7th International Conference on Holography and String Theory in Da Nang

2024.8.22-26 Duy Tan University

Wheeler-deWitt states of AdS black holes

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Based on the ongoing work with Kyung Kiu Kim and Yunseok Seo

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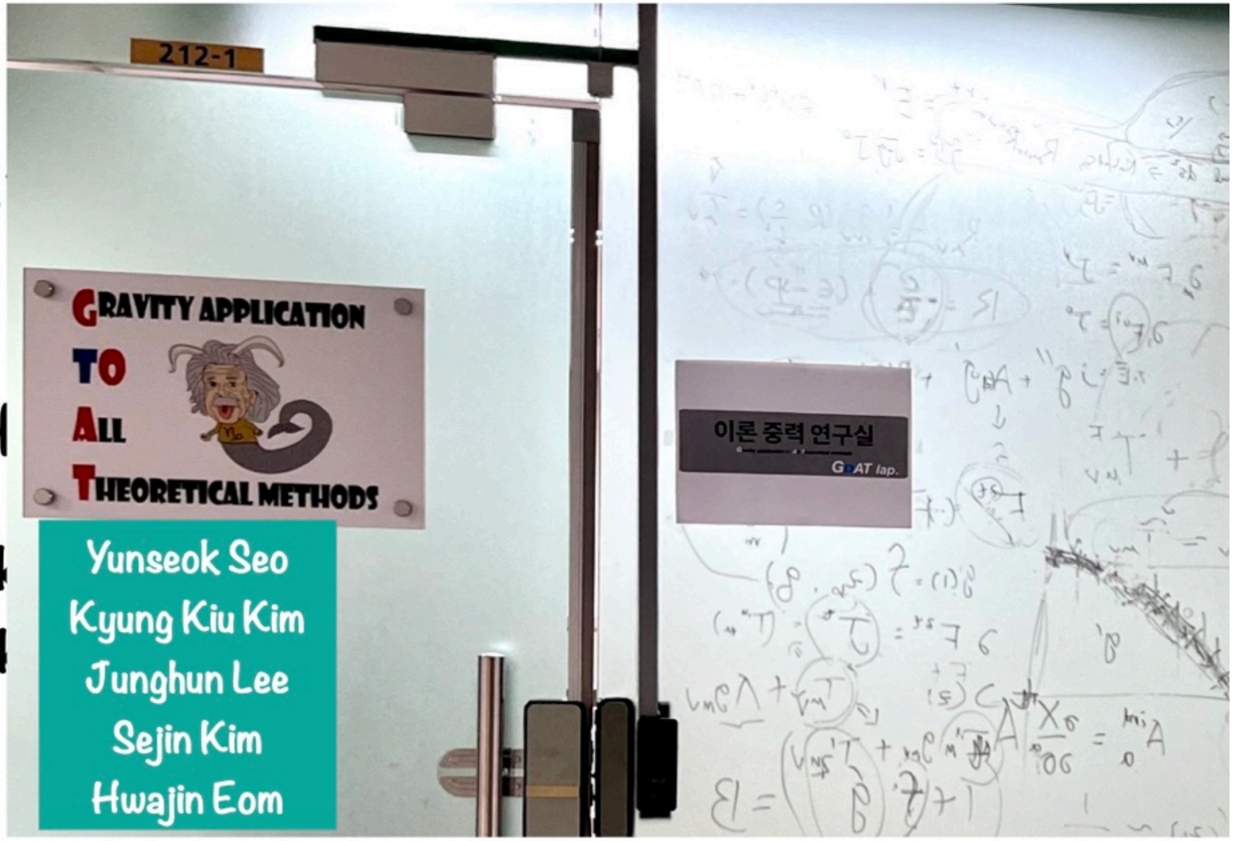


Introduction

- Hawking Radiation

- In the semiclassical approximations, the total number of particles emitted from black holes turned out to be the Planckian distribution of thermal radiation. [Hawking (1974,1975)]

-This discovery established a close relationship between gravity, thermodynamics, and statistical physics.

-There are several ways to study quantum radiation in a curved spacetime in the semiclassical regime.

-We tried to investigate this subject by using the functional Schrödinger equation which is derived from the Wheeler-deWitt equation. However, the results indicate that we cannot properly recognize the thermal properties of black holes.

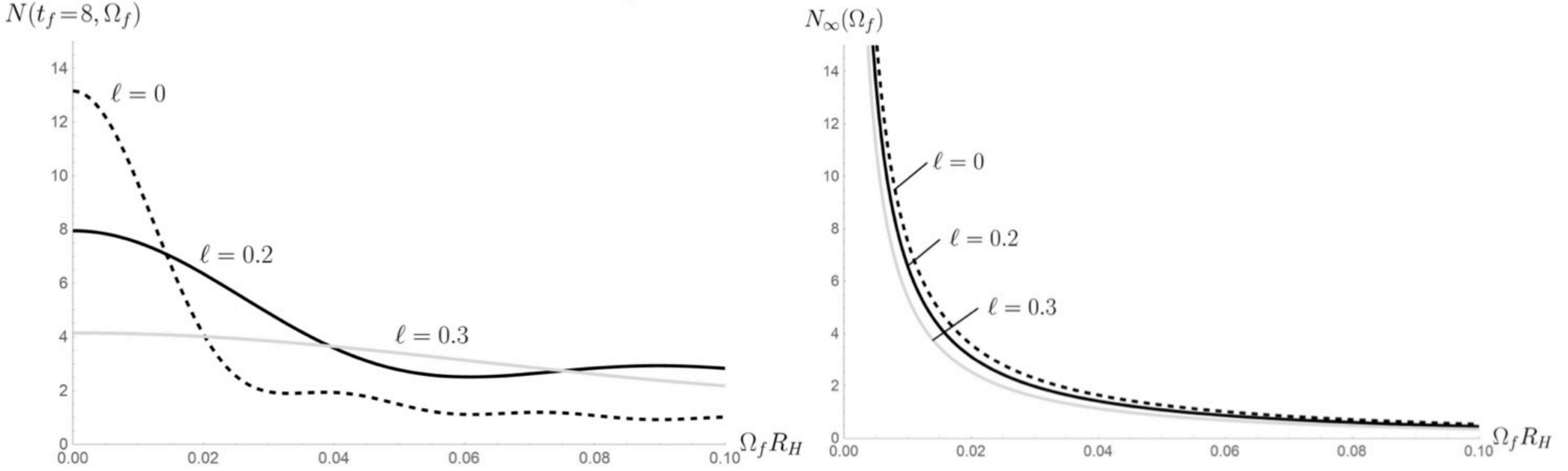
(e.g. The state of quantum radiation remains a pure state
although the energy spectrum is getting close to the thermal Hawking radiation)

[Vachaspati,Stojkovic,Krauss (2007)]
[Saini, Stojkovic (2018)]
[HE, Kim (2020,2021)]

Introduction

- HE, Kim (2020) - $4d$ regular black hole

Hayward black hole $\left(f(r) = 1 - \dfrac{2Mr^2}{r^3 + 2\ell^2 M}\right)$, quantum radiation seen by an observer at infinity



Before the formation of a black hole

When the black hole starts to form

- HE, Kim (2021) - BTZ black hole → The same result was obtained.

Introduction

- Return to the starting point: the Wheeler-deWitt equation.

- In the former approach, the functional Schrödinger equation(FSE), all degrees of freedom were ignored except the observer's time, so that the equations in various black holes were turned out to be a universal form:

$$\left[-\frac{f(R)}{2\alpha}\frac{\partial^2}{\partial b^2}+\frac{1}{2}\alpha\omega_0^2 b^2\right]\psi(t_{\rm obs},b)=i\frac{\partial}{\partial t_{\rm obs}}\psi(t_{\rm obs},b)$$

Quantized scalar field

- We decided to focus on the starting point, the Wheeler-deWitt equation instead of the FSE where the quantum fluctuation of geometry was turned off.
- We find that there are recent interesting works on the so-called the Wheeler-deWitt states corresponding to the explicit solutions of the Wheeler-deWitt equation. (especially, interior region of black holes) [Bouhmadi-Lopez,Brahma,Chene,Chene,Yeom (2020)] [Hartnoll (2022)] [Blacker,Ning (2023)]

arXiv > hep-th > arXiv:2208.04348

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High Energy Physics - Theory

[Submitted on 8 Aug 2022 (v1), last revised 24 Dec 2022 (this version, v3)]

Wheeler-DeWitt states of the AdS-Schwarzschild interior

Sean A. Hartnoll

We solve the Wheeler-DeWitt equation for the planar AdS-Schwarzschild interior in a minisuperspace approximation involving the volume and spatial anisotropy of the interior. A Gaussian wavepacket is constructed that is peaked on the classical interior solution. Simple observables are computed using this wavepacket, demonstrating the freedom to a choose a relational notion of `clock' in the interior and characterizing the approach to the spacelike singularity. The Wheeler-DeWitt equation may be extended out through the horizon, where it describes the holographic renormalization group flow of the black hole exterior. This amounts to the Hamilton-Jacobi evolution of the metric component g_{tt} from positive interior values to negative exterior values. The interior Gaussian wavepacket is shown to evolve into the Lorentizan partition function of the boundary conformal field theory over a microcanonical energy window.

arXiv > hep-th > arXiv:2308.00040

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High Energy Physics - Theory

[Submitted on 31 Jul 2023 (v1), last revised 2 Aug 2023 (this version, v2)]

Wheeler DeWitt States of a Charged AdS_4 Black Hole

Matthew J. Blacker, Sirui Ning

We solve the Wheeler DeWitt equation for the planar Reissner-Nordström-AdS black hole in a minisuperspace approximation. We construct semiclassical Wheeler DeWitt states from Gaussian wavepackets that are peaked on classical black hole interior solutions. By using the metric component g_{xx} as a clock, these states are evolved through both the exterior and interior horizons. Close to the singularity, we show that quantum fluctuations in the wavepacket become important, and therefore the classicality of the minisuperspace approximation breaks down. Towards the AdS boundary, the Wheeler DeWitt states are used to recover the Lorentzian partition function of the dual theory living on this boundary. This partition function is specified by an energy and a charge. Finally, we show that the Wheeler DeWitt states know about the black hole thermodynamics, recovering the grand canonical thermodynamic potential after an appropriate averaging at the black hole horizon.

- We would like to focus on the Wheeler-deWitt state itself.

  The ultimate goal is to describe the dynamics of gravity-radiation system observed for a certain observer.

Charged AdS4 black hole (classical)

- Hamilton-Jacobi equation

- In order to solve the Wheeler-de Witt equation, we first need to obtain the Hamilton-Jacobi function.

- Action : $I[g,A] = \frac{1}{\kappa^2}\int d^4x\sqrt{-g}\left(R+\frac{6}{L^2}\right) - \frac{1}{4}\int d^4x\sqrt{-g}F^2 + \frac{2}{\kappa^2}\int\sqrt{-h}K$

$$\kappa^2 = 16\pi G = 1, \quad F = dA$$

- We will restrict to metrics within the 'minisuperspace' ansatz :

gauge field

$$ds^2 = -N^2(r)dr^2 + g_{tt}(r)dt^2 + g_{xx}(r)\left(dx^2+dy^2\right), \qquad A = \phi_t(t)dt$$

$$\mathscr{L} = \frac{1}{2N}\left(-\frac{2}{\sqrt{t}}\dot{t}\dot{x} - \frac{\sqrt{t}}{x}\dot{x}^2 + \frac{x}{\sqrt{t}}\dot{\phi}^2\right) + \frac{6N}{L^2}\sqrt{t}x$$

Here, $\{g_{tt}, g_{xx}, \phi_t\}$ is simply written as $\{t, x, \phi\}$ and the overdot denotes d/dr.

- For the dynamical variables $\{t, x, \phi\}$, the conjugate momenta are given as

$$\pi_t = -\frac{\dot{x}}{\sqrt{t}N}, \quad \pi_x = -\frac{\dot{t}}{\sqrt{t}N} - \frac{\sqrt{t}\dot{x}}{xN}, \quad \pi_\phi = \frac{x\dot{\phi}}{\sqrt{t}N} .$$

Charged AdS4 black hole (classical)

- Hamilton-Jacobi equation

- Then we get the Hamiltonian density :

$$\mathscr{H} = \frac{\sqrt{t}N}{2x}\left(t\pi_t^2 - 2x\pi_t\pi_x + \pi_\phi^2 - \frac{12}{L^2}x^2\right)$$

N turns out to be a Lagrangian multiplier,
which imposes the Hamiltonian constraint of $\mathscr{H} = 0$.

- From $\pi_a = \partial_{g^a}S \quad (g^a = t, x, \phi)$, one can obtain the classical solution by solving the Hamilton-Jacobi equation:

$$t\left(\partial_t S\right)^2 - 2x\left(\partial_t S\right)\left(\partial_x S\right) + \left(\partial_\phi S\right)^2 - \frac{12}{L^2}x^2 = 0, \quad S(t,x,\phi) :$$ Hamilton-Jacobi function

more useful coordinate system to solve HJ eq

$$ds^2 = -N^2dr^2 + v^{2/3}\left(e^{4k/3}dt^2 + e^{-2k/3}\left[dx^2 + dy^2\right]\right)$$
$$g_{tt} = v^{2/3}e^{4k/3}, \qquad g_{xx} = v^{2/3}e^{-2k/3}$$

- The explicit solution is $S(t,x,\phi;k_0,c_0) = -\dfrac{e^{-k_0}\left(c_0^2 + 4x^2/L^2\right)}{2\sqrt{x}} + c_0\phi + 2t\sqrt{x}e^{k_0}$.

- Note that k_0, c_0 correspond to some constant momenta which satisfies

$$\partial_{k_0}S_1 = \epsilon_0, \qquad \partial_{c_0}S_1 = \mu_0$$

Charged AdS4 black hole (classical)

- Classical solution - interior charged AdS black hole

- The relations between the dynamical variables are obtained.

$$t = \frac{\epsilon_0 e^{-k_0}}{2\sqrt{x}} - \frac{e^{-2k_0}\left(c_0^2 + 4x^2/L^2\right)}{4x}, \qquad \phi = \frac{c_0 e^{-k_0}}{\sqrt{x}} + \mu_0$$

- From the equation of motion for N and the obtained relations, one finally get the classical solution describing the planar RN-AdS black hole.

$$ds^2 = \frac{1}{z^2}\left[\frac{dz^2}{f(z)} - e^{-2k_0} f(z)dt^2 + dx^2 + dy^2\right], \qquad \phi = c_0 e^{-k_0}\, z + \mu_0$$

$$f(z) = \frac{1}{L^2} - \frac{1}{2}\epsilon_0 e^{k_0}\, z^3 + \frac{1}{4}c_0^2 z^4, \qquad g_{xx} = 1/z^2$$

$z \to \infty$: singularity , $\qquad z_h = \left(\frac{2e^{-k_0}}{L^2\epsilon_0}\right)^{1/3}$: horizon

Wheeler-deWitt state

- Wheeler-deWitt equation

- From the deWitt metric, the Lagrangian is $\mathscr{L} = \frac{1}{2N} G_{ab} \dot{g}^a \dot{g}^b - NV(g)$,

  and the Hamiltonian constraint is written as $\frac{1}{2} G^{ab} \pi_a \pi_b + V(g) = 0.$

- Under the usual quantization rule ($\pi_a \to -i \nabla_{g^a}$), the Wheeler-deWitt(WdW) equation is

$$\left(-\frac{1}{2} \nabla^2 + V(g) \right) \Psi(g) = 0 .$$

- Then $e^{\pm iS(t,x,\phi;k,c)}$ form a basis of the semiclassical solution to the WdW eq, so the we get the general solution as

$$\Psi(t, x, \phi) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \int_{-\infty}^{\infty} \frac{dc}{2\pi} \beta(k, c) e^{iS(t,x,\phi;k,c)}$$

- Conserved norm (on the surface of $g^a = (\mathrm{const})$) can be defined as

$$|\Psi|^2_{g^a} = \int \sqrt{-G}\ \varepsilon_{abc}\ dx^b \wedge dx^c \left(\Psi^* \nabla_{g^a} \Psi - \Psi \nabla_{g^a} \Psi^* \right) .$$

g^a can be considered as a clock!

Wheeler-deWitt state

- Charged AdS black hole

- In our case, $G^{ab} = \begin{pmatrix} t & -x & 0 \\ -x & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad V(g) = -\frac{6}{L^2}x^2$

- $$\frac{\partial}{\partial t}\left(t\frac{\partial \Psi}{\partial t} - 2x\frac{\partial \Psi}{\partial x}\right) + \frac{\partial^2 \Psi}{\partial \phi^2} + \frac{12}{L^2}x^2\Psi = 0$$

- $$|\Psi|_x^2 = \frac{i}{2}\int \sqrt{-G}\ dtd\phi\ G^{xt}\left(\Psi^*\partial_t\Psi - \Psi\partial_t\Psi^*\right) = -\frac{i}{2}\int dtd\phi\left(\Psi^*\partial_t\Psi - \Psi\partial_t\Psi^*\right).$$

- Gaussian wavepacket

$$\beta(k,c) = N_\beta\ \exp\left[-i\epsilon_0\left(k-k_0\right) - \frac{\Delta_k^2}{2}\left(k-k_0\right)^2\right]\exp\left[-i\mu_0\left(c-c_0\right) - \frac{\Delta_c^2}{2}\left(c-c_0\right)^2\right],\ N_\beta = \sqrt{4\pi\Delta_k\Delta_c}$$

$\Delta_k, \Delta_c \to \infty$: classical limit

Wheeler-deWitt state (Gaussian)

- Gaussian wavepacket

- The wavepacket $\beta(k,c)$ defined in the momentum space (k,c).
- We impose the semiclassical regime such that $\Delta_k, \Delta_c \gg 1$,
  which indicates Ψ is strongly peaked on values of the classical solutions

$$k = k_0 \text{ and } c = c_0 .$$

- Note that, by using the Fourier transform $(k,c) \to (\epsilon, \mu)$

$$\alpha(\epsilon,\mu) = \int \frac{dk}{2\pi} \int \frac{dc}{2\pi} \beta(k,c) e^{-i\epsilon k} e^{-i\mu c} = \sqrt{\frac{4\pi}{\Delta_k \Delta_c}} \exp\left[i\epsilon k_0 - \frac{(\epsilon-\epsilon_0)^2}{2\Delta_k^2} \right] \exp\left[i\mu c_0 - \frac{(\mu-\mu_0)^2}{2\Delta_c^2} \right],$$

  one can find that Ψ is peaked on

$$\epsilon = \epsilon_0 \text{ and } \mu = \mu_0 .$$

- For strongly peaked Ψ also in (ϵ,μ)-space,

$$\epsilon_0 \gg \Delta_k \text{ and } \mu_0 \gg \Delta_c . \longrightarrow$$ will also be shown in the result

- Semiclassical regime : $\epsilon_0 \gg \Delta_k \gg 1$ and $\mu_0 \gg \Delta_c \gg 1$

[result] quantum expectation values

From now on, we recover the expression of the dynamical variables $\{g_{tt}, g_{xx}, \phi_t\}$ except for 'clock' (subscript).

- The expectation value of an operator is defined by

Hermitian ordering

$$\langle \hat{O}(g) \rangle_x = {}_x\langle \Psi | \hat{O}(g) | \Psi \rangle_x = -\frac{i}{2} \int dg_{tt} d\phi_t \left(\Psi^* \hat{O}(g) \partial_t \Psi - \Psi \left(\hat{O}(g) \partial_t \Psi \right)^* \right).$$

- $\langle g_{tt} \rangle_x$

$$\langle g_{tt} \rangle_x = -\frac{e^{-2k_0}}{z^2} \left[\left(\frac{1}{4} c_0^2 z^4 + \frac{1}{L^2} + \frac{z^4}{8\Delta_c^2} \right) e^{\frac{1}{\Delta_k^2}} - \frac{\epsilon_0 e^{k_0}}{2} z^3 e^{\frac{1}{4\Delta_k^2}} \right]$$

$$= -\frac{e^{-2k_0}}{z^2} \left[f(z) + \left\{ \left(\frac{1}{4} c_0^2 z^4 + \frac{1}{L^2} \right) - \frac{\epsilon_0 e^{k_0}}{2} z^3 \times \frac{1}{4} \right\} \frac{1}{\Delta_k^2} + \frac{z^4}{8\Delta_c^2} + \mathcal{O}\left(\frac{1}{\Delta^4} \right) \right],$$

recovers the exact classical solution in the limit of $\Delta_k, \Delta_c \gg 1$

We assume $\Delta_k \sim \Delta_c \sim \Delta$

- In neutral case $c_0 \to 0,\ \Delta_c \to \infty$

$$\langle g_{tt} \rangle_x = -\frac{e^{-2k_0}}{z^2} \left[\frac{1}{L^2} e^{\frac{1}{\Delta_k^2}} - \frac{\epsilon_0 e^{k_0}}{2} z^3 e^{\frac{1}{4\Delta_k^2}} \right]$$

$$z_{h,\text{deformed}} = \left(\frac{2e^{-k_0}}{L^2 \epsilon_0} \right)^{1/3} e^{\frac{1}{4\Delta_k^2}} = z_h e^{\frac{1}{4\Delta_k^2}} = z_h + \frac{z_h}{4\Delta_k^2} + \mathcal{O}\left(\frac{1}{\Delta_k^4} \right)$$

[result] quantum expectation values

- Quantum effect in $\langle g_{tt}^2 \rangle_x$ appears explicitly even at the leading order.

$$\langle g_{tt}^2 \rangle_x = -e^{-4k_0+\frac{4}{\Delta_k^2}}\left[\left(\frac{1}{4}c_0^2 z^4 + \frac{1}{L^2} + \frac{1}{8x\Delta_c^2}\right)^2 + \frac{c_0^2 z^4}{8\Delta_c^2} + \frac{z^4}{32\Delta_c^4}\right] + e^{-2k_0+\frac{1}{\Delta_k^2}}\left[\frac{\Delta_k^2 z^2}{8} - \frac{\epsilon_0 z^3}{8\Delta_c^2} - \epsilon_0 z\left(\frac{1}{4}c_0^2 z^4 + \frac{1}{L^2}\right) + \frac{\epsilon_0^2 z^2}{4}\right]$$

$$= \frac{e^{-2k_0}\Delta_k^2 z^2}{8} + \left\{-e^{-4k_0}\left(\frac{1}{4}c_0^2 z^4 + \frac{1}{L^2}\right)^2 + e^{-2k_0}\left((-\epsilon_0 z)\left(\frac{1}{4}c_0^2 z^4 + \frac{1}{L^2}\right) + \frac{\epsilon_0^2 z^2}{4}\right)\right\} + \mathcal{O}\left(\frac{1}{\Delta^0}\right)$$

- We can also check the semiclassical regime again.

In order for convincing the result in the semiclassical regime,

we could impose the variance is small compared to its value :

$$\text{var}\,(g_{tt})_x = \langle g_{tt}^2 \rangle_x - \langle g_{tt} \rangle_x^2 \ll \langle g_{tt} \rangle_x^2$$

Then, $$\text{var}\,(g_{tt})_x = \frac{z^2}{8}e^{-2k_0}\Delta_k^2 + \mathcal{O}\left(\frac{1}{\Delta^0}\right) = \langle g_{tt} \rangle_x^2 \frac{e^{2k_0} z^6}{8f(z)^2}\Delta_k^2 + \mathcal{O}\left(\frac{1}{\Delta^0}\right)$$

In neutral case $f(z) \to \left(L^{-2} - \epsilon_0 e^{k_0} z^3/2\right)$, [remind] $f(z) = \frac{1}{L^2} - \frac{1}{2}\epsilon_0 e^{k_0} z^3 + \frac{1}{4}c_0^2 z^4$

$$\frac{\text{var}\,(g_{tt})_x}{\langle g_{tt} \rangle_x^2} \to \frac{\Delta_k^2}{2\epsilon_0^2}\left[1 + \mathcal{O}\left(\frac{1}{\Delta^0}\right)\right]$$ (as $z \to \infty$) $\epsilon_0 \gg \Delta_k$ should be valid (even at the singularity)

[result] quantum expectation values

- Likewise, other expectation values can be calculated.

$$\frac{\mathrm{var}\left(\pi_{g_{tt}}\right)_x}{\langle \pi_{g_{tt}} \rangle_x^2} \ll 1 \text{ gives } \Delta_k \gg 1$$

$$\langle \pi_{g_{tt}} \rangle_x = \frac{2}{z} e^{k_0 + \frac{1}{4\Delta_k^2}}, \quad \langle \pi_{g_{tt}}^2 \rangle_x = \frac{4}{z^2} e^{2k_0 + \frac{1}{\Delta_k^2}}, \quad \mathrm{var}\left(\pi_{g_{tt}}\right)_x = \langle \pi_{g_{tt}} \rangle_x^2 \left[\frac{1}{2\Delta_k^2} + \mathcal{O}\left(\frac{1}{\Delta^4}\right) \right],$$

$$\langle \phi_t \rangle_x = \mu_0 + c_0 z e^{-k_0 + \frac{1}{4\Delta_k^2}}, \quad \langle \phi_t^2 \rangle_x = \frac{\Delta_c^2}{2} + \mu_0^2 + e^{-2k_0 + \frac{1}{\Delta_k^2}} \left[c_0^2 z^2 + \frac{z^2}{2\Delta_c^2} \right] + 2 c_0 \mu_0 z \, e^{-k_0 + \frac{1}{4\Delta_k^2}},$$

$$\mathrm{var}\left(\phi_t\right)_x = \frac{\Delta_c^2}{2} + \mathcal{O}\left(\frac{1}{\Delta^0}\right) = \langle \phi_t \rangle_x^2 \frac{\Delta_c^2}{2\left(\mu_0 + c_0 z e^{-k_0}\right)^2} + \mathcal{O}\left(\frac{1}{\Delta^0}\right), \quad \longrightarrow \quad \frac{\mathrm{var}\left(\phi_t\right)_x}{\langle \phi_t \rangle_x^2} \ll 1 \text{ gives } \mu_0 \gg \Delta_c$$

$$\langle \pi_{\phi_t} \rangle_x = c_0, \quad \langle \pi_{\phi_t}^2 \rangle_x = c_0^2 + \frac{1}{2\Delta_c^2}, \quad \mathrm{var}\left(\pi_{\phi_t}\right)_x = \frac{1}{2\Delta_c^2} = \langle \pi_{\phi_t} \rangle_x^2 \frac{1}{2 c_0^2 \Delta_c^2}. \quad \longrightarrow \quad \frac{\mathrm{var}\left(\pi\phi_t\right)_x}{\langle \pi_{\phi_t} \rangle_x^2} \ll 1 \text{ gives } \Delta_c \gg 1$$

- Also, we can check the uncertainty principles.

$$\mathrm{var}\,(g_{tt})_x \times \mathrm{var}\left(\pi_{g_{tt}}\right)_x = \left(\frac{z^2}{8} e^{-2k_0} \Delta_k^2\right)\left(\frac{2}{z^2 \Delta_k^2} e^{2k_0}\right) + \mathcal{O}\left(\frac{1}{\Delta^2}\right) = \frac{1}{4} + \mathcal{O}\left(\frac{1}{\Delta^2}\right),$$

$$\mathrm{var}\,(\phi_t)_x \times \mathrm{var}\left(\pi_{\phi_t}\right)_x = \left(\frac{\Delta_c^2}{2}\right)\left(\frac{1}{2\Delta_c^2}\right) = \frac{1}{4} + \mathcal{O}\left(\frac{1}{\Delta^2}\right).$$

[result] quantum expectation values

- [Caution] Since we followed the usual quantization rule, $\pi_{g_{xx}}$ is defined by $\pi_{g_{xx}} = -i\partial_{g_{xx}}$. However the expectation value are computed on a constant g_{xx} surface, so it should be considered carefully. Here, the WdW eq for Ψ can be used.

$$\partial_{g_{tt}}\left(g_{tt}\partial_{g_{tt}}\Psi - 2g_{xx}\partial_{g_{xx}}\Psi\right) + \partial^2_{\phi_t}\Psi + \frac{12}{L^2}g^2_{xx}\Psi = 0 \;\rightarrow\; \partial_{g_{xx}}\partial_{g_{tt}}\Psi = \frac{1}{2g_{xx}}\left[\partial_{g_{tt}}\left(g_{tt}\partial_{g_{tt}}\Psi\right) + \partial^2_{\phi_t}\Psi + \frac{12}{L^2}g^2_{xx}\Psi\right]$$

For example...

$$\langle\pi_{g_{xx}}\rangle_x = -\frac{i}{2}\int dg_{tt}d\phi_t\left(\Psi^*(-i\partial_{g_{xx}})\partial_{g_{tt}}\Psi - \Psi(+i\partial_{g_{xx}})\partial_{g_{tt}}\Psi^*\right)$$
$$= -\frac{1}{2}\int dg_{tt}d\phi_t\left(\Psi^*\partial_{g_{xx}}\partial_{g_{tt}}\Psi + \Psi\partial_{g_{xx}}\partial_{g_{tt}}\Psi^*\right)$$
$$= -\frac{1}{4g_{xx}}\int dg_{tt}d\phi_t\left(\Psi^*\left[\partial_{g_{tt}}\left(g_{tt}\partial_{g_{tt}}\Psi\right) + \partial^2_{\phi_t}\Psi + \frac{12}{L^2}g^2_{xx}\Psi\right] + \Psi\left[\partial_{g_{tt}}\left(g_{tt}\partial_{g_{tt}}\Psi^*\right) + \partial^2_{\phi_t}\Psi^* + \frac{12}{L^2}g^2_{xx}\Psi^*\right]\right)$$
$$= \int dg_{tt}d\phi_t\left(\frac{g_{tt}}{2g_{xx}}|\partial_{g_{tt}}\Psi|^2 + \frac{z^2}{4g_{xx}}|\partial_{\phi_t}\Psi|^2 - \frac{6}{L^2}g^2_{xx}|\Psi|^2\right) \qquad \text{(integration by parts)}$$

[result] quantum expectation values - Schwarzschild-AdS

- For simplicity, we calculate the neutral case (Schwarzschild-AdS).

$\frac{\mathrm{var}(\pi_{g_{xx}})_x}{\langle \pi_{g_{xx}} \rangle_x^2} \ll 1$ gives $\epsilon_0 \gg \Delta_k$

$$\langle \pi_{g_{xx}} \rangle_x = \frac{\epsilon_0 z^2}{2} - \frac{4}{L^2 z} e^{-k_0 + \frac{1}{4\Delta_k^2}}, \quad \mathrm{var}\left(\pi_{g_{xx}}\right)_x = \langle \pi_{g_{xx}} \rangle_x^2 \left[\frac{{\Delta_k}^2 e^{2k_0} z^6}{2\left(8L^{-2} - e^{k_0} z^3 \epsilon_0\right)^2} + \mathcal{O}\left(\frac{1}{\Delta^4}\right) \right],$$

- In the minisuperspace ansatz, the Ricci scalar can be written as

$$R = -\frac{t\pi_t^2 - 2x\pi_t\pi_x}{4x^2} - \frac{9}{L^2}$$

$$\begin{aligned}
\langle R \rangle_x &= -\frac{1}{4x^2}\langle t\pi_t^2 \rangle_x + \frac{1}{2x}\langle \pi_t \pi_x \rangle_x - \frac{9}{L^2} \\
&= -\frac{1}{4x^2}\left(-\frac{4x^2}{L^2} + 2\epsilon_0\sqrt{x}e^{k_0 + \frac{1}{4\Delta_k^2}}\right) + \frac{1}{2x}\left(-\frac{8x}{L^2} + \frac{\epsilon_0}{\sqrt{x}}e^{k_0 + \frac{1}{4\Delta_k^2}}\right) - \frac{9}{L^2} = -\frac{12}{L^2}
\end{aligned}$$

The AdS radius is unchanged !

The quantum fluctuation deforms the black hole mass .

$$\epsilon_0' = \epsilon_0 e^{\frac{3}{4\Delta_k^2}} \quad (= GM)$$

[result] quantum expectation values - Schwarzschild-AdS

- the Weyl curvature squared $W_{abcd}W^{abcd} = \frac{\pi_{g_{tt}}^2\left(-2g_{tt}\pi_{g_{tt}} + g_{xx}\pi_{g_{xx}}\right)^2}{3g_{xx}^4}$

$$\langle W^2 \rangle_x = 3z^6\epsilon_0^2 e^{2k_0 + \frac{1}{\Delta_k^2}} + \mathcal{O}\left(\frac{\Delta_k^2}{\epsilon_0^2}, \frac{1}{\Delta_k^2}\right)$$

→ The Gaussian wavepacket can probe the singularity ($z \to \infty$).

$$\langle W^2 \rangle_x = 3z^6\epsilon_0^2 e^{2k_0 + \frac{1}{\Delta_k}^{2}} + \mathcal{O}\left(\frac{\Delta_k^2}{\epsilon_0^2}, \frac{1}{\Delta_k^2}\right), \quad \mathrm{var}\left(W^2\right)_x = \frac{2}{\Delta_k^2} + \mathcal{O}\left(\frac{\Delta_k^2}{\epsilon_0^2}, \frac{1}{\Delta_k^2}\right)$$

→ The variance remains small in the semiclassical regime we adopted ($\epsilon_0 \gg \Delta_k \gg 1$).

→ The quantum uncertainty due to the wavepacket is not enough to 'resolve' the singularity, at least within this minisuperspace description.

Summary and Discussion

- The Wheeler-de Witt equation in the planar AdS black hole interior was investigated.
- As the Wheeler-de Witt states, the Gaussian wavepacket strongly peaked on the classical solution was adopted, and we calculated the expectation values explicitly.
  It turned out that the black hole mass is deformed due to quantum fluctuation and the AdS radius remains unchanged.
  Also, we checked the condition for the semiclassical regime in our calculation.
- In this minisuperspace description and semiclassical regime, the Wheeler-de Witt state can prove the singularity as the classical geometry.
- We would extend this method to the gravity-radiation system to explore dynamical quantum radiation in the future work.
  By using suitable coordinate for a 'clock', the wave functional Ψ could be considered the quantum radiation observed for a particular observer.

Thank you for listening!