Holographic Kibble-Zurek Mechanism with Discrete Symmetry Breaking

Hai-Qing Zhang
Beihang University

7TH INTERNATIONAL CONFERENCE ON HOLOGRAPHY AND STRING THEORY

Da Nang
22.08.2024

DUY TÂN UNIVERSITY

Zhi-Hong Li, Han-Qing Shi, HQZ, 2207.10995
Tian-Chi Ma, Han-Qing Shi, Adolfo del Campo, HQZ, 2406.05167
Han-Qing Shi, Tian-Chi Ma, HQZ, 2311.07017

Contents

- **Brief review of Kibble-Zurek mechanism and motivations**
- **Holographic kinks in 1+1-dim**
- **Holographic domain walls 2+1-dim**
- **Summary**

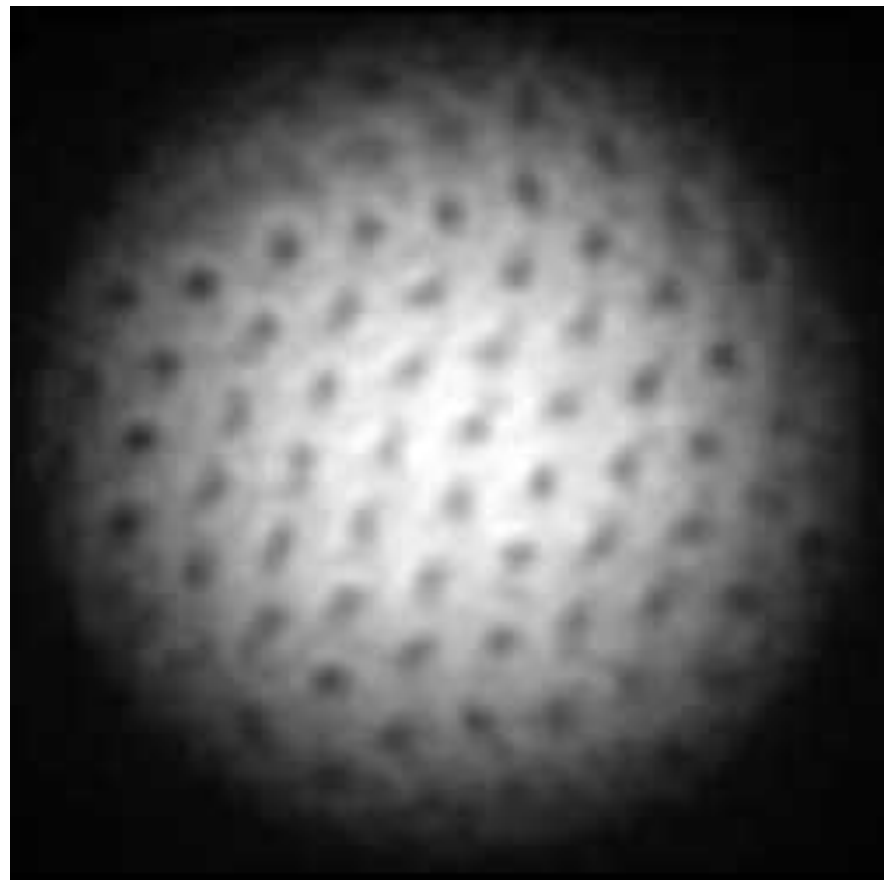
History of KZM

- KZM was first proposed in cosmology by Kibble in 1976.
- Cooling of the early universe will finally result in *topological defects*, such as cosmic string, monopoles, vortices, domain walls …
- However, not found to date.

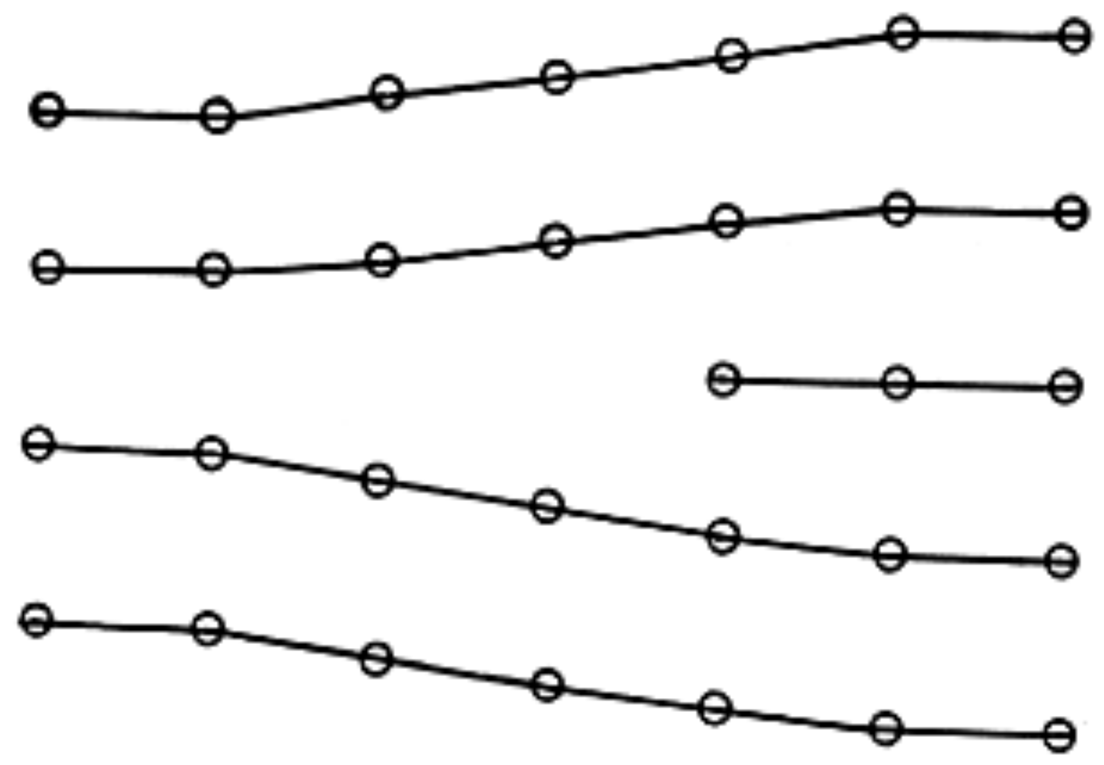
Tom W.B. Kibble (1932-2016)

Wojciech H. Zurek

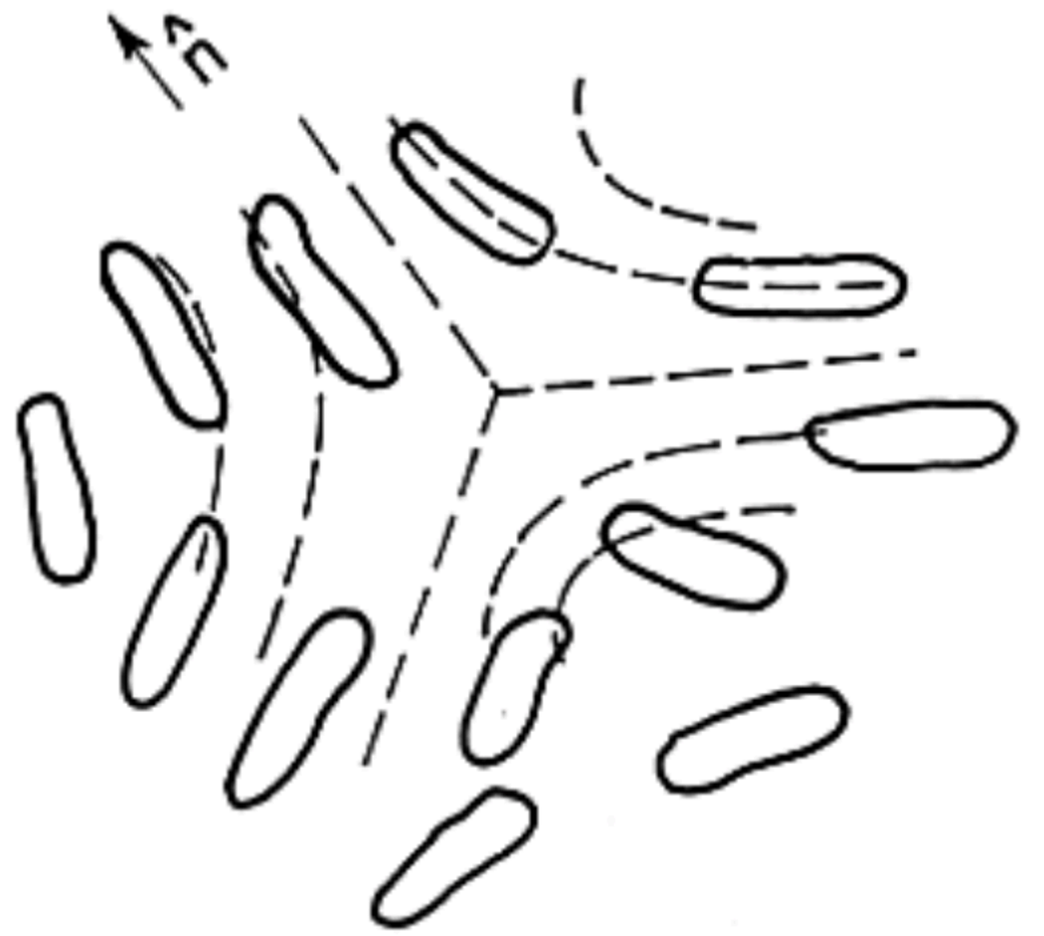
- Zurek extended this idea into superfluid in 1985.
- Phase transition from normal fluid helium to superfluid helium will induce vortices or vortex lines.
- Confirmed by various experiments.



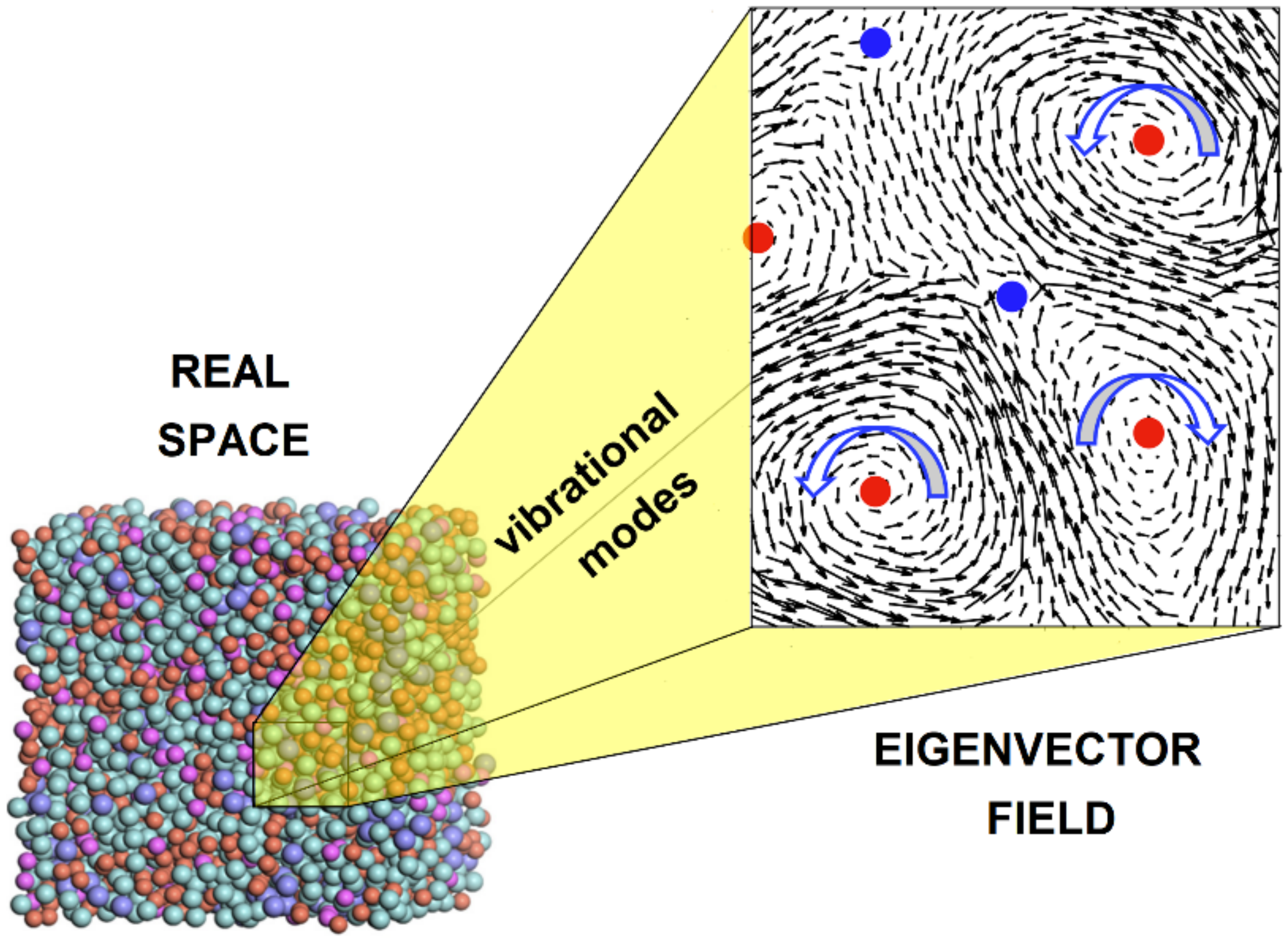
Superfluid vortices



Dislocation in crystal



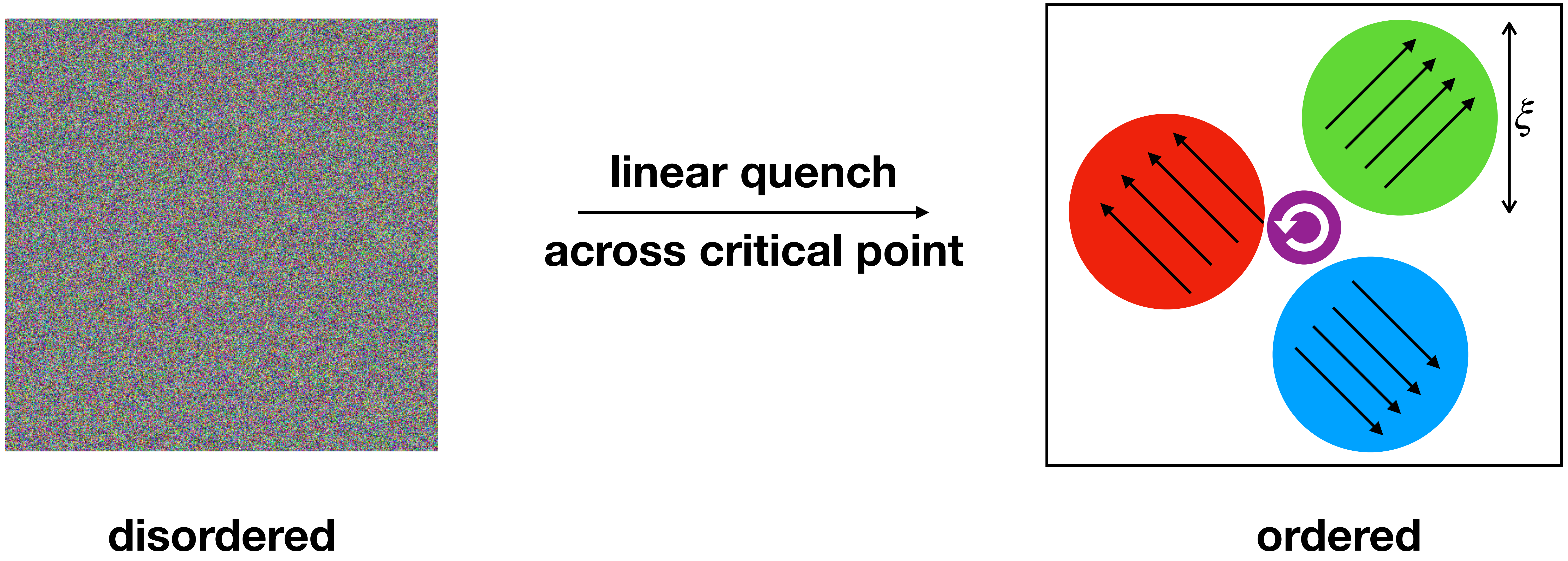
Defect lines in nematic liquid crystal



Defects in the vibration modes in glasses (Thanks to Matteo Baggioli)

- **Kibble-Zurek mechanism (KZM): Topological defects will turn out, when a system with higher symmetry quenched *across the critical point* to a system with lower symmetry.**

- **Vortices as topological defects in superfluid**



- KZM requires continuous phase transition

$$\xi \propto |\epsilon|^{-\nu}, \qquad \tau \propto |\epsilon|^{-z\nu}. \qquad\qquad \epsilon = 1 - T/T_c = t/\tau_Q$$

coherence length

relaxation time

- KZM predicts a power law relation between the *number density of topological defects* and the *quench rate* τ_Q

$$n \propto \left(\tau_Q\right)^{\frac{-(D-d)\nu}{1+z\nu}}$$

D: dimension of space
d: dimension of defects

Confirmed by various experiments

- Liquid crystals: Chuang, et.al., Science 251 (1991) 1336; Bowick, et.al.,Science 263 (1994) 943; Digal, et.al., PRL 83 (1999) 5030

- He-3 superfluids: Baeuerle,et.al.,Nature 382 (1996) 332; Ruutu et al. , Nature 382 (1996) 334

- Thin-film superconductors: Maniv,et.al., PRL 91 (2003) 197001; PRL 104, 247002 (2010).

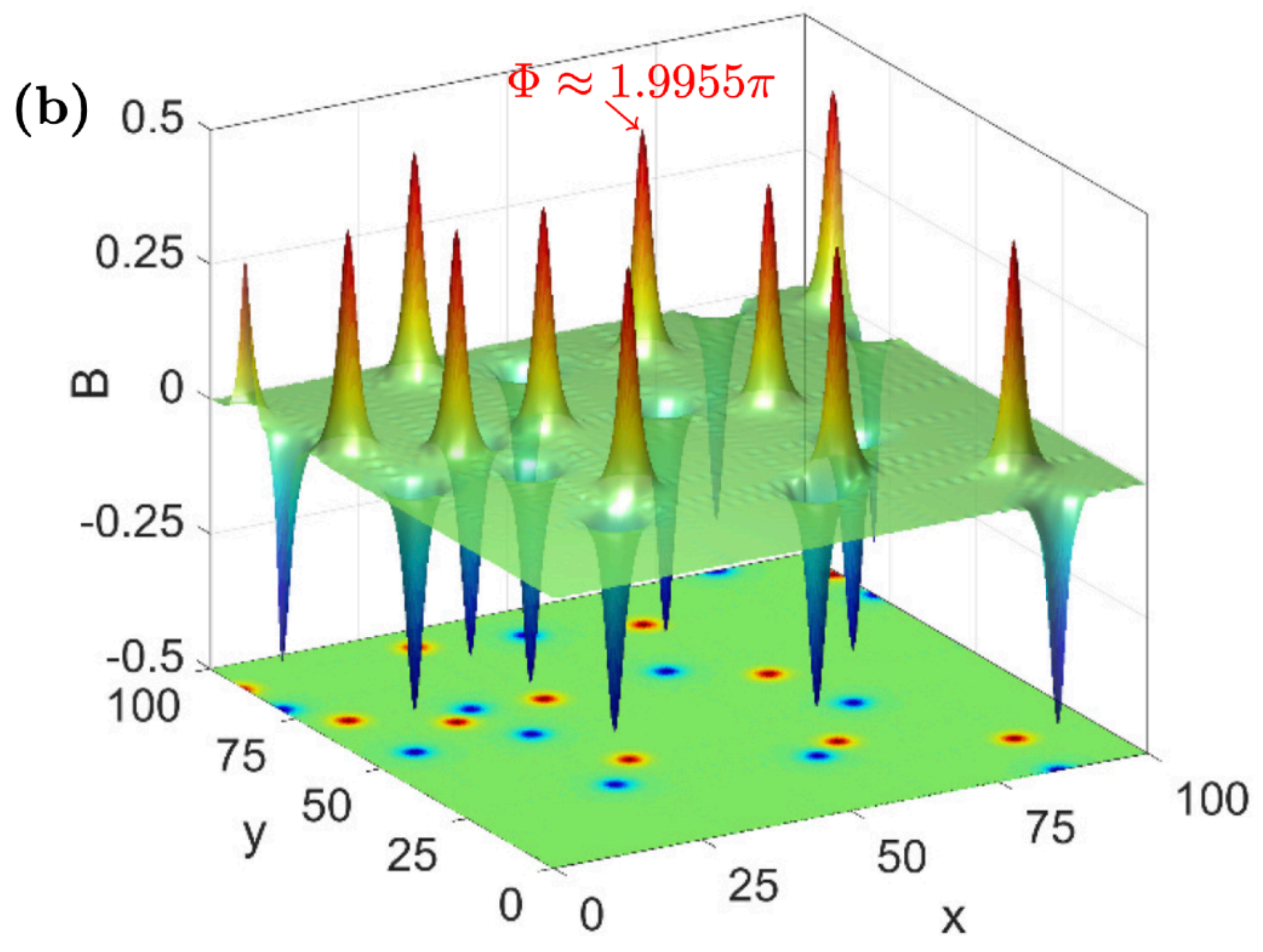
- Quantum optics: Xu, et.al., PRL,112, 035701(2014)

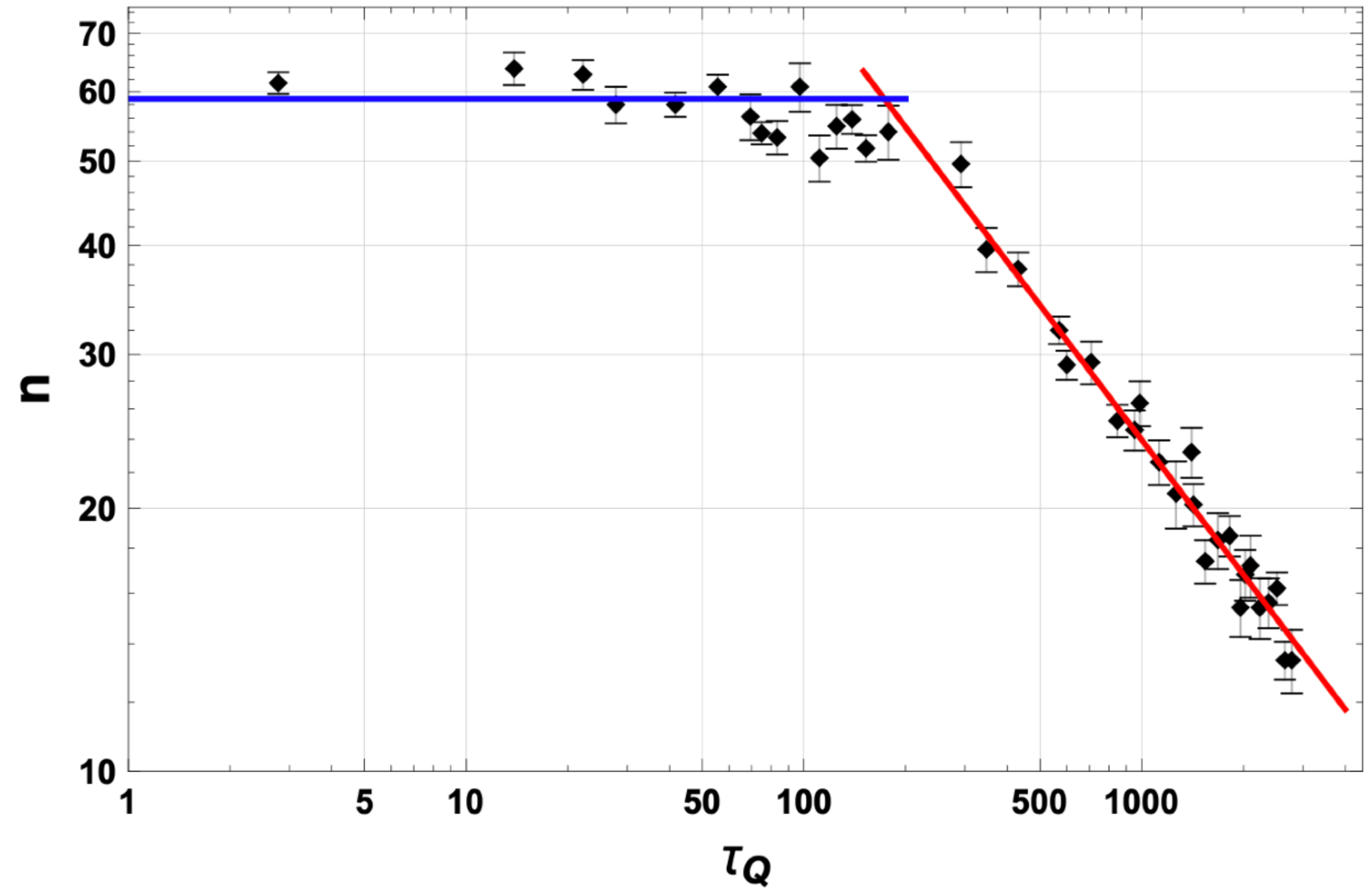
…

Holographic KZM with U(1) symmetry breaking

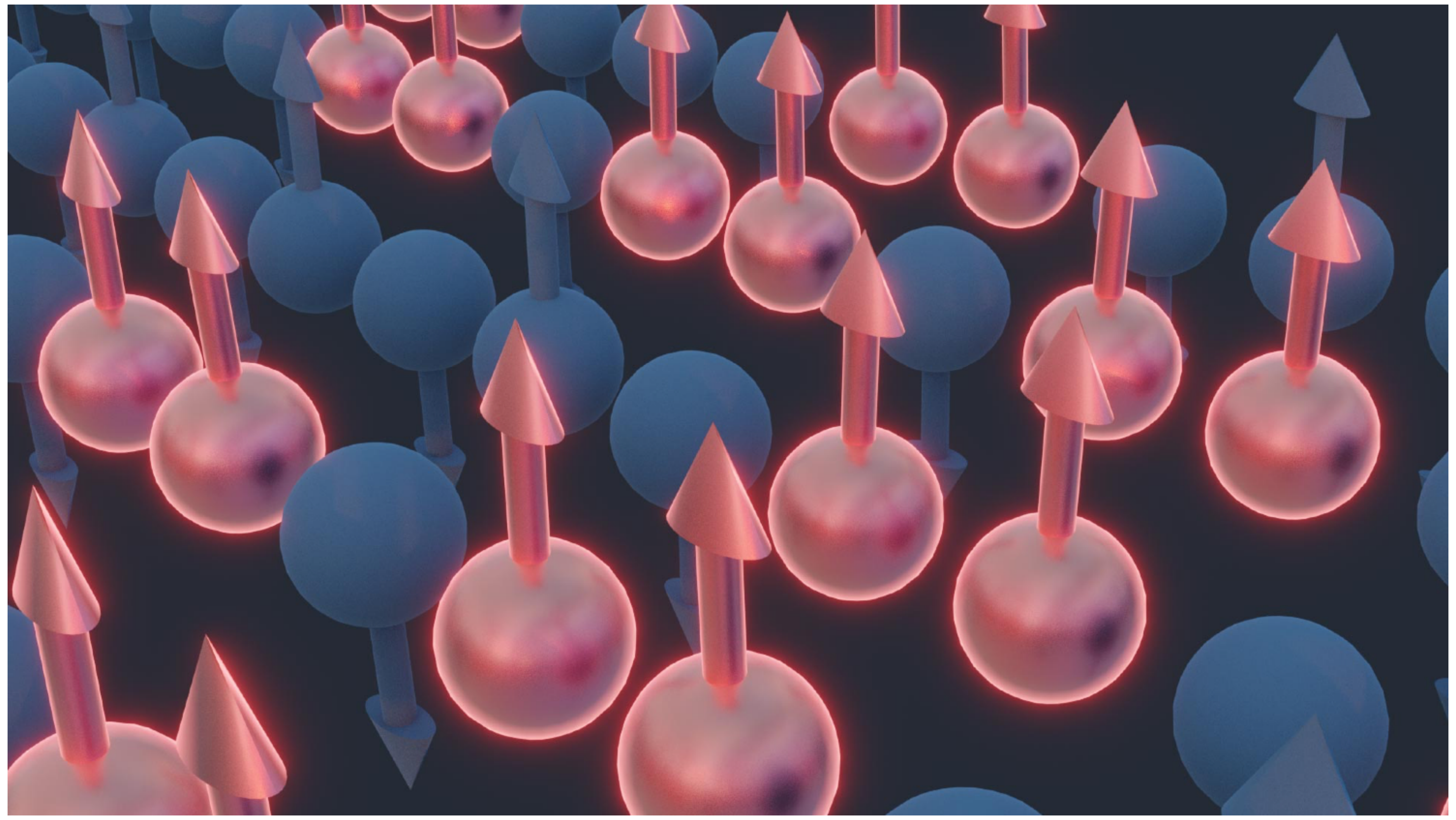
- Winding numbers in 1+1 dim holographic superfluid: Sonner, del Campo and Zurek, 1406.2329
- Vortices in 2+1 dim holographic superfluid: Chesler, Garcia-Garcia and Liu, 1407.1862
- Magnetic vortices in 2+1 dim holographic superconductors: Zeng, Xia, HQZ, 1912.08332

...





To realize discrete symmetry breaking in holography? 🤔

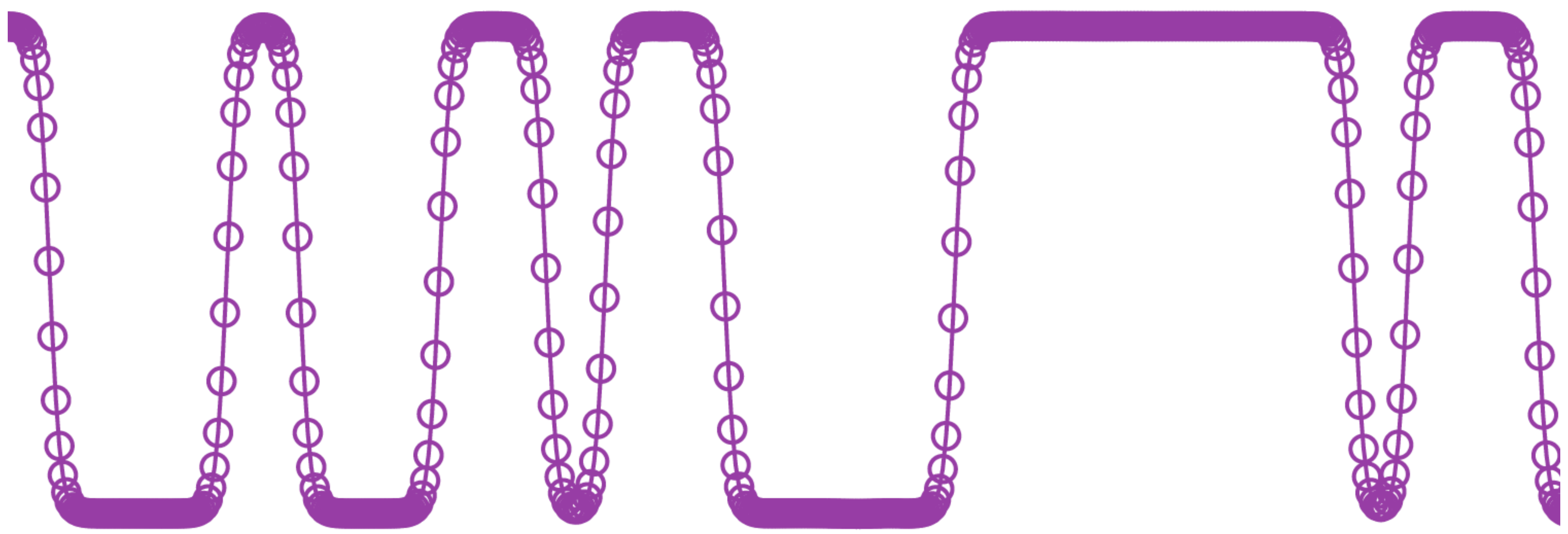


Simulate the kinks (1+1 dim) or domain wall (2+1 dim) in spin chain with strong couplings

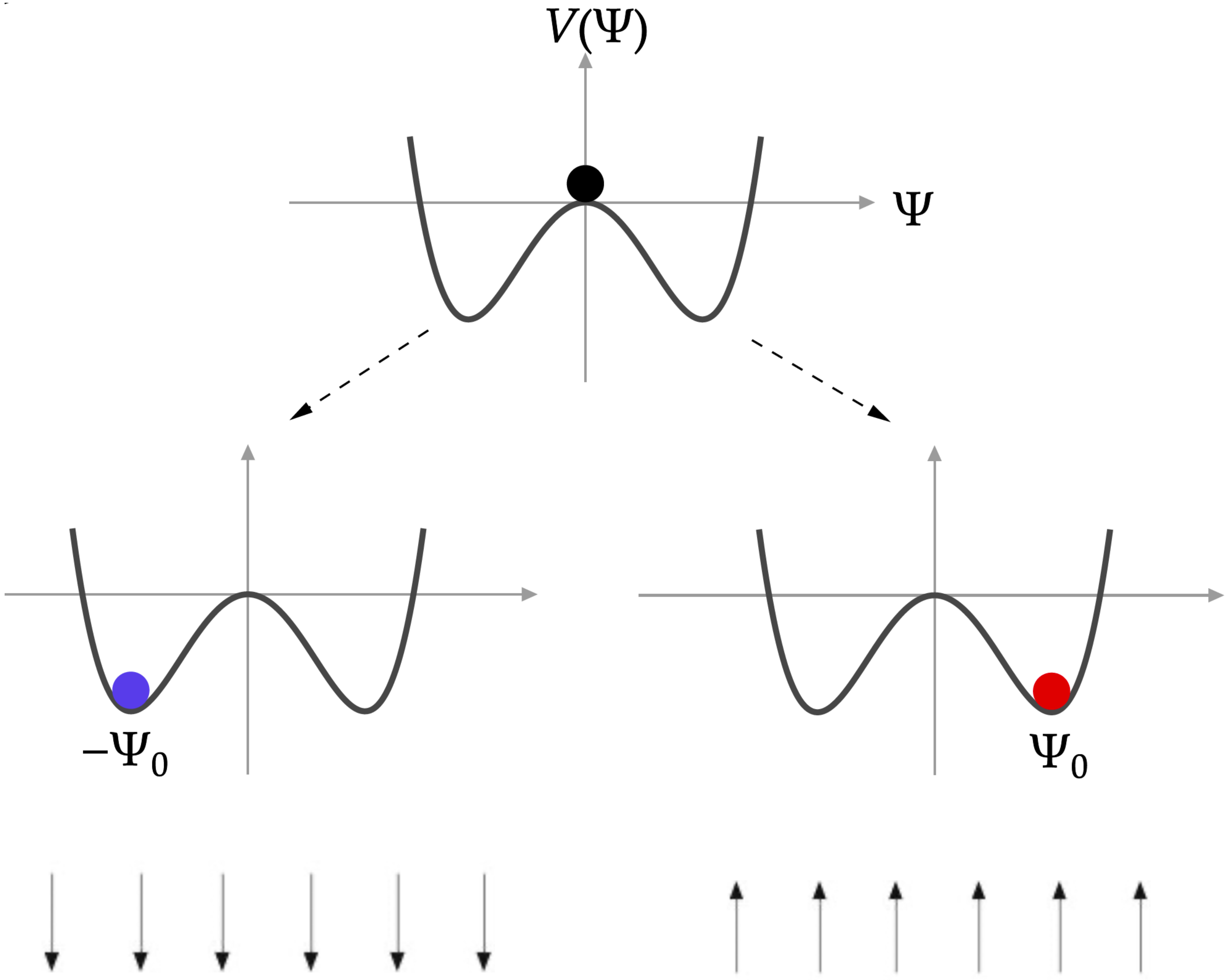
- Need to have *real scalar hairs* with Z_2 symmetry breaking in the bulk; i.e., kink hairs (domain wall hairs) near the horizon

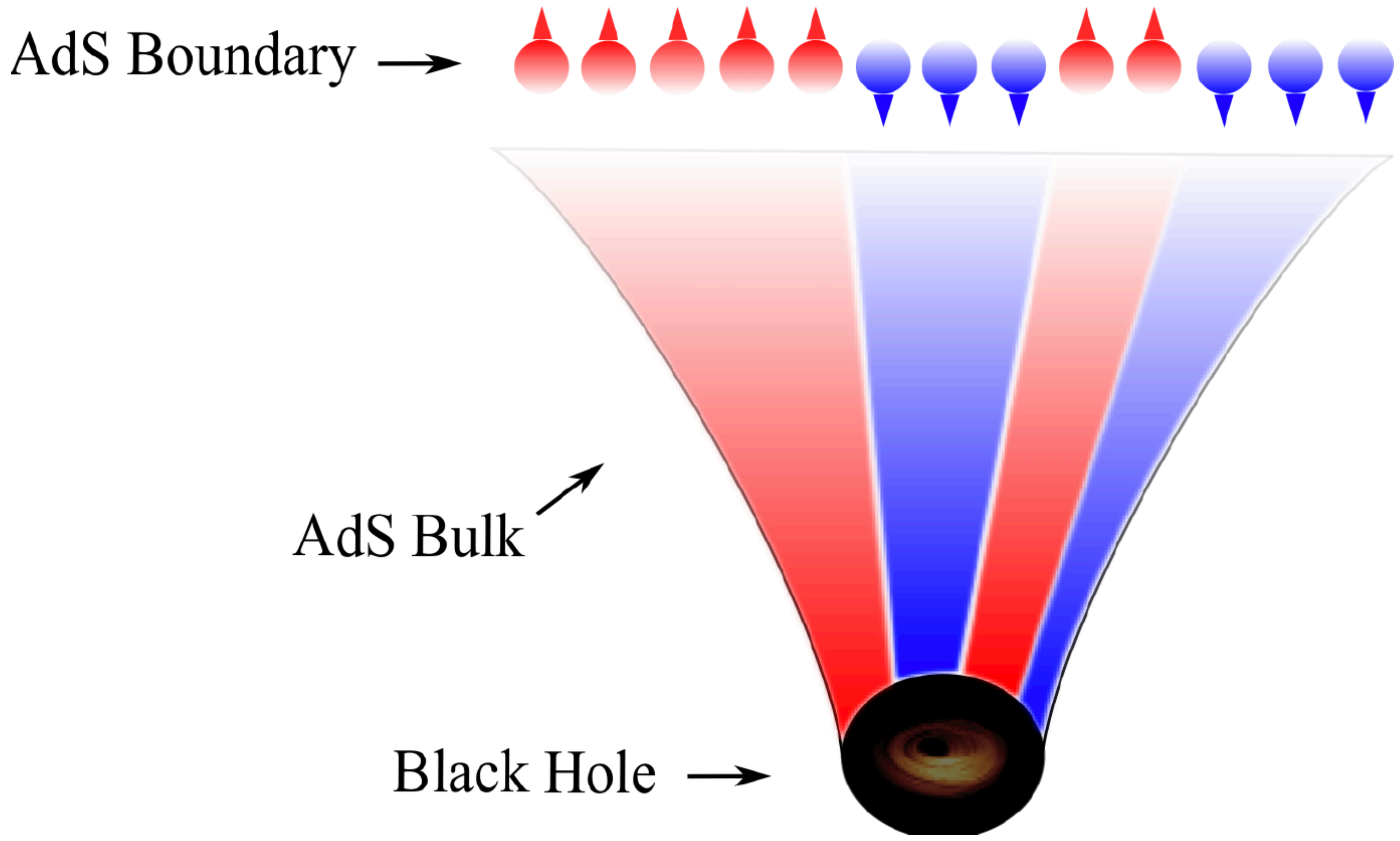
Zhi-Hong Li, Han-Qing Shi, HQZ, 2207.10995

Holographic Kinks in 1+1-dim



•Simulate a holographic spin chain





- Start with complex scalar fields + U(1) gauge fields

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D_\mu\tilde{\Psi}|^2 - m^2|\tilde{\Psi}|^2$$

$$D_\mu = \nabla_\mu - iA_\mu$$

- Gauge-like transformation

$$\tilde{\Psi} = \Psi e^{i\lambda}, \qquad A_\mu = M_\mu + \partial_\mu\lambda,$$

- EoMs of real functions

$$\left(\nabla_\mu - iM_\mu\right)\left(\nabla^\mu - iM^\mu\right)\Psi - m^2\Psi = 0, \qquad \nabla_\mu F^{\mu\nu} = 2M^\nu\Psi^2.$$

Z_2 symmetry: $+\Psi \leftrightarrow -\Psi$

- **Eddington-Finkelstein coordinates**

$$ds^2 = \frac{1}{z^2}\left[-f(z)dt^2 - 2dtdz + dx^2 + dy^2\right] \qquad f(z) = 1 - (z/z_h)^3$$

- **Ansatz of fields (turning off y-direction)**

$$\Psi = \Psi(t,z,x),\ M_t = M_t(t,z,x),\ M_z = M_z(t,z,x),\ M_x = M_x(t,z,x)$$

Note: must include M_z, 4 independent equations to solve 4 fields

$$\begin{aligned} \nabla_\mu \nabla^\mu \Psi - M_\mu M^\mu \Psi - m^2 \Psi &= 0, \\ (\nabla_\mu M^\mu)\Psi + 2M^\mu \nabla_\mu \Psi &= 0, \\ \nabla_\mu F^{\mu\nu} &= 2M^\nu \Psi^2. \end{aligned}$$

$$0 \equiv \nabla_\nu\left(\nabla_\mu F^{\mu\nu}\right) \Rightarrow \nabla_\nu\left(2M^\nu \Psi^2\right) = 0$$

$$\Rightarrow (\nabla_\nu M^\nu)\Psi + 2M^\nu \nabla_\nu \Psi = 0.$$

- **Initial condition**

Static, x-independent: EoMs of gauge fields becomes

$$0 = -\frac{2\Psi^2 M_t}{z^2} + f\partial_z^2 M_t,$$

$$0 = -\frac{2\Psi^2 M_z}{z^2} + \partial_z^2 M_t,$$

$$\Rightarrow \quad M_z = \frac{M_t}{f}$$

$$0 = -\frac{2\Psi^2 M_x}{z^2} + f'\partial_z M_x + f\partial_z^2 M_x. \quad \Rightarrow \quad M_x = 0$$

In normal state $\Psi = 0$, $M_t = \mu - \mu z$, $M_z = (\mu - \mu z)/f$

•Boundary conditions (set $m^2 = -2/L$)

$$
z \to 0 \quad \begin{cases} \Psi \sim \Psi_1(t,x)z + \Psi_2(t,x)z^2 + \mathcal{O}(z^3), \\ M_t \sim \mu(t,x) - \rho(t,x)z + \mathcal{O}(z^3), \\ M_z \sim a_z(t,x) + b_z(t,x)z + \mathcal{O}(z^3), \\ M_x \sim a_x(t,x) + b_x(t,x)z + \mathcal{O}(z^3) \end{cases}
$$

$\Psi_1 \equiv 0;\ \Psi_2 = \langle O \rangle$

μ: chemical potential

ρ: charge density

$a_z = \mu$

$a_x = 0$ **: velocity of gauge field**

b_x**: current of gauge field**

$$
z \to z_h \equiv 1: \quad M_t = 0
$$

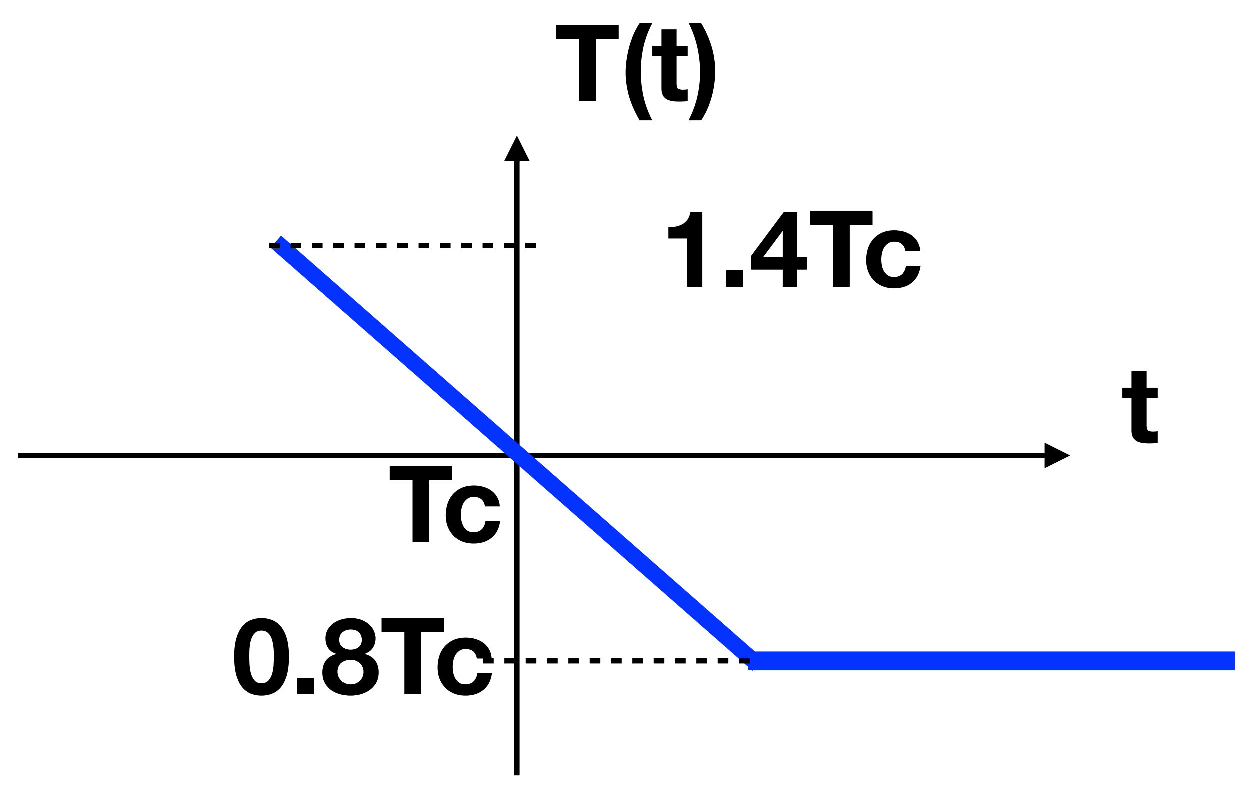
Other fields are finite

- **Quench chemical potential = quench temperature**

$$T(t)/T_c = 1 - t/\tau_Q$$

⬇

$$\mu(t) = \mu_c/(1 - t/\tau_Q)$$



$\mu_c \approx 4.06$ **is the critical chemical potential in static case**

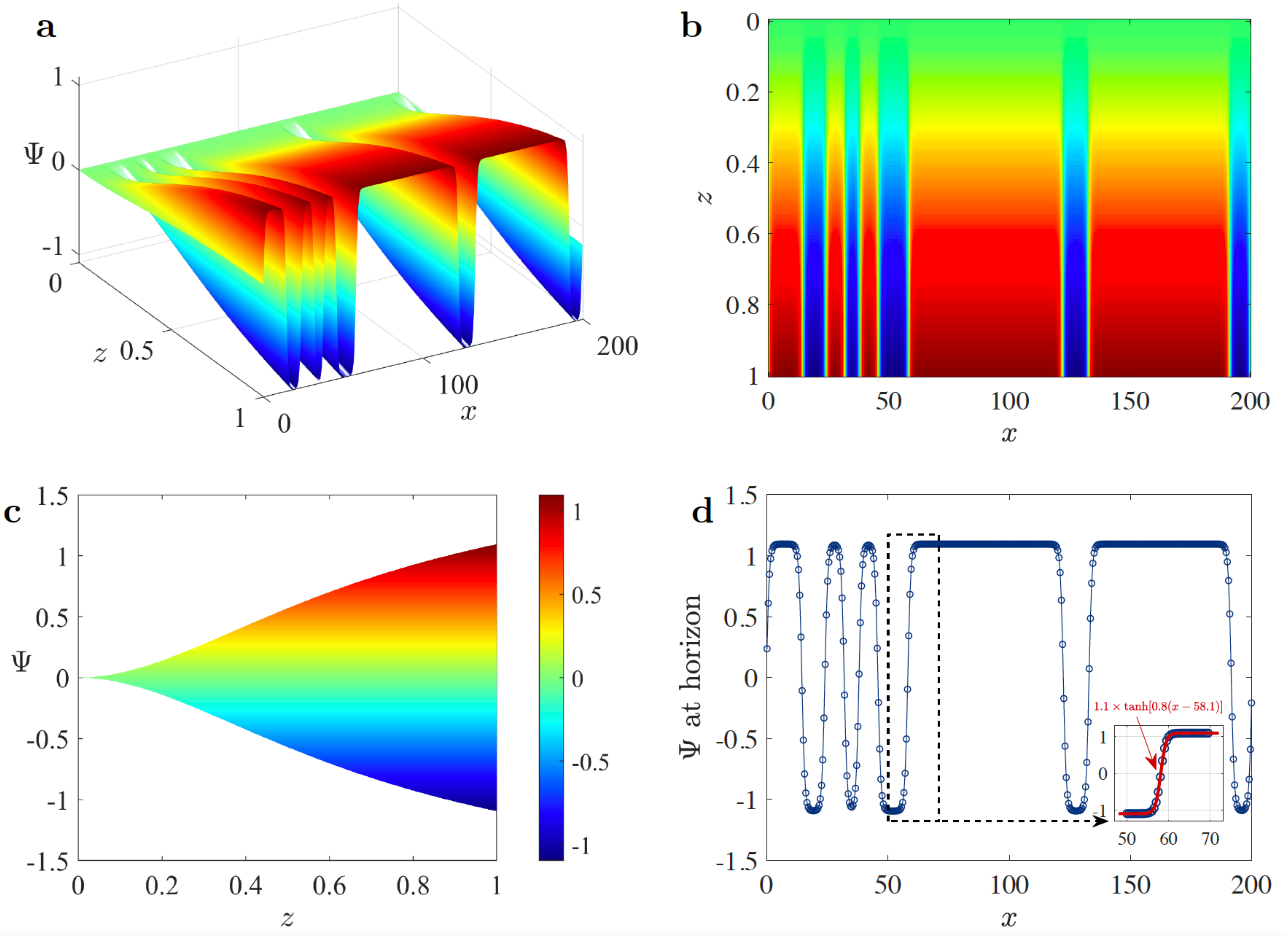
- **Small fluctuations of scalar field at initial time**

Gaussian white noise $\zeta(x_i, t)$: $\langle \zeta(x_i, t) \rangle = 0$

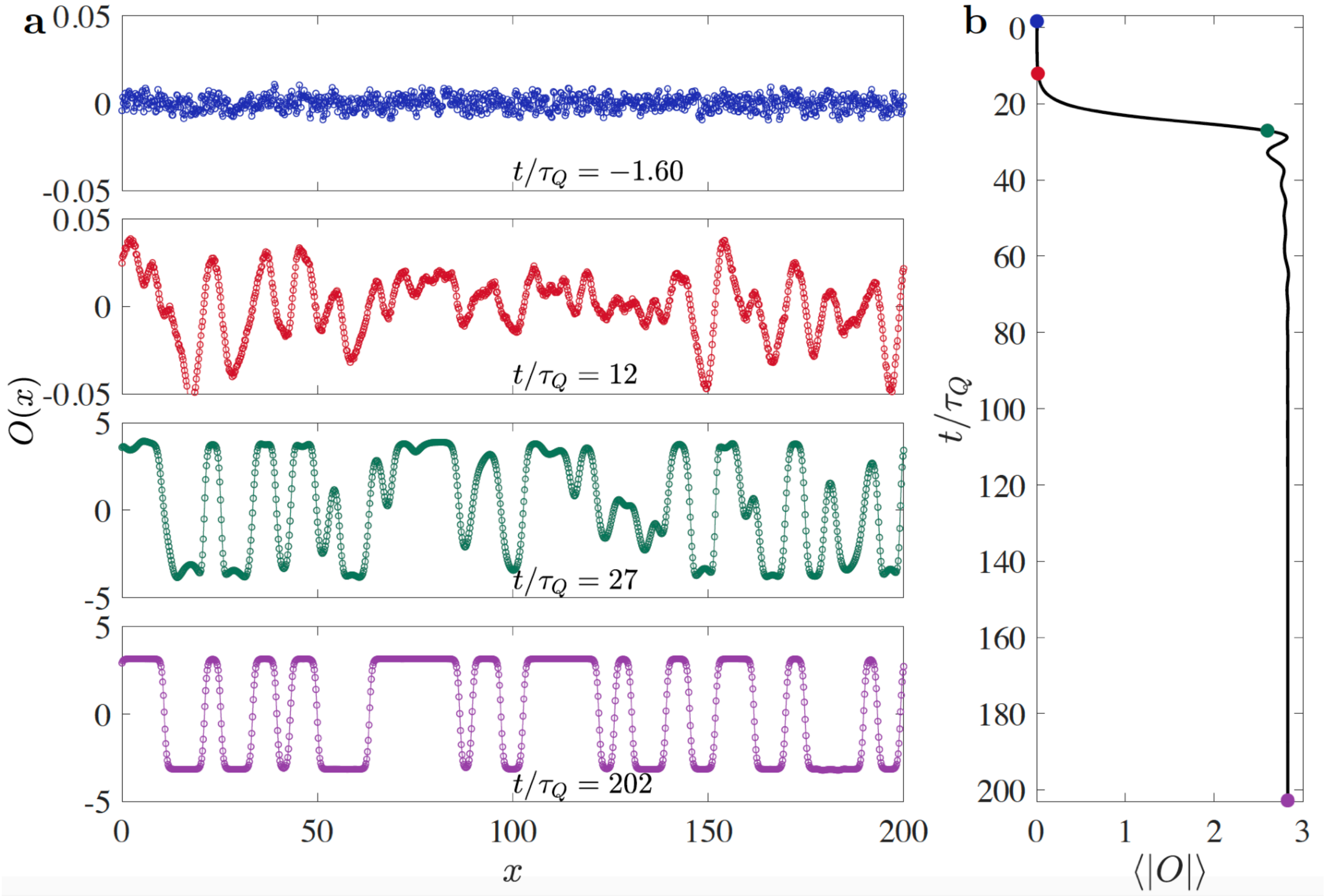
$$\langle \zeta(x_i, t) \zeta(x_j, t') \rangle = h\delta(t - t')\delta(x_i - x_j)$$

$$h = 0.001$$

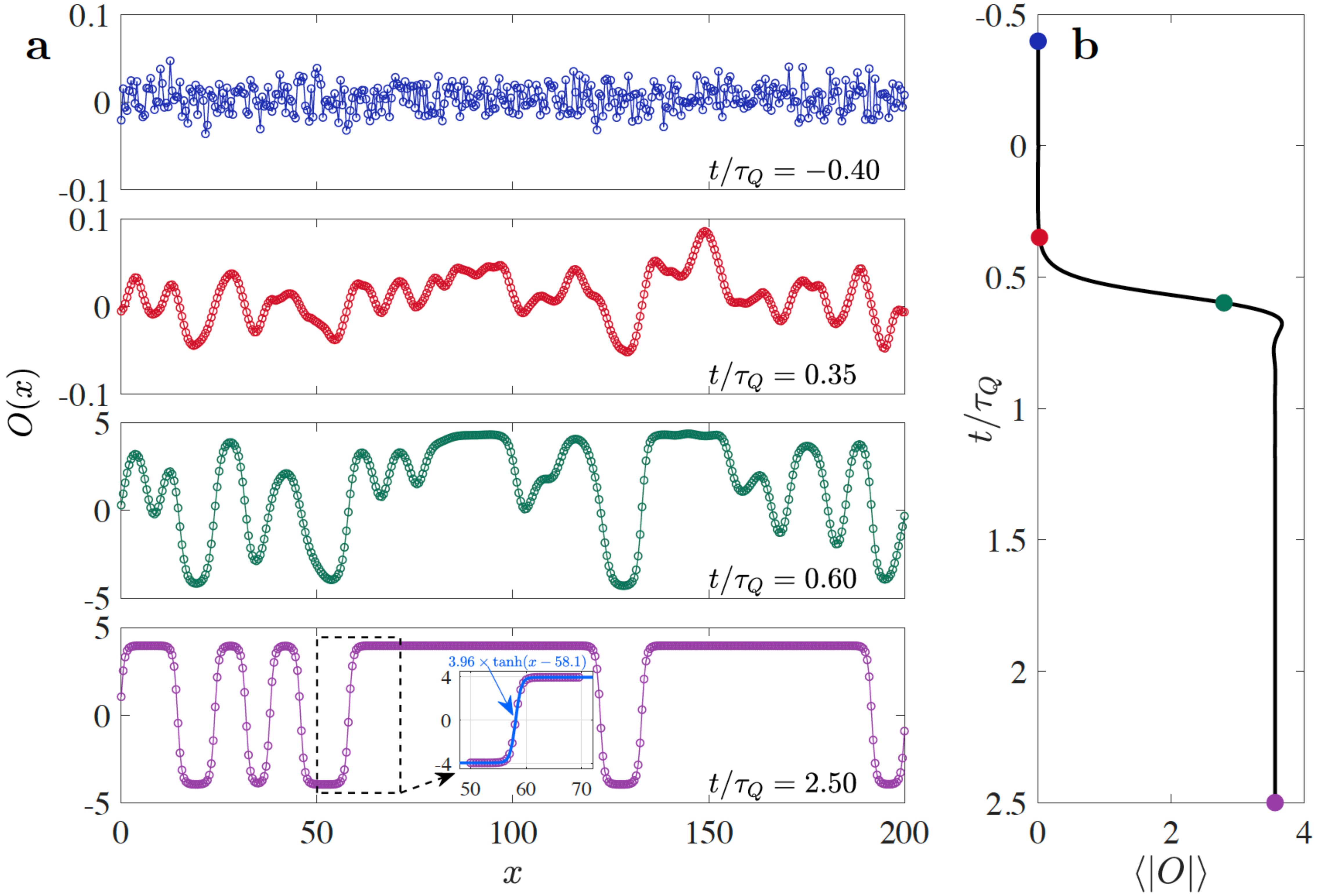
•Kink hairs in the bulk



• Time evolution of kinks



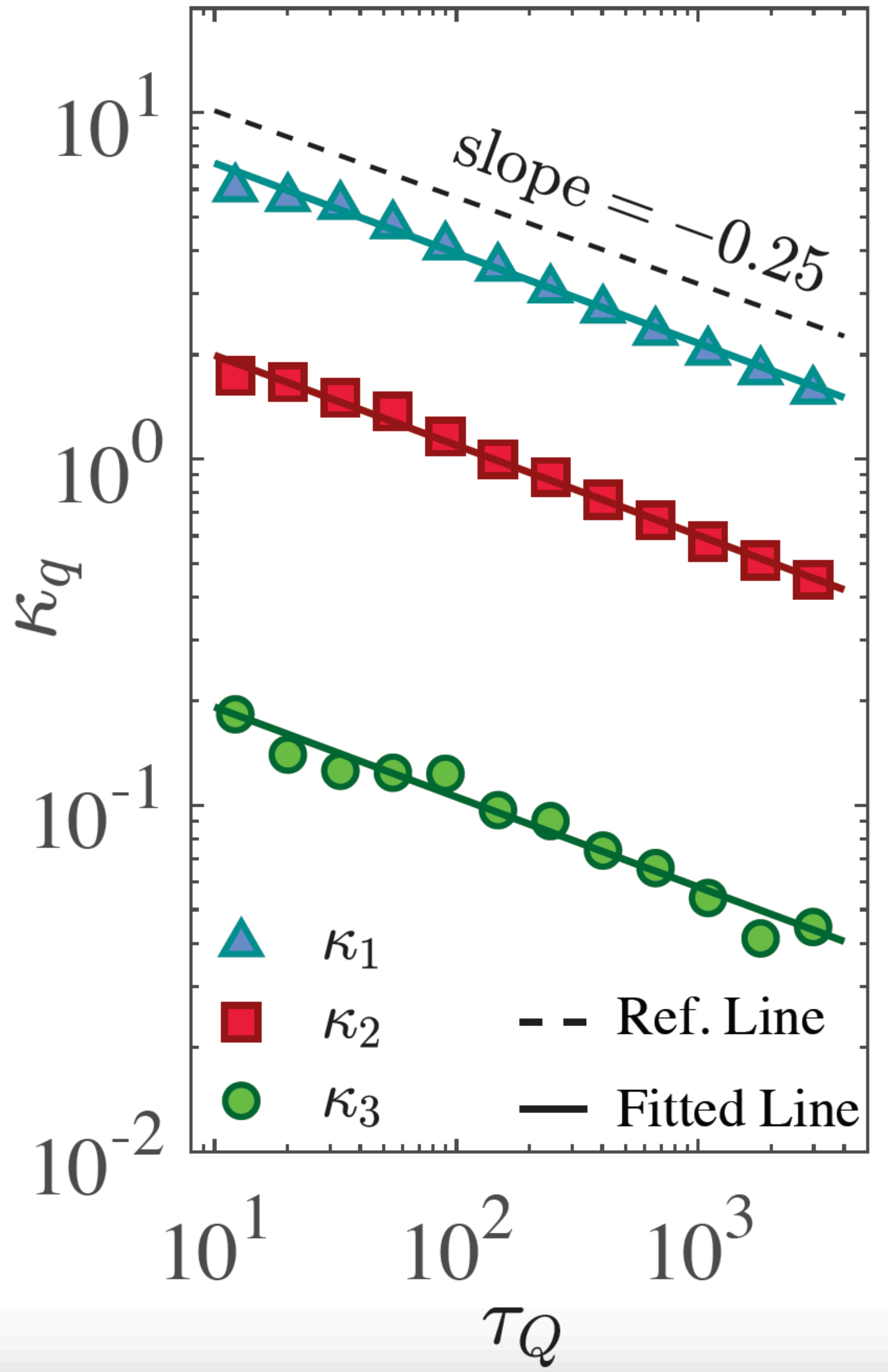
GL model



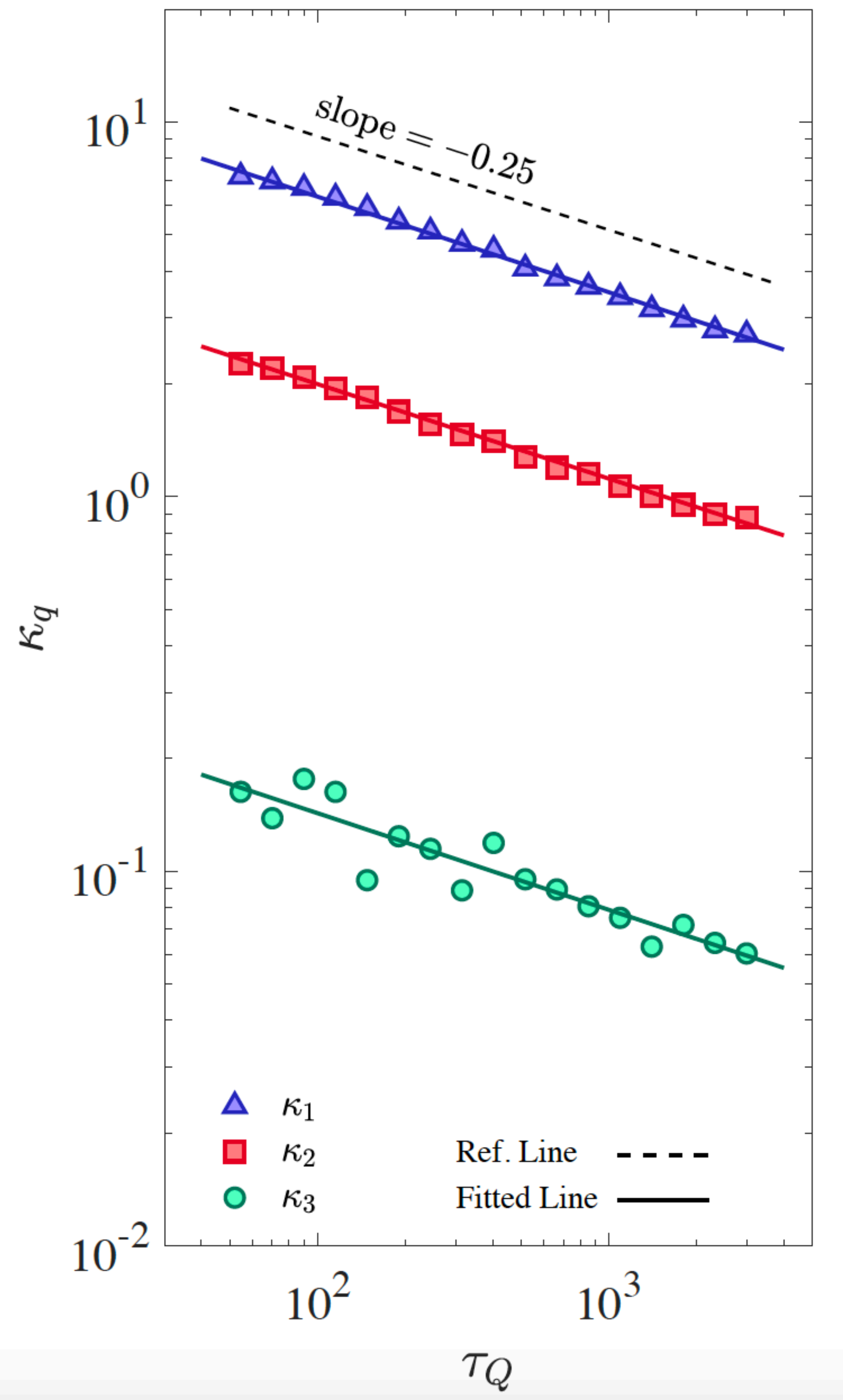
AdS boundary

- **Average kink number vs. quench rate (KZ scaling relation)**

$$\langle n \rangle \propto \tau_Q^{-(D-d)\nu/(1+z\nu)} \quad (D=1, d=0,\ \nu=1/2,\ z=2) \quad \langle n \rangle = \kappa_1 \propto \tau_Q^{-1/4}$$



GL model



AdS/CFT

- **Beyond KZ scaling relation**

del Campo, 1806.10646

One dimensional transverse-field quantum Ising model

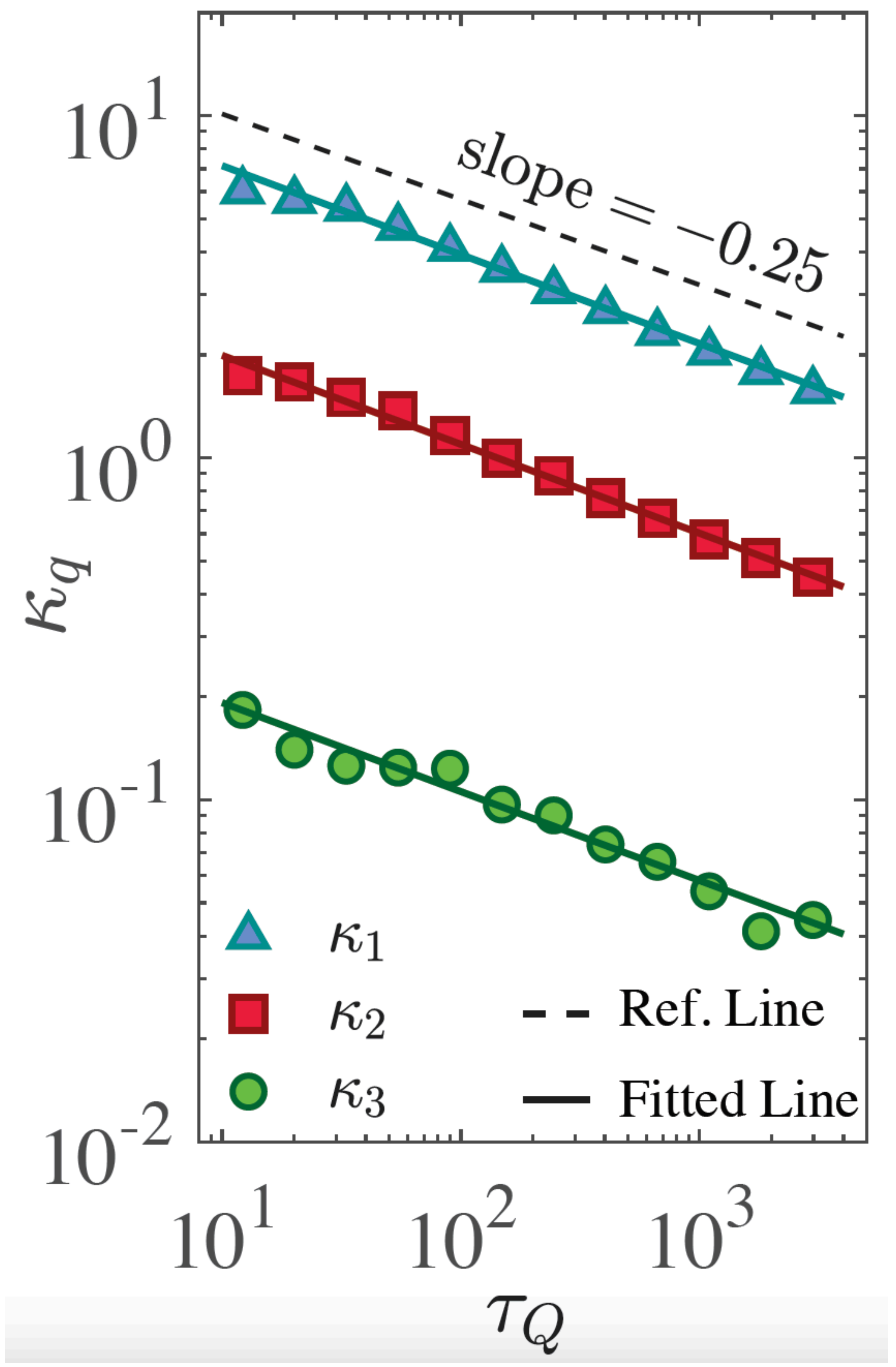
$$\mathcal{H} = -J\sum_{m=1}^{N}(\sigma_m^z\sigma_{m+1}^z + g\sigma_m^x).$$

***Poisson binomial distribution function*: N-independent Bernouilli trials, at each point kink has a possibility p to form a kink, and a possibility $1-p$ not to form a kink**

$$\kappa_2 = \langle n^2\rangle - \langle n\rangle^2 = \frac{2-\sqrt{2}}{2}\kappa_1 \approx 0.29\kappa_1$$

$$\kappa_3 = \langle (n - \langle n\rangle)^3\rangle = (1 - 3\sqrt{2} + 2/\sqrt{3})\kappa_1 \approx 0.033\kappa_1$$

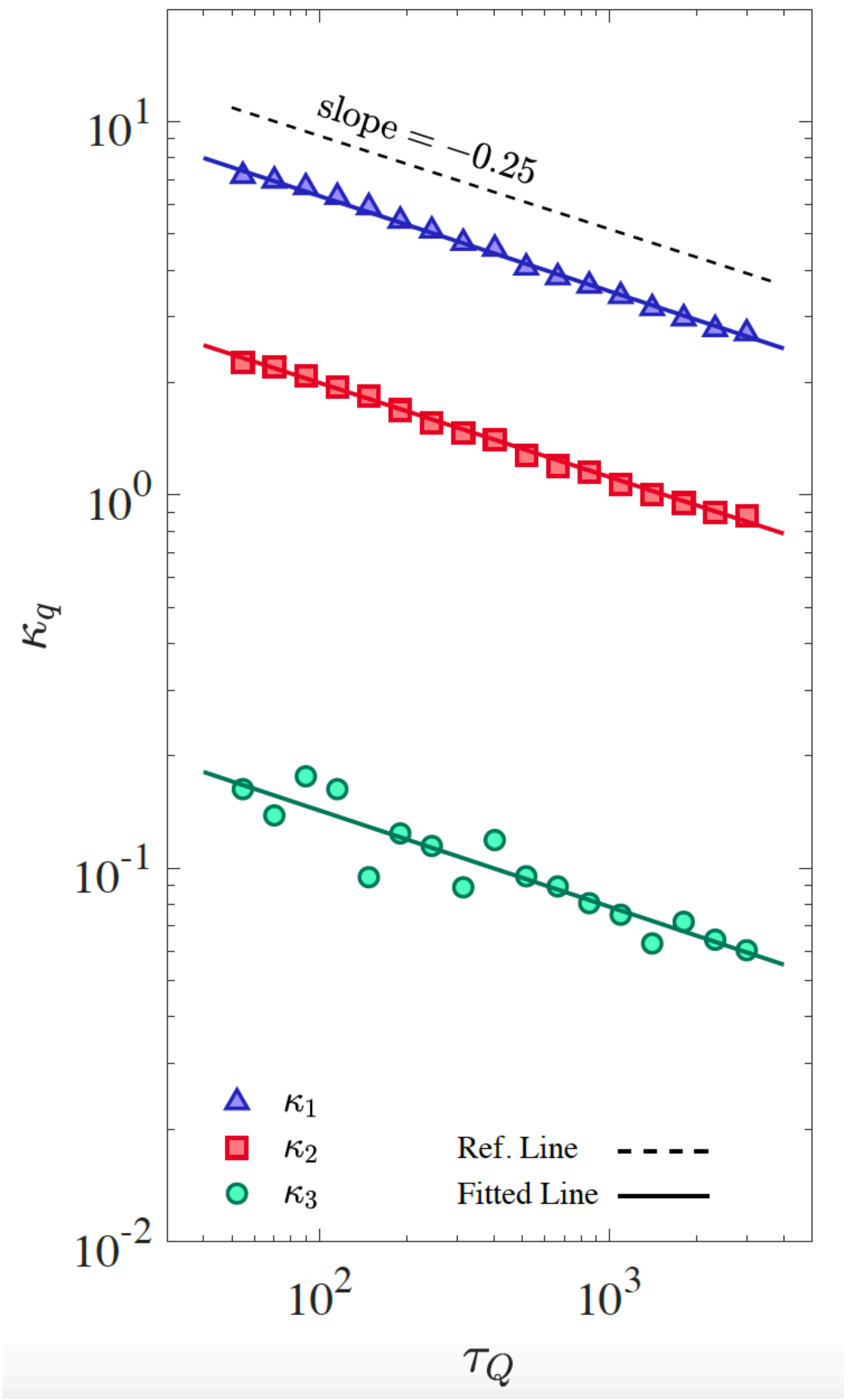
•Beyond KZ scaling relation, cumulants vs. quench rate



$\kappa_2/\kappa_1 \approx 0.294$

$\kappa_3/\kappa_1 \approx 0.023$

GL model



$\kappa_2/\kappa_1 \approx 0.312$

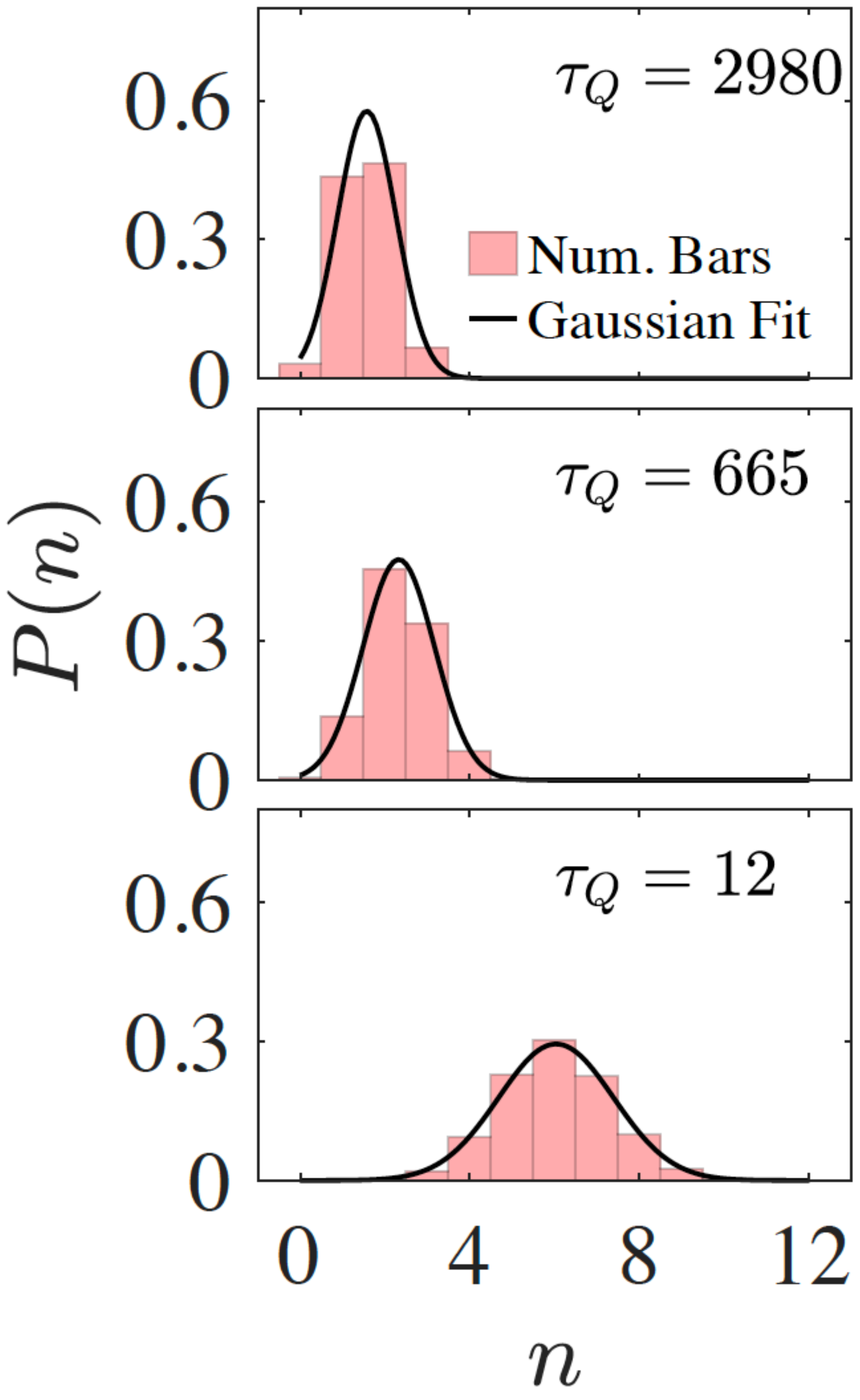
$\kappa_3/\kappa_1 \approx 0.023$

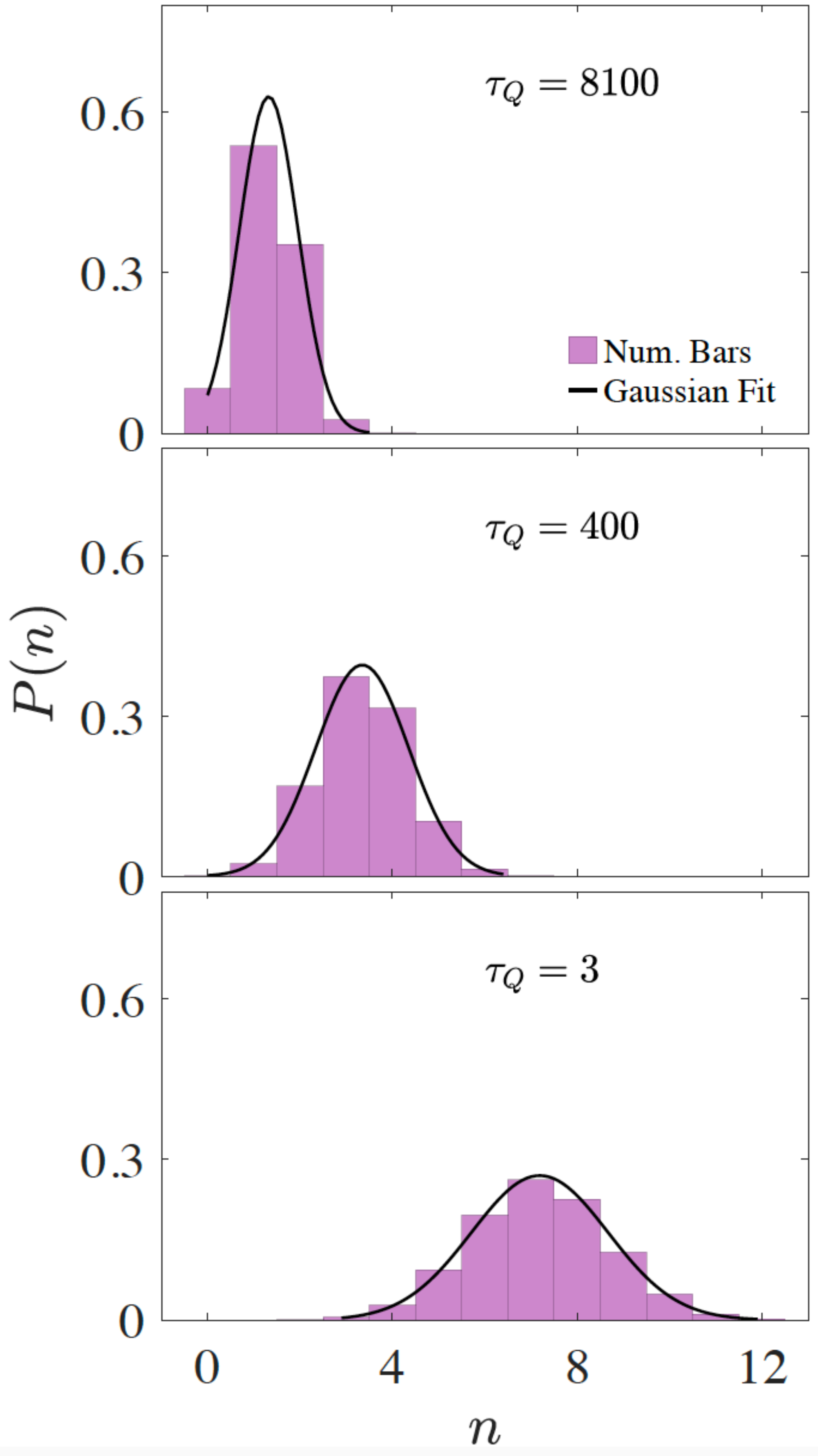
AdS/CFT

•**Gaussian distribution in large trial number**

In the limit of large trial number with fixed average probability, distribution becomes Gaussian （Central limit theorem）

$$P(n) \approx \frac{1}{\sqrt{2\pi\kappa_2}} \exp\left[-\frac{(n-\langle n\rangle)^2}{2\kappa_2}\right]$$

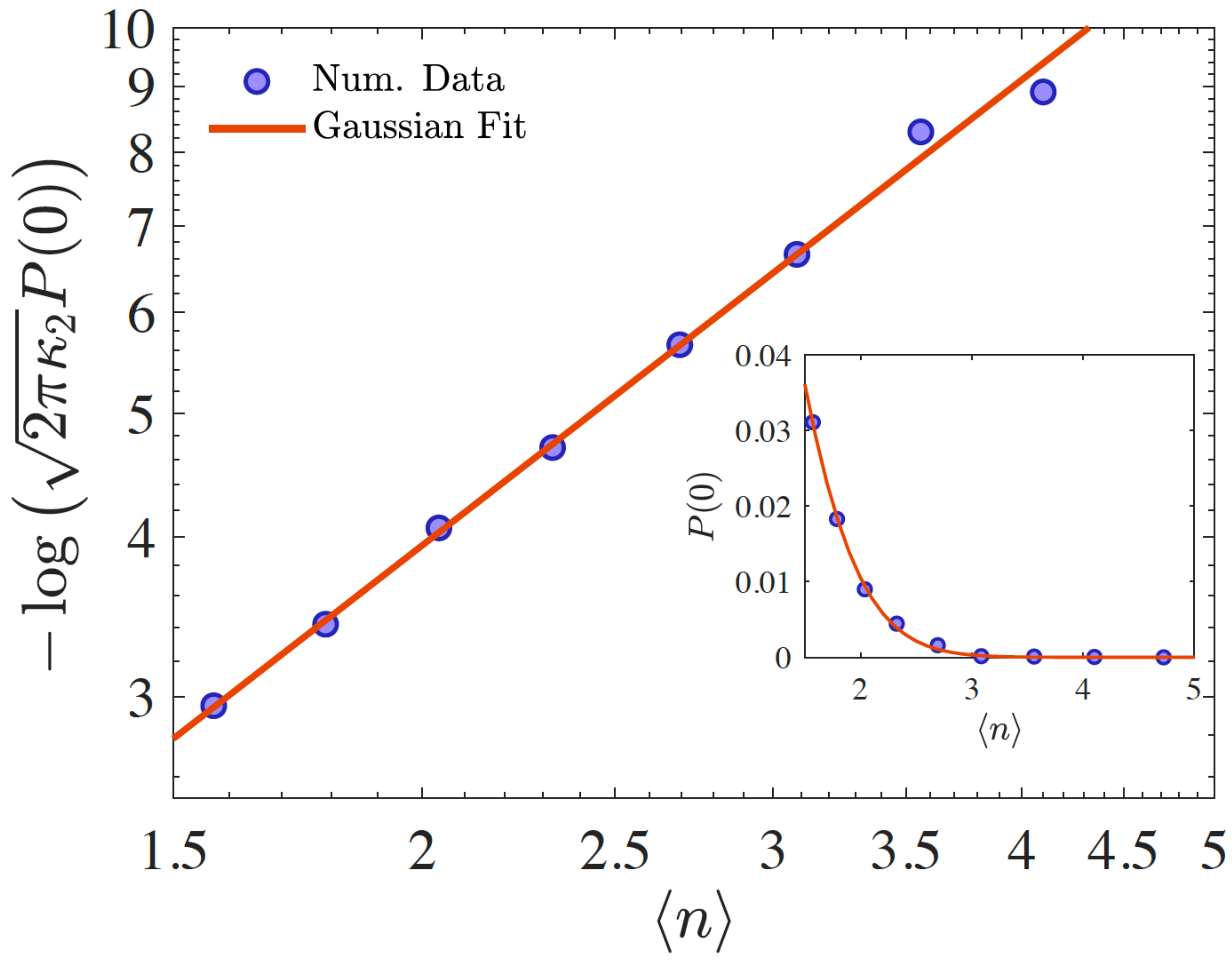




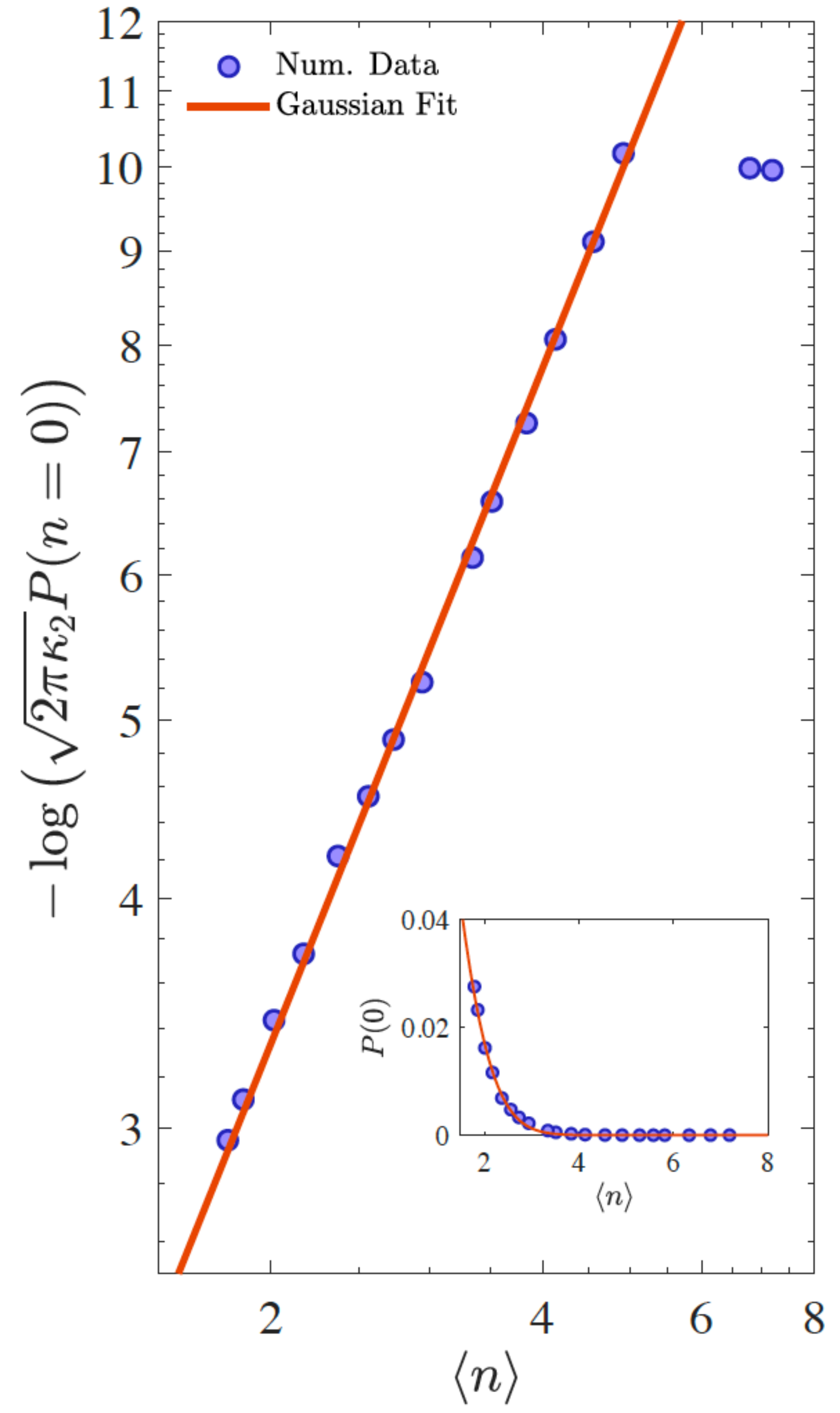
GL model **AdS/CFT**

- **Adiabaticity limit: P(n=0)**

$$P(n=0) \approx \frac{1}{\sqrt{2\pi\kappa_2}} \exp^{-\frac{\langle n\rangle^2}{2\kappa_2}}$$



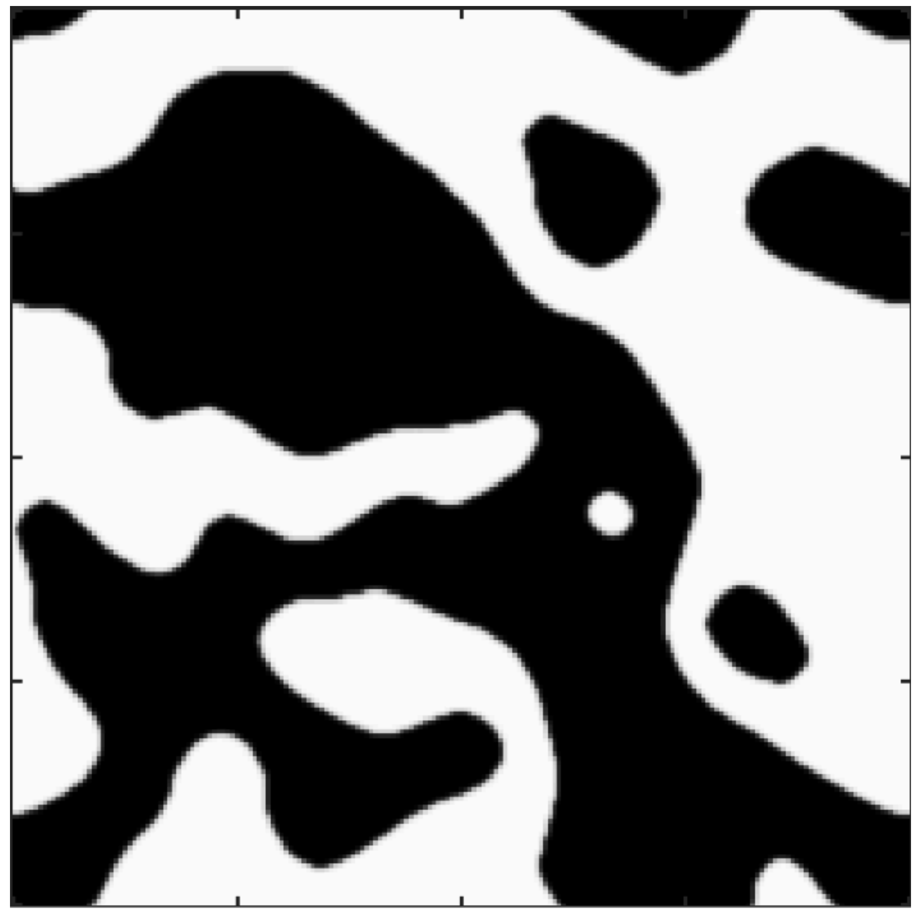
GL model

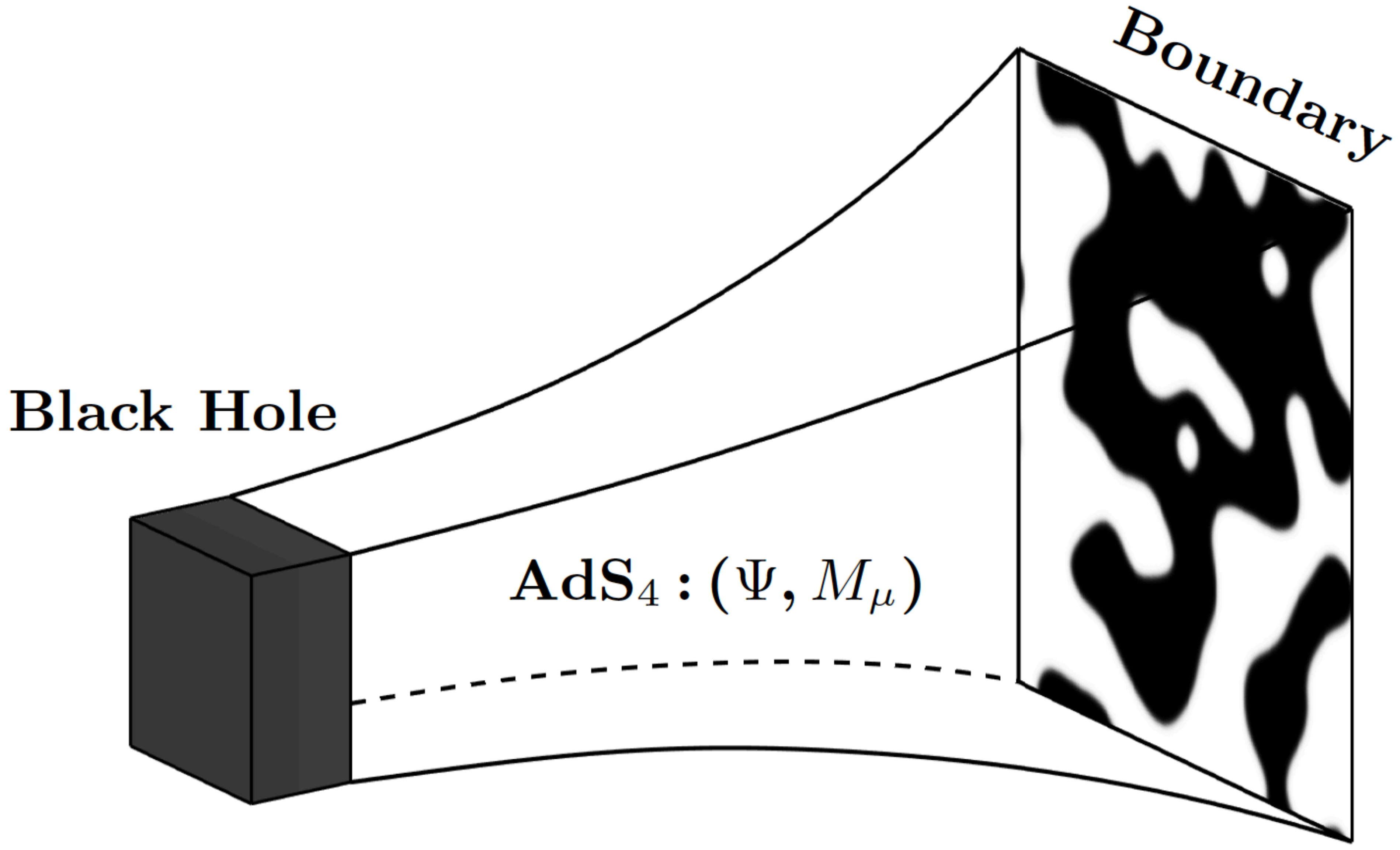


AdS/CFT

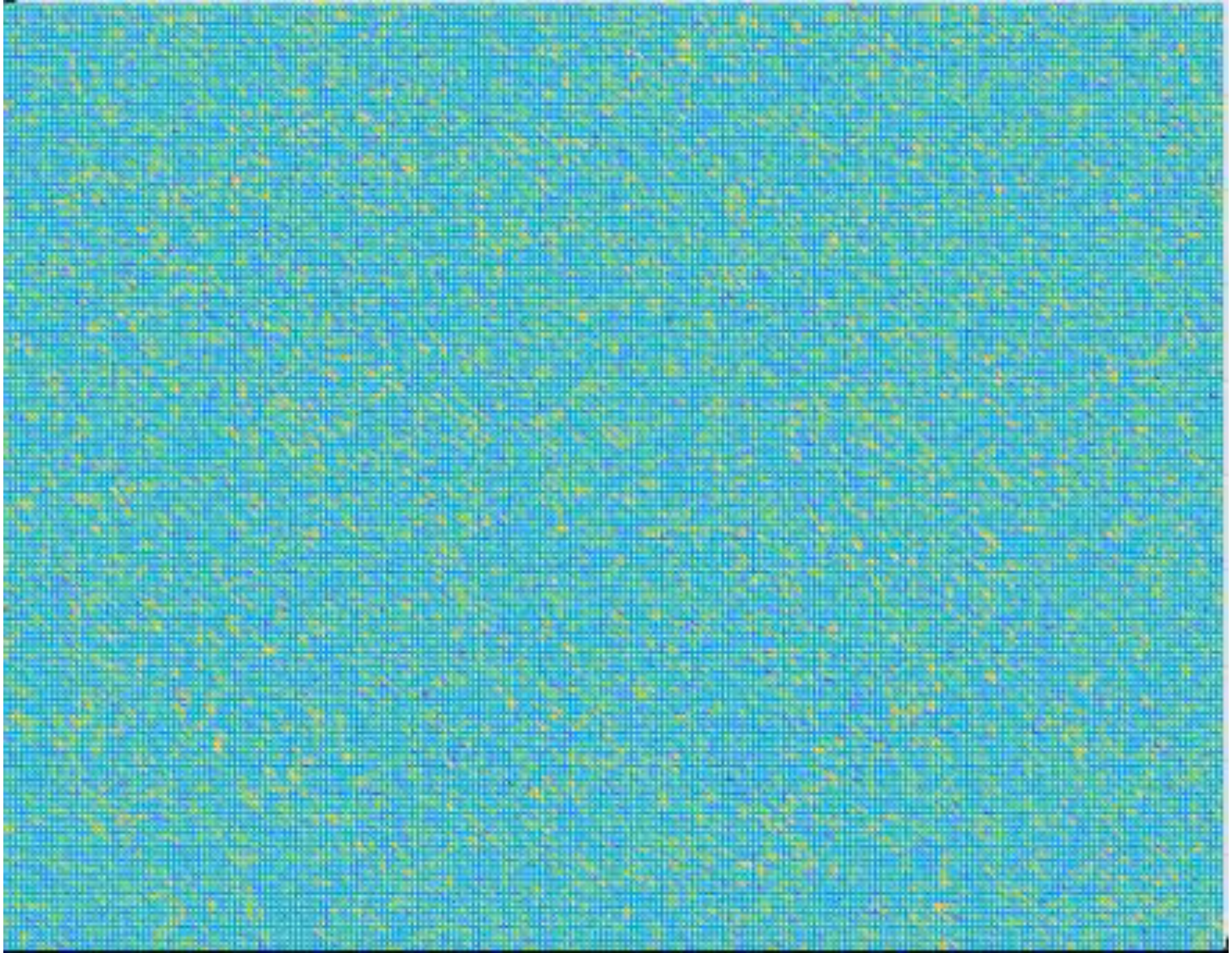
Tian-Chi Ma, Han-Qing Shi, Adolfo del Campo, HQZ, 2406.05167

Holographic Domain Walls in 2+1 dim

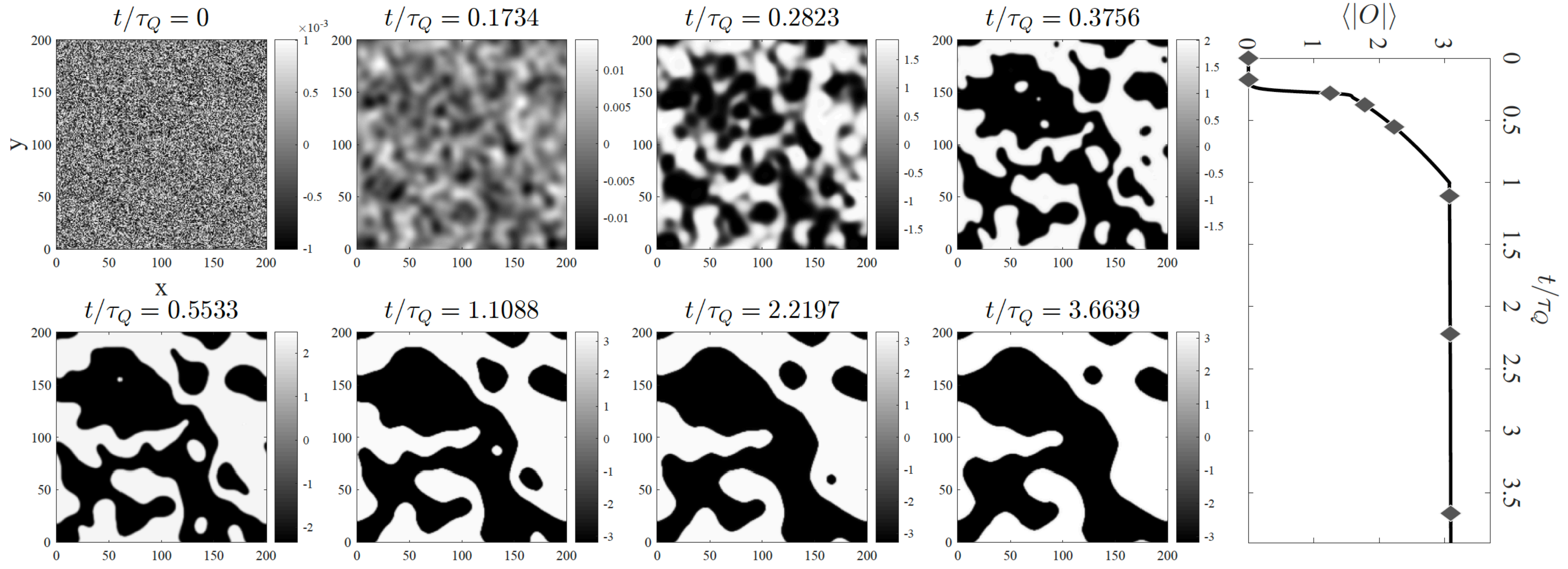




- **Actions, metric, EoMs, ansats of fields, quench profile are similar to holographic kinks;**
- **Only difference is adding y-direction and M_y gauge field;**
- **Numerically complicated**

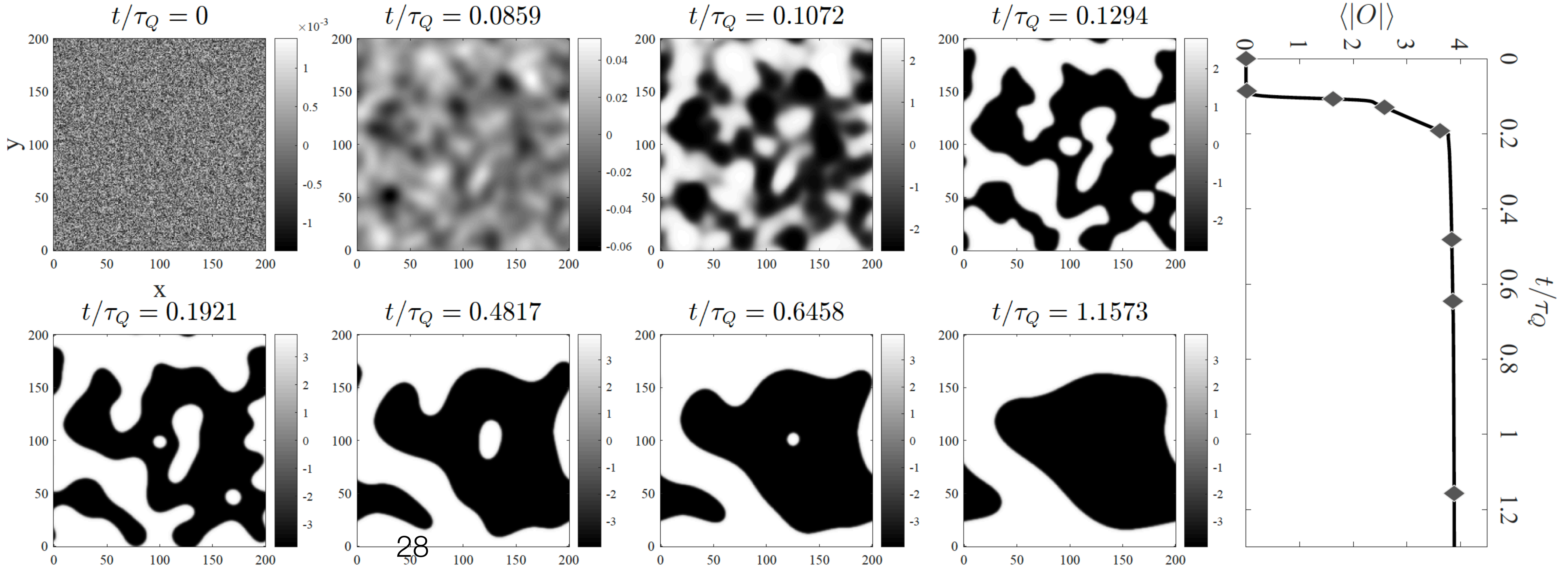


•Time evolution of domain walls



GL model

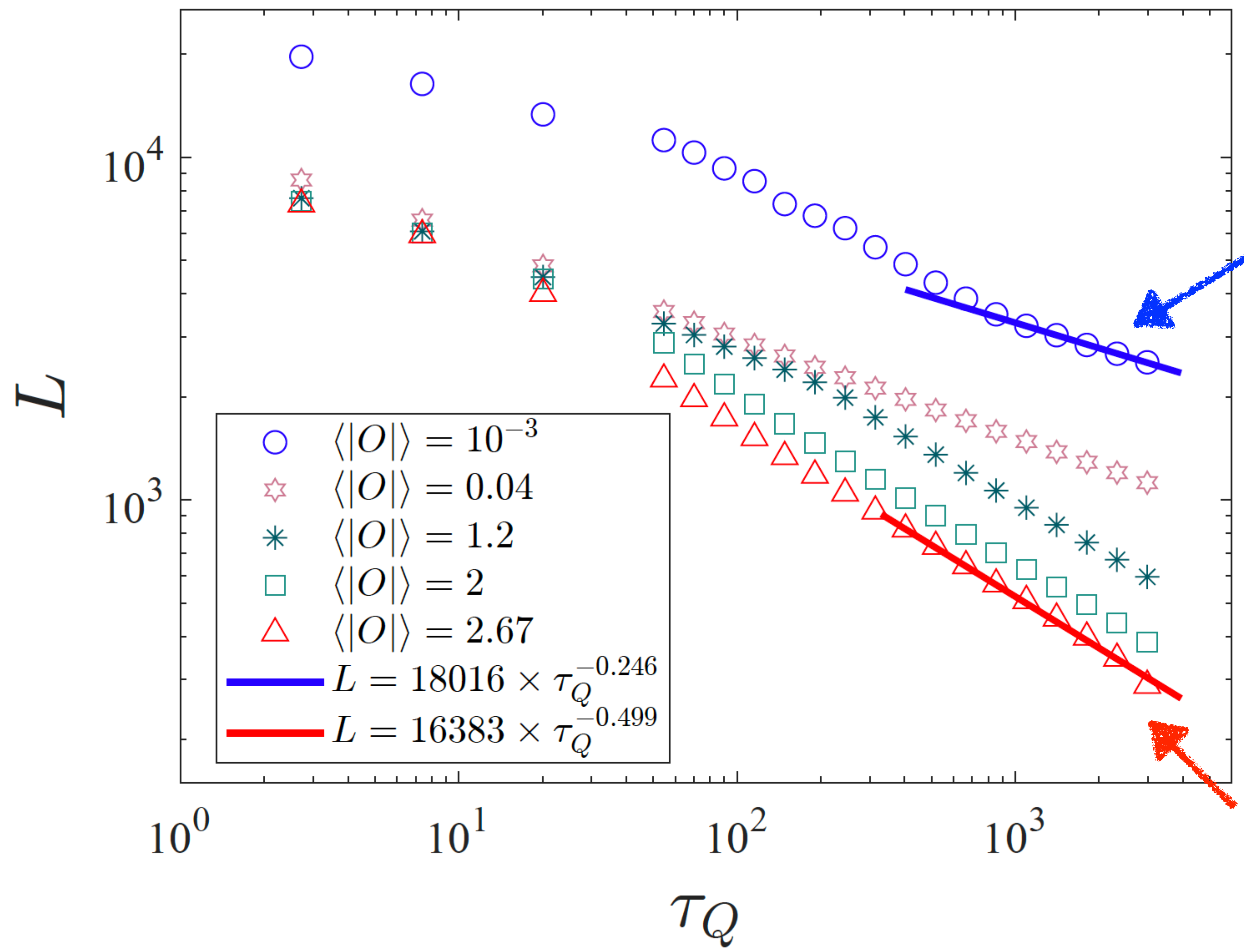
AdS/CFT



• Domain wall length vs. quench rate

$$L \propto \tau_Q^{-(D-d)\nu/(1+z\nu)}, \qquad (D=2, d=1, \nu=1/2, z=2)$$

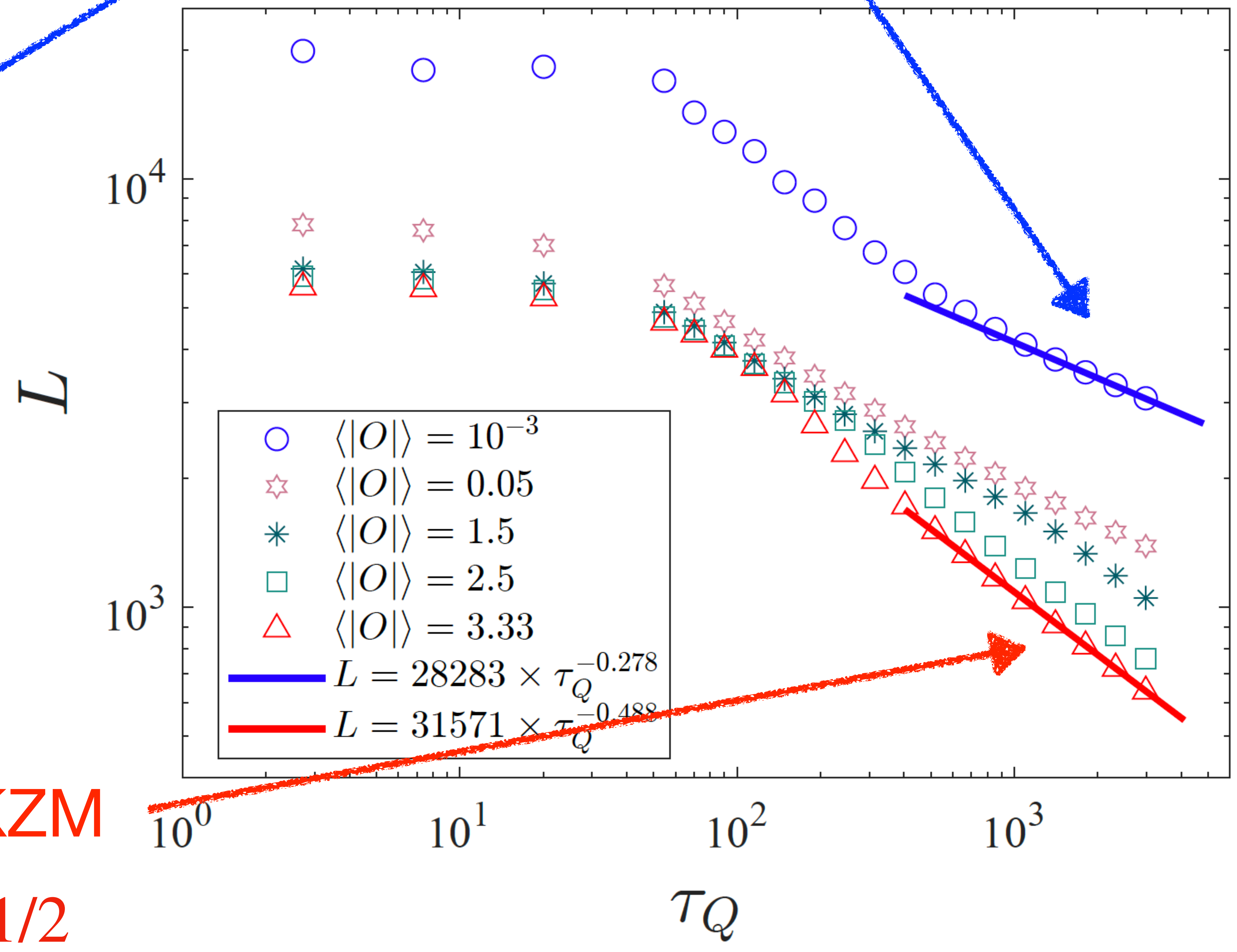
$L \propto \tau_Q^{-1/4}$ **KZM**



GL model

Beyond KZM

$L \propto \tau_Q^{-1/2}$

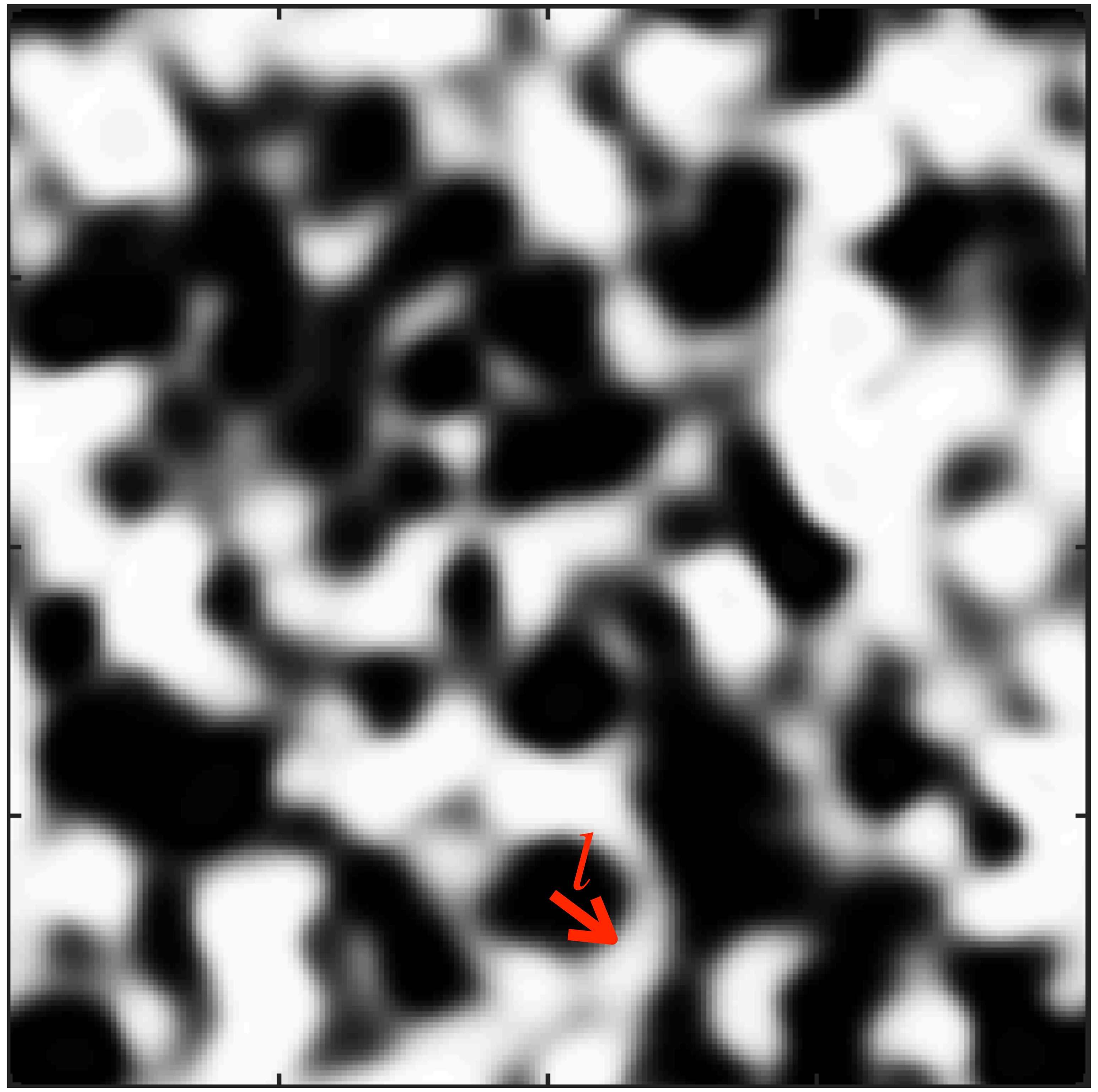


AdS/CFT

- **Coarsening domain wall length vs. time**

  A.J. Bray (1994), Advances in Physics, 43:3, 357-459

  the length scale $l \sim t^{1/2}$

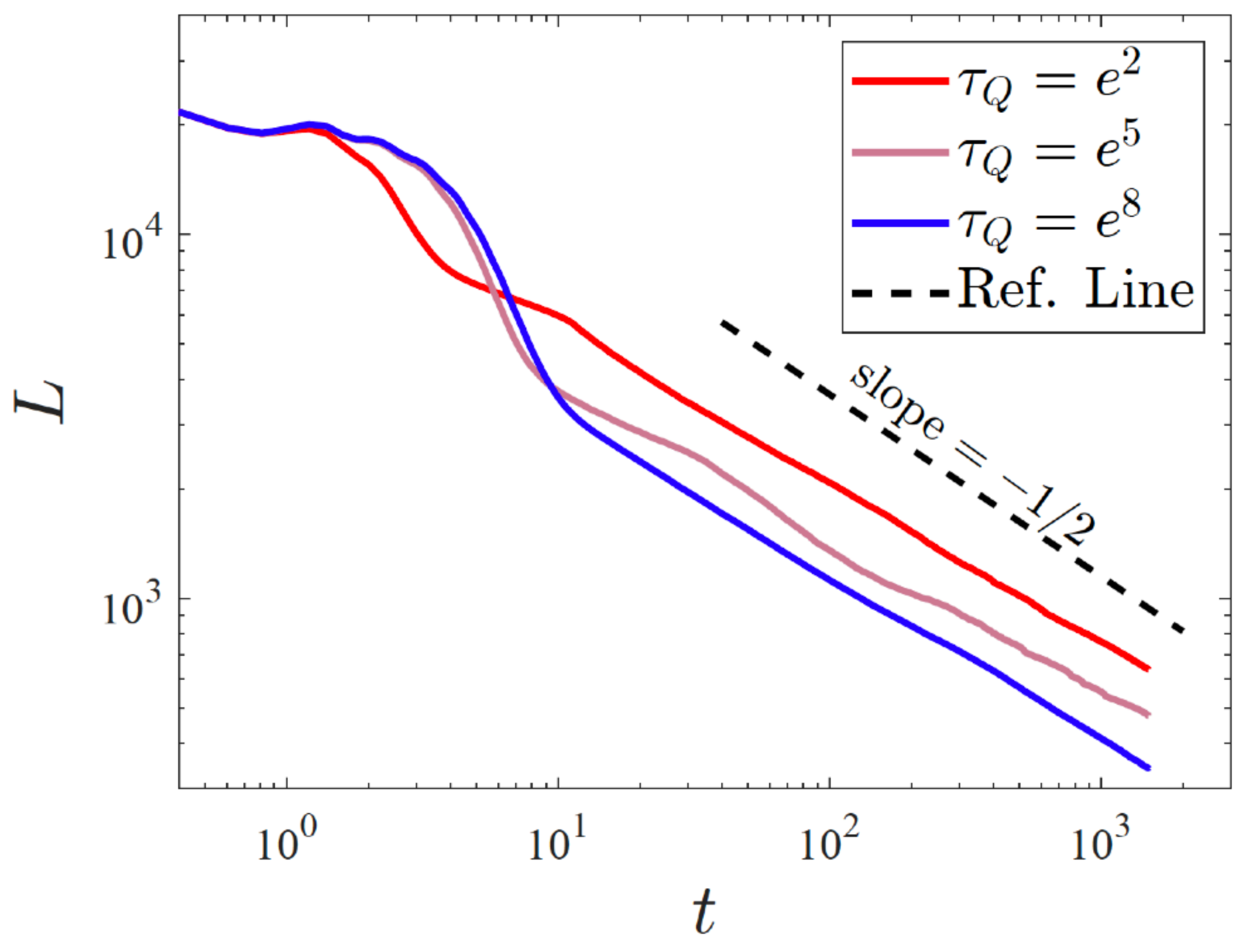


Area A

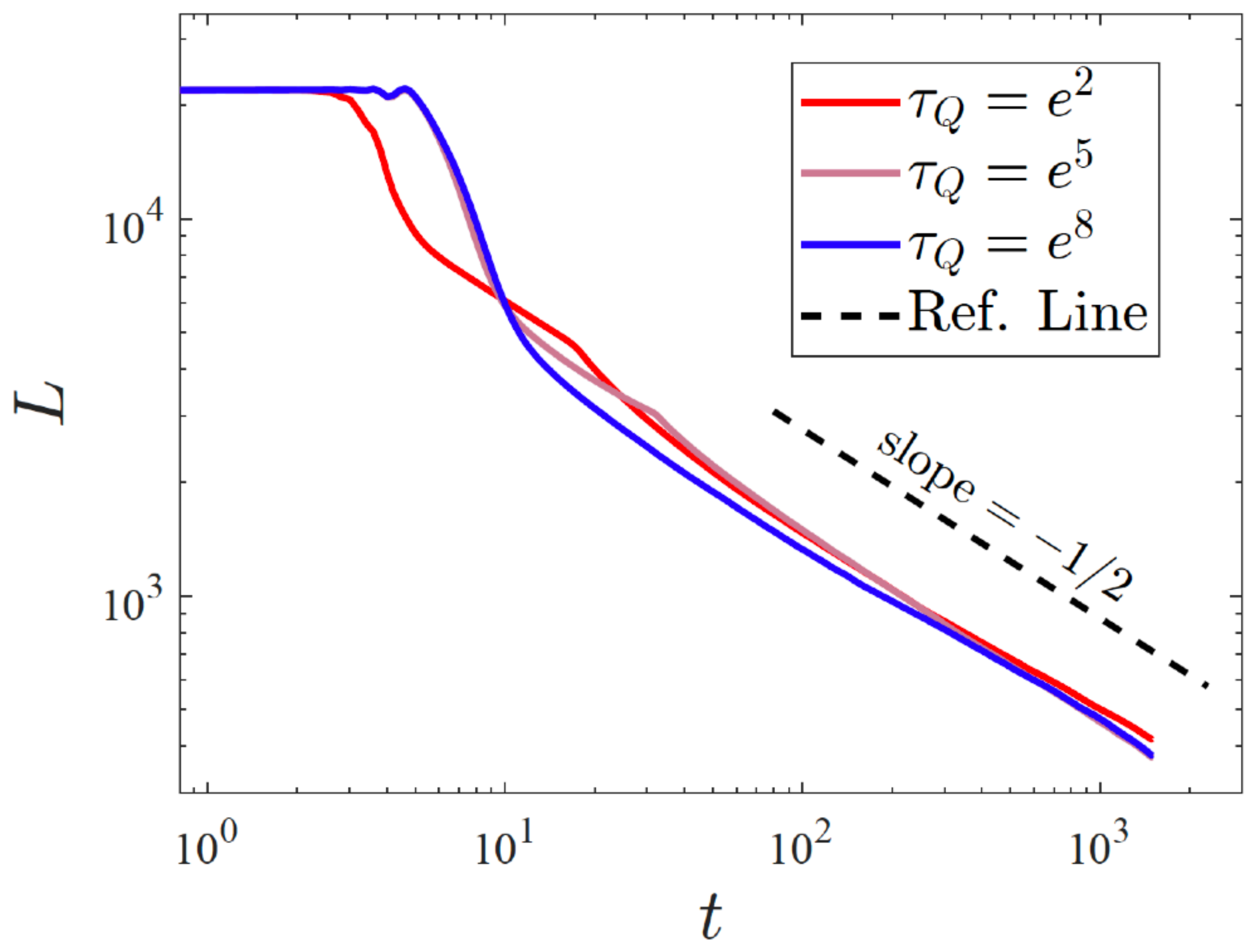
Number of domains: $n = A/\pi l^2$

Length of domain walls: $L \approx n \cdot 2\pi l = 2A/l$

$L \propto t^{-1/2}$

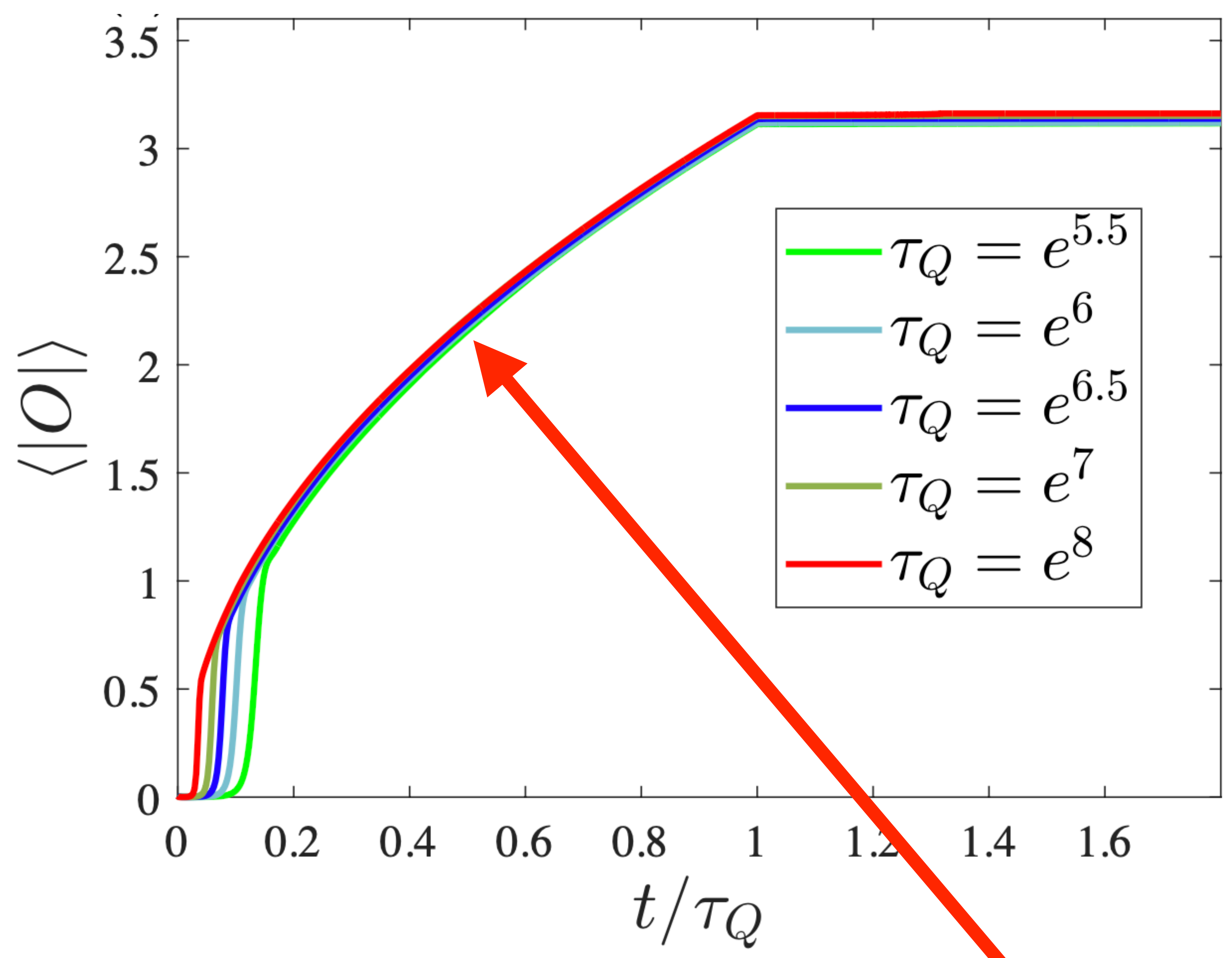


GL model

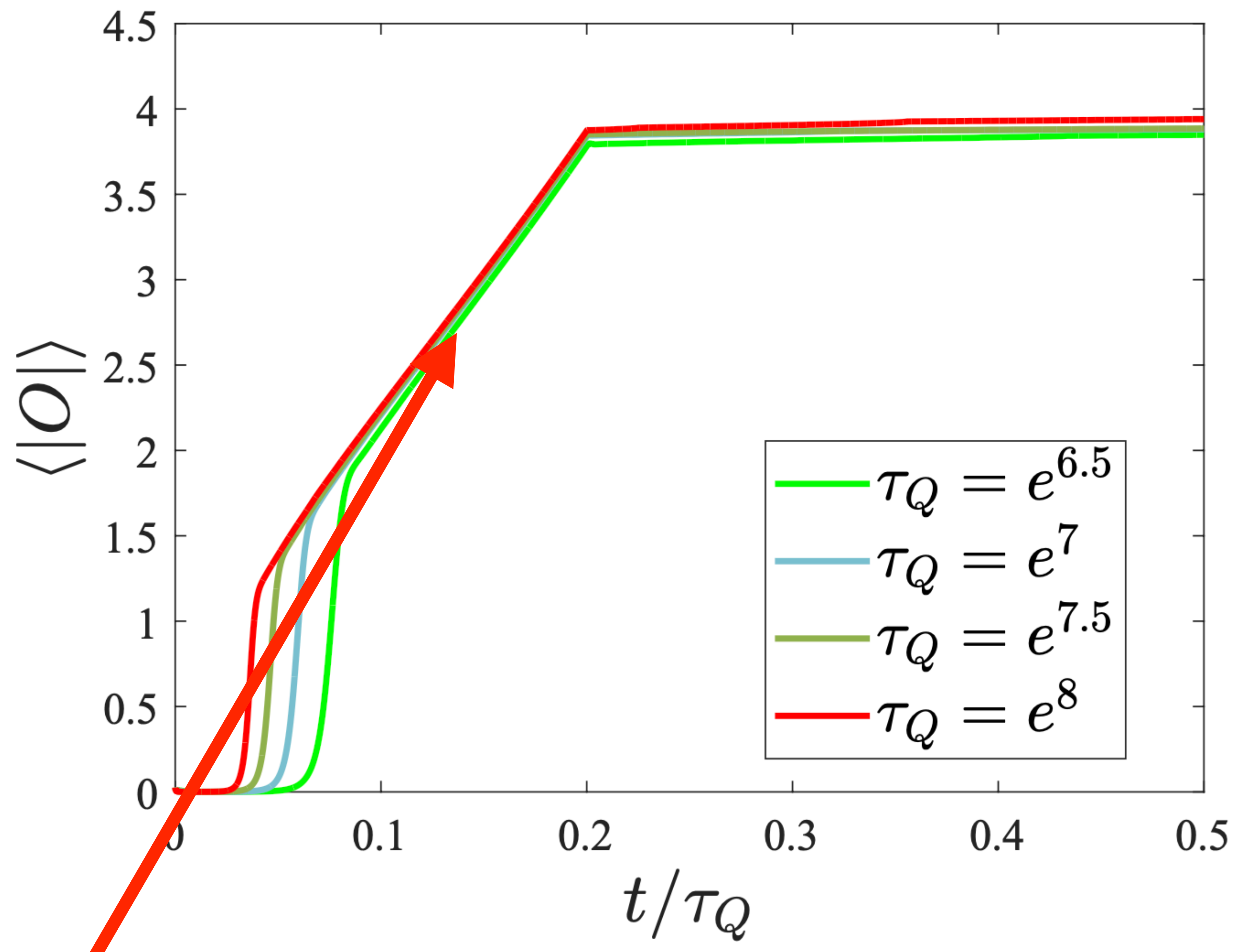


AdS/CFT

Condensate vs time



GL model



AdS/CFT

Adiabatic evolution at late time $t \propto \tau_Q$

$$L \propto t^{-1/2} \propto \tau_Q^{-1/2}$$

Summary

- We have realized the kink hairs in the bulk, whose holographic dual can be interpreted as a one-dimensional spin chain. They are consistent with KZM;

- We have realized the domain wall structures holographically; However, due to the coarsening dynamics, the KZ scalings are only satisfied nearby the critical point; away from the critical point, this relation would be destroyed, and satisfy another power-law

Thank you very much!