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Effects of fluctuations on higher-dimensional black holes

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Based on [Han, H., Gwak, B. "Metric fluctuations in higher-dimensional black holes." J. High Energ. Phys. 2023, 102 (2023)]

Contents

Introduction

▶ The classical theory of gravity predicts that a black hole absorbs any matter and energy falling onto its surface, called an event horizon. However, quantum mechanical effects lead to a thermal energy transition from the black hole to the asymptotic regions.

▶ Quantum fluctuations near the event horizon affect the properties of black hole geometry and radiation. This influence has been studied using a classical approach.

[J.W. York Jr., Dynamical Origin of Black Hole Radiance, Phys. Rev. D 28 (1983) 2929]

[C. Barrabès, V.P. Frolov and R. Parentani, Metric fluctuation corrections to Hawking radiation, Phys. Rev. D 59 (1999) 124010]

▶ A model where the quantum fluctuations are approximated by small perturbations in mass of a spherical black hole was proposed. It is described by an ingoing Vaidya-type metric with an oscillating mass.

Introduction

▶ We investigated the influence of mass fluctuations on the higher-dimensional black hole geometry by generalizing the model to arbitrary $D \geq 4$ dimensions.

▶ We examined how small oscillations in mass affect the event horizon, thermodynamic variables, and propagating rays.

▶ It has been found that complex gravitational phenomena can be greatly simplified in the limit where the number D of dimensions becomes extremely large.

[R. Emparan, R. Suzuki and K. Tanabe, The large d limit of general relativity, Journal of High Energy Physics 2013 (2013) 1]

We demonstrate that the complete solution to the perturbed ray can be analytically obtained for a large D.

Model of fluctuating black hole in higher dimensions

$c = G = \hbar = k_B = 1$

A spherically symmetric and neutral black hole with a fluctuating mass in higher dimensions

▶ Higher-dimensional Vaidya metric

$$ds^2 = -\left(1 - \frac{m}{r^{D-3}}\right) dv^2 + 2dvdr + r^2 d\Omega_{D-2}^2$$

▶ Mass function

$$m = m(v, \theta) = M\left[1 + \sum_l (2l+1)\mu_l \, sin(\omega_l v) \, Y_l(\theta)\right] \vartheta(v) \qquad \theta = (\theta_1, \cdots, \theta_{D-3})$$

Mass parameter $M = \frac{16\pi}{(D-2)\Omega_{D-2}} M_B$ Amplitude parameter $\mu_l = \alpha_l \frac{M_P}{M_B}$

M_B : Black hole mass (ADM) M_P : Planck mass

$\vartheta(v)$: Heaviside step function → A black hole is formed by a gravitational collapse of a null shell at $v = 0$

Model of fluctuating black hole in higher dimensions

▶ Spherical harmonics for $D > 3$

$$Y_l(m_k;\theta,\phi) = e^{\pm i m_{D-3}\phi}\prod_{k=0}^{D-4}(\sin\theta_{D-k-3})^{m_{k+1}}C_{m_k-m_{k+1}}^{m_{k+1}+\frac{D-k-3}{2}}(\cos\theta_{D-k-3})^{m_{k+1}}$$

We consider spherical oscillations $l = m_k = 0$ for a simple model.

$$m(v) = M[1+\mu_0\sin(\omega v)]\vartheta(v) \qquad \text{where} \quad \mu_0 \ll 1$$

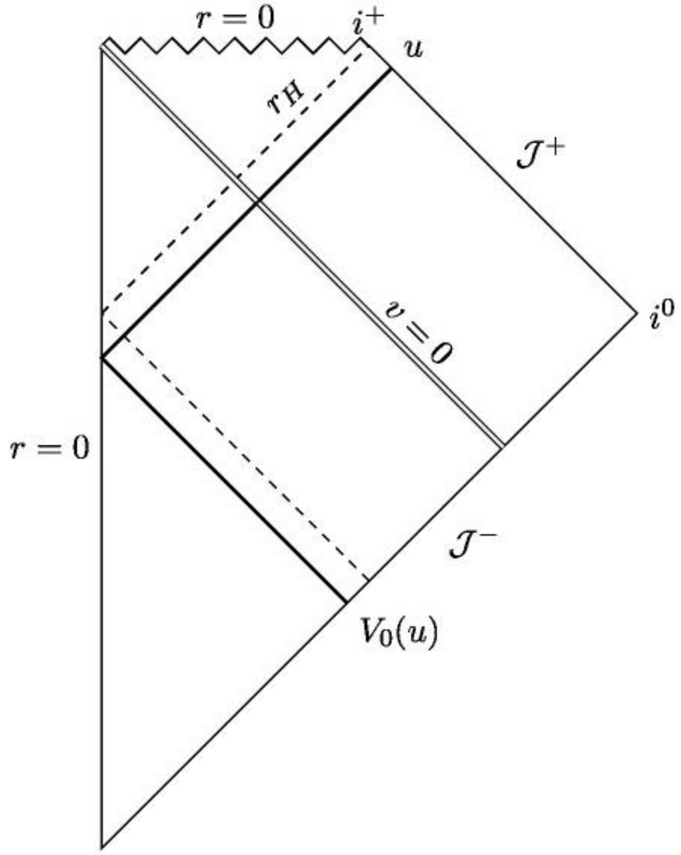
▶ Energy-momentum tensor

$$T_{\mu\nu} = \frac{(D-2)}{16\pi r^{D-2}}[M(1+\mu_0\sin(\omega v))\delta(v) + M\mu_0\omega\cos(\omega v)\,\vartheta(v)]l_\mu l_\nu$$

where $l_\mu = -\partial_\mu v$ is a null vector field tangent to the incoming null geodesic.

Model of fluctuating black hole in higher dimensions

▶ Conformal diagram (in the absence of fluctuations $\mu_0 = 0$)



Model of fluctuating black hole in higher dimensions

▶ To investigate the effect of fluctuations on the horizons and the rays propagating in the black hole geometry, we solve a null geodesic equation using a perturbation method.

$$2\frac{dr}{dv} = 1 - \frac{m(v)}{r^{D-3}}$$ for radially outgoing rays

$$r = r(v) = R(v) + \rho(v) + \sigma(v) + \cdots$$

We find

$$2\frac{dR}{dv} = 1 - \frac{M}{R^{D-3}}$$

$$2\frac{d\rho}{dv} - (D-3)\frac{M}{R^{D-2}}\rho = -\frac{M}{R^{D-3}}\mu$$

$$2\frac{d\sigma}{dv} - (D-3)\frac{M}{R^{D-2}}\sigma = (D-3)\frac{M}{R^{D-3}}\left[\frac{\rho\mu}{R} - \frac{D-2}{2}\frac{\rho^2}{R^2}\right]$$

Perturbed horizon and thermodynamic variables

▶ Event horizon $r_H = R_H + \rho_H + \sigma_H$

$$R_H = M^{\frac{1}{D-3}}$$

$$\kappa = \frac{D-3}{2R_H}$$: unperturbed surface gravity

$$\rho_H = \frac{\mu_0}{2\kappa} \frac{\Omega\, cos(\omega\, v) + sin(\omega\, v)}{1+\Omega^2}$$

$$\Omega \equiv \omega/\kappa$$

$$\sigma_H = \frac{\mu_0^2}{4\kappa}\left[\frac{2\Omega^2(2-\Omega^2)\,cos(2\,\omega v) + \Omega(1-5\Omega^2)\,sin(2\,\omega v)}{(1+\Omega^2)^2(1+4\Omega^2)} - \frac{D-4}{D-3}\frac{(1-5\Omega^2)\,sin^2(\omega\, v) + \Omega(2-\Omega^2)\,sin(2\,\omega v) + \Omega^2(5+2\Omega^2)}{(1+\Omega^2)^2(1+4\Omega^2)}\right]$$

- The position of an event horizon changes periodically due to fluctuations.
- The second-order perturbation contains an additional term in higher dimensions.

Perturbed horizon and thermodynamic variables

▶ Surface area

$$\overline{\mathcal{A}} = \Omega_{D-2}\,\overline{r_H^{D-2}(v)} = \Omega_{D-2}\,R_H^{D-2}\left[1+\frac{\mu_0^2(D-2)}{4(D-3)^2(1+\Omega^2)}\right]$$

▶ Surface gravity

$$\overline{\kappa} = \frac{D-3}{2}M\,\overline{\left(\frac{1+\mu_0\,sin(\omega v)}{r_H^{D-2}(v)}\right)} = \kappa\left[1+\frac{\mu_0^2(D-2)}{4(D-3)^2(1+\Omega^2)}\right]$$

▶ Hawking temperature

$$\overline{T}_H = \frac{\overline{\kappa}}{2\pi}$$

$$\frac{\overline{\mathcal{A}}-\mathcal{A}}{\mathcal{A}} = \frac{\overline{T_H}-T_H}{T_H}$$

Perturbed horizon and thermodynamic variables

▶ Entropy

$$\bar{S} = \int \frac{2\pi}{\kappa} dE \left(1 + \frac{\mu_0^2 (D-2)}{4(D-3)^2(1+\Omega^2)}\right)^{-1} \simeq \frac{\bar{\mathcal{A}}}{4}\left[1 - \frac{\mu_0^2(D-2)}{2(D-3)^2(1+\Omega^2)}\right]$$

- The usual relation between the entropy and the surface area is modified.
- The correction terms in these variables decrease as the number of dimensions increases.

Propagating rays in fluctuating black hole geometry

- General solution $R \neq R_H$

$$x = \frac{R - R_H}{R_H}, \qquad \tilde{u} = \kappa u, \qquad \tilde{f} = \frac{2}{D-3}\kappa f$$

First-order perturbation

$$\tilde{\rho}(x) = \left[1 - \frac{1}{(1+x)^{D-3}}\right]\int_x^\infty \frac{(1+\xi)^{D-3}}{\{(1+\xi)^{D-3}-1\}^2}\mu(\xi)\, d\xi$$

Second-order perturbation

$$\tilde{\sigma}(x) = -\left[1 - \frac{1}{(1+x)^{D-3}}\right]\int_x^\infty \frac{(1+\xi)^{D-3}}{\{(1+\xi)^{D-3}-1\}^2}(D-3)\left\{\frac{\tilde{\rho}(\xi)\mu(\xi)}{(1+\xi)} - \frac{D-2}{2}\frac{\tilde{\rho}(\xi)^2}{(1+\xi)^2}\right\} d\xi$$

where

$$\mu(\xi) = \mu_0\, sin[\Omega(\tilde{u} + 2\kappa R_*(\xi))] = \mu_0\, sin\left[\Omega\left(\tilde{u} + (D-3)\xi + \sum_{j=0}^{D-4} e^{i\frac{2\pi}{D-3}j}\, ln\left(\frac{1+\xi}{e^{i\frac{2\pi}{D-3}j}} - 1\right)\right)\right]$$

Fluctuations in large D dimensions

▶ We introduce a near-horizon coordinate

$$\hat{\mathrm{R}} \equiv \left(\frac{R}{R_H}\right)^{D-3} \qquad \text{where} \quad ln\,\hat{\mathrm{R}} \ll D-3$$

▶ Unperturbed tortoise coordinate $R_* = \dfrac{R_H}{D-3} ln(\hat{\mathrm{R}}-1)$

▶ Fluctuating part of the mass function $\mu(\hat{\mathrm{R}}) = \mu_0 \mathrm{Im}\left[e^{i\Omega\tilde{u}}(\hat{\mathrm{R}}-1)^{i\Omega}\right]$

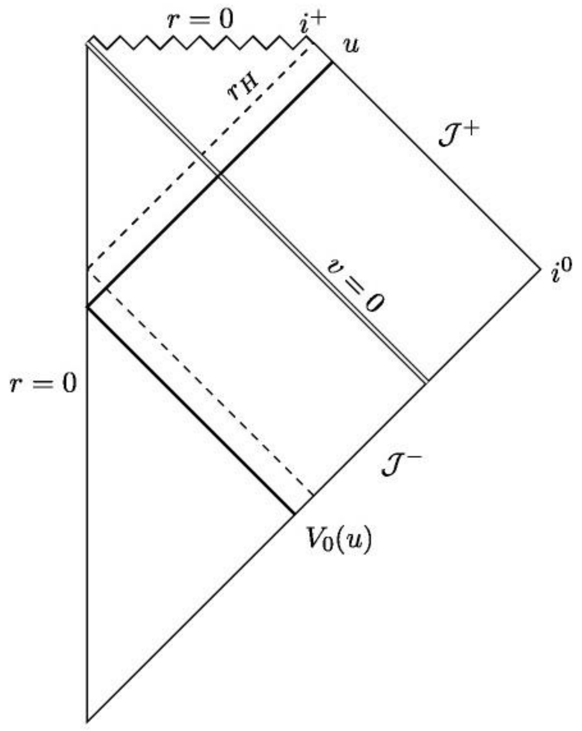
$$\tilde{\rho}(\hat{\mathrm{R}}) = \left(1-\frac{1}{\hat{\mathrm{R}}}\right) I(\hat{\mathrm{R}}) \qquad \tilde{\sigma}(\hat{\mathrm{R}}) = -\frac{1}{2}\frac{(\hat{\mathrm{R}}-1)^2}{\hat{\mathrm{R}}^2} I^2(\hat{\mathrm{R}})$$

where $I(\hat{\mathrm{R}}) = \mu_0 \mathrm{Im}\left[\dfrac{1+i\Omega}{1+\Omega^2} e^{i\Omega\tilde{u}}(\hat{\mathrm{R}}-1)^{i\Omega-1}\right]$

Fluctuations in large D dimensions

$V = -2[R_0 + \rho_0 + \sigma_0]$ from $v < 0$ region

$u = -2R_{*0}$ from $v > 0$ region



$$-\tilde{V}(\tilde{u}) = -\tilde{V}_0 + \mu_0 \frac{\Omega}{1+\Omega^2}\frac{1}{1+e^{-\tilde{u}}} - \frac{\mu_0^2}{2}\frac{\Omega^2}{(1+\Omega^2)^2}\frac{1}{(1+e^{-\tilde{u}})^2}$$

where $-\tilde{V}_0 = (D-3) + ln\left(1+e^{-\tilde{u}}\right)$

is the value in the absence of fluctuations.

- It illustrates the influence of fluctuations on a large D-dimensional black hole spacetime.

Summary

- ▶ We investigated how fluctuations affect higher-dimensional black hole geometry.
- ▶ The solutions describing the locations of the event horizon exhibit small and periodic changes depending on the amplitude and frequency of fluctuations.
- ▶ The correction terms in the time-averaged values of thermodynamic variables tend to decrease as the number of dimensions increases.
- ▶ We employed the large D limit and obtained a complete solution describing the perturbed ray propagating in the near-horizon zone.
- ▶ As a future work, non-spherical oscillations can be considered. Our model can be generalized to describe a rotating black hole.

Thank you for listening