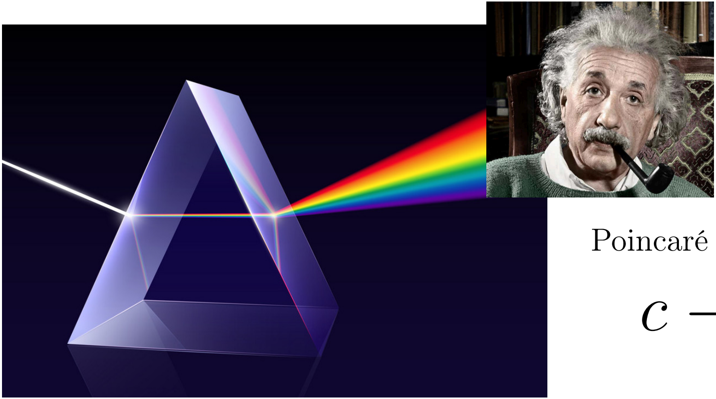
Light speed as a local observable for soft hairs



Poincaré $\to$ BMS

$$c \to ?$$

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Introduction

Theory

Theory: a Lagrangian $\mathcal{L}$ and Boundary Conditions (B.C)

Symmetry

Symmetry: a transformation such that Lagrangian is deformed by a surface term

$$\mathcal{L} \to \mathcal{L} + \nabla_\mu \mathcal{K}^\mu$$

Examples

- All covariant Lagrangians $\mathcal{L}$ — diffeomorphism $x^\mu \to x^\mu + \xi^\mu$
- Maxwell theory $\mathcal{L} \sim F_{\mu\nu}F^{\mu\nu}$ — U(1) gauge: $A_\mu \to A_\mu + \partial_\mu \lambda$
- QCD $\mathcal{L} \sim F^a_{\mu\nu}F^{a\mu\nu}$ — SU(3) gauge symmetry
- $\vdots$

Noether Theorem

$$\text{Symmetry} \quad \xrightarrow{\text{Noether}} \quad \text{Conserved charge } Q$$

Examples

- Rotations — Angular momenta
- Translations — Energy and linear momenta
- $\partial_\mu \lambda = 0 \quad \Rightarrow \quad A_\mu \to A_\mu$ — Electric charge
- $\vdots$

Trivial Symmetries

Trivial symmetries $\quad \sim \quad$ Pure gauge symmetries $\quad \sim \quad Q = 0$

Trivial symmetries $\to$ Constraints

Asymptotic Symmetry Group (ASG)

Asymptotic Symmetry Group

1) Symmetry $\mathcal{L} \to \mathcal{L} + \nabla_\mu \mathcal{K}^\mu$
2) Respect boundary conditions $\mathrm{B.C} \to \mathrm{B.C}$
3) Non-trivial and finite charges $Q \neq 0$

Examples

- Asymptotic flat in 3-dim BMS_3
- Asymptotic AdS_3 $\mathrm{Vir}\otimes\mathrm{Vir}$
- Asymptotic flat in 4-dim BMS_4
- Near horizon of extremal Kerr $\mathrm{Vir}\otimes\mathrm{U(1)}$
- $\vdots$

- Soft charges/hairs: ASG charges

Asymptotic Symmetry Group

1) Symmetry — $\mathcal{L} \to \mathcal{L} + \nabla_\mu \mathcal{K}^\mu$
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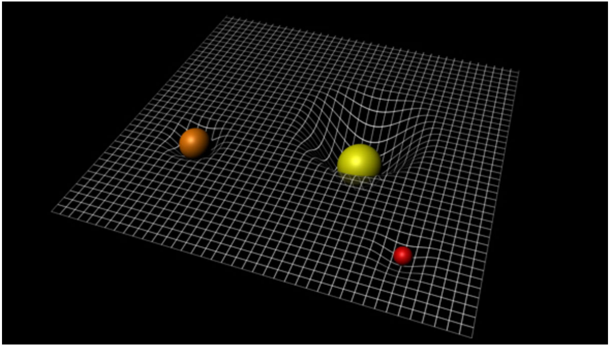
Examples

- Asymptotic flat in 3-dim — BMS_3
- Asymptotic AdS_3 — $\mathrm{Vir} \otimes \mathrm{Vir}$
- Asymptotic flat in 4-dim — BMS_4
- Near horizon of extremal Kerr — $\mathrm{Vir} \otimes \mathrm{U}(1)$
- $\vdots$

► Soft charges/hairs: ASG charges

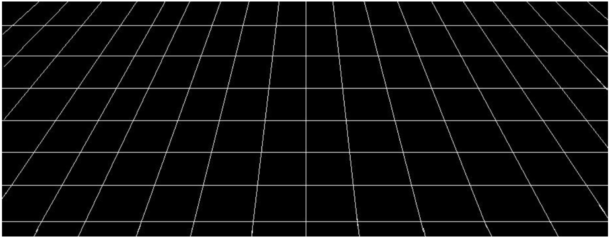
BMS_4



Asymptotic flat 4-dim space-time

ASG of 4-dimensional null asymptotically flat space-time $\rightarrow$ BMS_4

- H. Bondi, M. G. J. van der Burg and A. W. K. Metzner, R. K. Sachs (1962).
- G. Barnich and C. Troessaert, (2009).



4-dim flat space-time

$$ds^2 = -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)$$

- New coordinates:

$$(\theta, \varphi) \to (z, \bar{z}) : \qquad z = \cot(\theta/2)e^{i\varphi}, \quad \bar{z} = \cot(\theta/2)e^{-i\varphi}$$

$$ds^2 = -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\mathrm{d}z\mathrm{d}\bar{z}, \qquad \gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}.$$

Boundary conditions at null infinity

$$g_{\mu\nu} - \bar{g}_{\mu\nu} \sim \begin{pmatrix} \mathcal{O}(1/r) & \mathcal{O}(1/r^2) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1/r^2) & 0 & 0 & 0 \\ \mathcal{O}(1) & 0 & \mathcal{O}(r) & 0 \\ \mathcal{O}(1) & 0 & 0 & \mathcal{O}(r) \end{pmatrix}, \qquad \delta_\xi g_{\mu\nu} \sim \text{same}$$

ASG: BMS$_4$

- Super translations:

$$\xi_f = f\partial_u + D^z\partial_z f\partial_r - \frac{1}{r}(D^z f\partial_z + D^{\bar{z}} f\partial_{\bar{z}}) \qquad f = f(z,\bar{z})$$

- Super rotations:

$$\xi_h = \frac{u}{2}D_z h\partial_u - \frac{r+u}{2}D_z h\partial_r + (1+\frac{u}{2r})h\partial_z \qquad h = h(z)$$

$$\xi_{\bar{h}} = \frac{u}{2}D_{\bar{z}}\bar{h}\partial_u - \frac{r+u}{2}D_{\bar{z}}\bar{h}\partial_r + (1+\frac{u}{2r})\bar{h}\partial_{\bar{z}} \qquad \bar{h} = \bar{h}(\bar{z})$$

Boundary conditions at null infinity

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ASG: BMS$_4$

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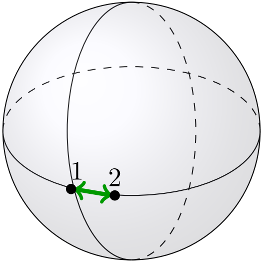
Super translation on (asymptotic) 4-dim flat metric:
$x'^{\mu} = x^{\mu} + \xi_f^{\mu}$
$g'_{\mu\nu} = \bar{g}_{\mu\nu} - \nabla_{\mu}\xi_{f\nu} - \nabla_{\nu}\xi_{f\mu}$

BMS metric

$$\begin{aligned} \mathrm{d}s^2 = &- \mathrm{d}u'^2 - 2\mathrm{d}u'\mathrm{d}r' + 2r'^2\gamma_{z\bar{z}}\mathrm{d}z'\mathrm{d}\bar{z}' \\ &+ r'\delta C_{zz}\mathrm{d}z'^2 + r'\delta C_{\bar{z}\bar{z}}\mathrm{d}\bar{z}'^2 + D^z\delta C_{zz}\mathrm{d}u'\mathrm{d}z' + D^{\bar{z}}\delta C_{\bar{z}\bar{z}}\mathrm{d}u'\mathrm{d}\bar{z}' + \cdots . \end{aligned}$$

$$\delta C_{ab} \equiv \begin{pmatrix} \delta C_{zz} & 0 \\ 0 & \delta C_{\bar{z}\bar{z}} \end{pmatrix}, \qquad \delta C_{zz} = 2D_z D_z f, \qquad \delta C_{\bar{z}\bar{z}} = 2D_{\bar{z}} D_{\bar{z}} f.$$

- Soft charges are related to δC_{zz} and $\delta C_{\bar{z}\bar{z}}$.
- How can one observe them?



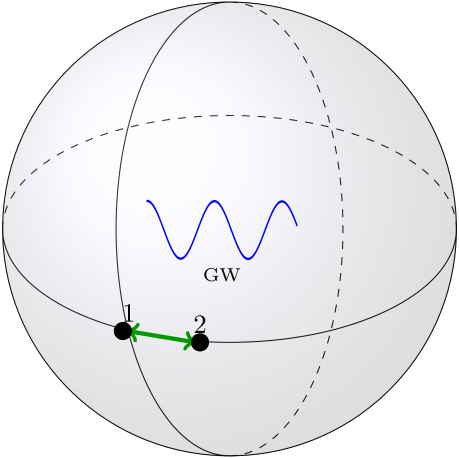
Initial setup

- Two free particle on the same radius r_0 in angles z_0 and $z_0 + \Delta z$:

$$X_1^\mu = (u, r_0, z_0, \bar{z}_0), \quad X_2^\mu = (u, r_0, z_0 + \Delta z, \bar{z}_0 + \Delta \bar{z})$$

- Asymptotic limit: $\Delta z \to 0, \qquad r_0 \to \infty, \qquad r_0 \Delta z \sim \text{constant}.$
- Initial distance:

$$\Delta s = \sqrt{\Delta X . \Delta X} = \sqrt{2 r_0^2 \gamma_{z_0 \bar{z}_0} \Delta z \Delta \bar{z}} \equiv L_0$$


GW
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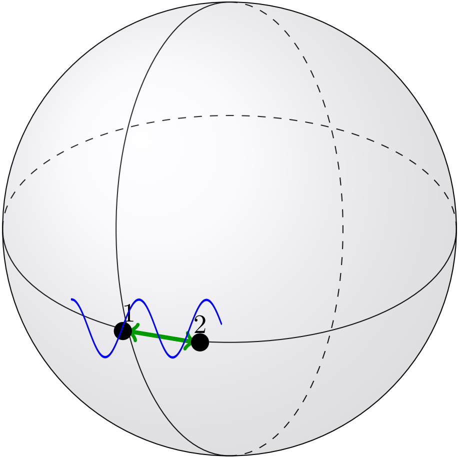
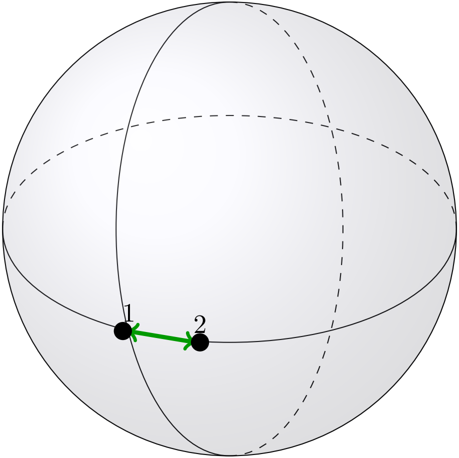
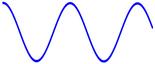
time: $u = u_0$



Memory Effect



time: $u = u_0 + \delta u$



After gravitational wave $u = u_0 + \delta u$

- Two particles remain approx. in the same places:

$$\Delta X^\mu \approx (0, 0, \Delta z, \Delta \bar{z}) \quad \Rightarrow \quad \delta \Delta X^\mu \approx 0.$$

- The metric changes by a super-translation:

$$\begin{aligned} \mathrm{d}s^2 =& -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\mathrm{d}z\mathrm{d}\bar{z} \\ &+ r\delta C_{zz}\mathrm{d}z^2 + r\delta C_{\bar{z}\bar{z}}\mathrm{d}\bar{z}^2 + D^z\delta C_{zz}\mathrm{d}u\mathrm{d}z + D^{\bar{z}}\delta C_{\bar{z}\bar{z}}\mathrm{d}u\mathrm{d}\bar{z} + \cdots . \end{aligned}$$

$$\begin{aligned} \delta\Delta s = \sqrt{\Delta X \cdot \Delta X}|_{u_0+\delta u} - L_0 &= \sqrt{L_0^2 + r_0\delta C_{zz}\Delta z^2 + r_0\delta C_{\bar{z}\bar{z}}\Delta\bar{z}^2} - L_0 \\ &\approx \frac{r_0}{2L_0}(\delta C_{zz}\Delta z^2 + \delta C_{\bar{z}\bar{z}}\Delta\bar{z}^2). \end{aligned}$$

- A. Strominger and A. Zhiboedov, JHEP **1601**, 086 (2016).
- A. Strominger, "Lectures on the Infrared Structure of Gravity and Gauge Theory," arXiv:1703.05448.

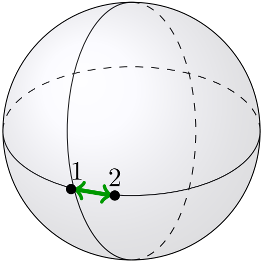
- δC_{zz} is related to the gravitational wave by:

$$\delta C_{zz}(z,\bar{z}) = 2\int \mathrm{d}^2\hat{z}\ \gamma_{\hat{z}\hat{\bar{z}}}\, D_z^2 G(z,\bar{z};\hat{z},\hat{\bar{z}}) \times \Big[\int_{u_0}^{u_0+\delta u} \mathrm{d}u \Big(T_{uu}(u,\hat{z},\hat{\bar{z}}) - \frac{1}{4\pi}\int \mathrm{d}^2\tilde{z}\ \gamma_{\tilde{z}\tilde{\bar{z}}} T_{uu}(u,\tilde{z},\tilde{\bar{z}})\Big)\Big],$$

where G is the Green's function satisfying

$$D_z^2 D_{\bar{z}}^2 G(z,\bar{z};\hat{z},\hat{\bar{z}}) = -\gamma_{z\bar{z}}\delta^2(z-\hat{z}).$$

- $\delta\Delta s \neq 0$ is called Memory Effect



Summary

- Soft hair:

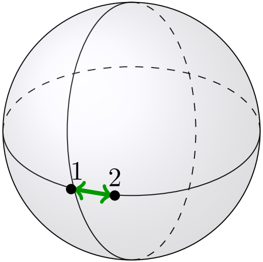
$$\mathrm{d}s^2 = -\,\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\mathrm{d}z\mathrm{d}\bar{z} \quad \rightarrow \quad \mathrm{d}s^2 = -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\mathrm{d}z\mathrm{d}\bar{z}$$
$$+ r\delta C_{zz}\mathrm{d}z^2 + r\delta C_{\bar{z}\bar{z}}\mathrm{d}\bar{z}^2 + D^z\delta C_{zz}\mathrm{d}u\mathrm{d}z + D^{\bar{z}}\delta C_{\bar{z}\bar{z}}\mathrm{d}u\mathrm{d}\bar{z} + \cdots .$$

- Particle memory:

$$\Delta s = L_0 \quad \rightarrow \quad L_0 + \frac{r_0}{2L_0}(\delta C_{zz}\Delta z^2 + \delta C_{\bar{z}\bar{z}}\Delta\bar{z}^2).$$

2 Light speed as a memory

Light speed as a memory



Bottom-line

- Soft hair:

$$\mathrm{d}s^2 = -\,\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\mathrm{d}z\mathrm{d}\bar{z} \quad \rightarrow \quad \mathrm{d}s^2 = -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\mathrm{d}z\mathrm{d}\bar{z}$$
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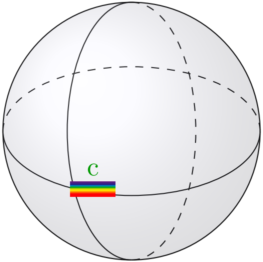
Light speed as a memory



Bottom-line

- Soft hair:

$$
\begin{aligned}
\mathrm{d}s^2 = -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\mathrm{d}z\mathrm{d}\bar{z} \quad \rightarrow \quad \mathrm{d}s^2 &= -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\mathrm{d}z\mathrm{d}\bar{z} \\
&+ r\delta C_{zz}\mathrm{d}z^2 + r\delta C_{\bar{z}\bar{z}}\mathrm{d}\bar{z}^2 + D^z\delta C_{zz}\mathrm{d}u\mathrm{d}z + D^{\bar{z}}\delta C_{\bar{z}\bar{z}}\mathrm{d}u\mathrm{d}\bar{z} + \cdots .
\end{aligned}
$$

- Light speed memory:

$$
c = 1 \quad \rightarrow \quad 1 - \frac{1}{2r_0}(D^z\delta C_{zz} + D^{\bar{z}}\delta C_{\bar{z}\bar{z}}).
$$

Disclaimer

Light propagates on the light cone, irrespective to any diffeomorphism including Poincaré and BMS.

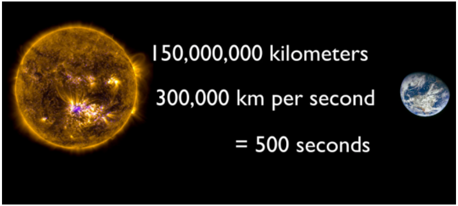
Claim

On flat space-time, Poincaré keeps c invariant, while BMS can change c.

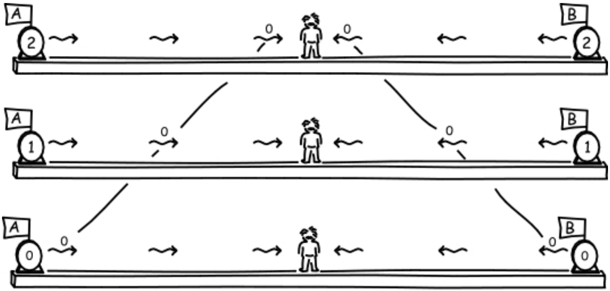
Question: what do we mean by c?

$$\vec{v} = \frac{\mathrm{d}\vec{x}}{\mathrm{d}t}$$
$$v = |\vec{v}|$$

$$\vec{c} = \frac{\mathrm{d}\vec{x}}{\mathrm{d}t}$$
$$c = |\vec{c}|$$

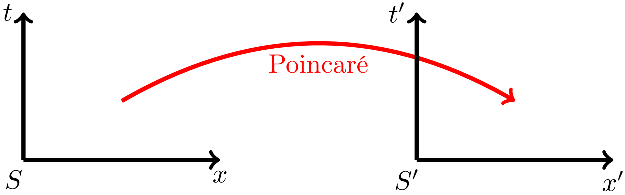


- Einstein method of simultaneity



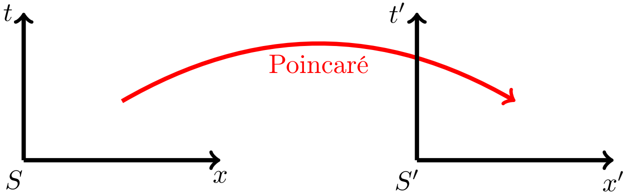
Einstein observers

- After synchronization, $\vec{v} = \frac{\mathrm{d}\vec{x}}{\mathrm{d}t}$ is meaningful and observable.
- After synchronization, $c = |\vec{c}| = 1$ in all directions by construction.



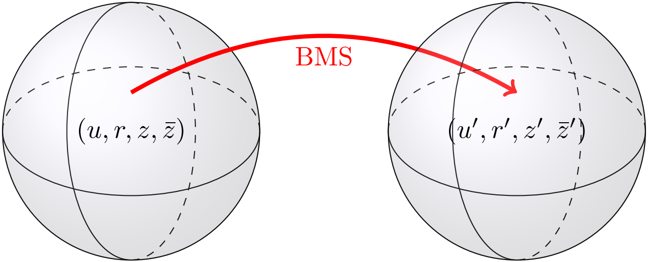
Poincaré transformations

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu} + a^{\mu}, \qquad \Lambda^{\mu}{}_{\nu} = \begin{pmatrix} \beta & \gamma\beta & 0 & 0 \\ \gamma\beta & \beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \beta = \frac{v_x}{c},\ \gamma = \frac{1}{\sqrt{1-\beta^2}}$$



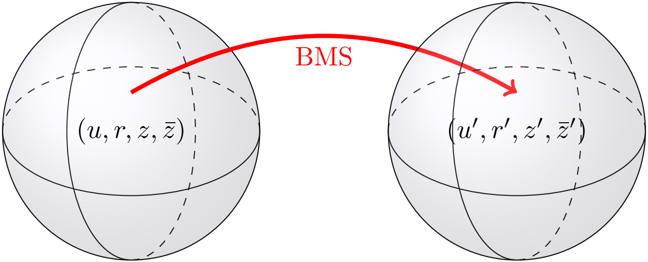
Poincaré transformations

1. (t, x) & Poincaré $\Rightarrow$ (t', x')
2. (t', x') is a physical coordinate system: clocks and rulers which move with relative velocity $\vec{v}$, or are shifted by a constant a^μ.
3. So, $\vec{c'} = \frac{\mathrm{d}\vec{x'}}{\mathrm{d}t'}$ is well-defined.
4. $c' = 1$. So, Poincaré transformations keep the speed of light invariant.



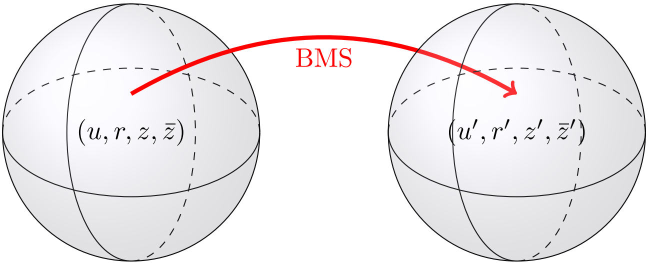
BMS transformations

$$\begin{aligned}
\mathrm{d}s^2 =& -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\mathrm{d}z\mathrm{d}\bar{z} \qquad \to \\
\mathrm{d}s^2 =& -\mathrm{d}u'^2 - 2\mathrm{d}u'\mathrm{d}r' + 2r'^2\gamma_{z\bar{z}}\mathrm{d}z'\mathrm{d}\bar{z}' \\
& + r'\delta C_{zz}\mathrm{d}z'^2 + r'\delta C_{\bar{z}\bar{z}}\mathrm{d}\bar{z}'^2 + D^z\delta C_{zz}\mathrm{d}u'\mathrm{d}z' + D^{\bar{z}}\delta C_{\bar{z}\bar{z}}\mathrm{d}u'\mathrm{d}\bar{z}' + \cdots .
\end{aligned}$$



BMS transformations

1. $(u, r, z, \bar{z})$ & BMS $\quad \Rightarrow \quad (u', r', z', \bar{z}')$
2. $(u', r', z', \bar{z}')$ is a physical coordinate system: clocks and rulers after a gravitational wave indicate $(u', r', z', \bar{z}')$.
3. So, $\vec{c'}$ is well-defined.
4. $c' \neq 1$. So, BMS transformations change the speed of light.



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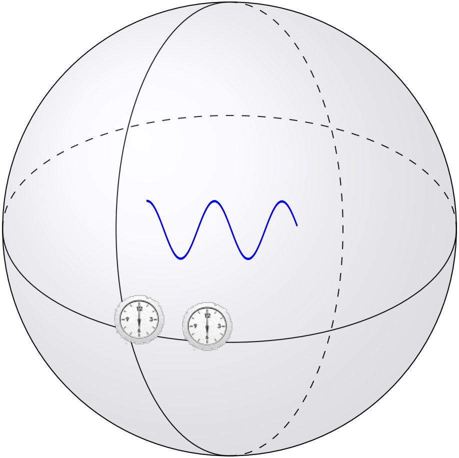
BMS time is physical



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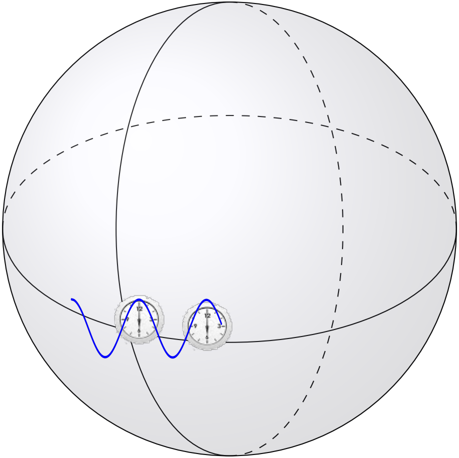
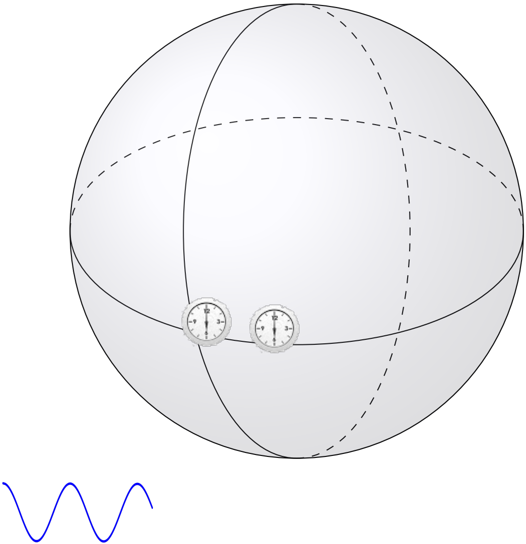




BMS time is physical

Einstein time

After gravitational wave

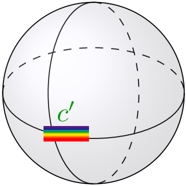
BMS time

Einstein time

After gravitational wave

Other time coordinates

- K. Hajian, ArXiv:2001.07540 [gr-qc].



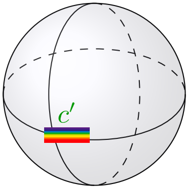
c' in BMS is not 1

$$
\begin{aligned}
\mathrm{d}s^2 = &- \mathrm{d}u'^2 - 2\mathrm{d}u'\mathrm{d}r' + 2r'^2\gamma_{z\bar{z}}\mathrm{d}z'\mathrm{d}\bar{z}' \\
&+ r'\delta C_{zz}\mathrm{d}z'^2 + r'\delta C_{\bar{z}\bar{z}}\mathrm{d}\bar{z}'^2 + D^z\delta C_{zz}\mathrm{d}u'\mathrm{d}z' + D^{\bar{z}}\delta C_{\bar{z}\bar{z}}\mathrm{d}u'\mathrm{d}\bar{z}' + \cdots .
\end{aligned}
$$

- On the equator $z\bar{z} = 1$. So, $\gamma_{z\bar{z}} = \frac{1}{2}$.
- Tangential motion $\Rightarrow$ $\mathrm{d}r' = 0$ and $\mathrm{d}t' = \mathrm{d}u'$.
- Spatial line element:

$$
\mathrm{d}r' = 0 \quad \& \quad \mathrm{d}u' = 0 \quad \Rightarrow \quad \mathrm{d}\ell' = \sqrt{r'^2\mathrm{d}z'\mathrm{d}\bar{z}' + r'\delta C_{zz}\mathrm{d}z'^2 + r'\delta C_{\bar{z}\bar{z}}\mathrm{d}\bar{z}'^2}
$$

- $c' = \frac{\mathrm{d}\ell'}{\mathrm{d}u'}$ and $\mathrm{d}s^2 = 0 \quad \Rightarrow \quad c' = \sqrt{1 - D^z\delta C_{zz}\frac{\mathrm{d}z'}{\mathrm{d}u'} - D^{\bar{z}}\delta C_{\bar{z}\bar{z}}\frac{\mathrm{d}\bar{z}'}{\mathrm{d}u'}}$



c' in BMS is not 1

$$\begin{aligned}\mathrm{d}s^2 = &-\mathrm{d}u'^2 - 2\mathrm{d}u'\mathrm{d}r' + 2r'^2\gamma_{z\bar{z}}\mathrm{d}z'\mathrm{d}\bar{z}' \\ &+ r'\delta C_{zz}\mathrm{d}z'^2 + r'\delta C_{\bar{z}\bar{z}}\mathrm{d}\bar{z}'^2 + D^z\delta C_{zz}\mathrm{d}u'\mathrm{d}z' + D^{\bar{z}}\delta C_{\bar{z}\bar{z}}\mathrm{d}u'\mathrm{d}\bar{z}' + \cdots .\end{aligned}$$

- On the equator $z\bar{z} = 1$. So, $\gamma_{z\bar{z}} = \frac{1}{2}$.
- Tangential motion $\Rightarrow \quad \mathrm{d}r' = 0$ and $\mathrm{d}t' = \mathrm{d}u'$.
- Spatial line element:

$$\mathrm{d}r' = 0 \quad \& \quad \mathrm{d}u' = 0 \quad \Rightarrow \quad \mathrm{d}\ell' = \sqrt{r'^2\mathrm{d}z'\mathrm{d}\bar{z}' + r'\delta C_{zz}\mathrm{d}z'^2 + r'\delta C_{\bar{z}\bar{z}}\mathrm{d}\bar{z}'^2}$$

- $c' = \frac{\mathrm{d}\ell'}{\mathrm{d}u'}$ and $\mathrm{d}s^2 = 0 \quad \Rightarrow \quad c' \approx 1 - \frac{1}{2}\left(D^z\delta C_{zz}\frac{\mathrm{d}z'}{\mathrm{d}u'} + D^{\bar{z}}\delta C_{\bar{z}\bar{z}}\frac{\mathrm{d}\bar{z}'}{\mathrm{d}u'}\right)$

Conclusion

$$\mathrm{d}s^2 = -\,\mathrm{d}u'^2 - 2\mathrm{d}u'\mathrm{d}r' + 2r'^2\gamma_{z\bar{z}}\mathrm{d}z'\mathrm{d}\bar{z}' + r'\delta C_{zz}\mathrm{d}z'^2 + r'\delta C_{\bar{z}\bar{z}}\mathrm{d}\bar{z}'^2 + D^z\delta C_{zz}\mathrm{d}u'\mathrm{d}z' + D^{\bar{z}}\delta C_{\bar{z}\bar{z}}\mathrm{d}u'\mathrm{d}\bar{z}' + \cdots.$$

- Particle memory: $\frac{\delta\Delta s}{\Delta s} \approx \frac{1}{2r_0}(\delta C_{zz} + \delta C_{\bar{z}\bar{z}})$.
- Light speed memory: $\frac{\delta c}{c} \approx -\frac{1}{2r_0}(D^z\delta C_{zz} + D^{\bar{z}}\delta C_{\bar{z}\bar{z}})$.

Outlook

- Other asymptotics and asymptotic symmetry groups.
- Observation of light speed memory in the lab.

Thanks for your attention