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23rd Afternoon II

Thermodynamic relation on corrected de Sitter Black holes

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Based on
Ko, J., Gwak, B. "Universality on thermodynamic relation with corrections in de Sitter black holes."
JHEP 2024, 072 (2024).

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Introduction

- Weak gravity conjecture (WGC) = explanatory concept for charge-to-mass ratio

  [C. Vafa, *The string landscape and the swampland, hep − th/0509212*]

  [N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, *The string landscape, black holes and gravity as the weakest force, JHEP* **06** (2007)060]

- Goon and Penco investigated thermodynamic relation to prove WGC

  This relation is based on 4D charged anti- de Sitter black holes in extremal limit

  [G. Goon and R. Penco, *Universal Relation between Corrections to Entropy and Extremality, Phys. Rev. Lett.* **124** (2020) 101103]

  [S. W. Wei, K. Yang and Y. X. Liu, *Universal thermodynamic relations with constant corrections for rotating AdS black holes, Nucl. Phys. B* **962** (2021) 115279]

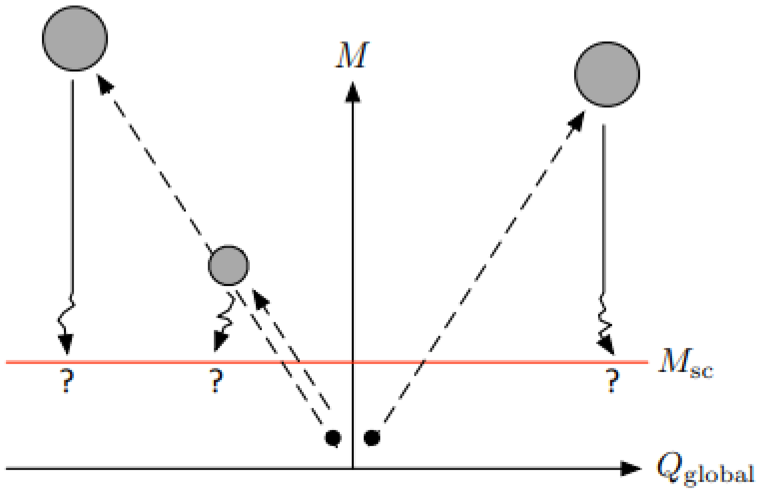
  [J. Sadeghi et al., *The emergence of universal relations in the AdS black holes thermodynamics, Phys. Scripta* **98** (2023) 025305]

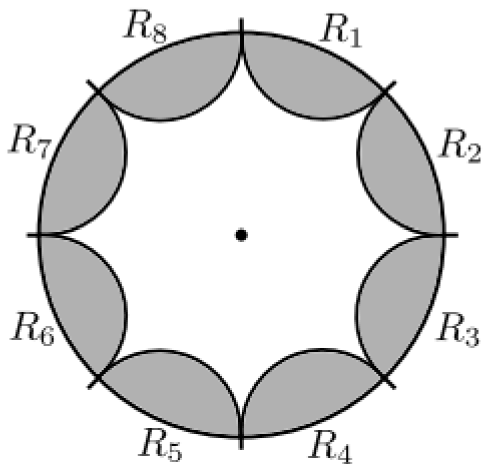
- applying this relation to various 4D black holes in de Sitter spacetime

  to find universality of relation

WGC?

- WGC in charged extremal Black Holes(BHs)
  simply "any gauge force must be stronger than gravity" or $M/Q < 1$
  (+ absence of global symmetries in quantum gravity)
  → BHs releasing particles regardless of global charges





[D. Harlow, B. Heidenreich, M. Reece and T. Rudelius, *Weak gravity conjecture*, *Rev. Mod. Phys.* 95 (2023)035003]

Goon and Penco relation

▶ Goon and Penco's relation(GP relation)
: one of the ways proving WGC's validity

perturbations in free energy (G) leading to perturbations in action → $\Delta G \propto \Delta I$

+ making relation btw <u>mass</u>, <u>temperature</u> and <u>entropy</u>(ϵ : perturbation parameter)

$$-T\left(\frac{\partial S}{\partial \epsilon}\right)_{M,\vec{Q}} = \Delta G\left[T\left(M,\vec{Q},\epsilon\right),\vec{\mu}(M,\vec{Q},\epsilon)\right]$$

$$\lim_{T\to 0}\left(\frac{\partial M}{\partial \epsilon}\right)_{T,\vec{Q}} = \lim_{T\to 0}\Delta G\left[T,\vec{\mu}(T,\vec{Q},\epsilon)\right]$$

▶ relation made with temperature and derivative of mass and entropy

$$\frac{\partial M_{ext}(\vec{Q},\epsilon)}{\partial \epsilon} = \lim_{M\to M_{ext}(\vec{Q},\epsilon)} -T\left(\frac{\partial S(M,\vec{Q},\epsilon)}{\partial \epsilon}\right)_{M,\vec{Q}}$$

→ leading to higher derivative form

$$\Delta M_{ext}\left(\vec{Q}\right) \approx -T_0(M,\vec{Q})\Delta S(M,\vec{Q})\Big|_{M\approx M^0_{ext}(\vec{Q})}$$

Application to various 4D black holes

▶ example in $Phys. Rev. Lett.$ **124** (2020)

$$\left(\frac{\partial M_E}{\partial \epsilon}\right)_{T,Q} = \lim_{M \to M_E} \left(-T \frac{\partial S}{\partial \epsilon}\right)_{M,Q} = \frac{l\left(-1 + \sqrt{1 + \frac{12(1+\epsilon)Q^2}{l^2}}\right)^{\frac{3}{2}}}{12\sqrt{6}(1+\epsilon)^{\frac{3}{2}}}$$

: originally based on 4D extremal Reissner-Nordström black holes

Application to various 4D black holes

▶ GP relation mainly based on 4D Reissner-Nordström anti-de Sitter BHs
→ our work focus on other situation, 4D de Sitter spacetime
expecting relation also satisfied in de Sitter spacetime

\+ check about two types of limit
: extremal($r_i = r_o$) and Nariai($r_o = r_c$) limit

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Schwarzschild-de Sitter black holes

$c = G = \hbar = k_B = 1$

- Action $I = \frac{1}{16\pi}\int dx^4 \sqrt{-g}(R - 2(1+\epsilon)\Lambda)\ \left(\Lambda = \frac{3}{l^2}\right)$
- Metric $ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$

$$f(r) = 1 - \frac{2M}{r} - \frac{(1+\epsilon)r^2}{l^2}$$

- Thermodynamic variables

$$S = \pi r_h^2 \rightarrow r_h = \frac{\sqrt{S}}{\sqrt{\pi}}$$

$$T = \frac{1}{4\pi}\left(\frac{\partial f}{\partial r}\right)_{r=r_h} = \frac{2M}{r^2} - \frac{2(1+\epsilon)r_h}{l^2} = \frac{1}{4\pi}\frac{\pi l^2 - 3(1+\epsilon)S}{\sqrt{\pi S}}$$

$$M\Big|_{r=r_h} = \frac{r_h}{2} - \frac{(1+\epsilon){r_h}^3}{2l^2} = \frac{\sqrt{S}}{2\sqrt{\pi}} - \frac{1+\epsilon}{2l^2}\left(\frac{S}{\pi}\right)^{\frac{3}{2}}$$

Schwarzschild-de Sitter black holes

- Extremality condition(set $T = 0$) $\quad S_N = \dfrac{\pi l^2}{3(1+\epsilon)}$

- Building relation $\quad$ Get function of epsilon from M + take partial derivative to function with S

$$\epsilon = \frac{-2l^2\pi^{\frac{3}{2}}M + \pi l^2 S^{\frac{1}{2}}}{S^{\frac{3}{2}}} \rightarrow \left(\frac{\partial \epsilon}{\partial S}\right)_M = \frac{\pi l^2 - 3(1+\epsilon)S}{2S^2}$$

$$-T\frac{\partial S}{\partial \epsilon} = -\frac{S^{\frac{3}{2}}}{2\pi^{\frac{3}{2}}l^2} \rightarrow \lim_{M\rightarrow M_N}\left(-T\frac{\partial S}{\partial \epsilon}\right)_M = -\frac{l}{6\sqrt{3}(1+\epsilon)^{\frac{3}{2}}}$$

$$M_N = \frac{l}{3\sqrt{3(1+\epsilon)}} \rightarrow \left(\frac{\partial M_N}{\partial \epsilon}\right)_T = -\frac{l}{6\sqrt{3}(1+\epsilon)^{\frac{3}{2}}}$$

$\therefore \left(\frac{\partial M_N}{\partial \epsilon}\right)_T = \lim_{M\rightarrow M_N} -T\left(\frac{\partial S}{\partial \epsilon}\right)_M$ confirmed

Reissner-Nordström-de Sitter black holes

- Action $I = \frac{1}{16\pi}\int dx^4\sqrt{-g}\left(R - 2(1+\epsilon)\Lambda - F_{\mu\nu}F^{\mu\nu}\right)$
- Metric $ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{(1+\epsilon)r^2}{l^2}$$

- Thermodynamic varialbes

$$S = \pi r_h^2 \rightarrow r_h = \frac{\sqrt{S}}{\sqrt{\pi}}$$

$$T = \frac{1}{4\pi}\left(\frac{\partial f}{\partial r}\right)_{r=r_h} = \frac{1}{4\pi}\frac{\pi l^2 S - 3(1+\epsilon)S^2 - (Q\pi l)^2}{l^2\sqrt{\pi}S^{\frac{3}{2}}}$$

$$M\Big|_{r=r_h} = \frac{r_h}{2} + \frac{Q^2}{2r^2} - \frac{(1+\epsilon){r_h}^3}{2l^2} = \frac{\sqrt{S}}{2\sqrt{\pi}} + \frac{Q^2\sqrt{\pi}}{2\sqrt{S}} - \frac{1+\epsilon}{2l^2}\left(\frac{S}{\pi}\right)^{\frac{3}{2}}$$

Reissner-Nordström-de Sitter black holes

- Extremality condition(set $T = 0$)

$$S_N = \frac{\pi l^2}{6(1+\epsilon)}\left(1+\sqrt{1-\frac{12(1+\epsilon)Q^2}{l^2}}\right)$$

- Building relation

$$\epsilon = \frac{-2\pi^{\frac{3}{2}} l^2 S^{\frac{1}{2}} M + \pi l^2 S + \pi^2 l^2 Q^2}{S^2} - 1 \;\rightarrow\; \left(\frac{\partial \epsilon}{\partial S}\right)_{M,Q} = \frac{\pi l^2 S - 3(1+\epsilon)S^2 - Q^2\pi^2 l^2}{l^2\sqrt{\pi} S^{\frac{3}{2}}}$$

$$-T\frac{\partial S}{\partial \epsilon} = -\frac{S^{\frac{3}{2}}}{2\pi^{\frac{3}{2}} l^2} \;\rightarrow\; \lim_{M\to M_N}\left(-T\frac{\partial S}{\partial \epsilon}\right)_M = -\frac{l}{12\sqrt{6}(1+\epsilon)^{\frac{3}{2}}}\left(1+\sqrt{1-\frac{12(1+\epsilon)Q^2}{l^2}}\right)^{\frac{3}{2}}$$

Reissner-Nordström-de Sitter black holes

$$M_N = \frac{12(1+\epsilon)Q^2 + l^2\left(1+\sqrt{1-\frac{12(1+\epsilon)Q^2}{l^2}}\right)}{3\sqrt{6(1+\epsilon)}l\sqrt{1+\sqrt{1-\frac{12(1+\epsilon)Q^2}{l^2}}}}$$

-> $\left(\frac{\partial M_N}{\partial \epsilon}\right)_{T,Q} = -\frac{l}{12\sqrt{6}(1+\epsilon)^{\frac{3}{2}}}\left(1+\sqrt{1-\frac{12(1+\epsilon)Q^2}{l^2}}\right)^{\frac{3}{2}}$

$\therefore \left(\frac{\partial M_N}{\partial \epsilon}\right)_{T,Q} = \lim_{M\to M_N} -T\left(\frac{\partial S}{\partial \epsilon}\right)_{M,Q}$ also confirmed in RNdS with Nariai limit

Reissner-Nordström-de Sitter black holes

▶ this time, we examine in extremal limit

$$S_E = \frac{\pi l^2}{6(1+\epsilon)}\left(1 - \sqrt{1 - \frac{12(1+\epsilon)Q^2}{l^2}}\right)$$

$$-T\frac{\partial S}{\partial \epsilon} = -\frac{S^{\frac{3}{2}}}{2\pi^{\frac{3}{2}} l^2} \rightarrow \lim_{M \to M_E}\left(-T\frac{\partial S}{\partial \epsilon}\right)_M = -\frac{l}{12\sqrt{6}(1+\epsilon)^{\frac{3}{2}}}\left(1 - \sqrt{1 - \frac{12(1+\epsilon)Q^2}{l^2}}\right)^{\frac{3}{2}}$$

$$\left(\frac{\partial M_E}{\partial \epsilon}\right)_{T,Q} = -\frac{l}{12\sqrt{6}(1+\epsilon)^{\frac{3}{2}}}\left(1 - \sqrt{1 - \frac{12(1+\epsilon)Q^2}{l^2}}\right)^{\frac{3}{2}}$$

extremal limit also confirmed

Kerr(-Newman)-de Sitter black holes

▶ Smarr formula

$$M = \left(\frac{A}{16\pi} + \frac{4\pi L^2}{A} + \frac{Q^2}{2} + \frac{\pi Q^4}{A}\right)^{\frac{1}{2}}$$ from Kerr-Newman black hole

[L. Smarr, *Mass formula for Kerr black holes, Phys. Rev. Lett.* **30** (1973) 71]

A : surface area, L : angular momentum, Q : electric charge

$$\rightarrow M^2 = \left(\frac{\pi}{S} - \frac{1}{l^2}\right)J^2 + \frac{S}{4\pi}\left(\frac{\pi Q^2}{S} + 1 - \frac{S}{\pi l^2}\right)^2$$

[M. M. Cardarelli, G. Cognola and D. Klemm, *Thermodynamics of Kerr − Newman − (anti −)de Sitter black hole with charged scalar field, Class. Quant. Grav.* **17** (2000) 399]

[Y.Sekiwa, *Thermodynamics of de Sitter black holes: Thermal cosmological constant, Phys. Rev.* D **73** (2006) 084009]

Kerr(-Newman)-de Sitter black holes

- Action $I = \frac{1}{16\pi}\int dx^4\sqrt{-g}(R - 2(1+\epsilon)\Lambda)$
- Metric $ds^2 = -\frac{\Delta_r}{\rho^2}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right)^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta\sin^2\theta}{\rho^2}\left(adt - \frac{r^2+a^2}{\Xi}d\phi\right)^2$

$$\Delta_r = (r^2+a^2)\left(1 - \frac{(1+\epsilon)}{3}\Lambda r^2\right) - 2mr \qquad \Delta_\theta = 1 + \frac{1}{3}\Lambda a^2\cos^2\theta$$

$$\rho^2 = r^2 + a^2\cos^2\theta \qquad \Xi = 1 + \frac{(1+\epsilon)}{3}\Lambda a^2$$

- Thermodynamic variables

can be obtained from Smarr formula

$$M = \sqrt{\frac{\pi l^2 - (1+\epsilon)S}{4\pi^3 l^4 S}\left(4\pi^3 l^2 J^2 + S^2(\pi l^2 - (1+\epsilon)S)\right)}$$

$$T = \left(\frac{\partial M}{\partial S}\right)_J = \frac{-4\pi^4 J^2 l^4 + S^2(\pi l^2 - (1+\epsilon)S)(\pi l^2 - 3(1+\epsilon)S)}{4\pi^{\frac{3}{2}} l^2\sqrt{S^3(\pi l^2 - (1+\epsilon)S)\left(4\pi^3 J^2 l^2 + S^2(\pi l^2 - (1+\epsilon)S)\right)}}$$

▶ Extremality condition

$$S_N = \frac{\pi l^2}{3(1+\epsilon)} + \frac{1}{2}\sqrt{\frac{2\pi^2 l^4}{9(1+\epsilon)^2} + P} - \frac{1}{2}\sqrt{\frac{4\pi^2 l^4}{9(1+\epsilon)^2} - P + \frac{4\pi^3 l^6}{27(1+\epsilon)^3\sqrt{\frac{2\pi^2 l^4}{9(1+\epsilon)^2} + P}}}$$

$$P = \frac{\pi^2\left(l^8 - 144J^2 l^4 (1+\epsilon)^2\right)}{9(1+\epsilon)Q^{\frac{1}{3}}} + \frac{\pi^2 Q^{\frac{1}{3}}}{9(1+\epsilon)^2}$$

$$Q = l^{12} - 432J^2 l^8 (1+\epsilon)^2 + 12\sqrt{3}\sqrt{-J^2 l^{20}(1+\epsilon)^2 + 288J^4 l^{16}(1+\epsilon)^4 + 6912J^6 l^{12}(1+\epsilon)^6}$$

Kerr(-Newman)-de Sitter black holes

▶ Building relation

$$\epsilon = \frac{\pi l^2\left(2\pi^2 J^2 + S^2 - 2\sqrt{\pi^4 J^4 + \pi M^2 S^3}\right)}{S^3} - 1 \;\rightarrow$$

$$\left(\frac{\partial \epsilon}{\partial S}\right)_{M,J} = \frac{-4\pi^4 J^2 l^2 + S^2(\pi l^2 - (1+\epsilon)S)(\pi l^2 - 3(1+\epsilon)S)}{4\pi^3 J^2 l^2 S^2 + 2S^2(\pi l^2 - (1+\epsilon)S)}$$

$$-T\left(\frac{\partial S}{\partial \epsilon}\right)_{M,J} = -\frac{4\pi^3 J^2 l^2 S^2 + 2S^2(\pi l^2 - (1+\epsilon)S)}{4\pi^{\frac{3}{2}} l^2 \sqrt{S^3(\pi l^2 - (1+\epsilon)S)\left(4\pi^3 J^2 l^2 + S^2(\pi l^2 - (1+\epsilon)S)\right)}} \rightarrow$$

$$\lim_{M\rightarrow M_N}\left(-T\frac{\partial S}{\partial \epsilon}\right)_{M,J} = -\frac{4\pi^3 J^2 l^2 S_N^2 + 2S_N^2(\pi l^2 - (1+\epsilon)S_N)}{4\pi^{\frac{3}{2}} l^2 \sqrt{S_N^3(\pi l^2 - (1+\epsilon)S_N)\left(4\pi^3 J^2 l^2 + S_N^2(\pi l^2 - (1+\epsilon)S_N)\right)}}$$

Kerr(-Newman)-de Sitter black holes

$$M_N = \sqrt{\frac{\pi l^2-(1+\epsilon)S_N}{4\pi^3 l^4 S_N}\left(4\pi^3 l^2 J^2 + S_N^2(\pi l^2 - (1+\epsilon)S_N)\right)} \rightarrow$$

$$\left(\frac{\partial M_N}{\partial \epsilon}\right)_{T,J} = -\frac{4\pi^3 J^2 l^2 S_N^2 + 2S_N^2(\pi l^2-(1+\epsilon)S_N)}{4\pi^{\frac{3}{2}} l^2 \sqrt{S_N^3(\pi l^2-(1+\epsilon)S_N)\left(4\pi^3 J^2 l^2 + S_N^2(\pi l^2-(1+\epsilon)S_N)\right)}}$$

$\therefore \left(\frac{\partial M_N}{\partial \epsilon}\right)_{T,J} = \lim_{M\rightarrow M_N} -T\left(\frac{\partial S}{\partial \epsilon}\right)_{M,J}$ confirmed

Kerr(-Newman)-de Sitter black holes

- Action $I = \frac{1}{16\pi}\int dx^4\sqrt{-g}\left(R - 2(1+\epsilon)\Lambda - F_{\mu\nu}F^{\mu\nu}\right)$
- Metric $ds^2 = -\frac{\Delta_r}{\rho^2}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right)^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta \sin^2\theta}{\rho^2}\left(adt - \frac{r^2+a^2}{\Xi}d\phi\right)^2$

$$\Delta_r = (r^2+a^2)\left(1-\frac{1}{3}\Lambda r^2\right) - 2Mr + Q^2, \quad \Delta_\theta = 1 + \frac{1}{3}\Lambda a^2\cos^2\theta$$

$$\rho^2 = r^2 + a^2\cos^2\theta \qquad \Xi = 1 + \frac{1}{3}\Lambda a^2$$

- Thermodynamic varialbes

  can be obtained from Smarr formula

$$M = \sqrt{\frac{1}{4\pi^3 l^4 S}\left(4\pi^3 l^2 J^2(\pi l^2 - (1+\epsilon)S) + \left(\pi^2 Q^2 l^2 + (\pi l^2 S - (1+\epsilon)S^2)\right)^2\right)}$$

$$T = \left(\frac{\partial M}{\partial S}\right)_J = \frac{-4\pi^4 J^2 l^4 - \pi^2 Q^2 l^2(\pi^2 Q^2 l^2 + 2S^2) + S^2(\pi l^2 - (1+\epsilon)S)(\pi l^2 - 3(1+\epsilon)S)}{4\pi^{\frac{3}{2}} l^2 \sqrt{S^3(4\pi^3 J^2 l^2(\pi l^2 - (1+\epsilon)S) + (\pi l^2(\pi Q^2 + S) - (1+\epsilon)S^2)^2)}}$$

Kerr(-Newman)-de Sitter black holes

- Extremality condition

$$S_N = \frac{\pi l^2}{3(1+\epsilon)} + \frac{1}{2}\sqrt{\frac{2\pi^2 l^4}{9(1+\epsilon)^2} + \frac{4\pi^2 Q^2 l^2}{9(1+\epsilon)^2} + X}$$

$$-\frac{1}{2}\sqrt{\frac{4\pi^2 l^4}{9(1+\epsilon)^2} - \frac{8\pi^2 Q^2 l^2}{9(1+\epsilon)^2} - X + \frac{\frac{16\pi^3 l^6}{27(1+\epsilon)^3} + \frac{4\pi^3 Q^2 l^4}{9(1+\epsilon)^3}}{27(1+\epsilon)^3\sqrt{\frac{2\pi^2 l^4}{9(1+\epsilon)^2} + \frac{4\pi^2 Q^2 l^2}{9(1+\epsilon)^2} + X}}}$$

$$X = \frac{(\pi^2 l^4 - 2\pi^2 Q^2 l^2)^2 - 36(1+\epsilon)^2(4\pi^4 J^2 l^4 + \pi^4 Q^4 l^4)}{9(1+\epsilon)^2 Y} + \frac{Y}{9(1+\epsilon)^2}$$

$$Y = \pi^2 l^6 \left(Z^3 - 108(7l^2 + 2Q^2)A + \sqrt{(-(Z^2 - 36A)^3 + (Z^3 - 216 l^2 A + 108 Z A)^2)}\right)^{\frac{1}{3}}$$

$$Z = l^2 - 2Q^2 \qquad A = (4J^2 + Q^4)(1+\epsilon)^2$$

Kerr(-Newman)-de Sitter black holes

▶ Building relation

$$\epsilon = \frac{\pi l^2 \left(2\pi^2 J^2 + \pi Q^2 S + S^2 - 2\sqrt{\pi^4 J^4 + \pi^3 Q^2 J^2 S + \pi M^2 S^3}\right)}{S^3} - 1 \rightarrow$$

$$\left(\frac{\partial \epsilon}{\partial S}\right)_{M,J} = \frac{-4\pi^4 J^2 l^2 - \pi^2 Q^2 J^2(\pi^2 Q^2 J^2 + 2S^2) + S^2(\pi l^2 - (1+\epsilon)S)(\pi l^2 - 3(1+\epsilon)S)}{4\pi^3 J^2 l^2 S^2 + 2\pi l^2 S^3(\pi Q^2 + S) - 2(1+\epsilon)S^5}$$

$$-T\left(\frac{\partial S}{\partial \epsilon}\right)_{M,J} = -\frac{4\pi^3 J^2 l^2 S^2 + 2\pi l^2 S^3(\pi Q^2 + S) - 2(1+\epsilon)S^5}{4\pi^{\frac{3}{2}} l^2 \sqrt{S^3(4\pi^3 J^2 l^2(\pi l^2 - (1+\epsilon)S) + (\pi l^2(\pi Q^2 + S) - (1+\epsilon)S^2)^2)}} \rightarrow$$

$$\lim_{M \to M_N}\left(-T\frac{\partial S}{\partial \epsilon}\right)_{M,J} = -\frac{4\pi^3 J^2 l^2 S_N^2 + 2\pi l^2 S_N^3(\pi Q^2 + S_N) - 2(1+\epsilon)S_N^5}{4\pi^{\frac{3}{2}} l^2 \sqrt{S_N^3(4\pi^3 J^2 l^2(\pi l^2 - (1+\epsilon)S_N) + (\pi l^2(\pi Q^2 + S_N) - (1+\epsilon)S_N^2)^2)}}$$

Kerr(-Newman)-de Sitter black holes

$$M_N = \sqrt{\frac{1}{4\pi^3 l^4 S_N}\left(4\pi^3 l^2 J^2(\pi l^2 - (1+\epsilon)S_N) + \left(\pi^2 Q^2 l^2 + (\pi l^2 S_N - (1+\epsilon)S_N^2)\right)^2\right)} \rightarrow$$

$$\left(\frac{\partial M_N}{\partial \epsilon}\right)_{T,J} = -\frac{4\pi^3 J^2 l^2 S_N^2 + 2\pi l^2 S_N^3(\pi Q^2 + S_N) - 2(1+\epsilon)S_N^5}{4\pi^{\frac{3}{2}} l^2 \sqrt{S_N^3(4\pi^3 J^2 l^2(\pi l^2 - (1+\epsilon)S_N) + (\pi l^2(\pi Q^2 + S_N) - (1+\epsilon)S_N^2)^2)}}$$

$\therefore \left(\frac{\partial M_N}{\partial \epsilon}\right)_{T,J} = \lim_{M \to M_N} -T\left(\frac{\partial S}{\partial \epsilon}\right)_{M,J}$ confirmed

Summary

- We investigated that the relation have universality in various black holes.
- The relation is not only valid to RNAdS with extremal limit also to SdS, RNdS, K(N)dS black holes with Nariai limit or extremal limit.
- The negative sign in result is closely connected to property of de Sitter spacetime.
- We can extend our analysis to version of higher-derivative corrections
- Proportional relationship between shifted mass and entropy can lead us to understand the WGC.
- Studies presented in this paper providing deeper comprehension for physics.

Thank you for listening