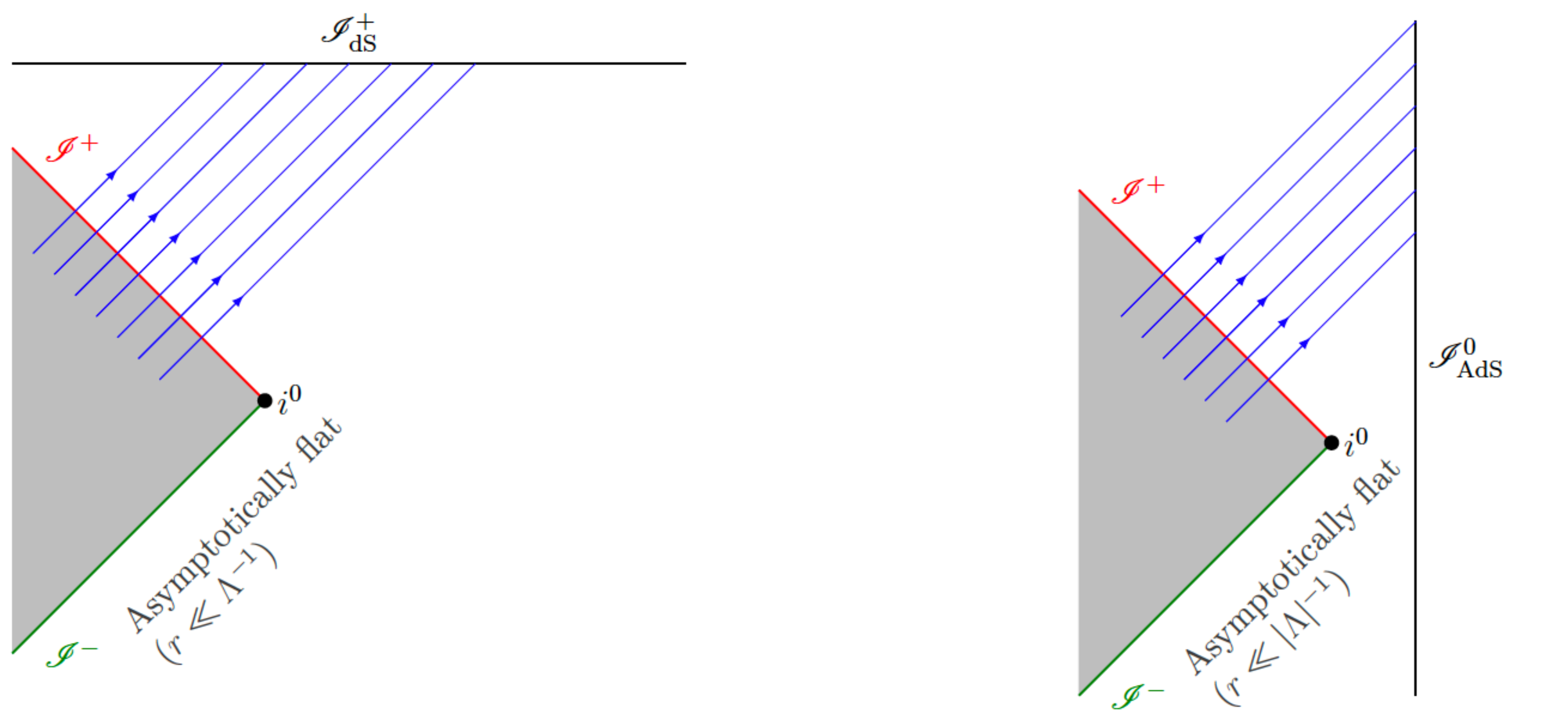
$$D_a^{(0)} T^{ab} = 0, \quad g^{(0)}_{ab} T^{ab} = 0.$$

$$\frac{\Lambda}{3} C_{AB} = e^{-2\beta_0}\Big[(\partial_u - l) q_{AB} + 2 D_{(A} U^0_{B)} - D^C U^0_C q_{AB}\Big].$$

$$(\partial_u + \frac{3}{2} l) M^{(\Lambda)} + \frac{\Lambda}{6} D^A N_A^{(\Lambda)} + \frac{\Lambda^2}{24} C_{AB} J^{AB} = 0.$$

$$(\partial_u + l) N_A^{(\Lambda)} - \partial_A M^{(\Lambda)} - \frac{\Lambda}{2} D^B J_{AB} = 0.$$

Why the Bondi mass differs from (A)dS and Minkowski?



(a) Asymptotically locally dS_4 case.

(b) Asymptotically locally AdS_4 case.

$$M^{(\Lambda)} = M + \frac{1}{16}(\partial_u + l)(C_{CD}C^{CD}),$$
$$N_A^{(\Lambda)} = N_A - \frac{3}{2\Lambda}D^B(N_{AB} - \frac{1}{2}lC_{AB}) - \frac{3}{4}\partial_A(\frac{1}{\Lambda}R[q] - \frac{3}{8}C_{CD}C^{CD}),$$

Solution space: free fields $\Lambda = 0$

SFG gauge

Bondi gauge

$$\frac{\Lambda}{3} e^{4\beta_0} du^2 + q_{AB}(dx^A - U_0^A du)(dx^B - U_0^B du)$$

$$M, N_A, C_{AB}, D_{AB}, E_{AB}, F_{AB}, \cdots$$

Constraints

$$\partial_u q_{AB} = l q_{AB} + 2 D_{(A} U^0_{B)} - D^C U^0_C q_{AB}$$

$$\partial_u M = -\frac{1}{8} N_{AB} N^{AB} + \frac{1}{4} D_A D_B N^{AB} + \frac{1}{8} D_A D^A \mathring{R},$$

$$\begin{aligned}\partial_u N_A = D_A M &+ \frac{1}{16} D_A (N_{BC} C^{BC}) - \frac{1}{4} N^{BC} D_A C_{BC} \\ &- \frac{1}{4} D_B (C^{BC} N_{AC} - N^{BC} C_{AC}) - \frac{1}{4} D_B D^B D^C C_{AC} \\ &+ \frac{1}{4} D_B D_A D_C C^{BC} + \frac{1}{4} C_{AB} D^B \mathring{R}.\end{aligned}$$

$$\partial_u D_{AB} = 0,$$

$$\partial_u E_{AB} = \cdots$$

$$\partial_u F_{AB} = \cdots$$

see [Barnich,Troessart, 2012]

Boundary gauge condition: Definition of Λ-BMS

$$g^{(0)}_{tt} = \frac{\Lambda}{3}, \qquad g^{(0)}_{tA} = 0, \qquad \det(g_{(0)}) = \frac{\Lambda}{3}\bar{q}.$$

$$ds^2_{(0)} = \frac{\Lambda}{3}dt^2 + q_{AB}dx^A dx^B$$

- Can always be reached
- Does not constraint the Cauchy problem

The residual diffeomorphisms in a given bulk gauge and in this boundary gauge form the Λ-BMS group.

Symmetry generators

Preserving SFG gauge: Diff(S³)xWeyl

$$\xi^u = f,$$
$$\xi^A = Y^A + I^A, \quad I^A = -\partial_B f \int_r^\infty dr' (e^{2\beta} g^{AB}),$$
$$\xi^r = -\frac{r}{2}(\mathcal{D}_A Y^A - 2\omega + \mathcal{D}_A I^A - \partial_B f U^B + \frac{1}{2} f g^{-1} \partial_u g),$$

$$g = \det(g_{AB})$$

$$\partial_r f = 0 = \partial_r Y^A$$

Preserving further boundary gauge:

$$\delta_\xi \sqrt{q} = 0 \longrightarrow \omega = 0.$$

$$\delta_\xi \beta_0 = 0 \longrightarrow \left(\partial_u - \frac{1}{2} l\right) f = \frac{1}{2} D_A Y^A,$$

$$l = l^A_A = \frac{1}{2} q^{AB} \partial_u q_{AB} = \partial_u \ln \sqrt{q}.$$

$$\delta_\xi U_0^A = 0 \longrightarrow \partial_u Y^A = -\frac{\Lambda}{3} \partial^A f.$$

Flat spacetime limit: $Y^A = V^A(x^B), \; f = T(x^A) + \frac{u}{2} D_A V^A$

(Soft) Algebra / Algebroid

$\bar{\xi} = f\partial_u + Y^A\partial_A$ $[\bar{\xi}_1, \bar{\xi}_2] = \hat{\bar{\xi}},$ $\hat{\bar{\xi}} = \hat{f}\partial_u + \hat{Y}^A\partial_A$

$$
\begin{aligned}
\hat{f} &= Y_1^A\partial_A f_2 + \frac{1}{2} f_1 D_A Y_2^A - (1 \leftrightarrow 2), \\
\hat{Y}^A &= Y_1^B\partial_B Y_2^A - \frac{\Lambda}{3} f_1 q^{AB}\partial_B f_2 - (1 \leftrightarrow 2).
\end{aligned}
$$

The structure constants of the Λ-BMS algebra are field-dependent.

In the flat limit, the structure constants are field-independent and reproduce the generalized BMS algebra.

The explicit generalized BMS algebra

$Y^A = Y^A(x^B)$ and $f = T(x^A) + \frac{u}{2} D_A Y^A$

$$
\begin{aligned}
[\xi(T_1), \xi(T_2)]_\star &= 0, \\
[\xi(Y_1), \xi(T_2)]_\star &= \xi(\hat{T}), \qquad \hat{T} = Y_1^A \partial_A T_2 - \frac{1}{2} D_A Y_1^A T_2, \\
[\xi(Y_1), \xi(Y_2)]_\star &= \xi(\hat{Y}), \qquad \hat{Y}^A = Y_1^B \partial_B Y_2^A - Y_2^B \partial_B Y_1^A.
\end{aligned}
$$

[Barnich-Troessaert]
[Campiglia-Laddha]

Their associated Noether charges are the Bondi mass and angular momentum aspects

Upon quantization, the equality of the total flux at scri+ and scri- upon antipodal map is equivalent to the leading and subleading soft theorems

Symplectic structure

Action with holographic counterterms

$$S = \frac{1}{16\pi G}\int_{\mathscr{M}} d^4x\sqrt{-g}\,(R[g]-2\Lambda) + \frac{1}{16\pi G}\int_{\mathscr{I}} d^3x\sqrt{|\gamma|}\,(2K + \frac{4}{\ell} - \ell R[\gamma]).$$

[Balasubramanian, Kraus]

Symplectic structure gets a contribution from the counterterm

$$\delta L = \frac{\delta L}{\delta g_{\mu\nu}}\delta g_{\mu\nu} + d\Theta, \qquad \omega = \delta\Theta$$

$$\omega = \omega_{EH}[\delta g, \delta g; g] - d\omega_{EH}[\delta\gamma, \delta\gamma; \gamma]$$

[G.C., Marolf]

At the boundary, the orthogonal component of the symplectic structure is

$$\omega^\rho = \frac{1}{2\ell^2}\int_{\mathscr{I}} d^3x\,\delta\Big(\sqrt{|g_{(0)}|}T^{ab}\Big)\wedge\delta g^{(0)}_{ab}.$$

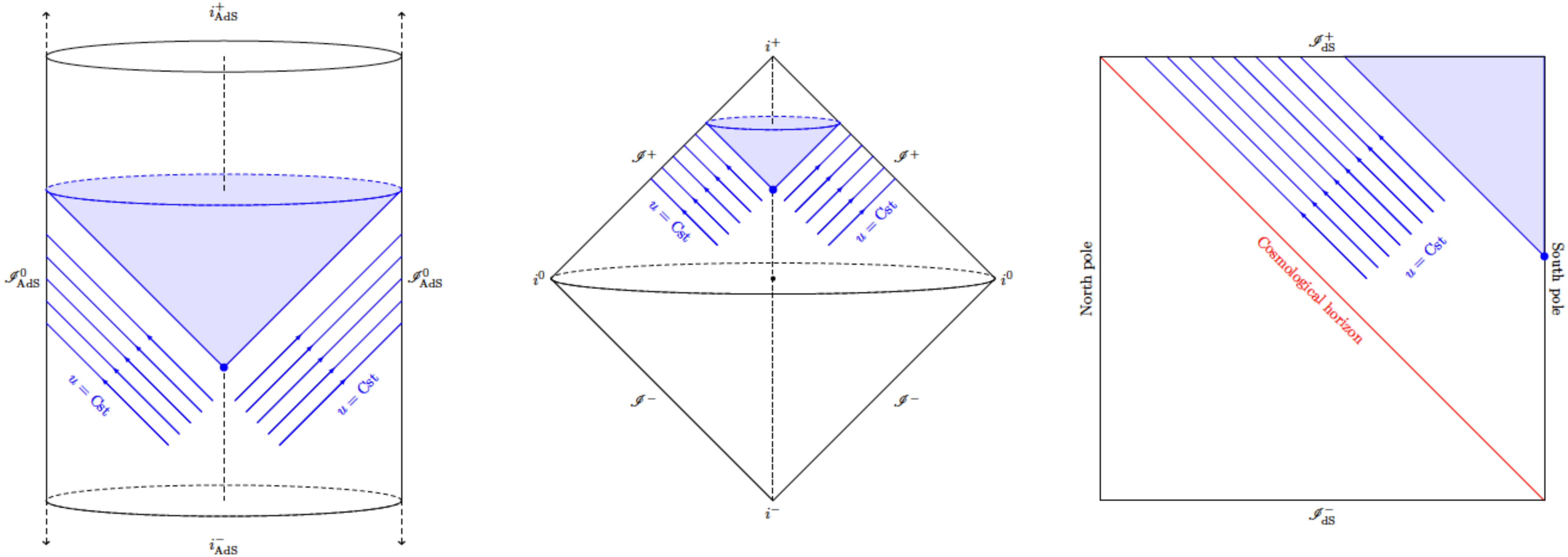
$$\omega^\rho = \frac{3}{32\pi G\ell^4}\int_{\mathscr{I}} d^3x\,\sqrt{\bar{q}}\,\delta J^{AB}\wedge\delta q_{AB}.$$

$$g^{(0)}_{tt} = \frac{\Lambda}{3}, \qquad g^{(0)}_{tA} = 0, \qquad \det(g_{(0)}) = \frac{\Lambda}{3}\bar{q}.$$

$$T_{ab} = \frac{\sqrt{3|\Lambda|}}{16\pi G}\begin{bmatrix} -\frac{4}{3}M^{(\Lambda)} & -\frac{2}{3}N^{(\Lambda)}_B \\ -\frac{2}{3}N^{(\Lambda)}_A & J_{AB} + \frac{2}{\Lambda}M^{(\Lambda)}q_{AB} \end{bmatrix}$$

Therefore, energy is transferred due to changes of both J^{AB} and q_{AB}. They are the analogue in dS/AdS of the Bondi news.

Symplectic flux fine for Minkowski and dS_4



AdS case $\Lambda < 0$. Flat case $\Lambda = 0$. dS case $\Lambda > 0$.

In AdS, the Cauchy problem requires a boundary condition

New boundary conditions for AdS_4

- Boundary gauge fixing = Dirichlet boundary condition
$$g^{(0)}_{uu} = \frac{\Lambda}{3}, \quad g^{(0)}_{uA} = 0, \quad \det(g_{(0)}) = \frac{\Lambda}{3}\bar{q}.$$
- Well-posed dynamics requires to cancel

$$\omega^{\rho} = \frac{3}{32\pi G \ell^4} \int_{\mathscr{I}} d^3x \, \sqrt{\bar{q}}\, \delta J^{AB} \wedge \delta q_{AB}.$$

  - Dirichlet boundary condition q_{AB} fixed gives SO(3,2). This leads to AdS/CFT.
  - Neumann boundary condition J^{AB}=0 gives new boundary conditions: deformation of AdS/CFT.

New boundary conditions for AdS_4

- Asymptotic symmetry group $\mathbb{R} \oplus \mathcal{A}$

Writing $\bar{\xi} = \bar{\xi}^a \partial_a = T\partial_t + \epsilon^{AB}\partial_B \Phi \partial_A$, we have $[\bar{\xi}_1, \bar{\xi}_2] = \hat{\bar{\xi}}$ where

$$\hat{T} = 0, \qquad \hat{\Phi} = \epsilon^{AB}\partial_A \Phi_2 \partial_B \Phi_1.$$

- Associated with finite and conserved charges, Dirac bracket of charges with no central extension.

$$-\delta_{\xi(T_1,\Phi_1)} Q_{\xi(T_2,\Phi_2)} = Q_{\xi(\hat{T},\hat{\Phi})}.$$

New boundary conditions for AdS_4

- Solutions: Schwarzschild but not Kerr

$$g^{(0)}_{ab}dx^a dx^b = -\ell^{-2}dt^2 + d\theta^2 + \sin^2\theta d\phi^2,$$
$$T^{ab} = T^{ab}_{\mathrm{Kerr}} \equiv -\frac{m\gamma^3\ell}{8\pi}(3u^a u^b + g^{ab}_{(0)}),$$

$$u^a\partial_a = \gamma\ell(\partial_t + \frac{a}{\ell^2}\partial_\phi), \qquad \gamma^{-1} \equiv \sqrt{1 - \frac{a^2}{\ell^2}\sin^2\theta}.$$

$$\Xi = 1 - a^2\ell^{-2}$$

[Awad, Johnson, 2000]

- Families of stationary solutions.

[Bhattacharyya, Lahiri, Loganayagam, Minwalla, 2000]

$$N^A = \epsilon^{AB}D_B\alpha(x^C), \quad \partial_A M = 0.$$

$$T^{tt} = -\frac{m\ell^3}{4\pi}, \qquad T^{t\phi} = -\frac{3am\ell\gamma^5}{8\pi}, \qquad T^{AB} = -\frac{m\ell}{8\pi}q^{AB}.$$

- If regular, they might bear on the cosmic censorship conjecture. No uniqueness theorem.

Back to Λ-BMS: Surface charges

$$\mathrm{d}\boldsymbol{k}_{\xi,\mathrm{ren}}[\delta\phi;\phi] = \boldsymbol{\omega}_{\mathrm{ren}}[\delta_\xi\phi,\delta\phi;\phi],$$

$$\boxed{\delta\!\!\!/ H_\xi[\phi] = \int_{S^2_\infty} 2(\mathrm{d}^2x)_{\rho t}\left[\delta\left(\sqrt{|g^{(0)}|}T^t_{(\mathrm{tot})b}\right)\xi^b_{(0)} - \frac{1}{2}\sqrt{|g^{(0)}|}\,\xi^t_{(0)}\,T^{bc}_{(\mathrm{tot})}\delta g^{(0)}_{bc}\right].}$$

- Charges are finite thanks to renormalization
- Charges are neither conserved or integrable $\delta\!\!\!/ H_\xi[\phi] = \delta H_\xi[\phi] + \Xi_\xi[\delta\phi;\phi],$
- Charges associated with Weyl are zero
- They obey the surface charge algebra

$$\boxed{\{H_\xi[\phi], H_\chi[\phi]\}_\star = H_{[\xi,\chi]_\star}[\phi].}$$

where

$$[\xi,\chi]_\star = [\xi,\chi] - \delta_\xi\chi + \delta_\chi\xi.$$

$$\{H_\xi[\phi], H_\chi[\phi]\}_\star \equiv \delta_\chi H_\xi[\phi] + \Xi_\chi[\delta_\xi\phi,\phi].$$

The flat limit leads to a divergence

$$\boldsymbol{\Theta}_{\text{ren}}[\delta\phi;\phi]|_{\mathscr{I}} = \frac{\mathrm{d}u\,\mathrm{d}^2\Omega}{16\pi G}\left[\frac{3}{2\Lambda}\partial_u(N^{TF}_{AB}\delta q^{AB}) + \frac{1}{2}\Big(N^{AB}_{TF} + \frac{1}{2}R[q]q^{AB}\Big)\delta C_{AB} - D_{(A}\mathcal{U}_{B)}\delta q^{AB}\right] + \frac{\delta \boldsymbol{L}_\circ}{\delta q^{AB}}\delta q^{AB} + \mathcal{O}(\Lambda),$$

It is cured by adding a corner term to the action

$$\int_{\mathscr{I}} \mathrm{d}^2x\,\mathrm{d}u\,L_\circ = \int_{\partial\mathscr{I}_+}\mathrm{d}^2x\,L_C - \int_{\partial\mathscr{I}_-}\mathrm{d}^2x\,L_C.$$

$$\boldsymbol{L}_\circ = \mathrm{d}\boldsymbol{L}_C \qquad \boldsymbol{L}_C = \frac{d^2\Omega}{\sqrt{q}}L_C$$

We define the corner presymplectic potential as

$$\delta L_\circ = \partial_u\left(\frac{\delta L_C}{\delta q_{AB}}\delta q_{AB}\right) + \partial_u\Theta^C_\circ,$$

And the corner presymplectic form as

$$\boldsymbol{\Theta}^C_\circ = \mathrm{d}^2x\,\Theta^C_\circ, \qquad \boldsymbol{\omega}^C_\circ(\delta_1 q_{AB},\delta_2 q_{AB};q_{AB}) = \delta_1\boldsymbol{\Theta}^C_\circ(\delta_2 q_{AB};q_{AB}) - (1\leftrightarrow 2).$$

Our prescription:

$$\boxed{\boldsymbol{\Theta}_{\text{ren,tot}}|_{\mathscr{I}} = \boldsymbol{\Theta}_{\text{ren}}|_{\mathscr{I}} - \delta\mathrm{d}\boldsymbol{L}_C + \mathrm{d}\boldsymbol{\Theta}^C_\circ.}$$

With our prescription for taking into account the corner boundary term, the divergence is cancelled

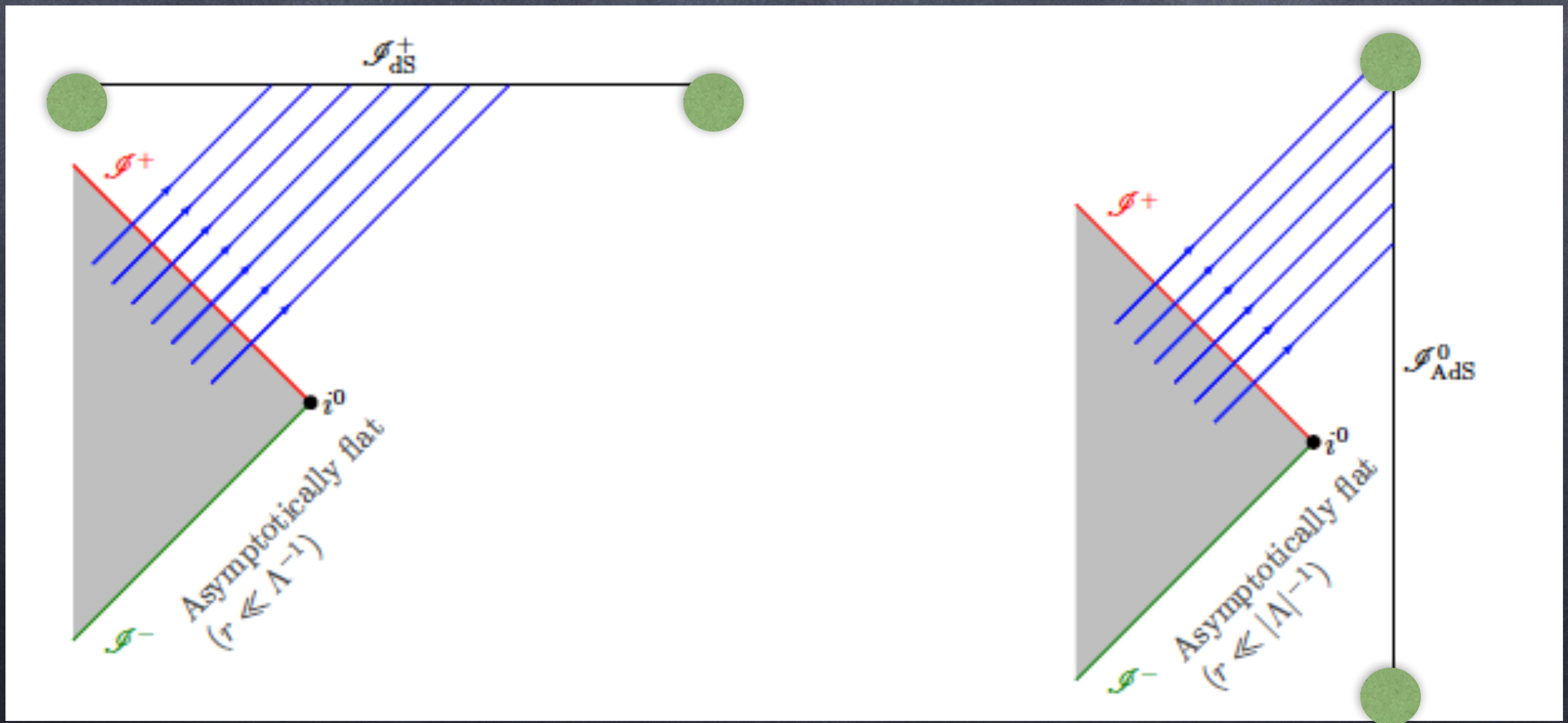
$$L_C[q_{AB}, \boldsymbol{T}, \mathring{q}] \equiv \frac{\sqrt{\mathring{q}}}{64\pi G} C^{AB} C_{AB} = \frac{\sqrt{\mathring{q}}\ell^4}{64\pi G} \partial_u q_{AB} q^{AC} q^{BD} \partial_u q_{CD}.$$

$$\boldsymbol{L}_\circ = \mathrm{d}\boldsymbol{L}_C$$

$$L_\circ = \frac{\sqrt{\mathring{q}}\ell}{64\pi G} \mathcal{L}_{\boldsymbol{T}} \left[\mathcal{L}_{\boldsymbol{T}} g^{(0)}_{ab} P^{ac} P^{bd} \mathcal{L}_{\boldsymbol{T}} g^{(0)}_{cd} \right].$$

$$P_{ab} = g^{(0)}_{ab} + \eta T_a T_b,$$

$$\eta \equiv -\mathrm{sgn}(\Lambda).$$



Flat Limit

- Solution space admits a smooth limit
- The asymptotic symmetry group Λ-BMS reduces to (generalized) BMS group
- Symplectic structure admits a smooth limit

$$\boxed{\boldsymbol{\omega}_{\mathrm{ren,tot}}[\delta_1\phi,\delta_2\phi;\phi]\Big|_{\mathscr{I}} \overset{(\Lambda\to 0)}{=} \frac{\mathrm{d}u\,\mathrm{d}^2\Omega}{16\pi G}\left[\frac{1}{2}\delta_1\left(N^{AB}+\frac{1}{2}R[q]q^{AB}\right)\wedge\delta_2 C_{AB}-\delta_1\left(D_{(A}\mathcal{U}_{B)}\right)\wedge\delta_2 q^{AB}\right].}$$

Corner term contribution to the Dirac bracket

$$\boldsymbol{\omega}_{\mathrm{ren,tot}}|_{\mathscr{I}} = \boldsymbol{\omega}_{\mathrm{ren}}|_{\mathscr{I}} + \mathrm{d}\boldsymbol{\omega}^{C}_{\circ}.$$

$$\mathrm{d}\boldsymbol{k}^{\mathrm{tot}}_{\xi,\mathrm{ren}}[\delta\phi;\phi]\Big|_{\mathscr{I}} = \boldsymbol{\omega}_{\mathrm{ren,tot}}[\delta_\xi\phi,\delta\phi;\phi]\Big|_{\mathscr{I}},$$

$$\delta\!\!\!/ H^{\Lambda\text{-BMS}}_{\xi,\mathrm{tot}}[\phi] = \delta H^{\Lambda\text{-BMS}}_{\xi}[\phi] + \Xi^{\Lambda\text{-BMS}}_{\xi}[\delta\phi;\phi] + \int_{S^2_\infty} \mathrm{d}^2\Omega\ \omega^{C}_{\circ}[\delta_\xi q_{AB}, \delta q_{AB}].$$

It leads to a non-trivial 2-cocycle

$$\begin{aligned}
&\{H^{\Lambda\text{-BMS}}_{\xi}[\phi], H^{\Lambda\text{-BMS}}_{\chi}[\phi]\}_{\star,\mathrm{tot}} \\
&\qquad = \delta_\chi H^{\Lambda\text{-BMS}}_{\xi}[\phi] + \Xi^{\Lambda\text{-BMS}}_{\chi}[\delta_\xi\phi;\phi] + \int_{S^2_\infty} \mathrm{d}^2\Omega\, \omega^{C}_{\circ}[\delta_\chi q_{AB}, \delta_\xi q_{AB}] \\
&\qquad = H^{\Lambda\text{-BMS}}_{[\xi,\chi]_\star}[\phi] + \underbrace{\int_{S^2_\infty} \mathrm{d}^2\Omega\, \omega^{C}_{\circ}[\delta_\chi q_{AB}, \delta_\xi q_{AB}]}_{K^{\Lambda\text{-BMS,tot}}_{\xi;\chi}[q_{AB}]},
\end{aligned}$$

Flat BMS surface charge algebra

Surface charges

$$\mathrm{d}\boldsymbol{k}_\xi^{\text{flat}}[\delta\phi;\phi] = \boldsymbol{\omega}_{\text{ren,tot}}[\delta_\xi\phi,\delta\phi;\phi]\Big|_{\Lambda=0}.$$

$$\not{\delta} H_\xi^{\text{flat}}[\phi] = \int_{S^2_\infty} \boldsymbol{k}_\xi^{\text{flat}}[\delta\phi;\phi] = \int_{S^2_\infty} 2(\mathrm{d}^2x)_{ru}\,(k_\xi^{\text{flat}})^{ru}[\delta\phi;\phi].$$

$$\boxed{\not{\delta} H_\xi^{\text{flat}}[\phi] = \delta H_\xi^{\text{flat}}[\phi] + \Xi_\xi^{\text{flat}}[\delta\phi;\phi],}$$

where

$$H_\xi^{\text{flat}}[\phi] = \frac{1}{16\pi G}\int_{S^2_\infty}\mathrm{d}^2\Omega\left[4fM + 2Y^A N_A + \frac{1}{16}Y^A\partial_A(C_{BC}C^{BC})\right],$$

$$\begin{aligned}\Xi_\xi^{\text{flat}}[\delta\phi;\phi] = \frac{1}{16\pi G}\int_{S^2_\infty}\mathrm{d}^2\Omega\Big[&\frac{1}{2}f\left(N^{AB}+\frac{1}{2}q^{AB}R[q]\right)\delta C_{AB} - 2\partial_{(A}f\mathcal{U}_{B)}\delta q^{AB}\\ &-fD_{(A}\mathcal{U}_{B)}\delta q^{AB} - \frac{1}{4}D_C D^C f C_{AB}\delta q^{AB}\Big].\end{aligned}$$

$$\mathcal{U}_A \equiv -\frac{1}{2}D^B C_{AB}.$$

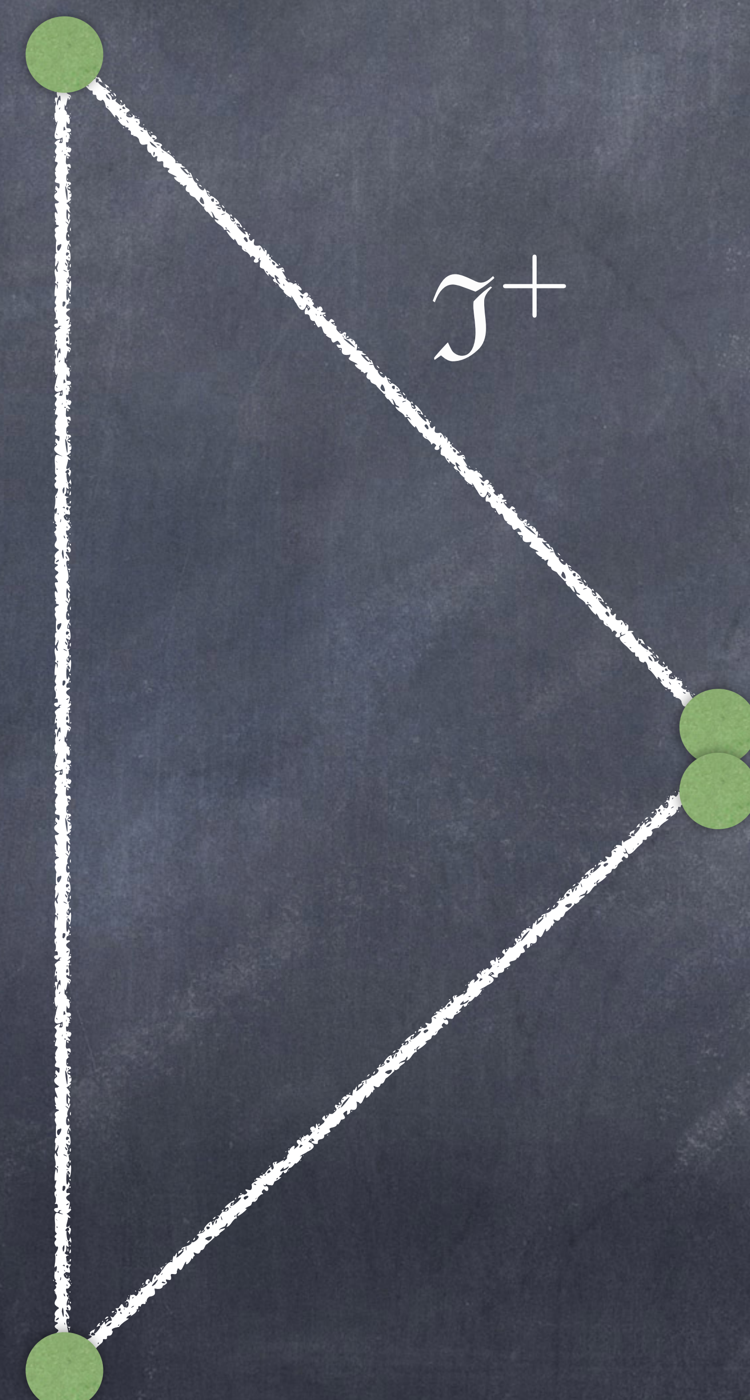
Dirac bracket

$$\boxed{\{H_\xi^{\text{flat}}[\phi], H_\chi^{\text{flat}}[\phi]\}_\star = H_{[\xi,\chi]_\star}^{\text{flat}}[\phi] + K_{\xi;\chi}^{\text{flat}}[\phi]}$$

Non-trivial 2-cocyle

$$K_{\xi_1;\xi_2}^{\text{flat}}[\phi] = \frac{1}{16\pi G}\int_{S^2_\infty}\mathrm{d}^2\Omega\left[\frac{1}{2}f_1 D_A f_2 D^A R[q] + \frac{1}{2}C^{BC}f_1 D_B D_C D_D Y_2^D - (1\leftrightarrow 2)\right].$$

Flat BMS surface charge algebra at the corners



BMS fluxes:

$$\bar{F}_\xi[\phi] = \bar{H}^{\text{flat}}_\xi[\phi]\Big|_{\mathscr{I}^+_+} - \bar{H}^{\text{flat}}_\xi[\phi]\Big|_{\mathscr{I}^+_-} = \int_{-\infty}^{+\infty} \mathrm{d}u\, \partial_u \bar{H}^{\text{flat}}_\xi[\phi].$$

Non-radiative asymptotic boundary conditions

Minkowski in an arbitrary super-Lorentz and supertranslation frame? Result:

$$ds^2 = -\frac{\mathring{R}}{2}\mathrm{d}u^2 - 2\mathrm{d}\rho\mathrm{d}u + (\rho^2 q_{AB} + \rho C^{\mathrm{vac}}_{AB} + \frac{1}{8}C^{\mathrm{vac}}_{CD}C^{CD}_{\mathrm{vac}}q_{AB})\mathrm{d}x^A\mathrm{d}x^B + D^B C^{\mathrm{vac}}_{AB}\mathrm{d}x^A\mathrm{d}u$$

where

$$q_{AB} = q^{\mathrm{vac}}_{AB} \equiv e^{-\Phi}\partial_A G^a \partial_B G^b \gamma_{ab}.$$

$$C^{\mathrm{vac}}_{AB}[\Phi, C] = (u + C)N^{\mathrm{vac}}_{AB} + C^{(0)}_{AB}, \qquad \begin{cases} N^{\mathrm{vac}}_{AB} &= \left[\frac{1}{2}D_A\Phi D_B\Phi - D_A D_B\Phi\right]^{\mathrm{TF}} \\ C^{(0)}_{AB} &= -2D_A D_B C + q_{AB}D^2 C. \end{cases}$$

$$\mathring{R} = D^2\Phi,$$

We therefore impose as boundary conditions at past/future of scri:

$$C_{AB} = (u + C_\pm)N^{\mathrm{vac}}_{AB} - 2D_A D_B C_\pm + q_{AB}D^E D_E C_\pm + o(u^0)$$

$$N^{AB} = N^{AB}_{\mathrm{vac}} + o(u^{-1}) \text{ as } u \to \pm\infty$$

Ambiguity shift in the Hamiltonian

$$\bar{H}^{\text{flat}}_{\xi(T,Y)}[\phi] \equiv H^{\text{flat}}_{\xi(T,Y)}[\phi] + \Delta H^{\text{flat}}_{\xi(T,Y)}[\phi]$$

This leads in particular to an ambiguity in the definition of angular momentum. This ambiguity can be fixed by requesting the BMS algebra to be realized without 2-cocyle at the corners.

Under the standard Dirac bracket,

$$\{\bar{H}^{\text{flat}}_{\xi}[\phi], \bar{H}^{\text{flat}}_{\chi}[\phi]\} = \bar{H}^{\text{flat}}_{[\xi,\chi]_\star}[\phi] + R_{\xi,\chi}[\phi],$$

$$R_{\xi,\chi}[\phi] \equiv K^{\text{flat}}_{\xi;\chi}[\phi] - \Delta H^{\text{flat}}_{[\xi,\chi]_\star}[\phi] + \delta_\chi \Delta H^{\text{flat}}_{\xi}[\phi] - \Xi_\chi[\delta_\xi\phi;\phi]$$

With hindsight we define the shift

$$\Delta H^{\text{flat}}_{\xi(T,Y)}[\phi] \equiv \int_{S^2_\infty} \frac{\mathrm{d}^2\Omega}{16\pi G}\left[\frac{1}{2} f C_{AB} N^{AB}_{\text{vac}} + \frac{1}{2} Y^A C_{AB} D_C C^{BC} + \frac{1}{8} Y^A \partial_A (C^B_C C^C_B)\right].$$

Final (nice) charges

$$\bar{H}^{\text{flat}}_{\xi(T,Y)}[\phi] = \int_{S^2_\infty} \frac{\mathrm{d}^2\Omega}{16\pi G}\Big[4T\bar{M} + 2Y^A\bar{N}_A\Big],$$

$$\bar{M} = M + \frac{1}{8}C_{AB}N^{AB}_{\text{vac}},$$
$$\bar{N}_A = N_A - u\partial_A\bar{M} + \frac{1}{4}C_{AB}D_C C^{BC} + \frac{3}{32}\partial_A(C_{BC}C^{BC}).$$

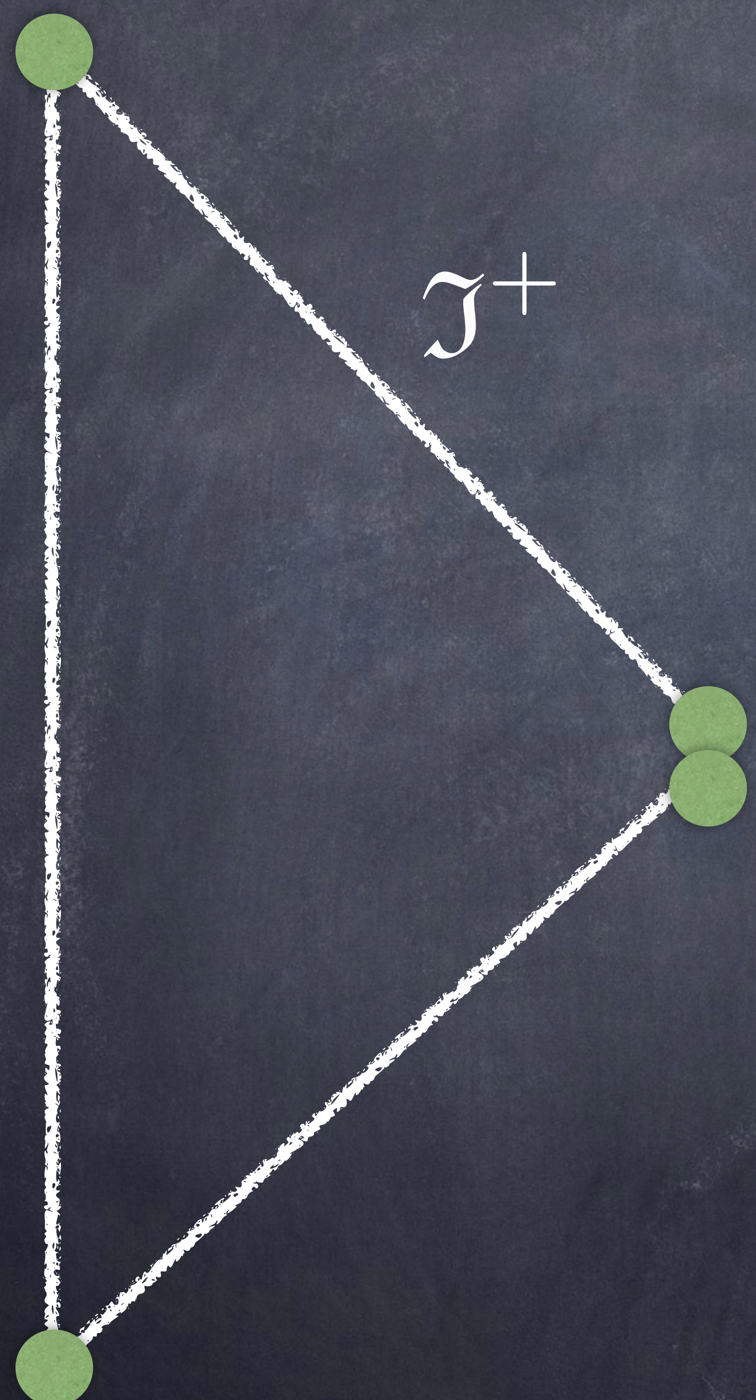
- All BMS charges are identically zero for Minkowski in an arbitrary BMS frame

- The BMS algebra is realized without central extension at the corners

$$\boxed{\{\bar{H}^{\text{flat}}_\xi[\phi], \bar{H}^{\text{flat}}_\chi[\phi]\}\Big|_{\mathscr{I}^+_\pm} = \bar{H}^{\text{flat}}_{[\xi,\chi]_\star}[\phi]\Big|_{\mathscr{I}^+_\pm}.}$$

$$R_{\xi,\chi}[\phi] = 0$$

BMS flux algebra



$$\bar{F}_\xi[\phi] = \bar{H}^{\mathbf{flat}}_\xi[\phi]\Big|_{\mathscr{I}^+_+} - \bar{H}^{\mathbf{flat}}_\xi[\phi]\Big|_{\mathscr{I}^+_-} = \int_{-\infty}^{+\infty} \mathrm{d}u\, \partial_u \bar{H}^{\mathbf{flat}}_\xi[\phi].$$

Supermomenta $\bar{P}_T \equiv \bar{F}_{\xi(T,0)}[\phi]$

Super-Lorentz $\bar{J}_Y \equiv \bar{F}_{\xi(0,Y)}[\phi],$

$$\{\bar{P}_{T_1}, \bar{P}_{T_2}\} = 0, \quad \{\bar{J}_{Y_1}, \bar{P}_{T_2}\} = \bar{P}_{Y_1(T_2)}, \quad \{\bar{J}_{Y_1}, \bar{J}_{Y_2}\} = \bar{J}_{[Y_1,Y_2]}$$

where $Y_1(T_2) \equiv (Y_1^A \partial_A - \frac{1}{2} D_A Y_1^A) T_2.$

[see also Campiglia, Peraza, 2020]

Conclusions

- The flat BMS group admits a natural extension to (A)dS: the Λ-BMS group(oid)/soft group. It is the asymptotic symmetry group of open Al(A)dS spacetimes without any boundary condition except a boundary gauge fixing condition (determined by a foliation and a measure) that does not constraint the Cauchy problem.
- The Λ-BMS_4 algebra is realized by the Dirac bracket of surface charges
- In AdS_4, alternative boundary conditions exist
- The flat limit of the Λ-BMS_4 algebra exists provided corner terms are included in the variational principle and in the symplectic structure. The inclusion of corner terms might be useful in other contexts.
- BMS surface charges were proposed with two nice properties: vanishing for Minkowski and obeying the standard Dirac bracket without central extension at the corners.