Holographic RG flows on Magnetic Brane geometry

Fast and ongoing works

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Photo from quantamagazine.org

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Introduction - Part.1

- Renormalization Group (*RG*) transformation

  - Coarse graining method

  - Result in an RG flow : $\mathcal{R}_t : \mathcal{T} \to \tilde{\mathcal{T}}$

  - RG equation : $\frac{d}{dt} \equiv -\beta^i(\lambda) \frac{\partial}{\partial \lambda^i}$

  - When acting the RG equation to a accessible quantity $\mathcal{C}(\mathcal{T}_*)$, we have the following three conjectures

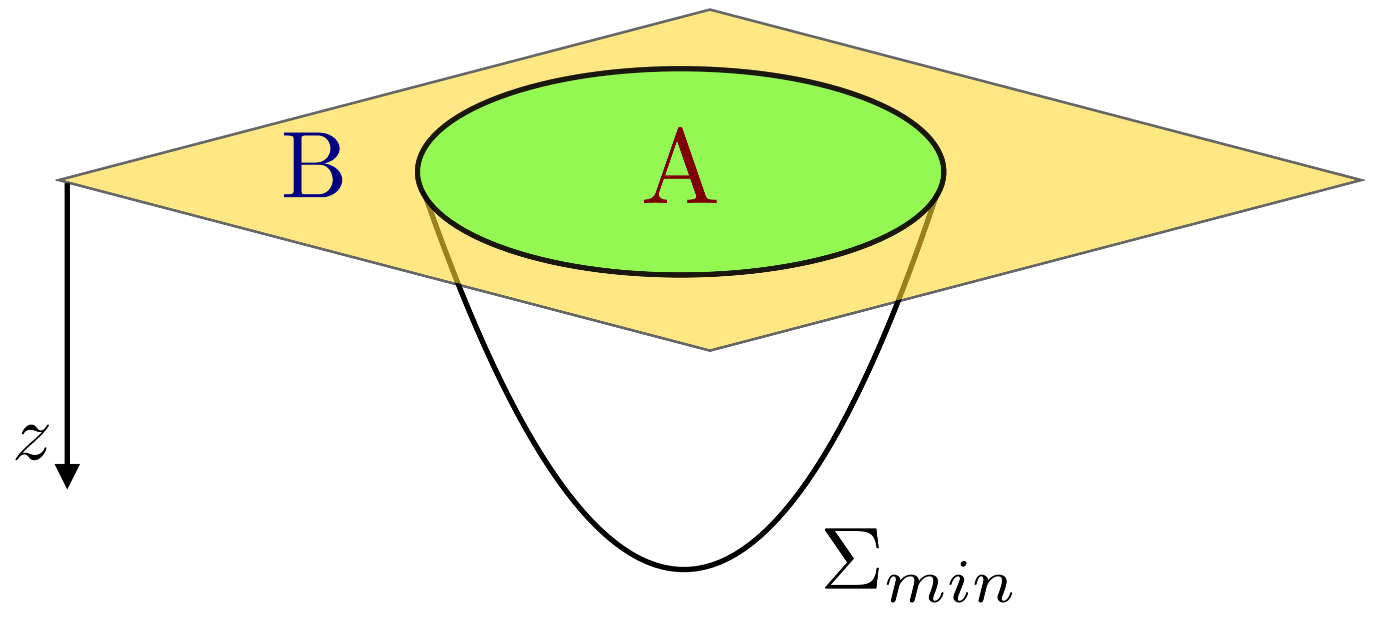
The weak conjecture	$\mathcal{C}_{\mathrm{UV}} \geq \mathcal{C}_{\mathrm{IR}}$
The strong conjecture	$\frac{d\mathcal{C}}{dt} \leq 0$
The strongest conjecture	$\beta_i(\lambda) = G_{ij}(\lambda) \frac{\partial \mathcal{C}}{\partial \lambda_{ij}}$

- 2D : c-theorem
- 3D : F-theorem
- 4D : a-theorem

Introduction - Part.1

- Entanglement entropy (*EE*) and holography

Von-Neumann entropy	RT entropy
$S = -\mathrm{Tr}_A \rho_A \log \rho_A$	$S_{BH} = \dfrac{A(\Sigma_{min})}{4G_N}$



- Depending on the dimension EE has different UV structures

$$S_A = \begin{cases} a_{d-2}(\frac{l_A}{\epsilon})^{d-2} + a_{d-4}(\frac{l_A}{\epsilon})^{d-4} + \cdots + a_1 \frac{l_A}{\epsilon} + (-1)^{(d-1)/2}\mathbf{S_A} + \mathcal{O}(\epsilon) & \mathrm{d : odd} \\ a_{d-2}(\frac{l_A}{\epsilon})^{d-2} + a_{d-4}(\frac{l_A}{\epsilon})^{d-4} + \cdots + (-1)^{(d-2)/2}\mathbf{S_A} \log \frac{l_A}{\epsilon} + \mathcal{O}(\epsilon^0) & \mathrm{d : even} \end{cases}$$

- The coefficient in front of the universal terms is related to the conformal anomalies

$$\langle T^\mu_\mu \rangle = \frac{(-1)^{d/2}}{2} A\, E_d - \sum_i B_i\, I_i$$

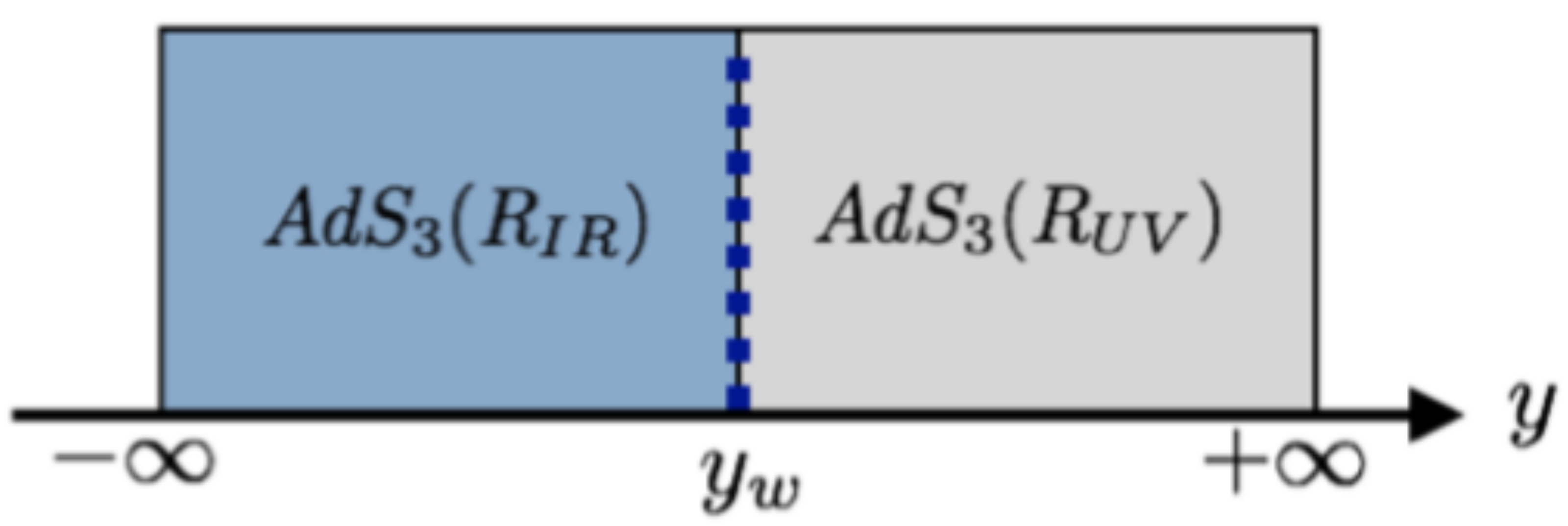
In 4D, $\mathbf{S_A} \propto \mathbf{A}[\chi]$ for $\Sigma = \mathbb{S}^2$ and $\mathbf{S_A} \propto \mathbf{B}$ for $\Sigma = \mathbb{S}^1 \times \mathbb{R}$

In 3D, $\mathbf{S_A} = \mathbf{F}$

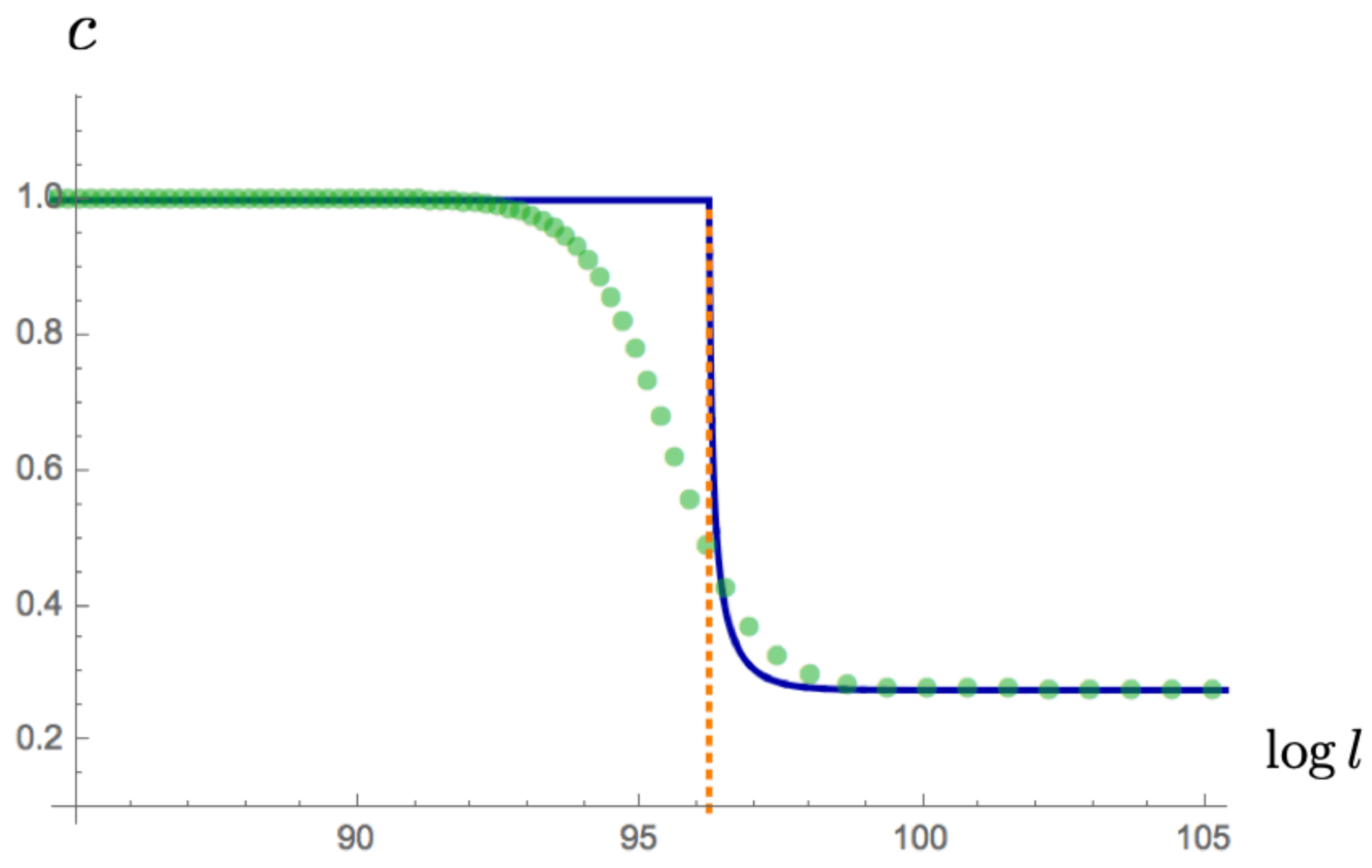
Introduction - Part.2 [Chanyong Park & J.H.L @JHEP11(2018)165]

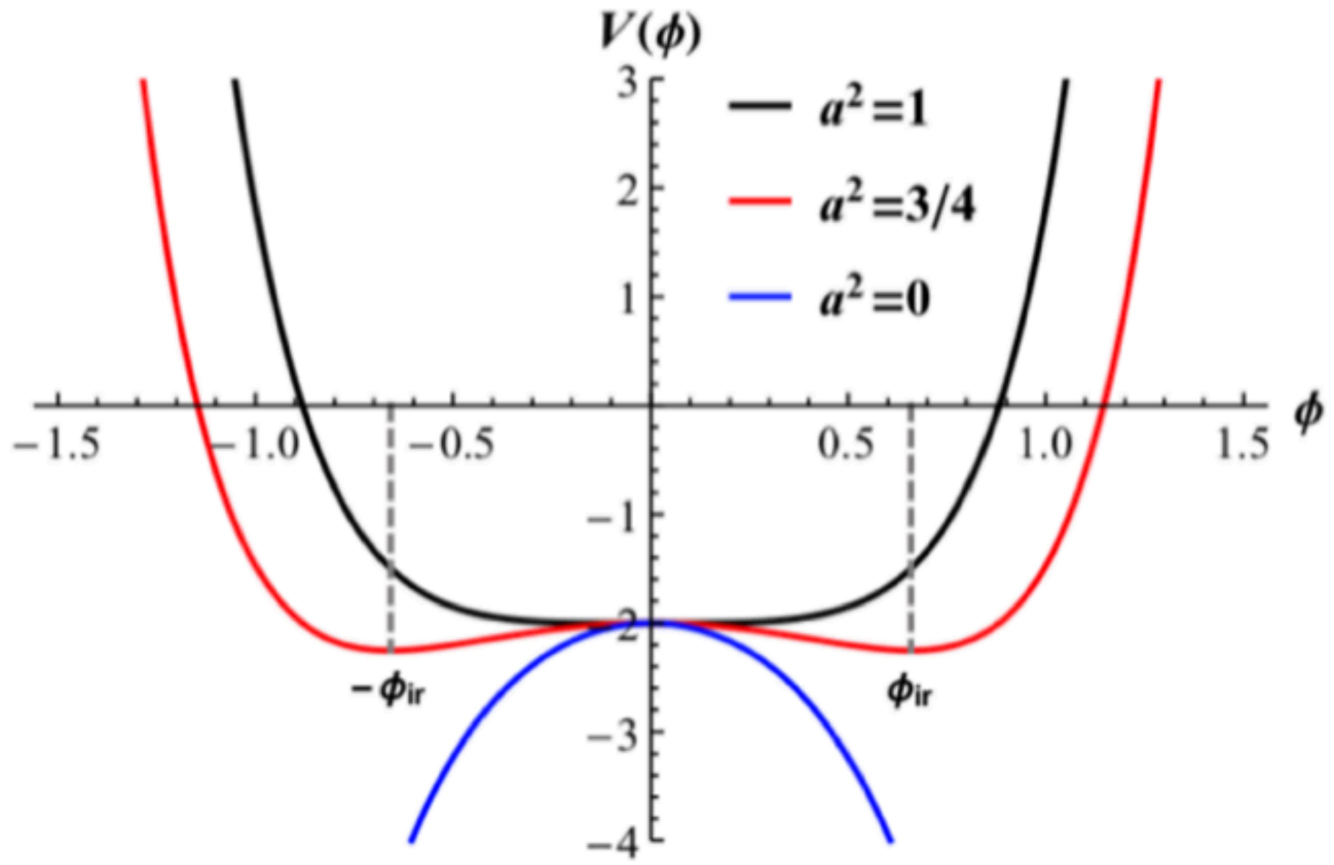
- N=2 gauged supergravity
- Holographic RG flow by relevant deformation
- Thin and Thick wall approximations

$$ds^2 = e^{2A(y)}\eta_{ij}dx^i dx^j + dy^2$$



- The scale function *A* defined the holographic c-function

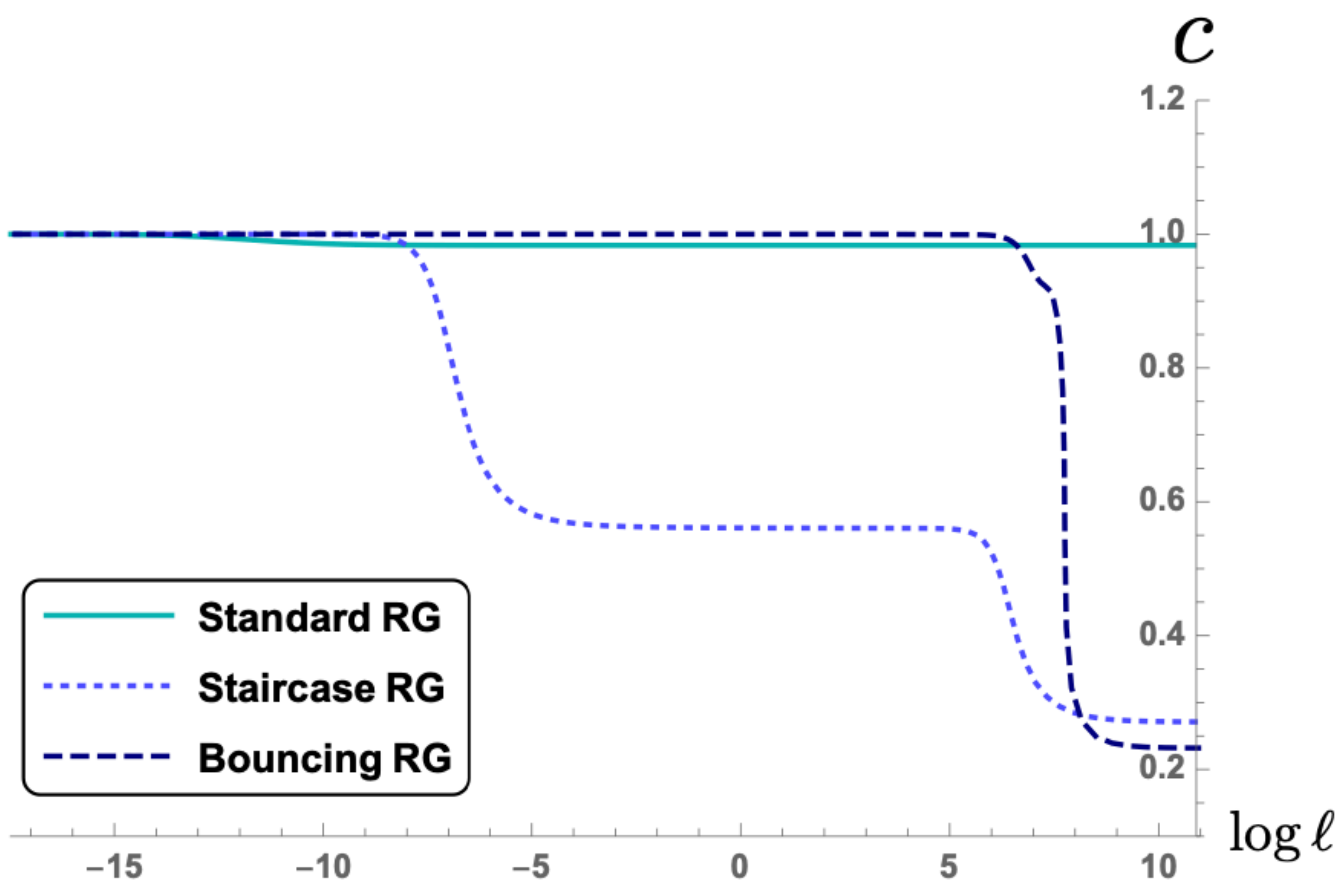


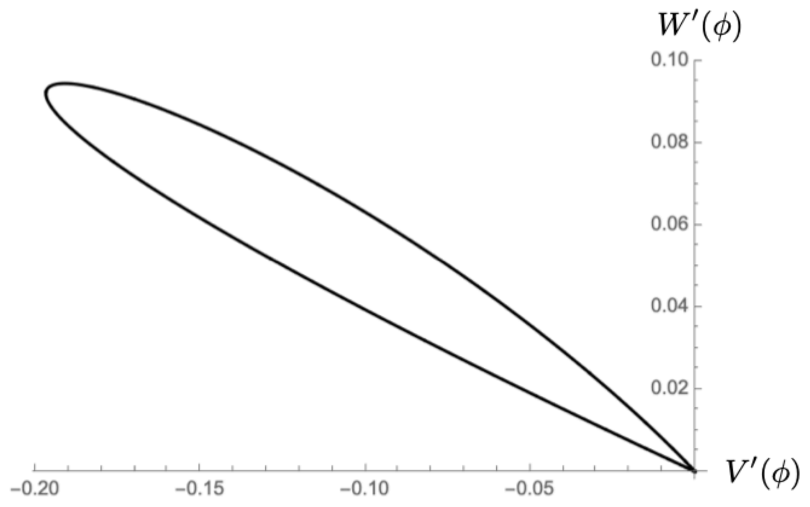


Introduction - Part.2 [Chanyong Park & J.H.L @PRD.101.086008]

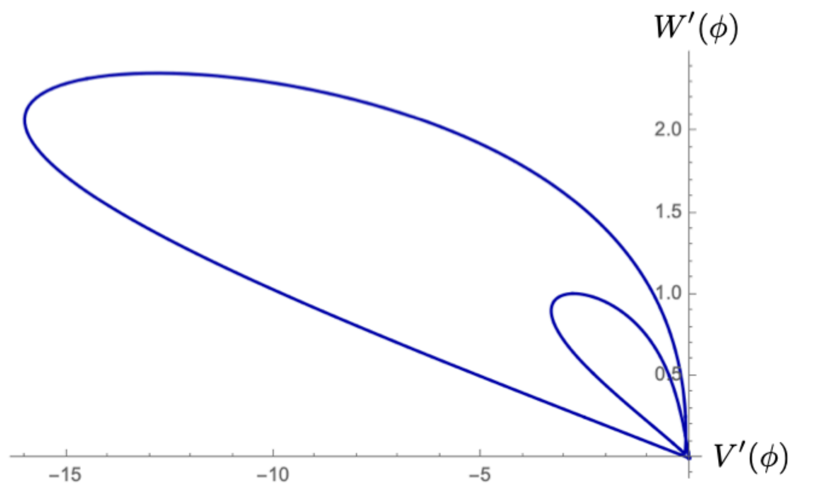
- 3D Einstein-dilaton gravity
- Introducing a new controllable paramter in the potential
- Exotic RG flows in the field space plane

$$\{V'(\phi), W'(\phi)\}$$

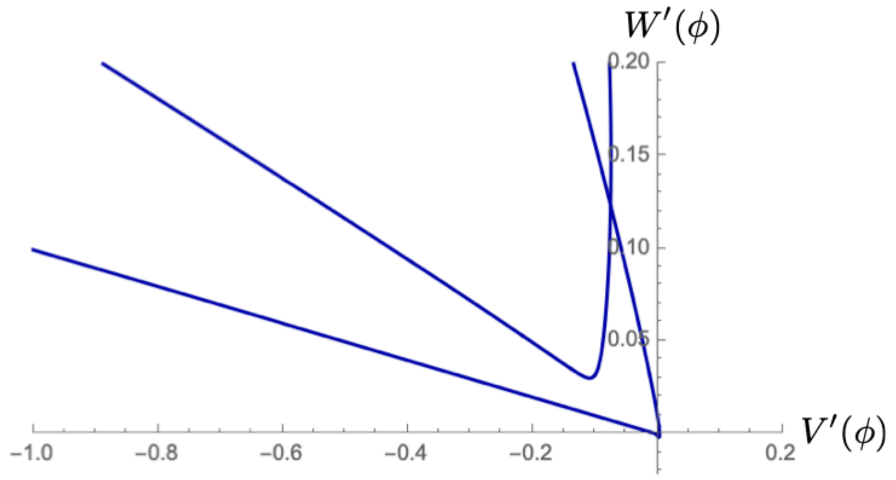




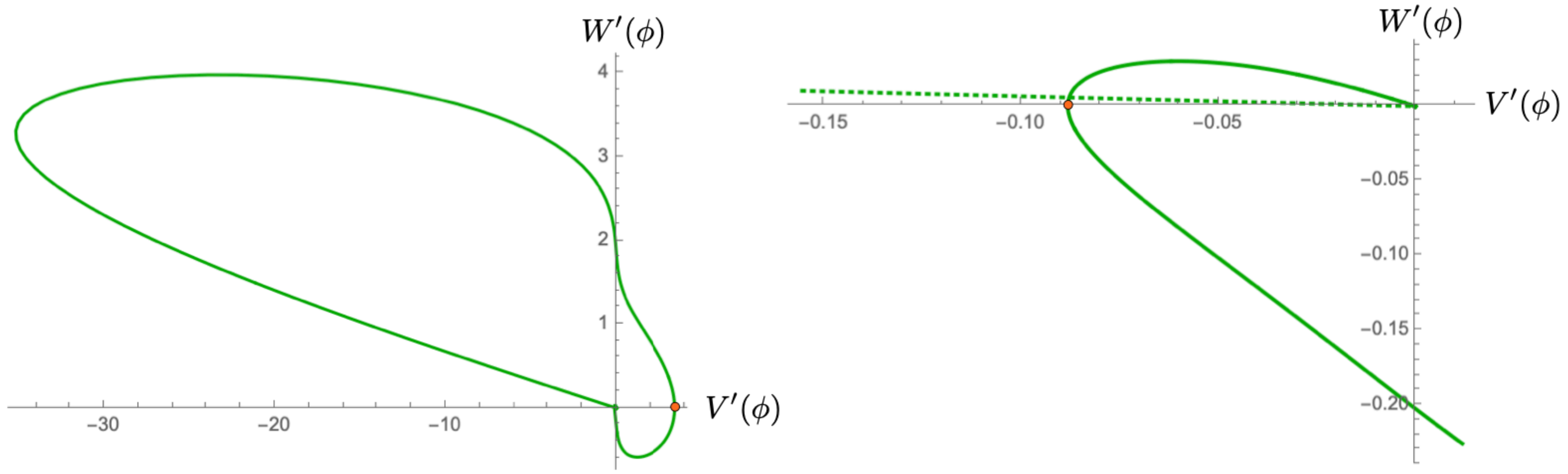
(a) Phase curve in the standard RG flows



(b) Phase curve in the staircase RG flows



(c) Phase curve near the origin of Fig. (b)



Phase curve in the bouncing RG flow

Introduction - Part.2 [Chanyong Park & J.H.L @arXiv:2406.17221]

- In the most recent work, we have studied the IR structure of the entanglement eintropy in the Einstein-scalar theory.
- The main motivation is to understand the critical behaviour of the two-dimensional transverse Ising model.
- Using the holographic dictionary we constructed a holographic correlation function and

$$\langle O(t,x_1)\ O(t,x_2)\rangle \sim e^{-\Delta\, L(t,x_1;t,x_2)/R_{UV}}$$

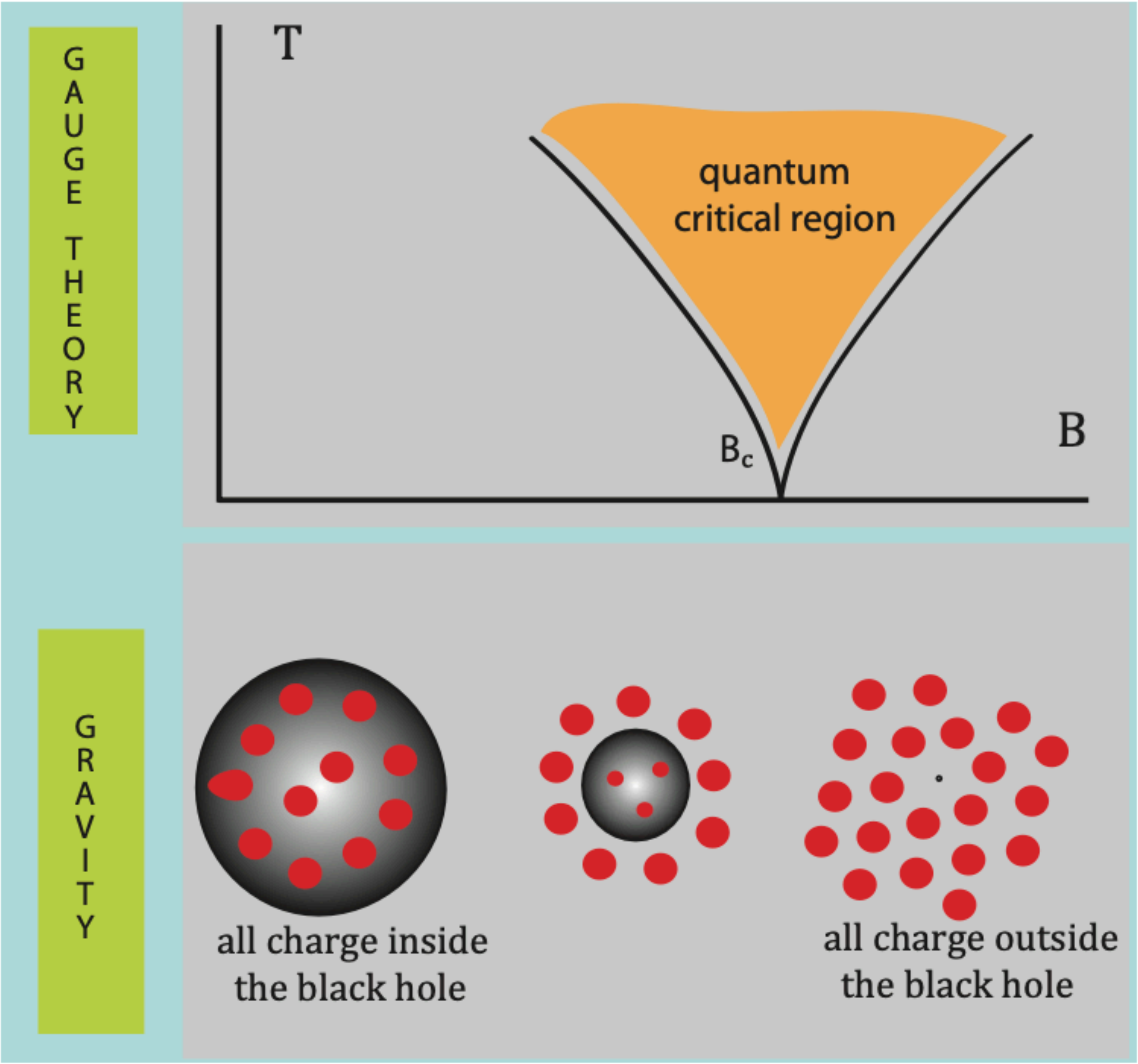
reinterpreted the RG equation in terms of the bulk quantities, $\lambda = \phi,\ t_{\rm RG} = \log z$.

- We also investigated the behaviour of the effective dimension along the RG flow.

For more details, Please watching the next talk of prof. Chanyong Park !

Ongoing work

- The main motivation is to understand the critical behaviours observed in condensed Matter physics based on the viewpoint of the *entanglement RG*.

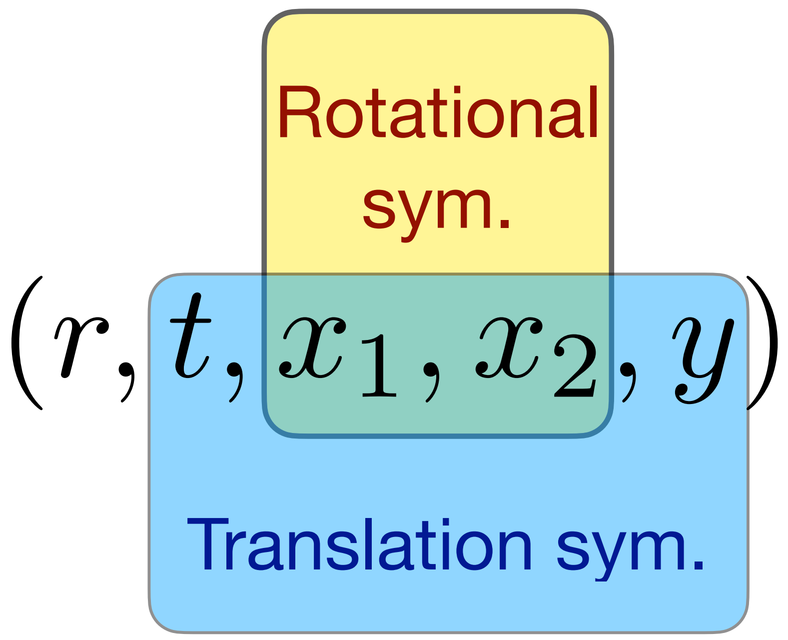


Ongoing work

- The main motivation is to understand the critical behaviours observed in condensed Matter physics based on the viewpoint of the *entanglement RG*.
- As a first step, we consider the 5D magnetic brane geometry and its dual QFT.

$$S = -\frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R + F^{MN}F_{MN} + 2\Lambda\right) + S_{\rm bdy} + S_{CS}$$

- To make AdS5 + a constant uniform magnetic field along the direction y + the finite temperature



- A metric ansatz is

$$ds^2 = -U(t)dt^2 + \frac{dr^2}{U(r)} + e^{2V(r)}((dx^1)^2 + (dx^2)^2) + e^{2W(r)}dy^2 \quad , F = Bdx^2 \wedge dx^2$$

Ongoing work

- A metric anf field strenth ansatz are again

$$ds^2 = -U(t)dt^2 + \frac{dr^2}{U(r)} + e^{2V(r)}((dx^1)^2 + (dx^2)^2) + e^{2W(r)}dy^2 \qquad , F = Bdx^2 \wedge dx^2$$

- From the metric ansatz, the Maxwell equation is autumatically satisfied, and the Einstein Equation reduced to

$$U(V'' - W'') + (U' + U(2V' + W'))(V' - W') = -2B^2 e^{-4V}$$

$$2V'' + W'' + 2(V')^2 + (W')^2 = 0$$

$$\frac{1}{2}U'' + \frac{1}{2}U'(2V' + W') = 4 + \frac{2}{3}B^2 e^{-4V}$$

$$2U'V' + U'W' + 2U(V')^2 + 4UV'W' = 12 - 2e^{-4V}B^2$$

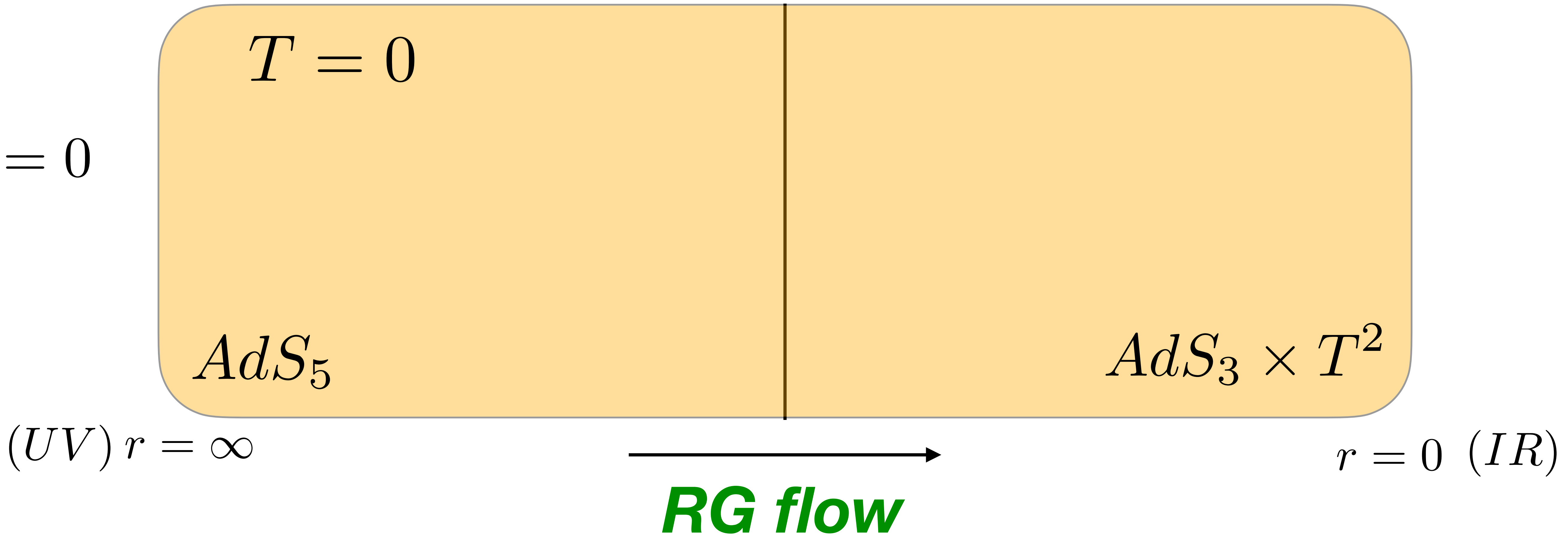
- With B=0, a solution to these equation is AdS_5 : $U = e^{2V} = e^{2W} = r^2$
- A main goal is to find *dual gravity solutions* that correspond to an *RG* flows between a ***D=3+1 CFT*** at short distance and a ***D=1+1 CFT*** at long distance.

Ongoing work

- ***A zero temperature solution*** preserving Lorentz invariance in the (t, y)

$$2V'' + W'' + 2(V')^2 + (W')^2 = 0$$

$$V'^2 + W'^2 + 4V'W' - 6e^{-2W} + e^{-4V-2W}B^2 = 0$$



- The holographic dual to this interpolating solution is a ***RG flow*** from 4-dimensional N=4 super Yang-Mills theory in the UV to 2-dimensional CFT in the IR.

Ongoing work

- Null energy condition (*NEC*) and the monotonicity
- For holographic domain wall flows across dimensions *NEC* yields a monotonic c-function along the *RG* flow from AdS_{D+1} to AdS_{d+1}.
- Rewritten the metric in terms of the Poincare coordinate

$$ds^2 = e^{2f(z)}(g_{\mu\nu}(x)dx^\mu dx^\nu + dz^2) + e^{2g(z)}g_{ij}dy^i dy^j$$

$$R_{MN}\zeta^M\zeta^N \geq 0 \Rightarrow -(d-1)(f'' - (f')^2) - (D-d)(g'' + g'(g' - 2f')) \geq 0 \quad : \ \mathrm{NEC}$$

- From the boundary conditions at conformal fixed points

$$\mathrm{UV} : (e^{-f})' = (e^{-g})' = 1/L_{UV} \Rightarrow c_{UV} = (L_{UV})^{D-1} \qquad \mathrm{IR} : (e^{-f})' = 1/L_{IR} \Rightarrow c_{IR} = \ell^{D-d} e^{(D-d)g_{IR}} (L_{IR})^{d-1}$$

- For the effective AdS radius along the RG flow in the five-dimension magetic brane solution

$$c(z) = \frac{\ell^2}{(e^{-\alpha(z)})'}, \quad \text{where } \alpha(z) \equiv f(z) + 2g(z).$$

- Using NEC, It is easy to show that the holographic c-function is monotonic : $c(z)' \leq 0.$

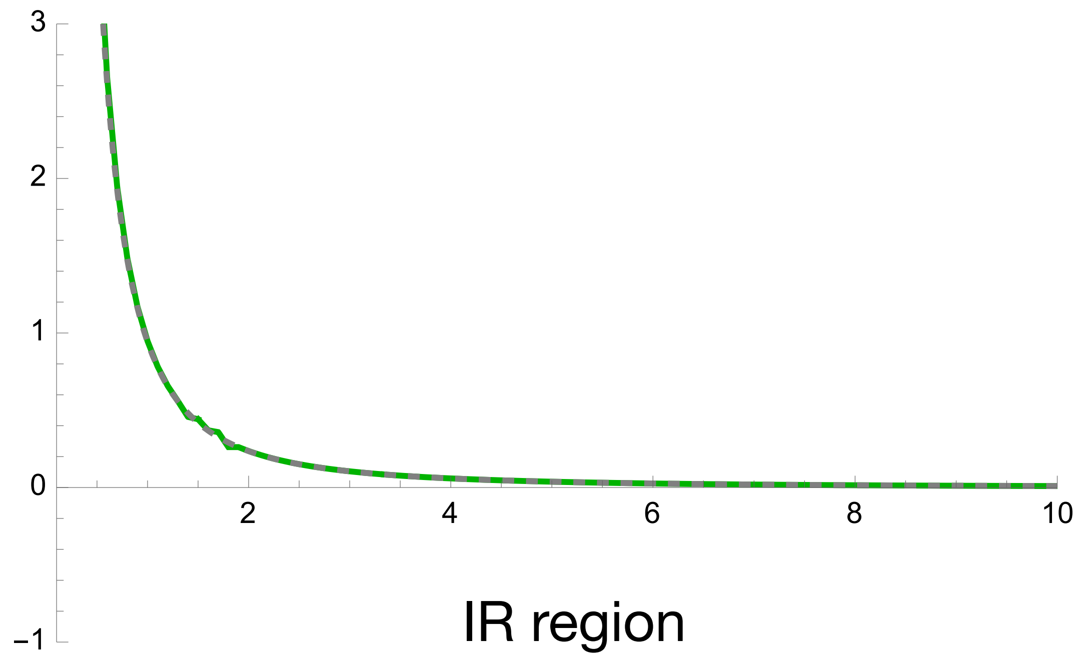
Ongoing work

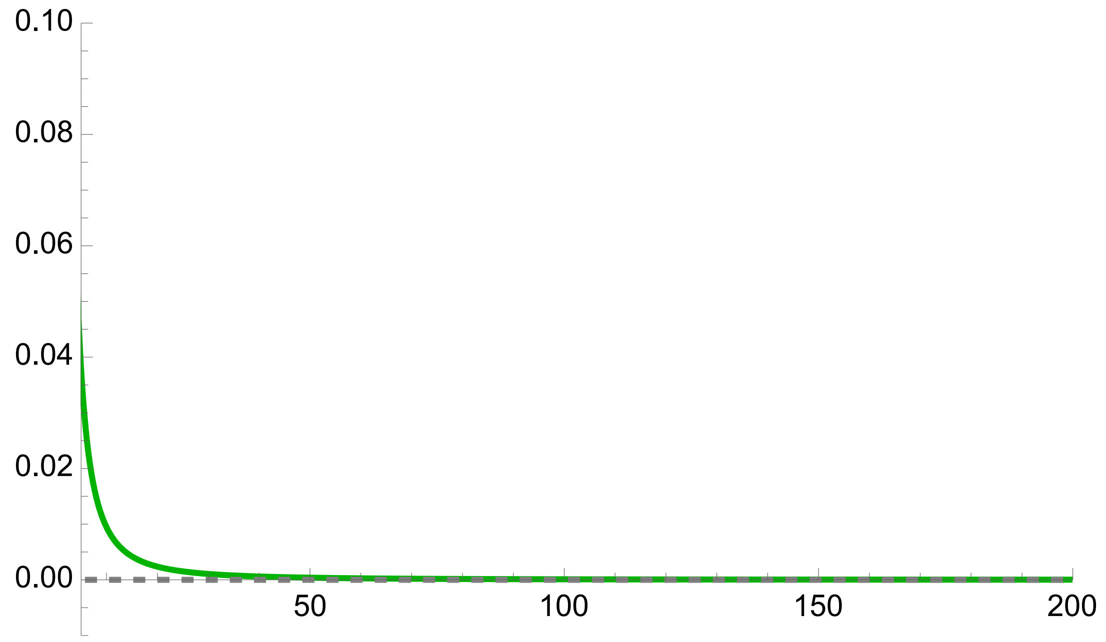
- Entanglement c-function can also quantify the RG flow.
- The Entanglement entropy has different forms of divergenct structure at UV and IR fixed points.

$$S_{EE} = \frac{\mathcal{A}}{4G_N^{(5)}}$$

$$\mathcal{A}_D^{UV}(\ell) = vL^2 R_{UV}\left(\frac{r_\infty^2}{R_{UV}^2} - \frac{4\pi^{3/2}\,\Gamma(5/6)^3}{3\sqrt{3}\Gamma(1/3)^3}\frac{R_{UV}^2}{\ell^2}\right)$$

For a strip shape





$$\mathcal{A}_D^{UV}(r_0) = 4\pi v R_{UV}^3\left(\frac{r_\infty}{r_0}\right)^2 - \pi v R_{UV}^3(1+4\log 2) - 4\pi v R_{UV}^3 \log\left(\frac{r_\infty}{r_0}\right)$$

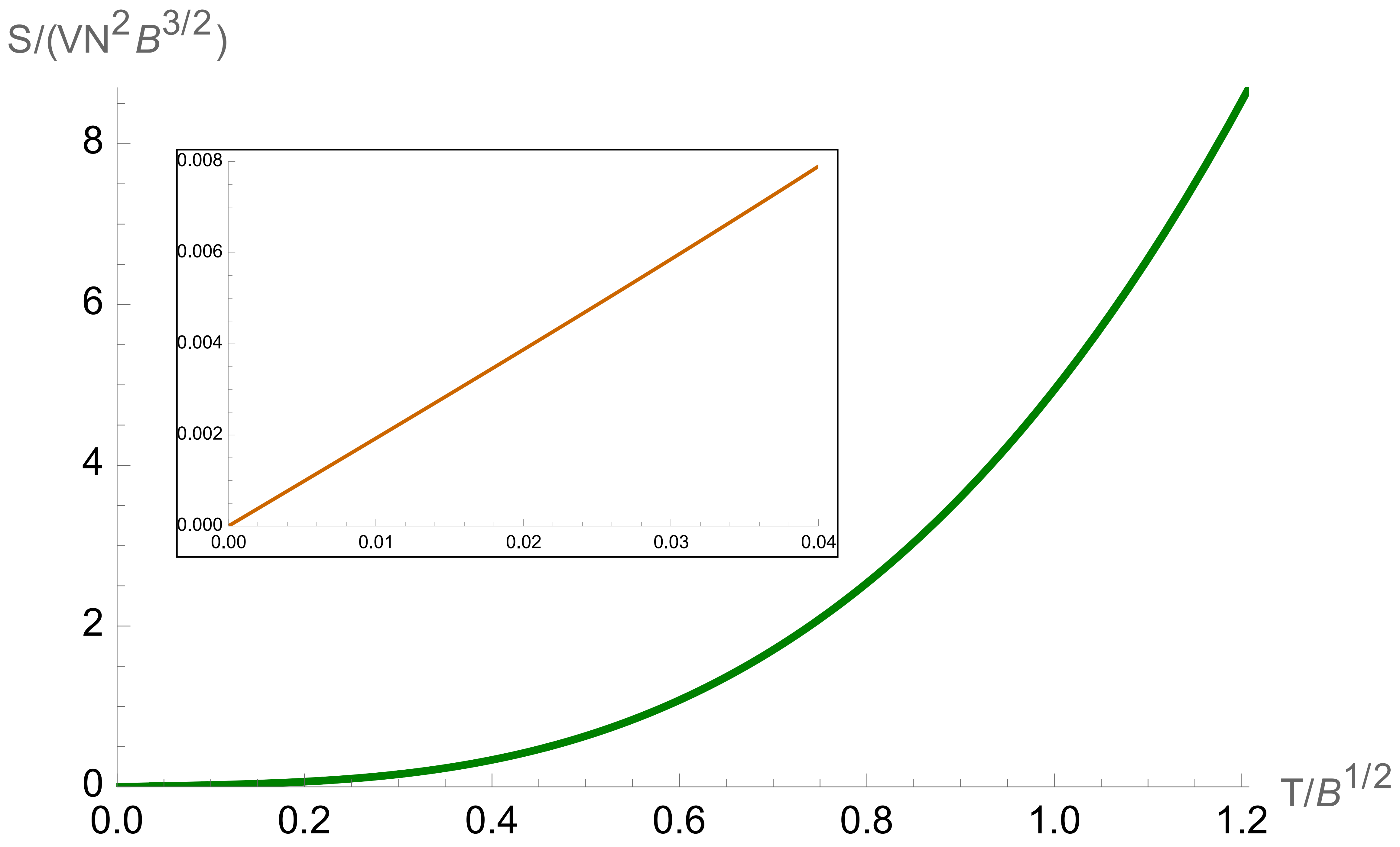
For a disk shape

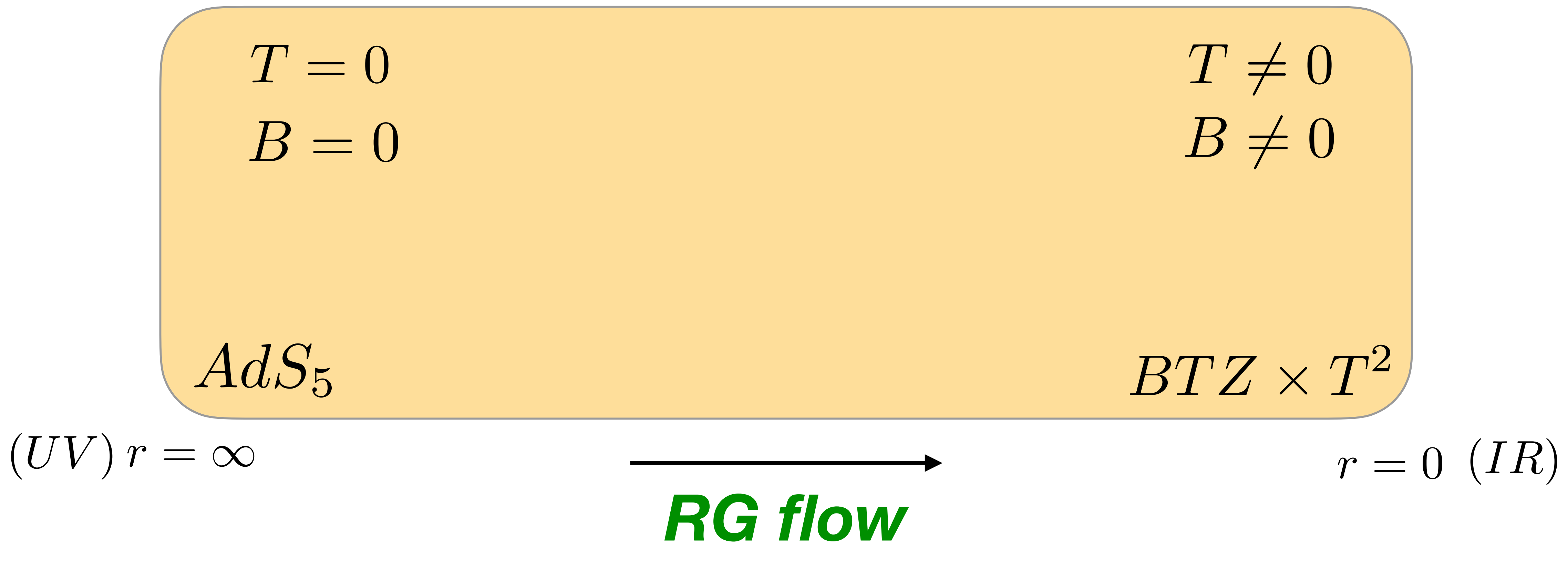
- Notice that for the IR region the minimal surface has only one unique shape.

$$\mathcal{A}^{IR}(\ell) = \boxed{\mathrm{Vol}(T^2)\times 2R_{IR}\log\left(\frac{r_\infty \ell}{R_{IR}^2}\right)}, \quad \mathrm{Vol}(T^2) \equiv L^2\left(\frac{B}{\sqrt{3}}\right)$$

Ongoing work

- ***A finite temperature solution***
- Found smooth interpoatiing flow solution through the numerics. [E.D'Hoker & P.Kraus '09]
- From the numeric solution they studied the behaviours of a temperature dependece of entorpy.
- At low T, the entropy density is linear in T.
- For high T, it is found to behave as $s \sim T^3$.





Ongoing work - Homework and future directions

- ***A list we have to do in the ongoing study***

1. Define an expression of a candidate c-function.
2. Survey c-theorem and its monotonicity.
3. Compute $\langle \mathcal{O}\mathcal{O} \rangle$ and $\Delta_{\rm eff.}$ along the *RG* flow.
4. Extend to the space different topology and higher dimensions.

Thank you for listening :):