DUY TÂN UNIVERSITY

7TH INTERNATIONAL CONFERENCE ON HOLOGRAPHY AND STRING THEORY IN DA NANG

Deep Learning Bulk Spacetime from Boundary quantum data

Keun-Young Kim

2024.08. 21

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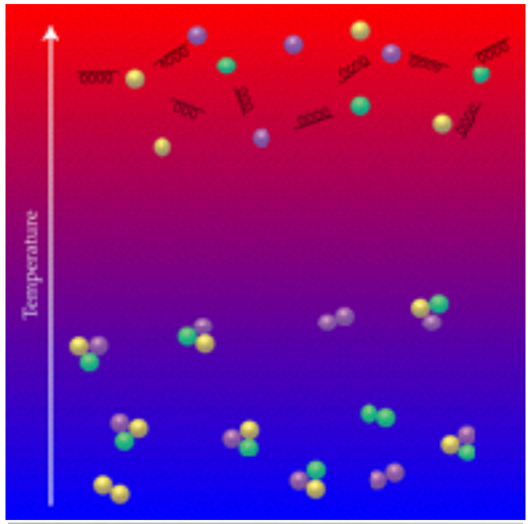
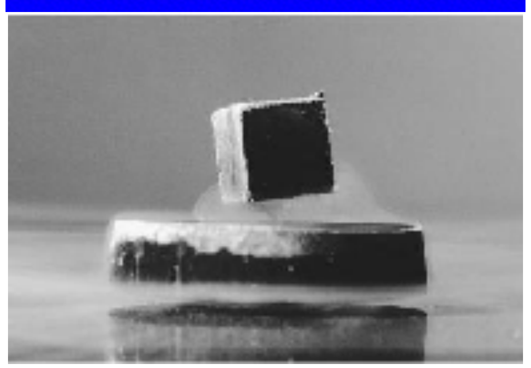
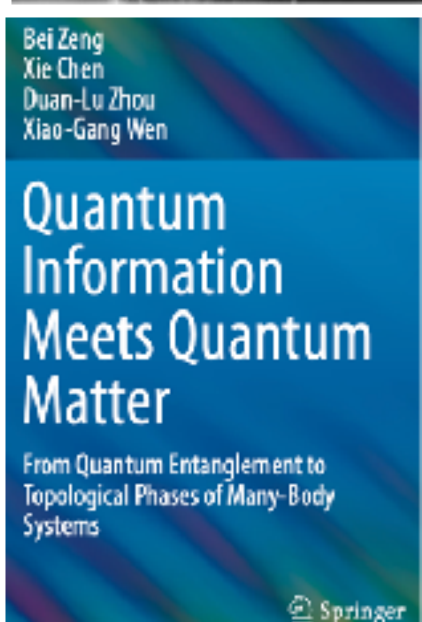
GIST Gwangju Institute of Science and Technology

Holography

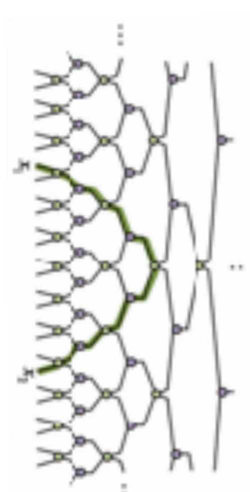
Deep Learning Bulk Spacetime from Boundary data

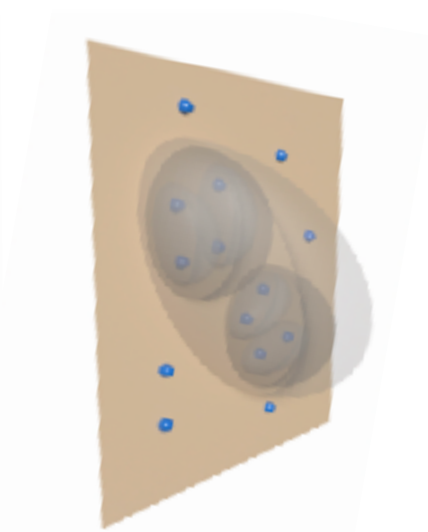




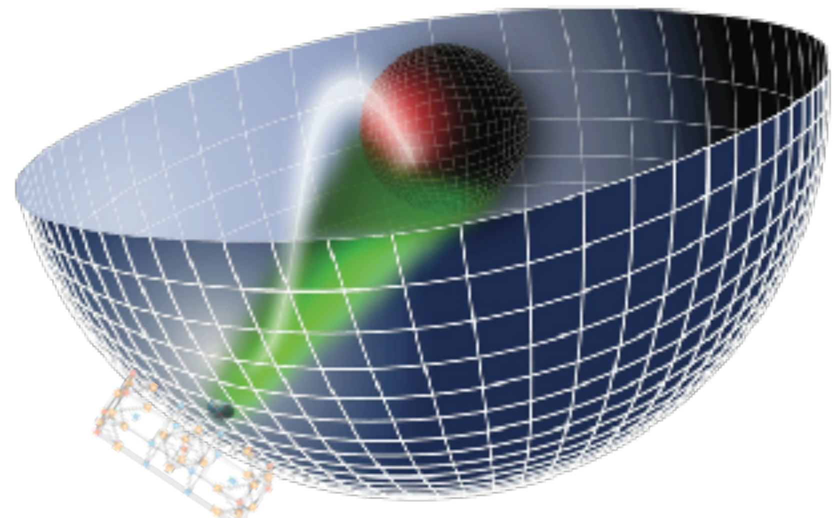


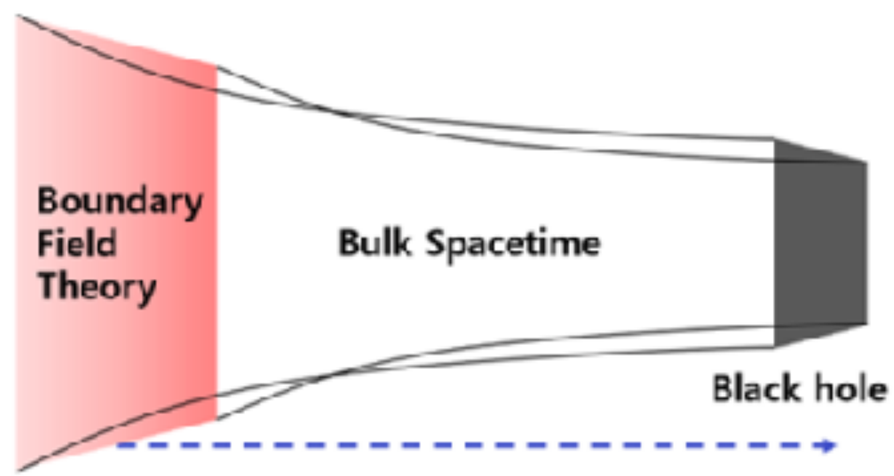
Entanglement structure
~Tensor network





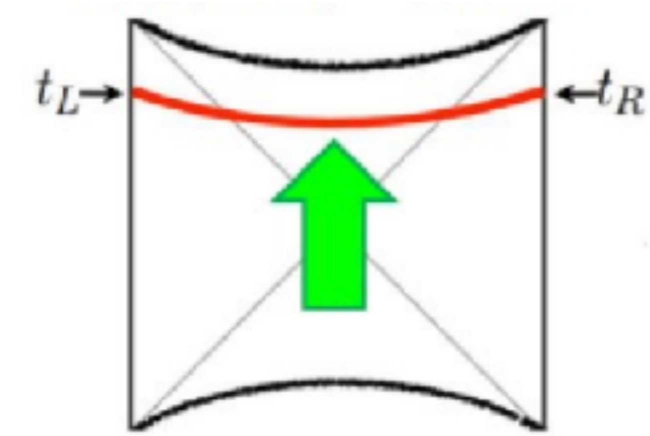
Quantum physics in 4D = Gravity in 5D

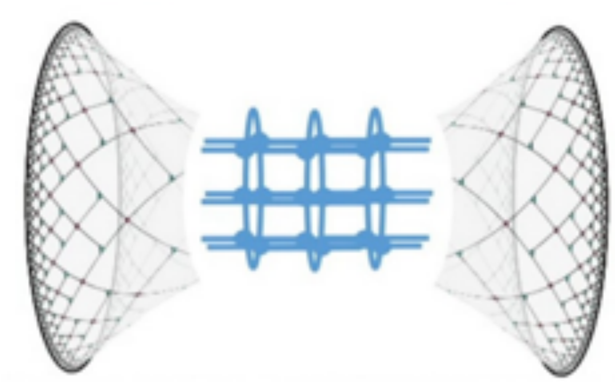




Quantum entanglement
= Minimal Area

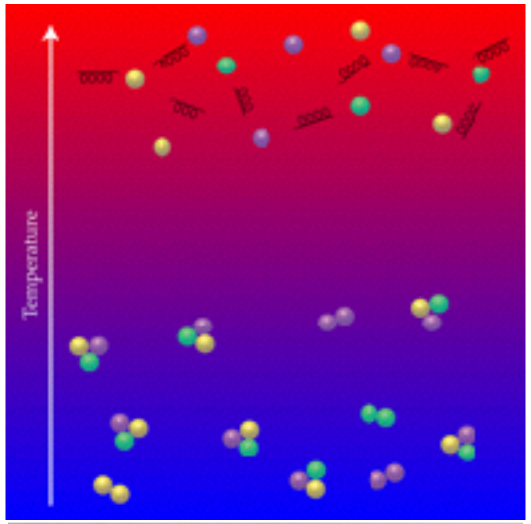
Complexity

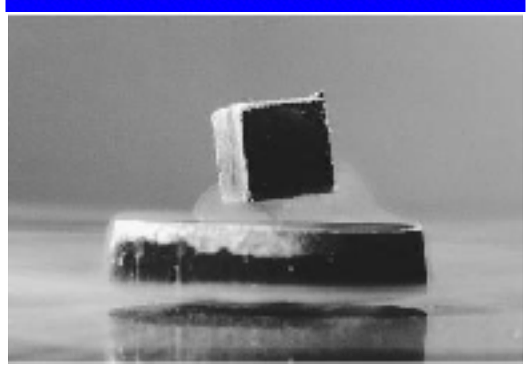




Holography

Deep Learning Bulk Spacetime from Boundary data





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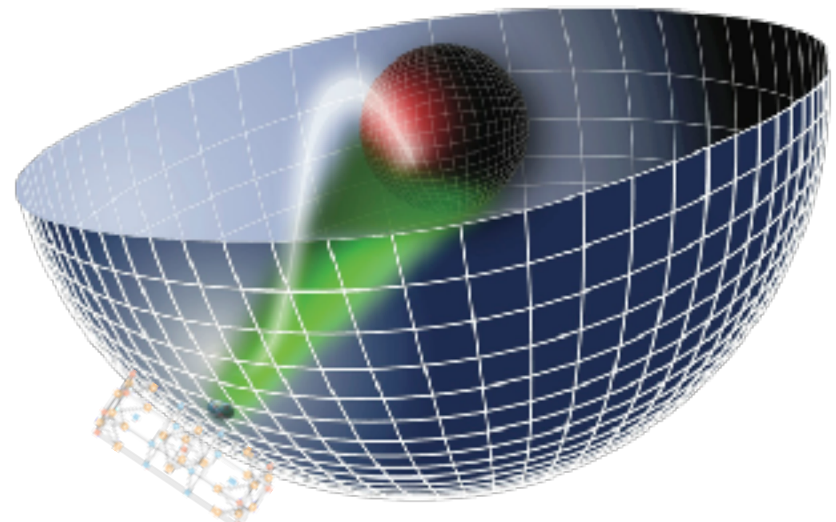
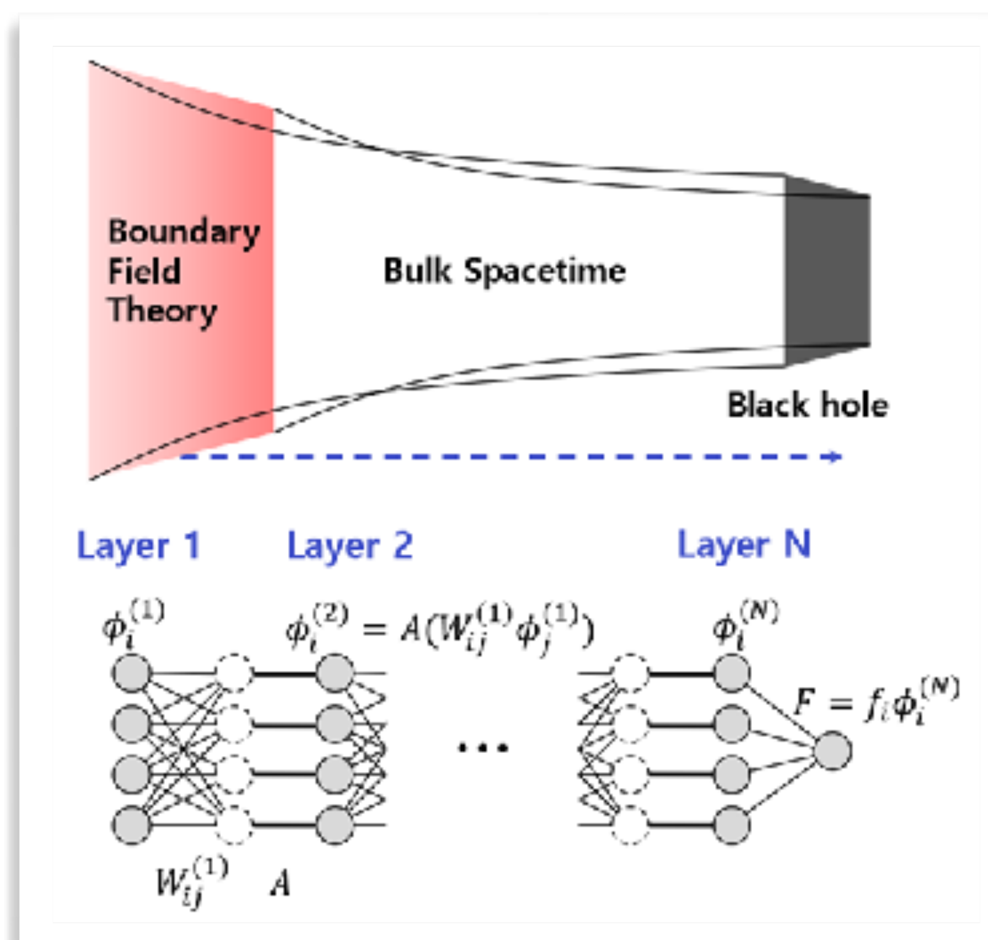




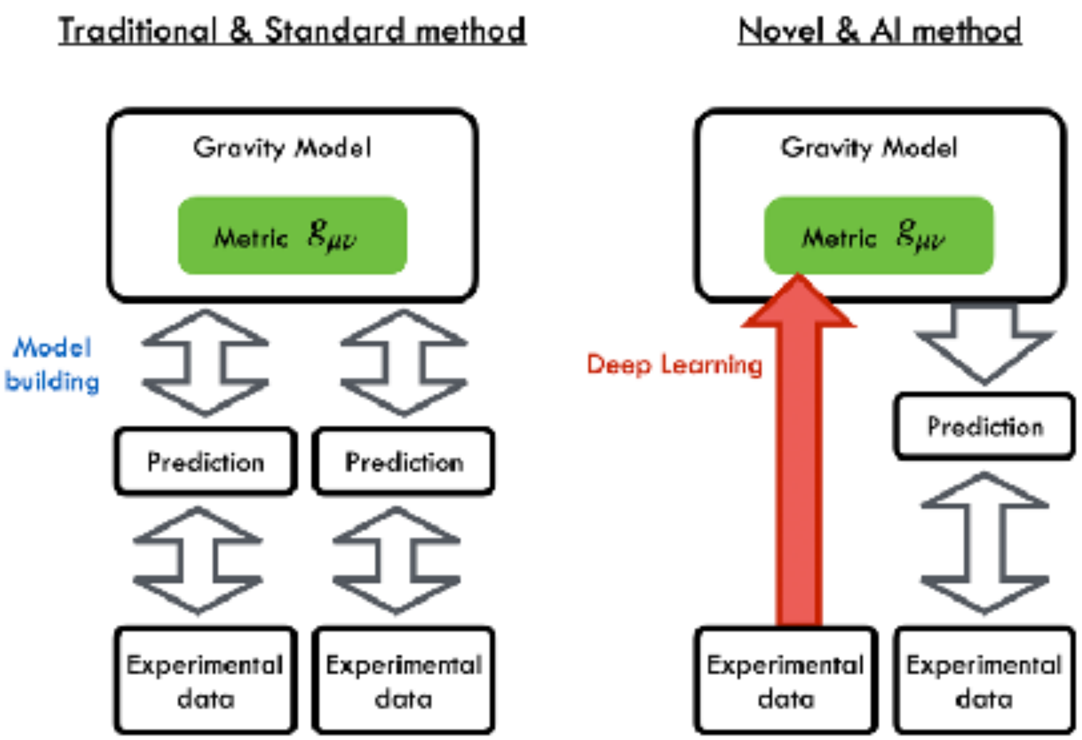
AI Question:

How to construct the extra (holographic) dimension?

as a deep neural network?

Holography

Deep Learning Bulk Spacetime from Boundary data



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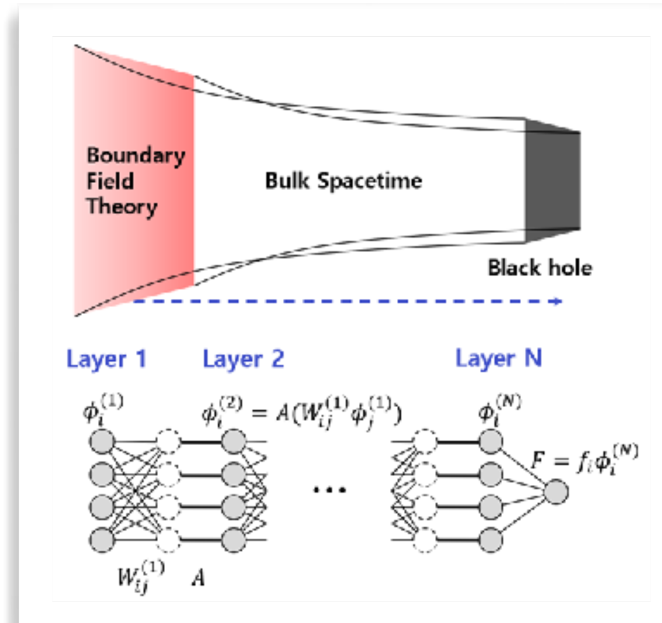

Traditional way: Bulk to Boundary
(physics intuition, principle (ex: symmetry), etc required)

Inverse Problem: from boundary to bulk
(physics intuition, symmetry(ex: symmetry), etc discovered)

For a difficult problem
Once we know a qualitative answer, we can understand its meaning more easily
(for example, "Linear T resistivity + T^2 Hall angle together")

Holography

Deep Learning Bulk Spacetime from Boundary data

2016 March

Why ML?

Surprisingly, there are still many new ways to play Go!

Machines may reveal unexpected new methods of understanding nature.

The role of ML

Knowing the answer (assisted by machines) is not the end of the story, but the beginning of human work.

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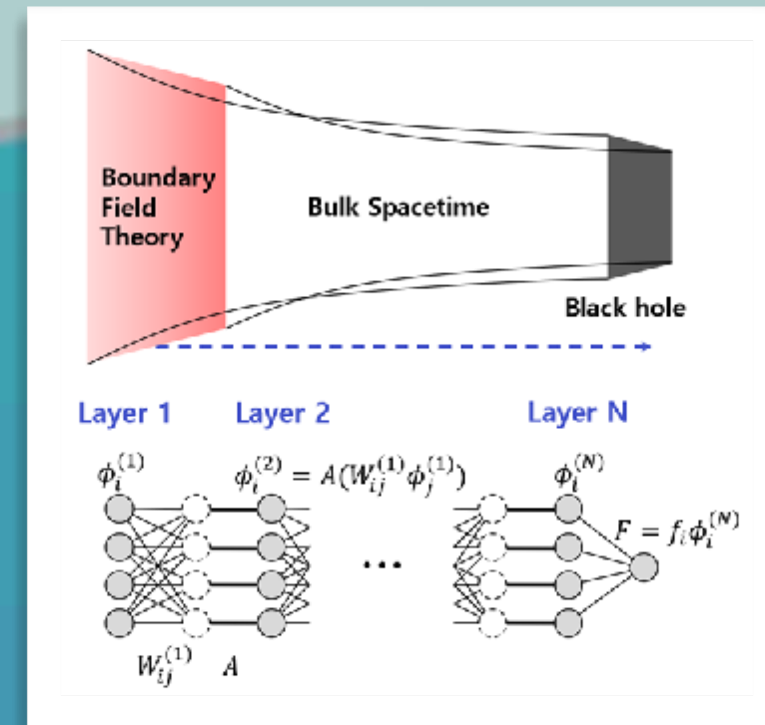
The status of ML as a "general" research tool

It's time to use machine learning as a toolbox.
You use Mathematica without fully understanding how it works.
You don't feel guilty using Mathematica, so using machine learning isn't cheating either.
Furthermore, I believe that ML will become as common as Python or C coding,
making it a must-learn 'language' for science majors.

My view point:
Someone must study machine learning for holography,
both for its fundamental understanding
and its practical benefits in solving difficult problems.



I would collaborate with Machine.

Original Idea

PHYSICAL REVIEW D **98,** 046019 (2018)

Deep learning and the AdS/CFT correspondence

Koji Hashimoto,[1] Sotaro Sugishita,[1] Akinori Tanaka,[2,3,4] and Akio Tomiya[5]

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[4]Interdisciplinary Theoretical & Mathematical Sciences Program (iTHEMS) RIKEN 2-1, Hirosawa, Wako, Saitama 351-0198, Japan
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We present a deep neural network representation of the AdS/CFT correspondence, and demonstrate the emergence of the bulk metric function via the learning process for given data sets of response in boundary quantum field theories. The emergent radial direction of the bulk is identified with the depth of the layers, and the network itself is interpreted as a bulk geometry. Our network provides a data-driven holographic modeling of strongly coupled systems. With a scalar ϕ^4 theory with unknown mass and coupling, in unknown curved spacetime with a black hole horizon, we demonstrate that our deep learning (DL) framework can determine the systems that fit given response data. First, we show that, from boundary data generated by the anti–de Sitter (AdS) Schwarzschild spacetime, our network can reproduce the metric. Second, we demonstrate that our network with experimental data as an input can determine the bulk metric, the mass and the quadratic coupling of the holographic model. As an example we use the experimental data of the magnetic response of the strongly correlated material $Sm_{0.6}Sr_{0.4}MnO_3$. This AdS/DL correspondence not only enables gravitational modeling of strongly correlated systems, but also sheds light on a hidden mechanism of the emerging space in both AdS and DL.

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AdS/Deep-Learning made easy: simple examples*

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[1]Gwangju Institute of Science and Technology (GIST), Department of Physics and Photon Science, Gwangju, South Korea
[2]University of California–Merced, Department of Physics, Merced, CA, USA

Abstract: Deep learning has been widely and actively used in various research areas. Recently, in gauge/gravity duality, a new deep learning technique called AdS/DL (Deep Learning) has been proposed. The goal of this paper is to explain the essence of AdS/DL in the simplest possible setups, without resorting to knowledge of gauge/gravity duality. This perspective will be useful for various physics problems: from the emergent spacetime as a neural network to classical mechanics problems. For prototypical examples, we choose simple classical mechanics problems. This method is slightly different from standard deep learning techniques in the sense that we not only have the right final answers but also obtain physical understanding of learning parameters.

Keywords: gauge/gravity duality, holographic principle, machine learning

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Deep learning bulk spacetime from boundary optical conductivity

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[b]*Instituto de Física Teórica UAM/CSIC, Calle Nicolás Cabrera 13-15, 28049 Madrid, Spain*
[c]*Departamento de Física Teórica, Universidad Autónoma de Madrid, 28049 Madrid, Spain*
[d]*Research Center for Photon Science Technology, Gwangju Institute of Science and Technology, 123 Cheomdan-gwagiro, Gwangju 61005, Korea*

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Reference III

Holographic reconstruction of black hole spacetime: machine learning and entanglement entropy

Byoungjoon Ahn,[a] **Hyun-Sik Jeong,**[b,c] **Keun-Young Kim**[a,d] **and Kwan Yun**[a]

[a] *Department of Physics and Photon Science, Gwangju Institute of Science and Technology, 123 Cheomdan-gwagiro, Gwangju 61005, Korea*

[b] *Instituto de Física Teórica UAM/CSIC, Calle Nicolás Cabrera 13-15, 28049 Madrid, Spain*

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ArXiv:2404:07395

Holography

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- Physics equation related Deep Learning I

  - Deep Learning for ODE: classical mechanics

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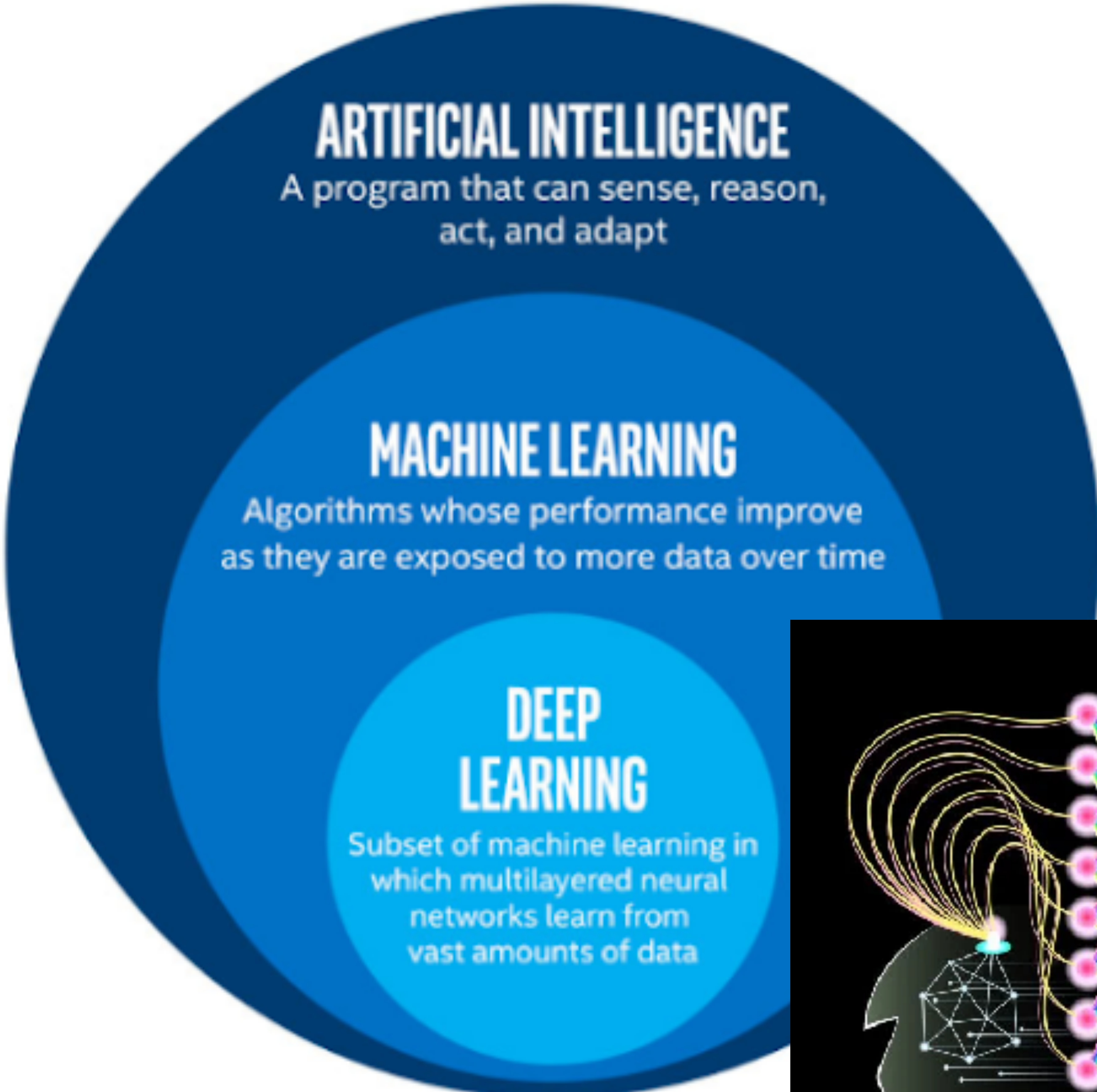
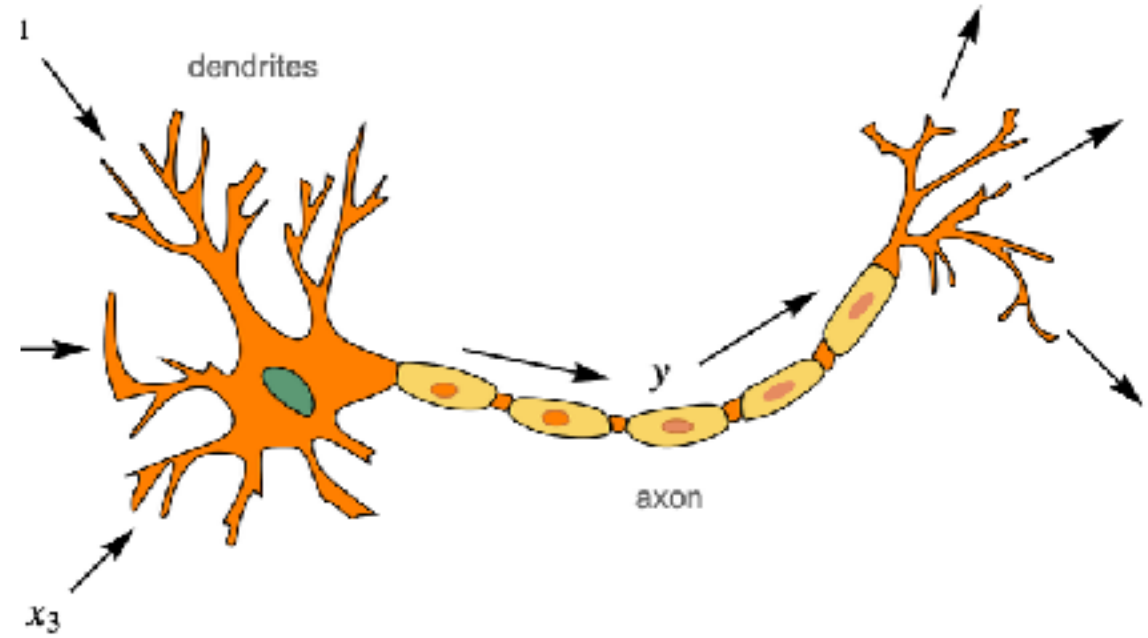
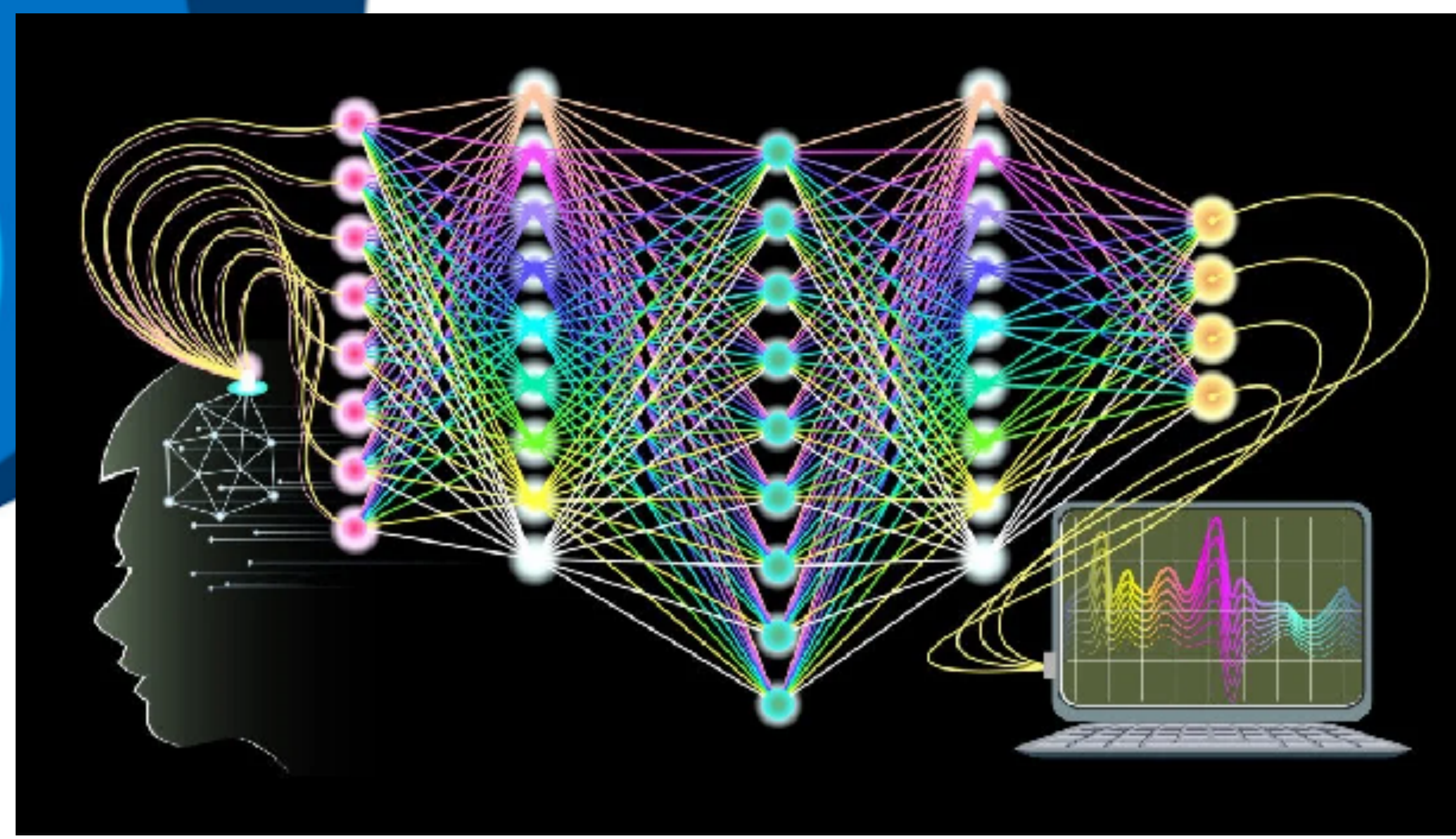
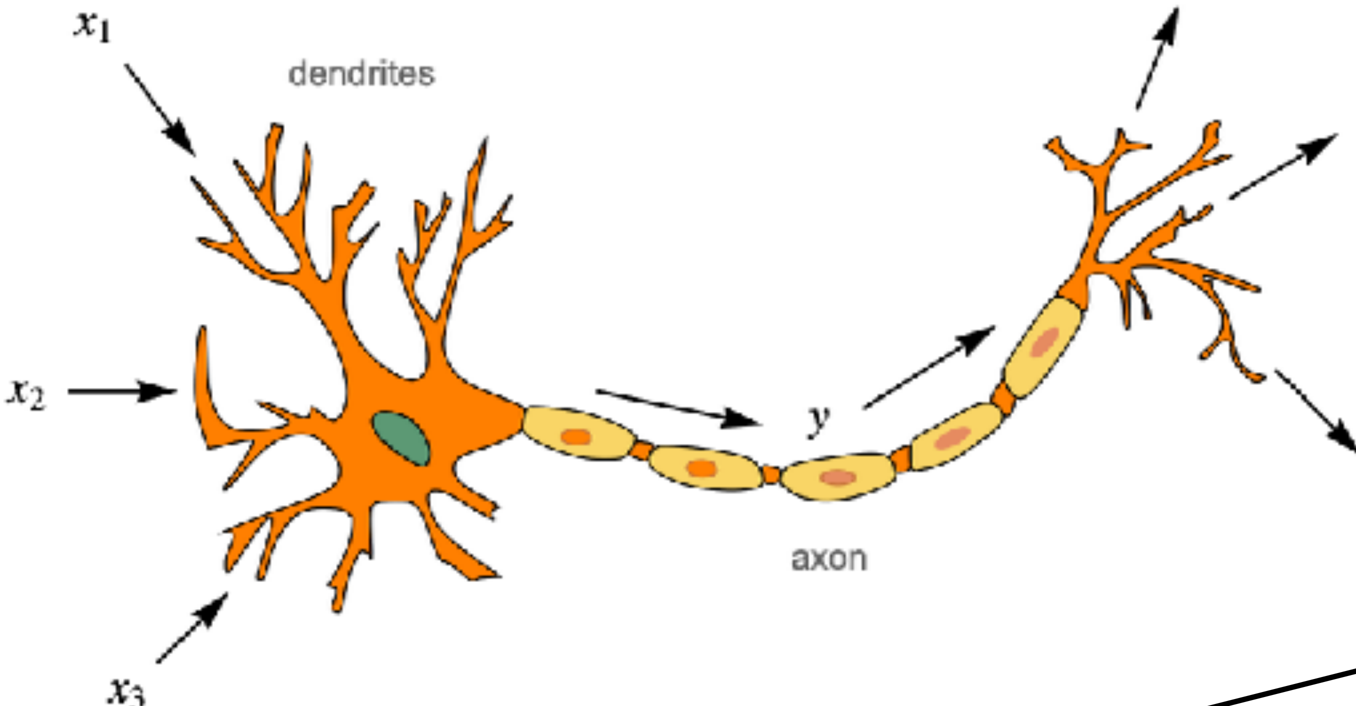
  - AdS/Deep Learning: Entanglement entropy

Deep Learning 101



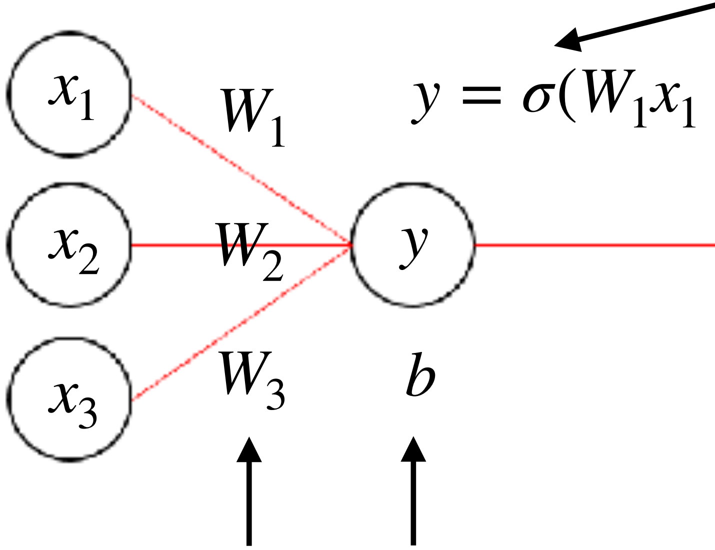




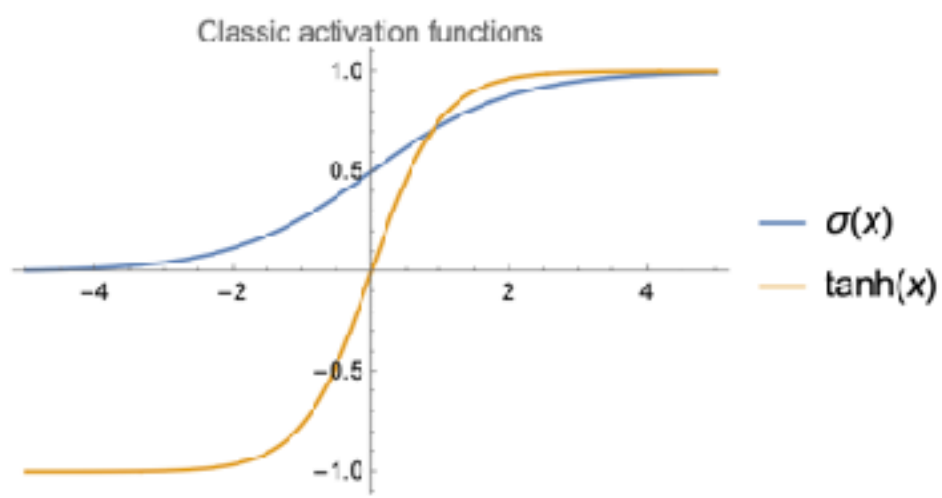


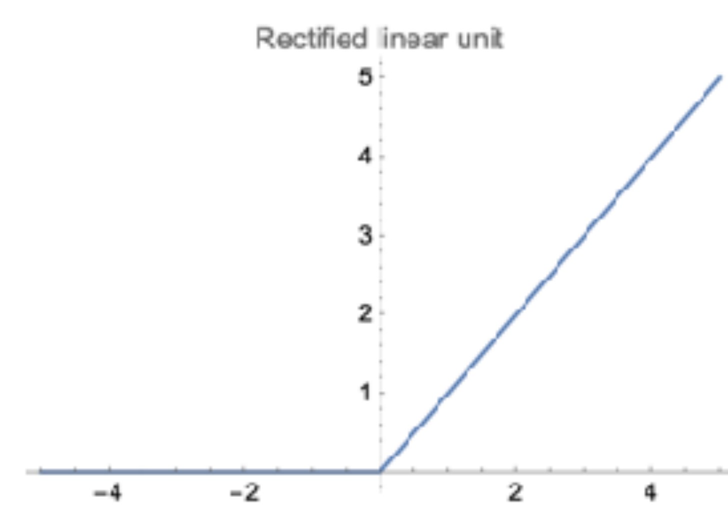
Activation function

$$y = \sigma(W_1x_1 + W_2x_2 + W_3x_3 + b)$$

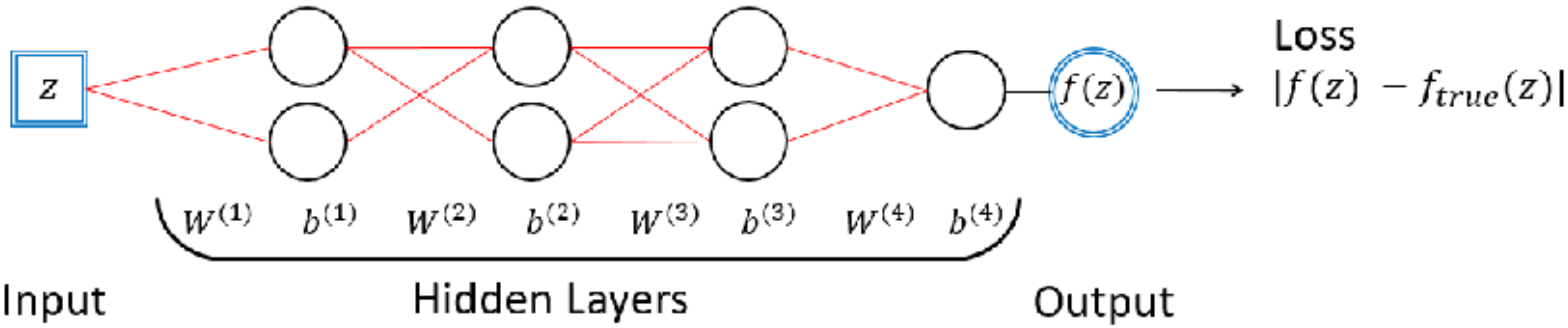


Weight Bias



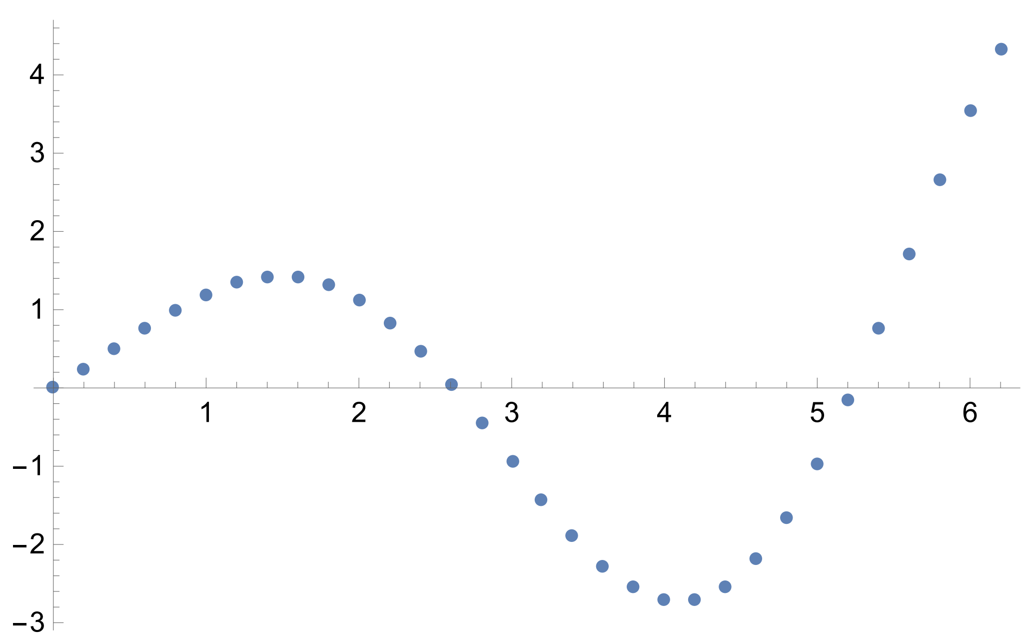


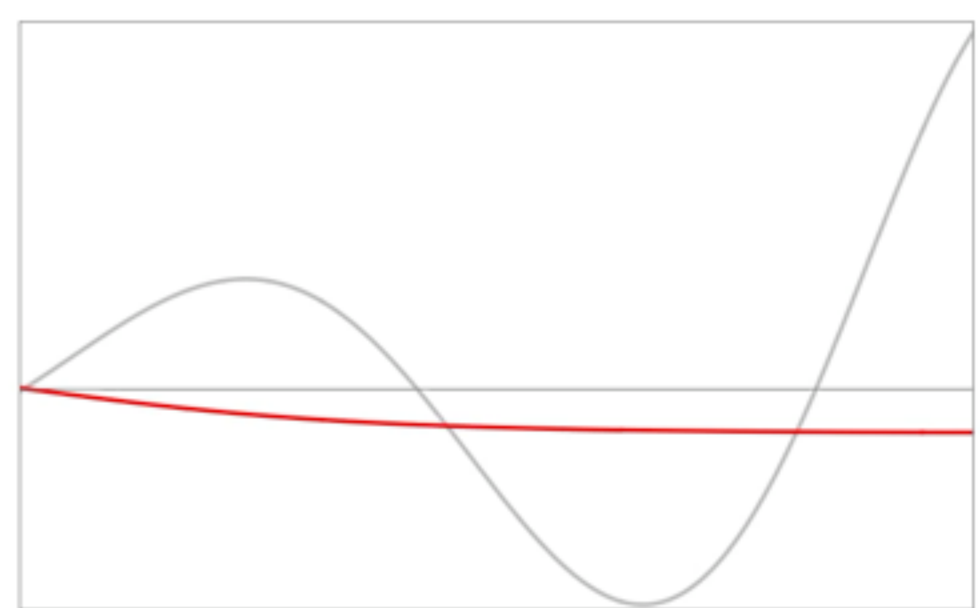
Standard Deep Learning

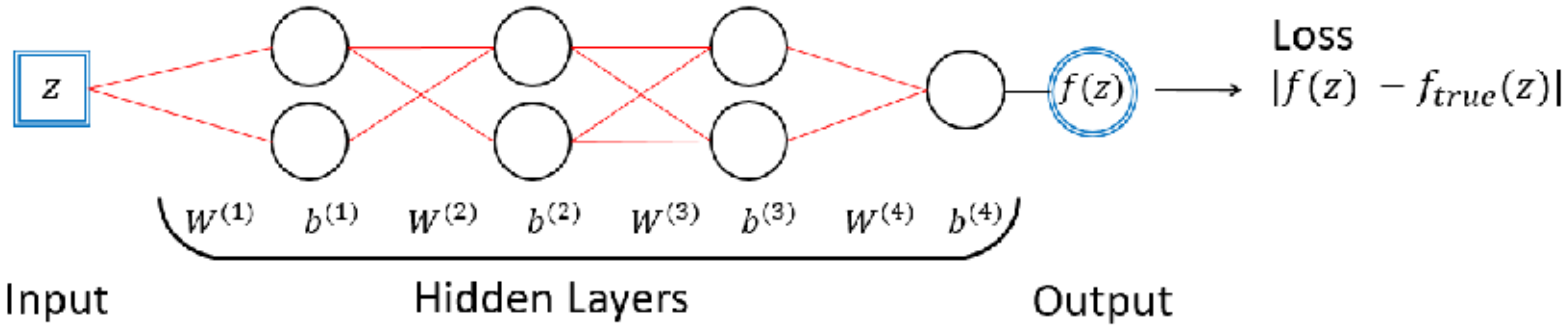


$$z_j^{(i+1)} = \sigma(W_{jk}^{(i)} \cdot z_k^{(i)} + b_j^{(i)})$$ ~2000 epoch

```
net = NetChain[{LinearLayer[2], Tanh, LinearLayer[2], Tanh, LinearLayer[2], Tanh, LinearLayer[]}]
      |네트워크 체인 |선형층              |쌍곡…  |선형층              |쌍곡…  |선형층              |쌍곡…  |선형층
solutions = NetTrain[net, data, <|"Property" → Function[#Net[Range[0, 2 π, 0.2]]], "Form" → "List",
            |네트워크 훈련                          |함수
   "Interval" → Quantity[30, "Rounds"]|>, (*LearningRate→0.001,*) TimeGoal → 600];
                |수량                                          |작업 시간 지정
```







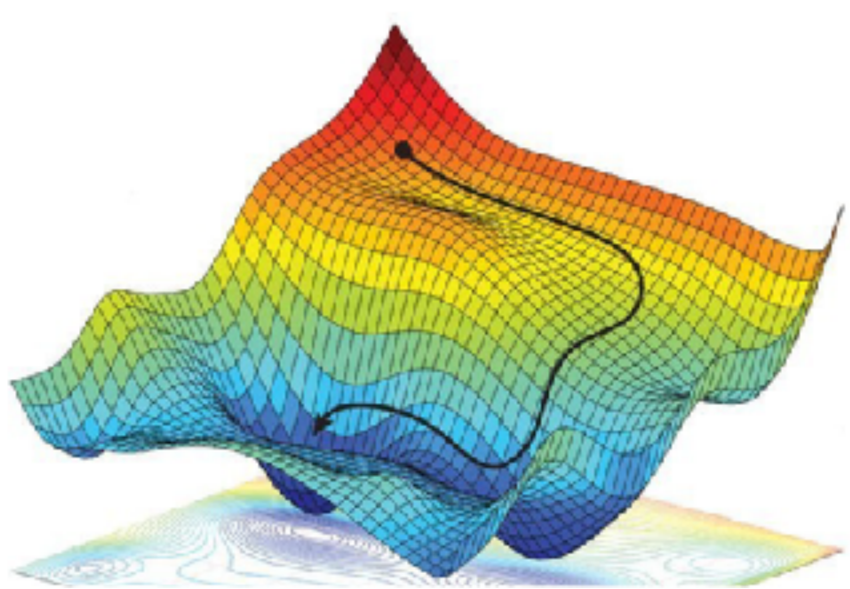
$$z_j^{(i+1)} = \sigma(W_{jk}^{(i)} \cdot z_k^{(i)} + b_j^{(i)})$$

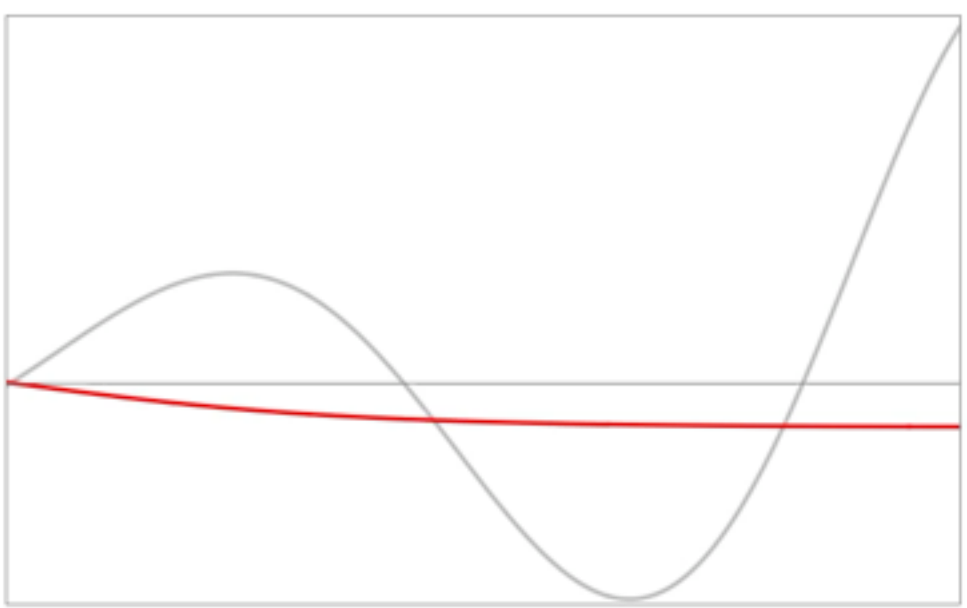
~2000 epoch

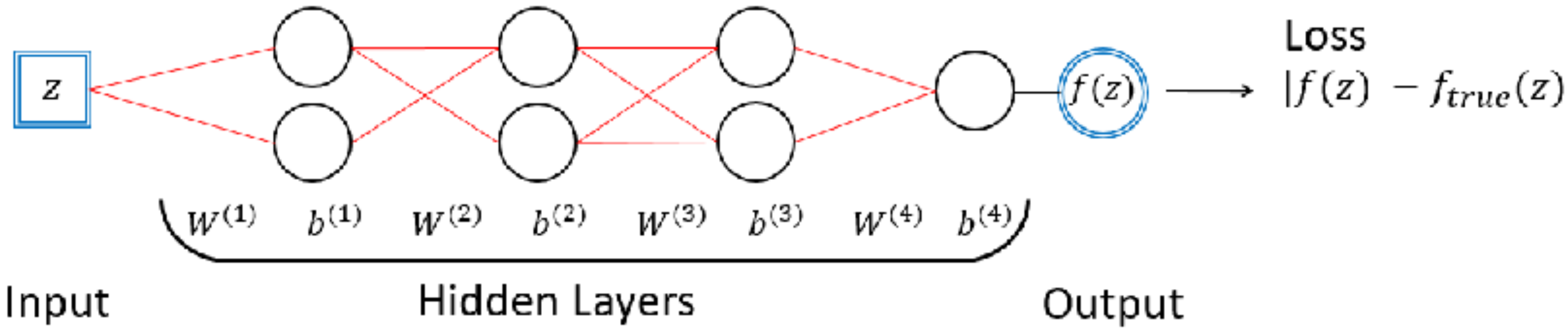
19 parameters

$$\tanh\left[\begin{pmatrix}\square & \square\end{pmatrix} \tanh\left[\begin{pmatrix}\square & \square \\ \square & \square\end{pmatrix}\left(\tanh\left[\begin{pmatrix}\square & \square \\ \square & \square\end{pmatrix}\left(\tanh\left[\begin{pmatrix}\square \\ \square\end{pmatrix} z + \begin{pmatrix}\square \\ \square\end{pmatrix}\right]\right) + \begin{pmatrix}\square \\ \square\end{pmatrix}\right] + \begin{pmatrix}\square \\ \square\end{pmatrix}\right)\right] + (\square)\right]$$

$$\begin{pmatrix}0.878421 & 6.21182\end{pmatrix} \begin{pmatrix}1.55725 & -2.14132 \\ -2.49736 & -1.0517\end{pmatrix} \begin{pmatrix}-1.21558 & -0.693247 \\ 2.86862 & 3.01908\end{pmatrix} \begin{pmatrix}0.317991 \\ 0.521212\end{pmatrix} \begin{pmatrix}-1.96854 \\ -0.020086\end{pmatrix} \begin{pmatrix}0.320358 \\ -0.614063\end{pmatrix} \begin{pmatrix}-0.442947 \\ 0.854547\end{pmatrix} \begin{pmatrix}1.33224\end{pmatrix}$$







$$z_j^{(i+1)} = \sigma(W_{jk}^{(i)} \cdot z_k^{(i)} + b_j^{(i)})$$ ~2000 epoch 19 parameters

tanh[(□ □) tanh[(□ □; □ □) (tanh[(□ □; □ □) (tanh[(□; □) z + (□; □)]) + (□; □)] + (□; □))] + (□)]

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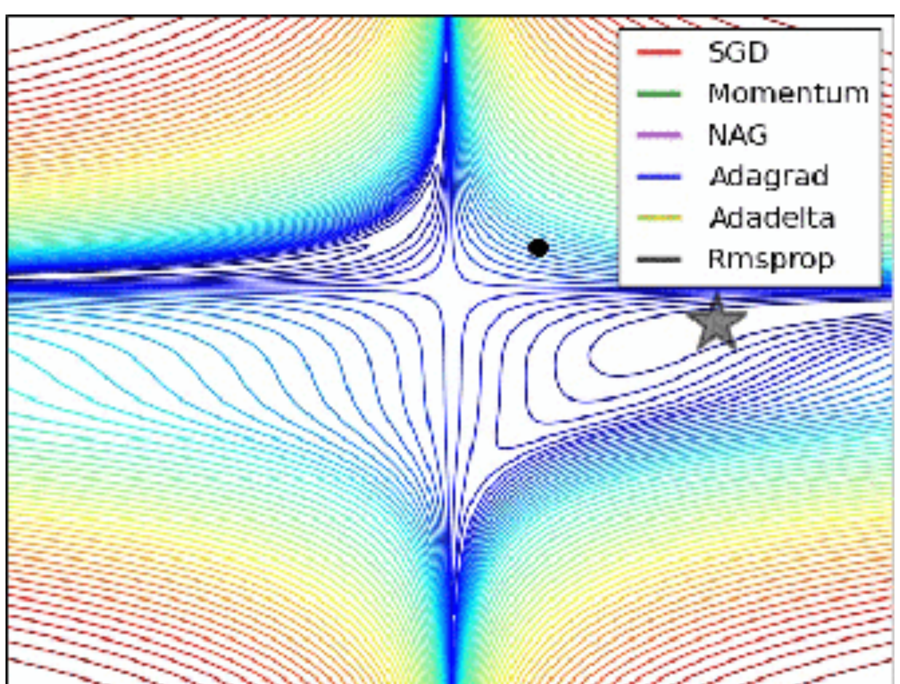
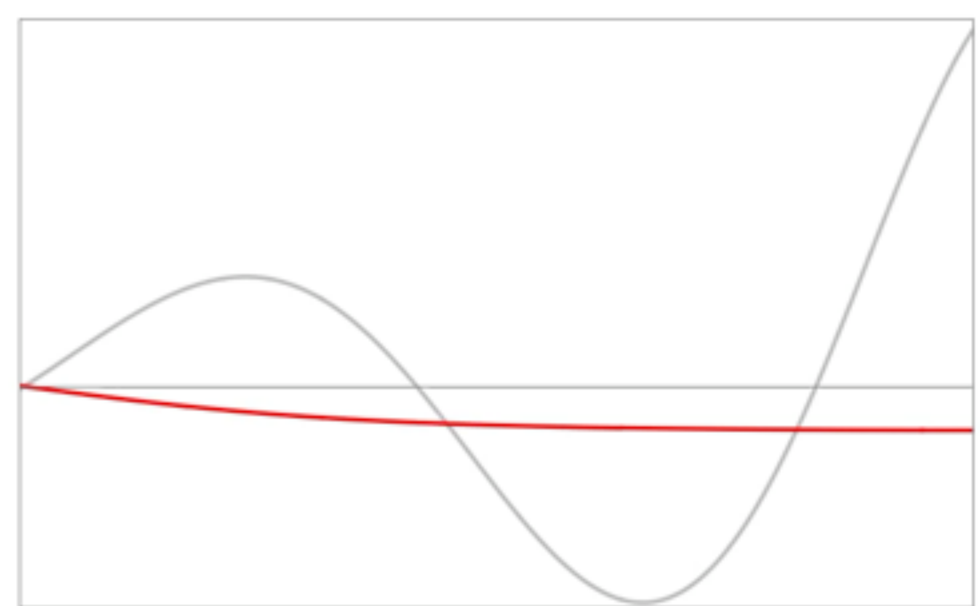
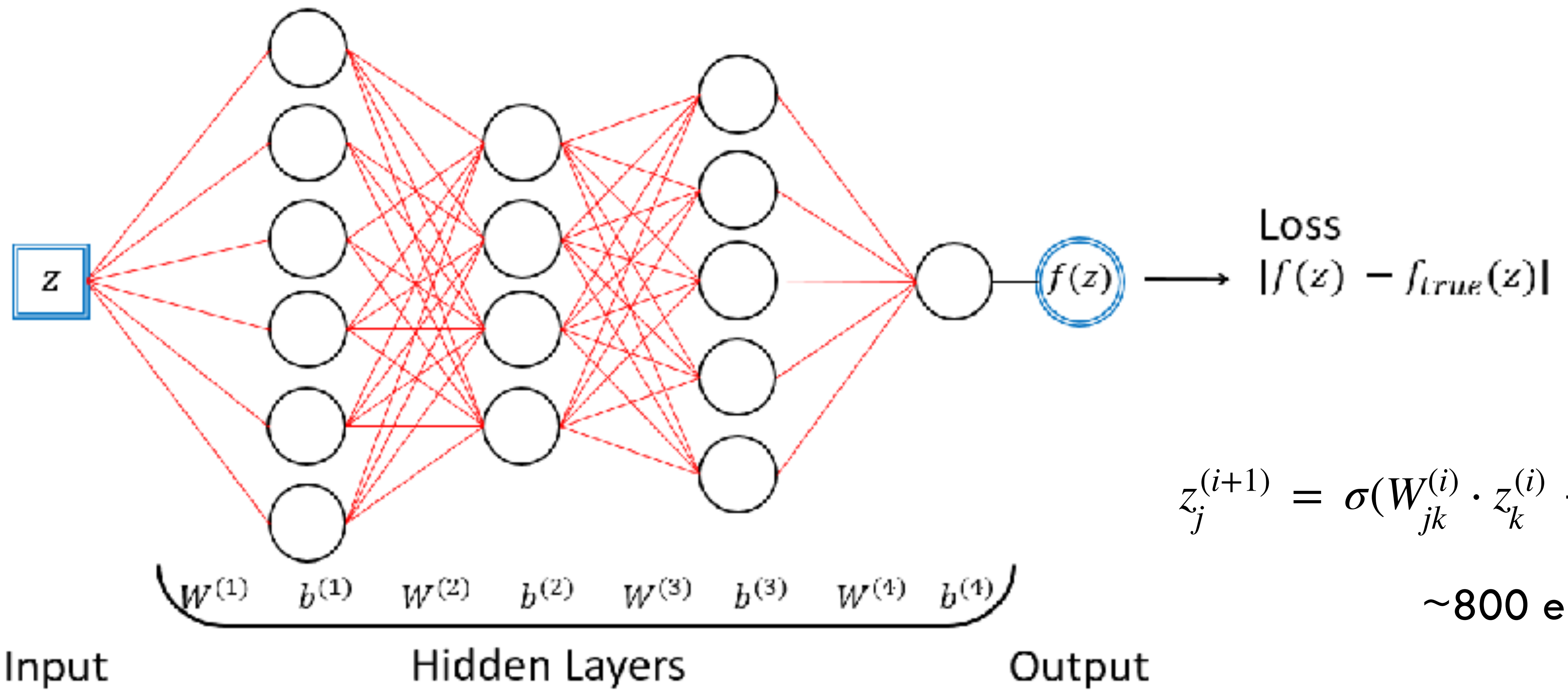




Standard Deep Learning



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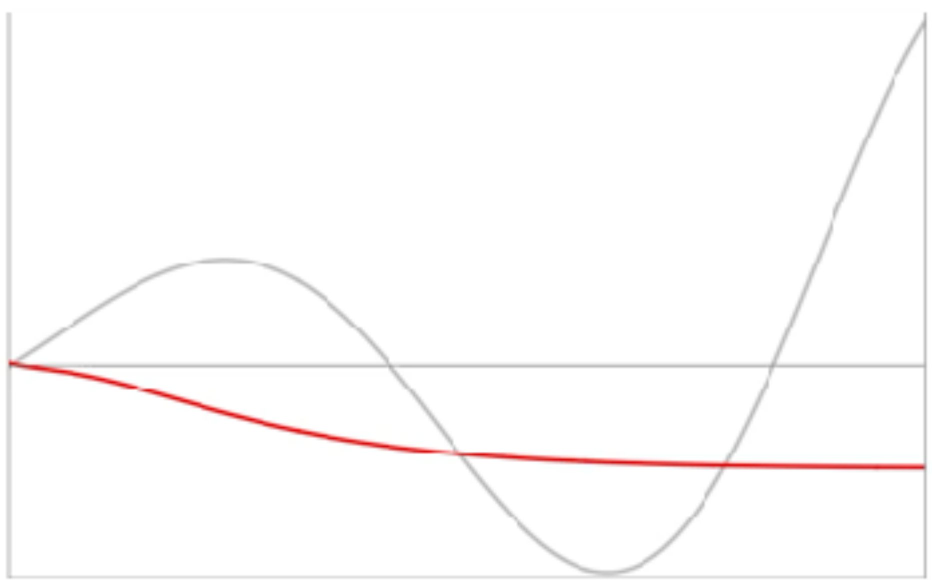
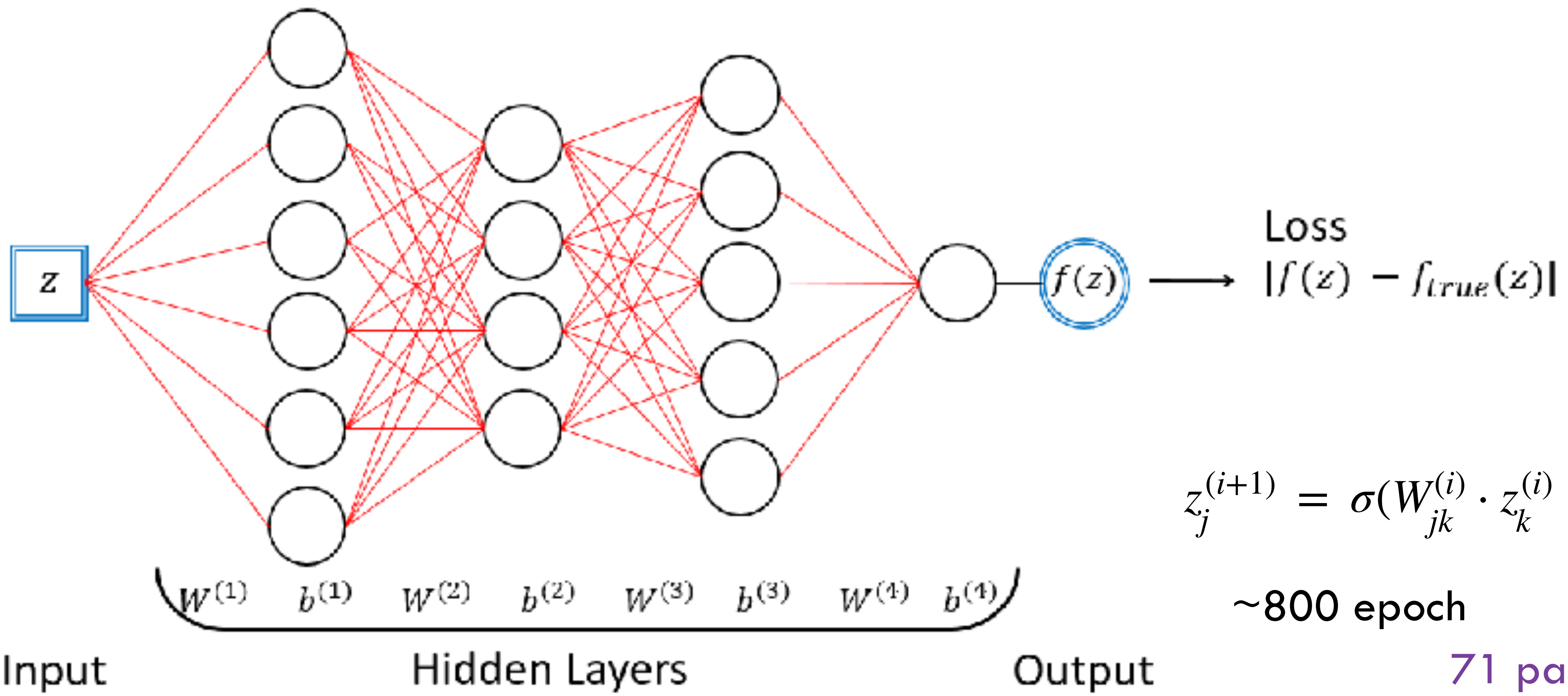
~800 epoch





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~800 epoch

71 parameters

$$\text{Show}\left[\left\{\begin{pmatrix}0.155467\\0.503477\\-0.19021\\-0.573821\\0.238013\\0.93589\end{pmatrix}, \begin{pmatrix}-0.421564\\-0.664526\\0.768171\\1.02467\\-1.01427\\-0.94677\end{pmatrix}, \begin{pmatrix}0.584066 & 0.331801 & -0.29404 & -0.586447 & 0.820878 & 1.04534\\0.208755 & 0.465765 & -1.21082 & -0.63667 & 1.73886 & -0.53486\\0.918991 & 0.328462 & -1.08441 & 0.474161 & -0.0382366 & -0.502777\\0.600695 & -0.32061 & -0.914458 & 0.73871 & 0.124301 & 0.402687\end{pmatrix},\right.\right.$$

$$\begin{pmatrix}0.172725\\-0.0946543\\0.0526899\\0.201293\end{pmatrix}, \begin{pmatrix}0.308126 & -1.0333 & 1.21593 & 1.62945\\-0.217696 & 0.696293 & -0.338222 & -2.68779\\0.0759481 & -0.664867 & 0.86521 & 1.7725\\0.445185 & 0.0762033 & -1.15522 & -2.01414\\-0.595479 & 0.916218 & -0.400813 & -2.12668\end{pmatrix}, \begin{pmatrix}0.621626\\-0.192633\\0.402367\\-0.14795\\-0.331163\end{pmatrix},$$

$$\left.\left.\begin{pmatrix}0.903621 & -2.09077 & 1.97153 & -1.85892 & -1.05424\end{pmatrix}, \begin{pmatrix}0.319326\end{pmatrix}\right\}\right]$$

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Original Idea

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AdS/Deep-Learning made easy: simple examples*

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Abstract: Deep learning has been widely and actively used in various research areas. Recently, in gauge/gravity duality, a new deep learning technique called AdS/DL (Deep Learning) has been proposed. The goal of this paper is to explain the essence of AdS/DL in the simplest possible setups, without resorting to knowledge of gauge/gravity duality. This perspective will be useful for various physics problems: from the emergent spacetime as a neural network to classical mechanics problems. For prototypical examples, we choose simple classical mechanics problems. This method is slightly different from standard deep learning techniques in the sense that we not only have the right final answers but also obtain physical understanding of learning parameters.

Keywords: gauge/gravity duality, holographic principle, machine learning

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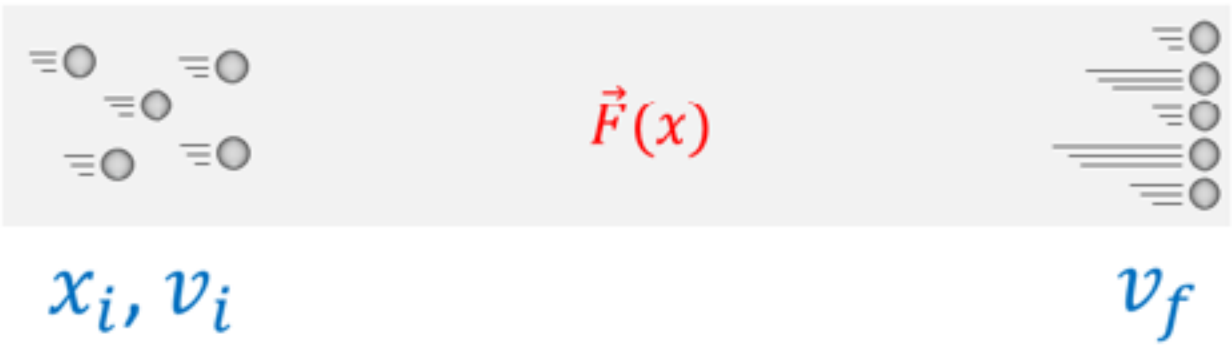
$$m\ddot{x} = F(x)$$

$$\dot{x} = v\,, \quad \dot{v} = \frac{1}{m}F$$



Deep Learning for ODE: classical mechanics

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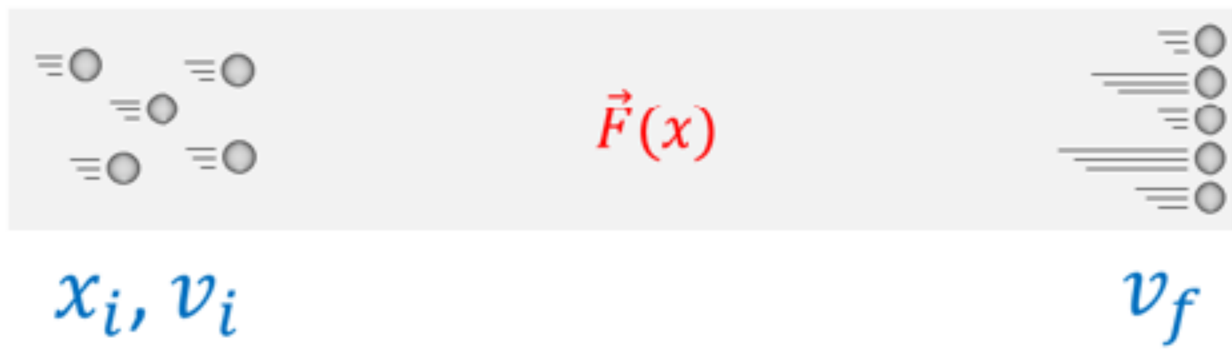

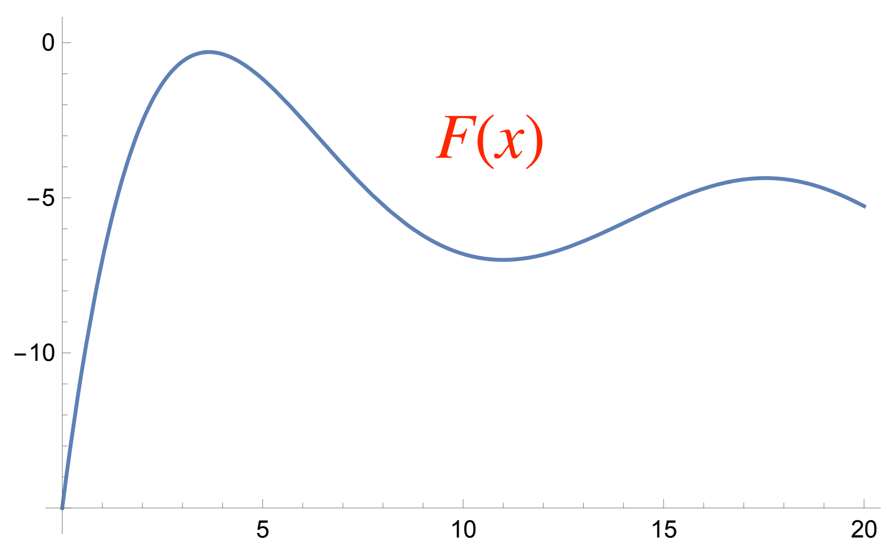

$$F_{2,\mathrm{True}}(x) = \frac{1}{8000}(x-1)(x-11)^2(x-23)^2 - 0.7$$

Deep Learning for ODE: classical mechanics

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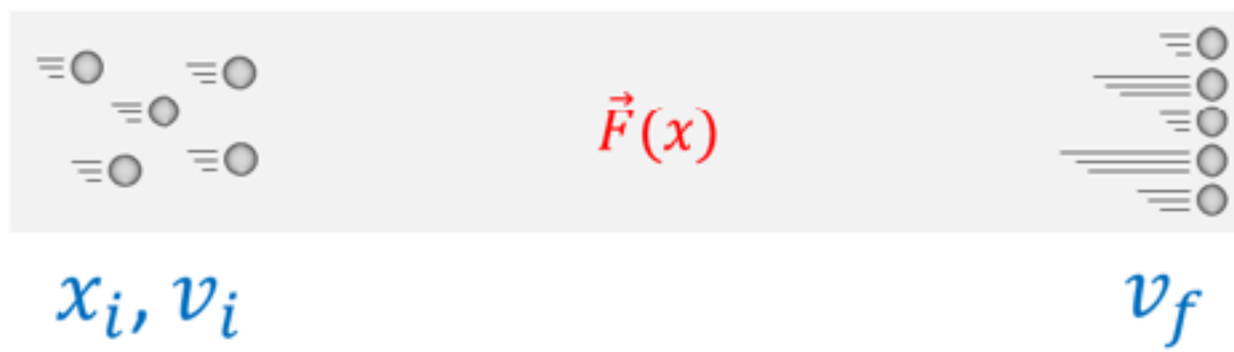

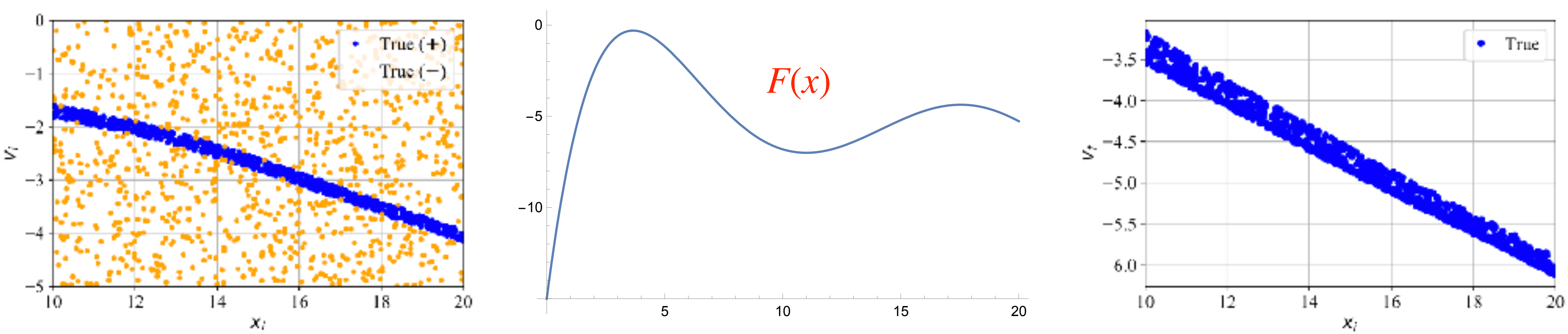

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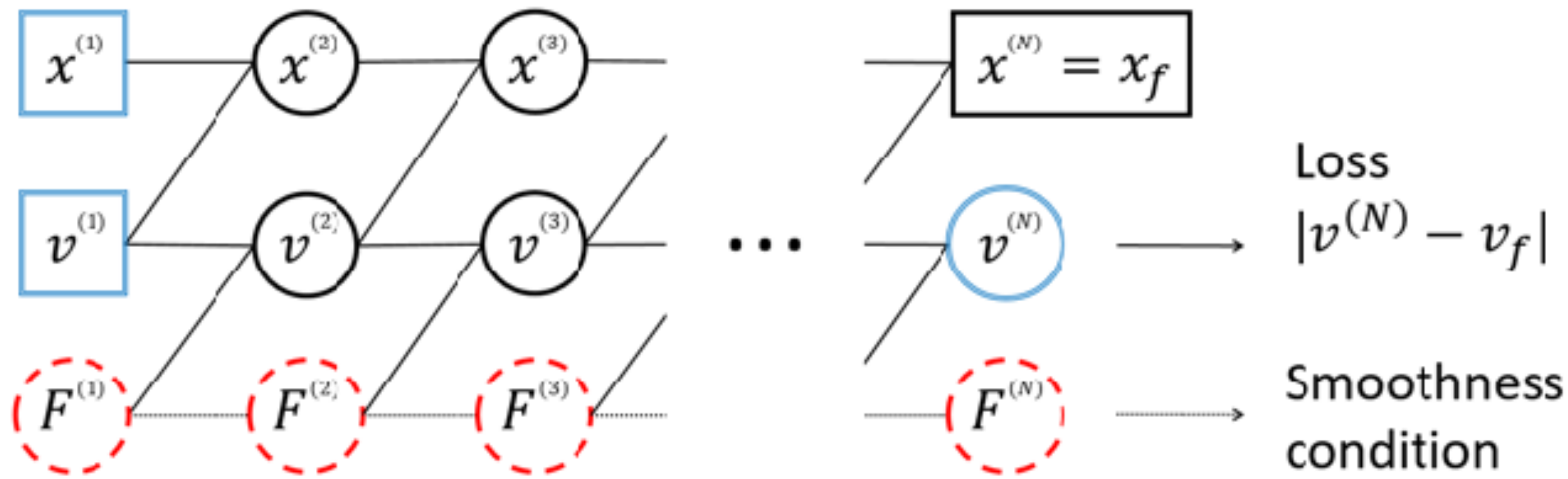
Deep Learning for ODE: classical mechanics

$$x^{(i+1)} = x^{(i)} + \Delta t \cdot v^{(i)}$$

$$v^{(i+1)} = v^{(i)} + \Delta t \cdot F(x^{(i)})$$



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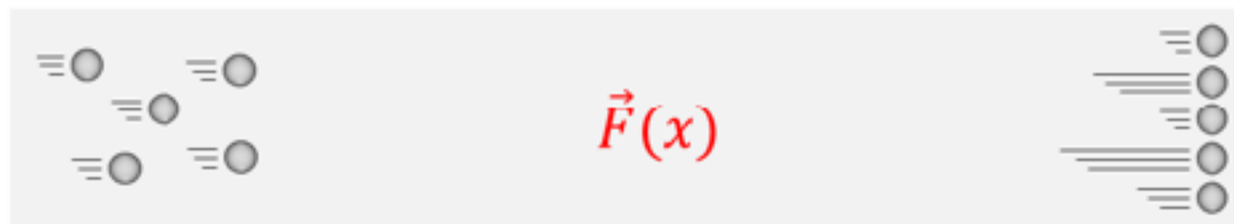
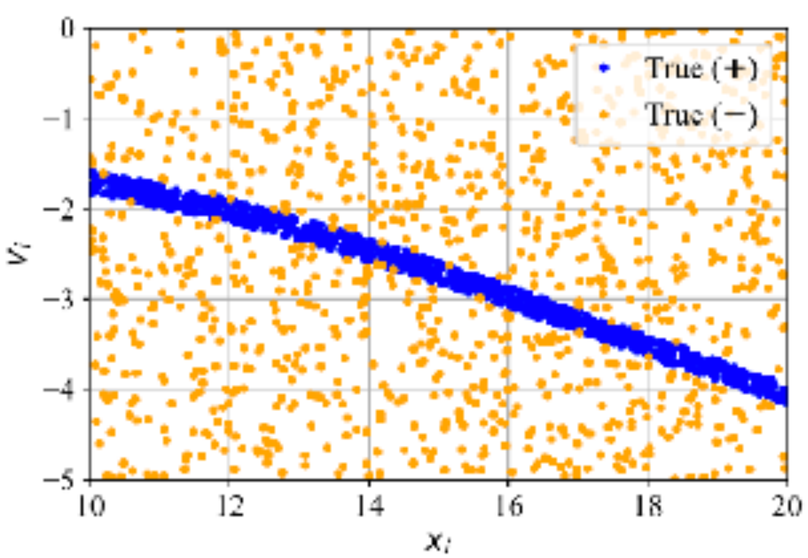
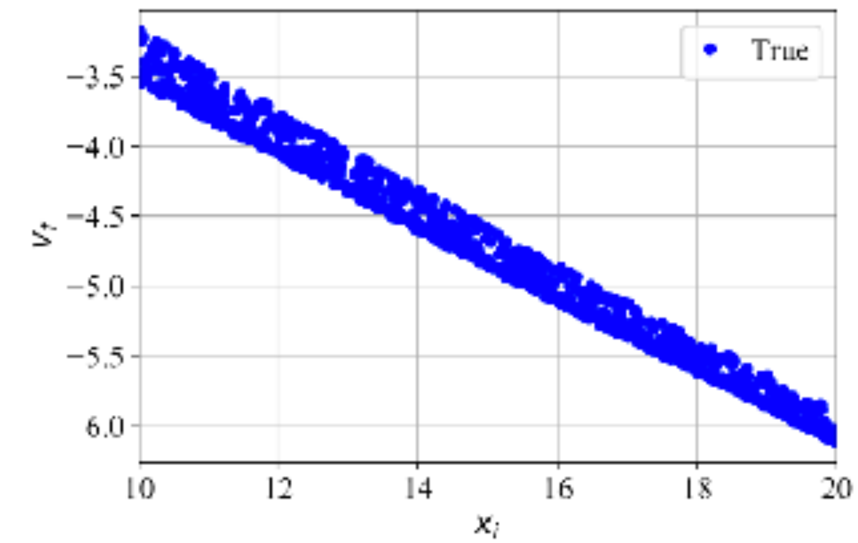


x_i, v_i v_f





Deep Learning for ODE: classical mechanics

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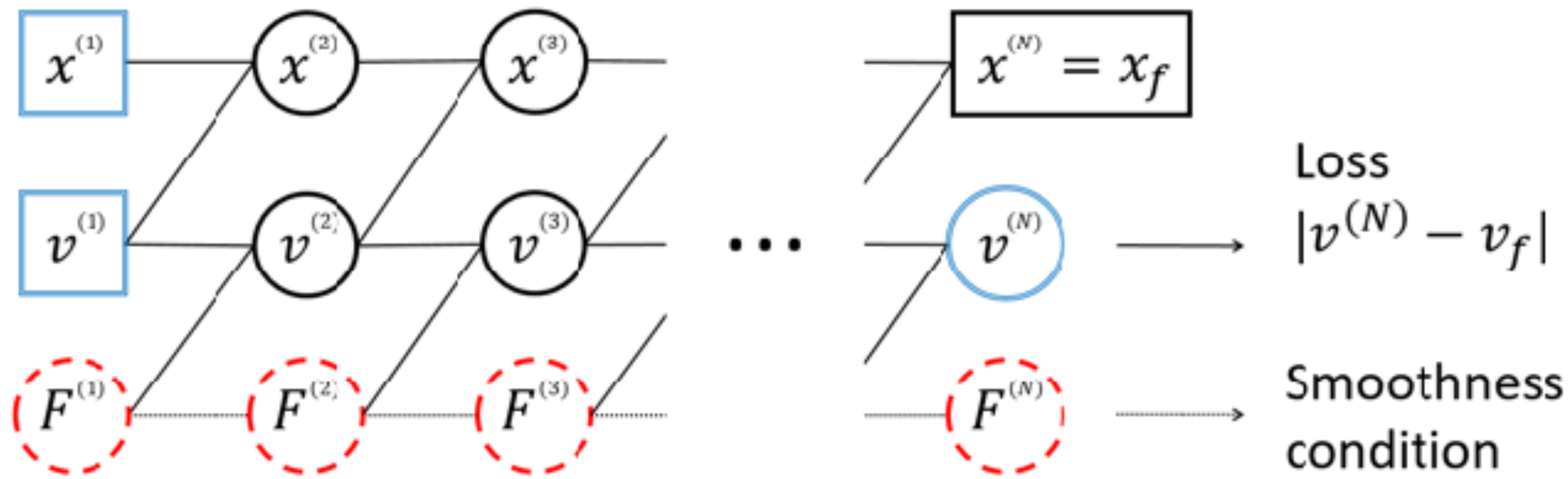

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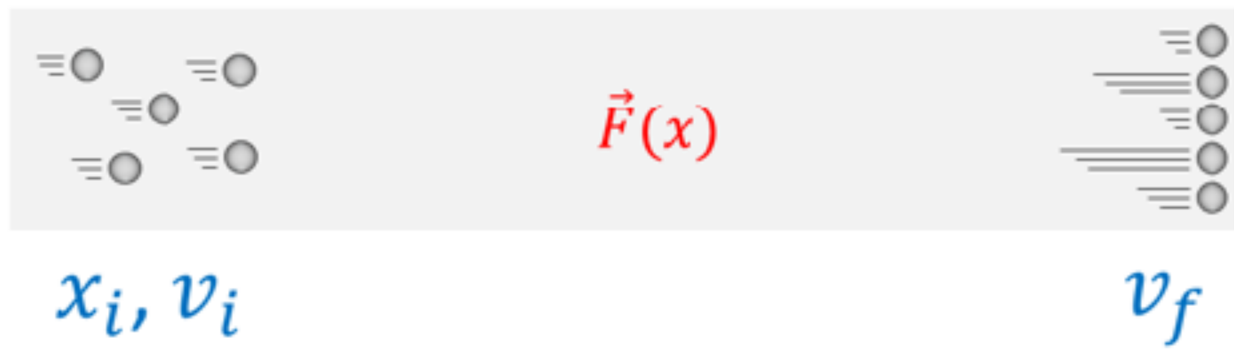

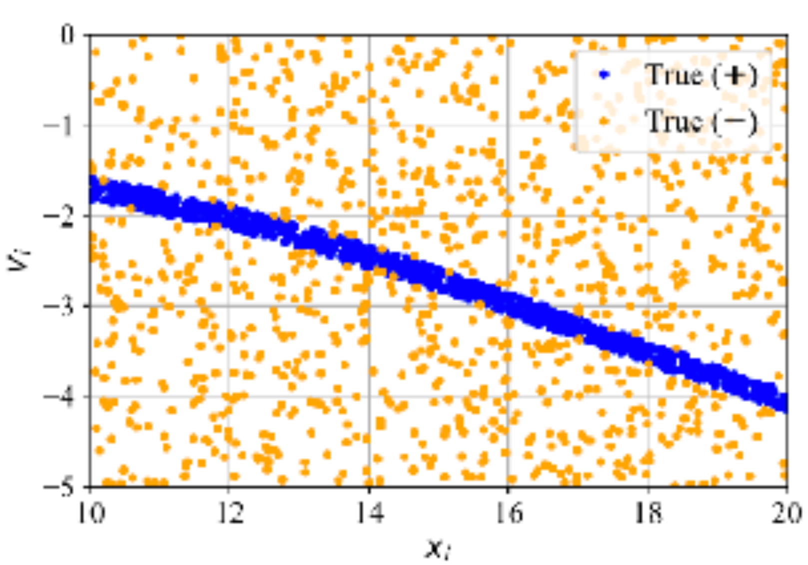

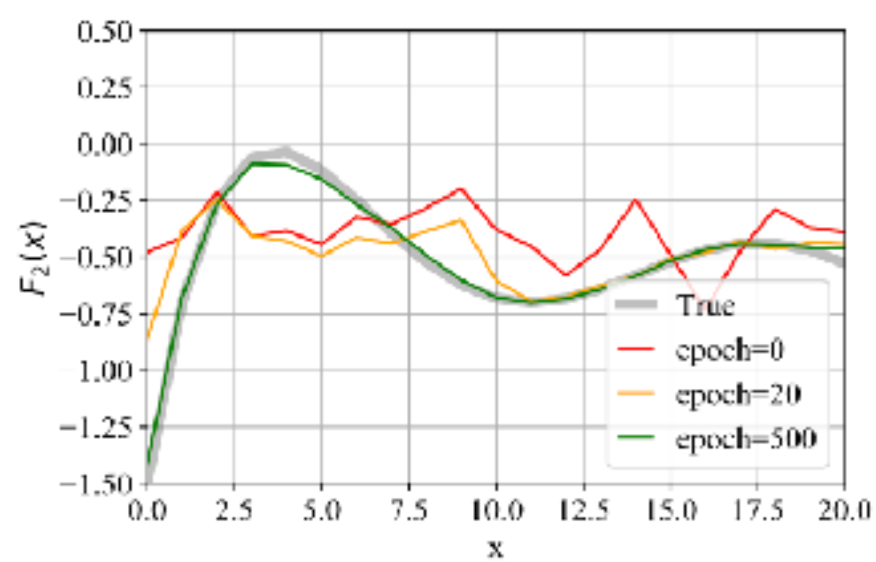

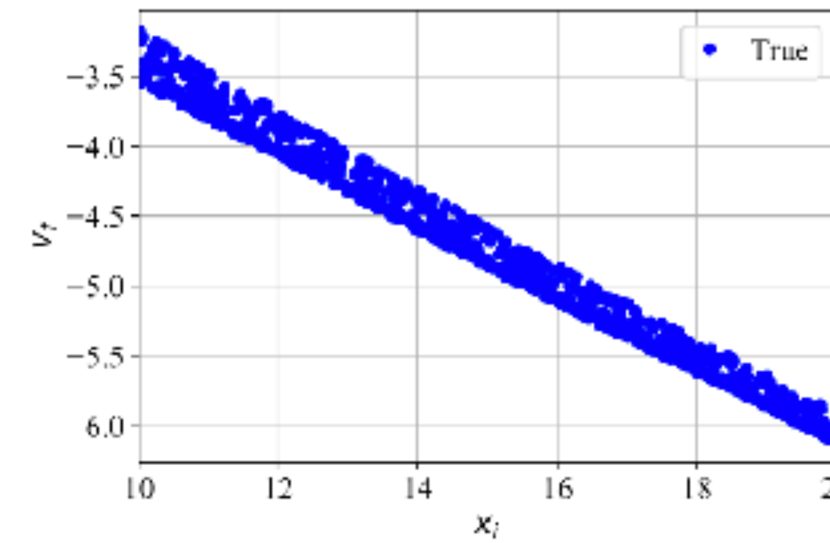

$$F_{2,\mathrm{True}}(x) = \frac{1}{8000}(x-1)(x-11)^2(x-23)^2 - 0.7$$

- What is “Machine Learning” & “Deep Learning”?
  - Deep Learning 101: standard story
- Physics equation related Deep Learning I
  - Deep Learning for ODE: classical mechanics
  - AdS/Deep Learning: optical conductivity
- Physics equation related Deep Learning II
  - Deep Learning for Integral: classical electrostatics
  - AdS/Deep Learning: Entanglement entropy

AdS/Deep learning: optical conductivity

$$m\ddot{x} = F$$

$$A(z)f''(z)+B(z)f'(z)+C(z)f(z) = F(z)$$

$$A(z)f''(z)+B(z)f'(z)+C(z)f(z) = D(z)g(z)$$

$$E(z)g''(z)+F(z)g'(z)+G(z)g(z) = H(z)h(z)$$

$$\partial_z^2 A_x = \zeta\,\partial_z A_x + \left(\frac{z^2\mu^2}{f} - \xi\right) A_x + \frac{iz\mu}{f}\Phi\,,$$

$$\partial_z^2 \Phi = \zeta\,\partial_z \Phi + \left(\frac{\alpha^2}{f} + \frac{f'}{zf} - \xi\right) \Phi - \frac{iz\alpha^2\mu}{f} A_x\,,$$

$$\zeta := \frac{2i\omega}{(1-z)f'(1)} - \frac{f'(z)}{f(z)}\,, \qquad \xi := \frac{\omega^2}{f(z)^2} + \frac{i\omega}{(1-z)f'(1)}\left(\frac{i\omega}{(1-z)f'(1)} - \frac{1}{1-z} - \frac{f'(z)}{f(z)}\right)$$

AdS/Deep learning: optical conductivity

Action

$$S=\int d^4x\sqrt{-g}\left(R+6-\frac{1}{4}F_{ab}F^{ab}-\frac{1}{2}\sum_{I=1}^{2}(\partial X_I)^2\right)$$

EOM

$$R_{ab}-\frac{1}{2}g_{ab}\left(R+6-\frac{1}{4}F_{ab}F^{ab}-\frac{1}{2}\sum_{I=1}^{2}(\partial X_I)^2\right)-F_{ac}F_b^c-\frac{1}{2}\sum_{I=1}^{2}\partial_a X_I\partial_b X_I=0\,,$$

$$\nabla^a F_{ab}=0\,,\qquad \nabla_a\nabla^a X_I=0\,,$$

Background

$$ds^2=\frac{1}{z^2}\left[-f(z)dt^2+\frac{dz^2}{f(z)}+dx^2+dy^2\right],\qquad f(z)=1-\frac{\alpha^2}{2}z^2-\left(1-\frac{\alpha^2}{2}+\frac{\mu^2}{4}\right)z^3+\frac{\mu^2}{4}z^4\,,$$

$$A=\mu(1-z)\,dt\,,\qquad X_1=\alpha\,x\,,\quad X_2=\alpha\,y$$

Flucutation EOM I

$$\delta g_{tx}=e^{-i\omega t}\frac{h_{tx}(z)}{z^2}\,,\qquad \delta A_x=e^{-i\omega t}a_x(z)\,,\qquad \delta X_1=e^{-i\omega t}\frac{\psi_x(z)}{\alpha}\,,$$

$$a_x''(z)+\frac{f'(z)}{f(z)}a_x'(z)+\left(\frac{\omega^2}{f(z)^2}-\frac{\mu^2z^2}{f(z)}\right)a_x(z)-\frac{i\mu z}{f(z)}\phi(z)=0\,,\qquad \phi(z):=-\frac{f(z)\psi_x'(z)}{\omega\,z}$$

$$\phi''(z)+\frac{f'(z)}{f(z)}\phi'(z)+\left(\frac{\omega^2}{f(z)^2}-\frac{\alpha^2}{f(z)}-\frac{f'(z)}{zf(z)}\right)\phi(z)+\frac{i\alpha^2\mu z}{f(z)}a_x(z)=0\,,$$

$$\sigma(\omega)=\frac{1}{i\omega}G^R_{j^xj^x}(\omega)=\frac{1}{i\omega}\frac{a_x^{(R)}}{a_x^{(S)}}=\frac{A_x'(z_{\rm fin})}{i\omega A_x(z_{\rm fin})}-\frac{1}{f'(1)}$$

Flucutation EOM II

$$A_x(z):=(1-z)^{-\frac{i\omega}{f'(1)}}a_x(z)\,,\qquad \Phi(z):=(1-z)^{-\frac{i\omega}{f'(1)}}\phi(z)\,,$$

$$\partial_z^2A_x=\zeta\,\partial_zA_x+\left(\frac{z^2\mu^2}{f}-\xi\right)A_x+\frac{iz\mu}{f}\Phi\,,$$

$$\partial_z^2\Phi=\zeta\,\partial_z\Phi+\left(\frac{\alpha^2}{f}+\frac{f'}{zf}-\xi\right)\Phi-\frac{iz\alpha^2\mu}{f}A_x\,,$$

$$\zeta:=\frac{2i\omega}{(1-z)f'(1)}-\frac{f'(z)}{f(z)}\,,\qquad \xi:=\frac{\omega^2}{f(z)^2}+\frac{i\omega}{(1-z)f'(1)}\left(\frac{i\omega}{(1-z)f'(1)}-\frac{1}{1-z}-\frac{f'(z)}{f(z)}\right)$$

AdS/Deep learning: optical conductivity

$$m\ddot{x} = F(x) \qquad \dot{x} = v\,, \quad \dot{v} = \frac{1}{m}F \qquad \begin{aligned} x^{(i+1)} &= x^{(i)} + \Delta t \cdot v^{(i)} \\ v^{(i+1)} &= v^{(i)} + \Delta t \cdot F(x^{(i)}) \end{aligned}$$

$$\partial_z^2 A_x = \zeta\, \partial_z A_x + \left(\frac{z^2\mu^2}{f} - \xi\right) A_x + \frac{iz\mu}{f}\Phi\,,$$

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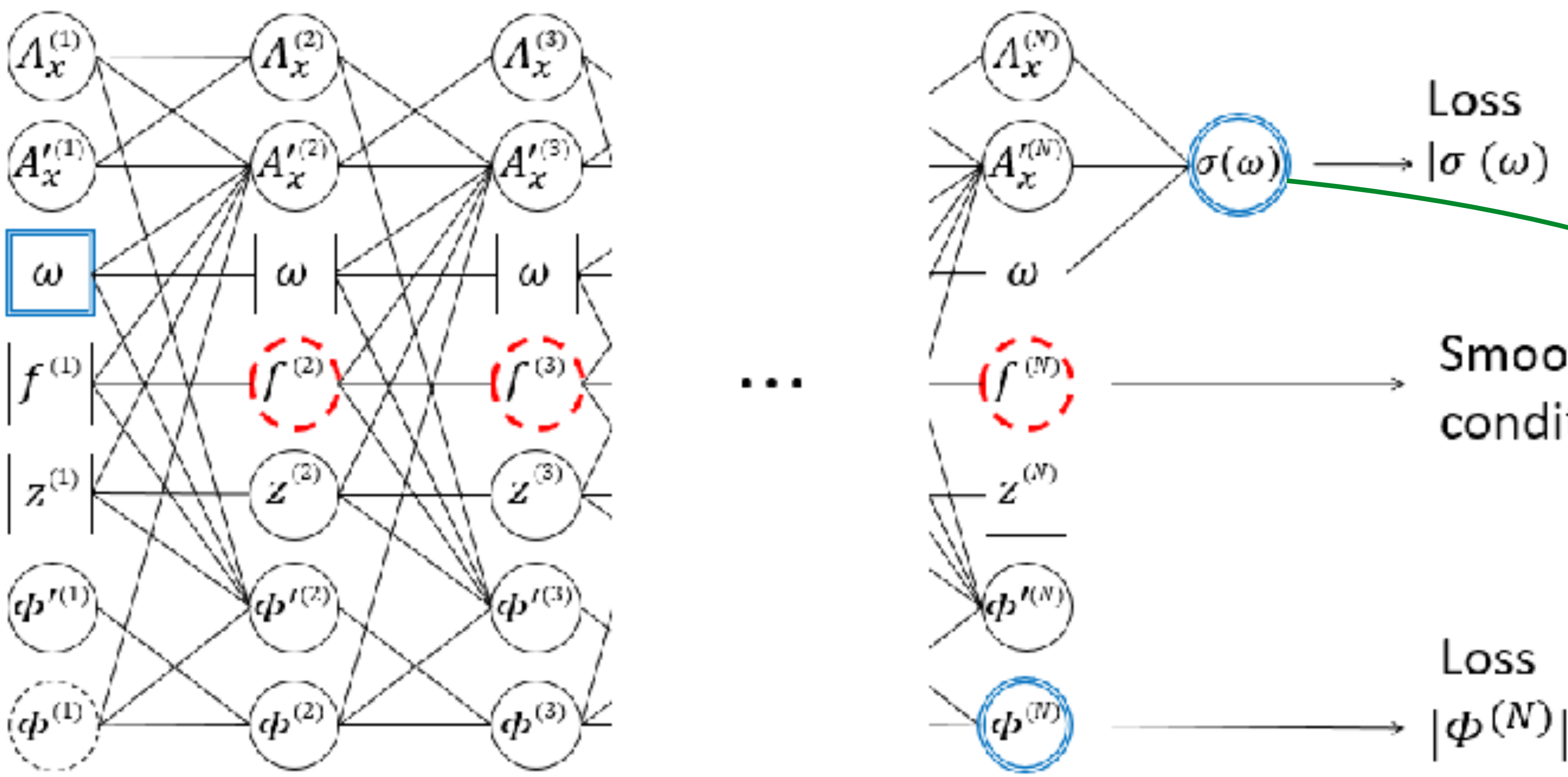
$$A_x^{(i+1)} = A_x^{(i)} + \Delta z \cdot A_x'^{(i)}$$
$$\Phi^{(i+1)} = \Phi^{(i)} + \Delta z \cdot \Phi'^{(i)}$$

$$A_x'^{(i+1)} = \left(\frac{z^2\mu^2}{f} - \xi\right) \Delta z A_x^{(i)} + \frac{iz\mu}{f} \Delta z \Phi^{(i)} + (1 + \zeta \Delta z)\, A_x'^{(i)}$$

$$\Phi'^{(i+1)} = -\frac{iz\alpha^2\mu}{f}\Delta z A_x^{(i)} + \left(\frac{\alpha^2}{f} + \frac{f'}{zf} - \xi\right) \Delta z \Phi^{(i)} + (1 + \zeta\Delta z)\, \Phi'^{(i)}$$

AdS/Deep learning: optical conductivity



$$\sigma(\omega) = \frac{1}{i\omega} G^R_{j^x j^x}(\omega) = \frac{1}{i\omega}\frac{a_x^{(R)}}{a_x^{(S)}}$$
$$= \frac{A'_x(z_{\rm fin})}{i\omega A_x(z_{\rm fin})} - \frac{1}{f'(1)}$$

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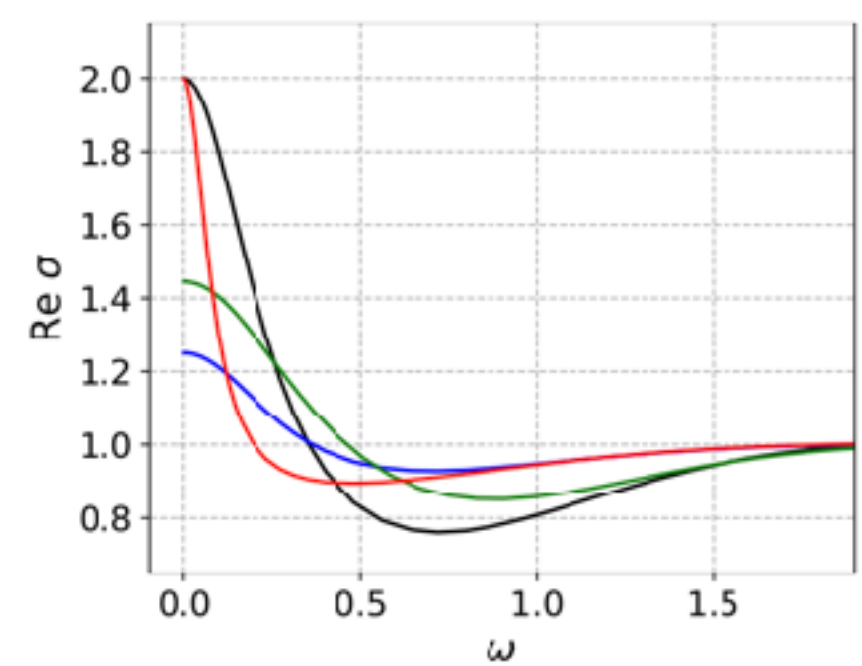
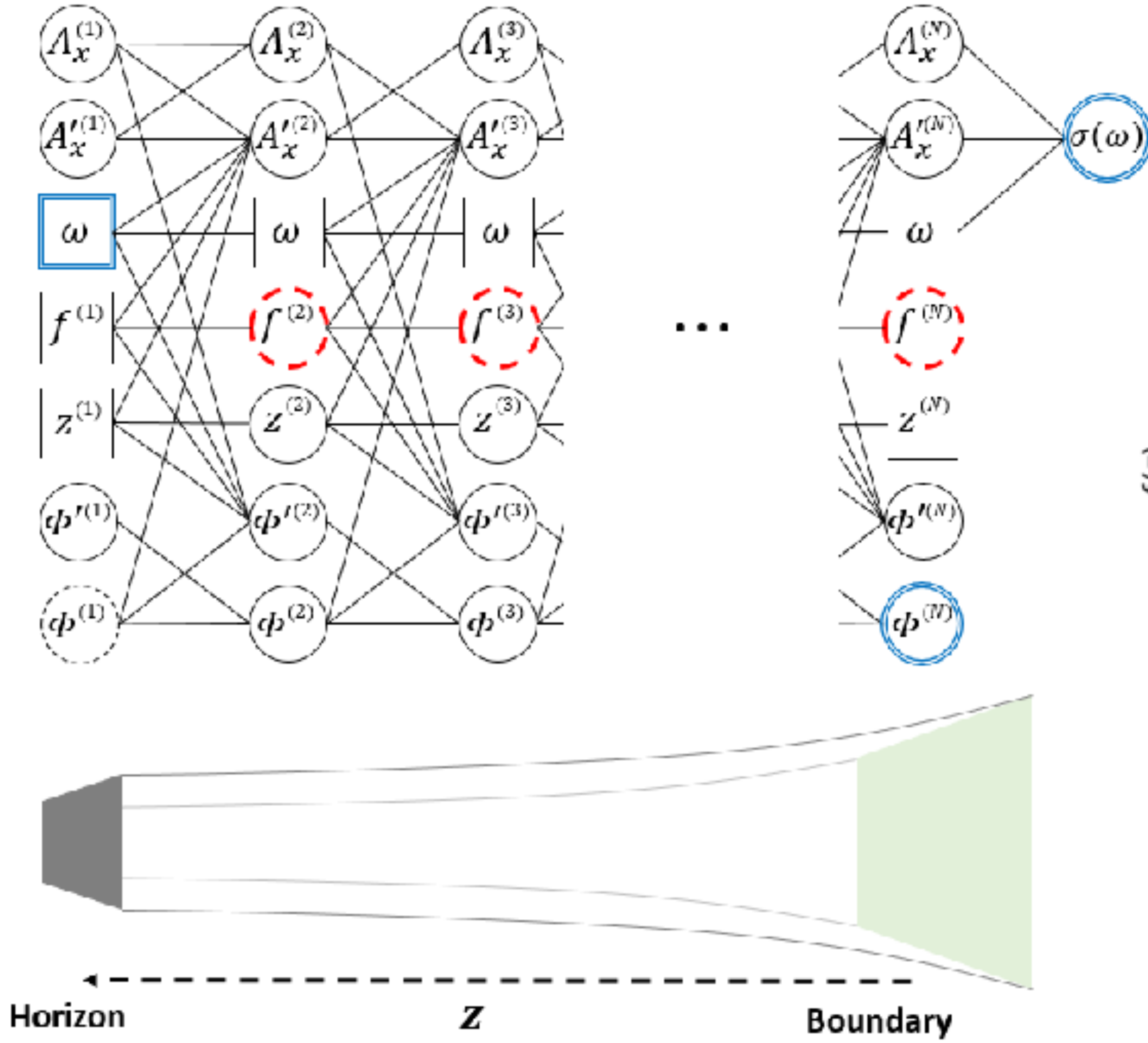
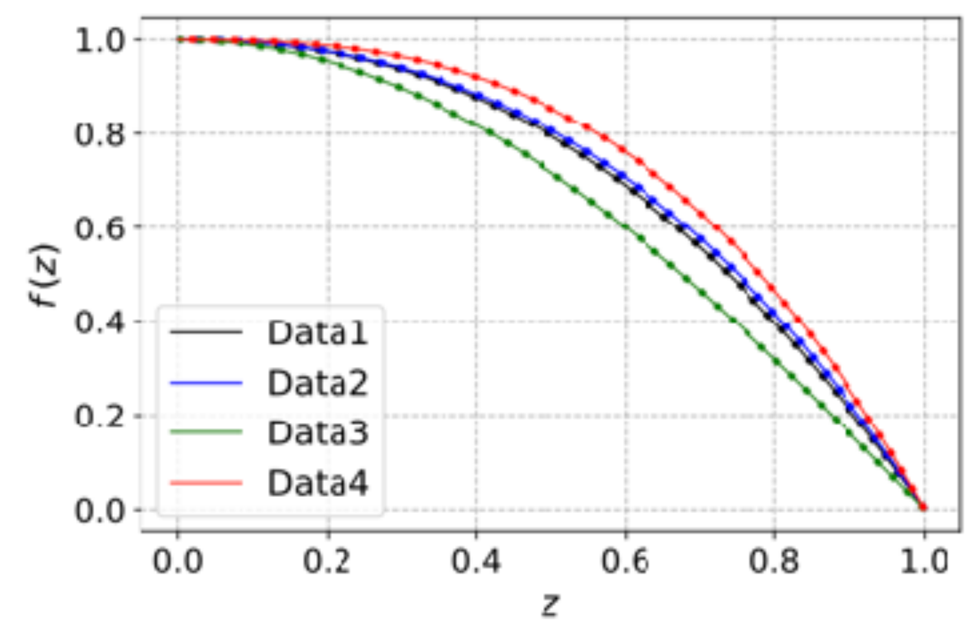


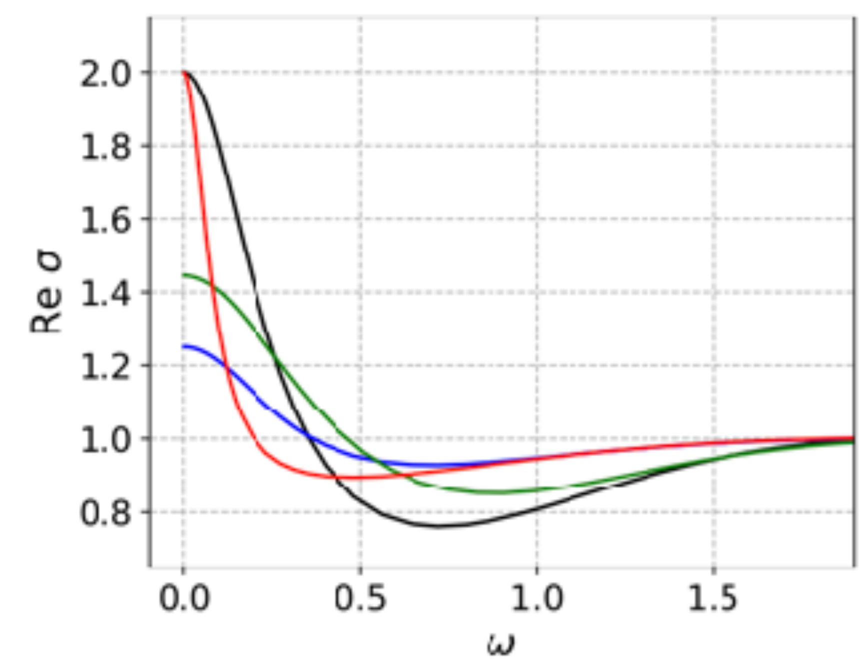
AdS/Deep learning: optical conductivity



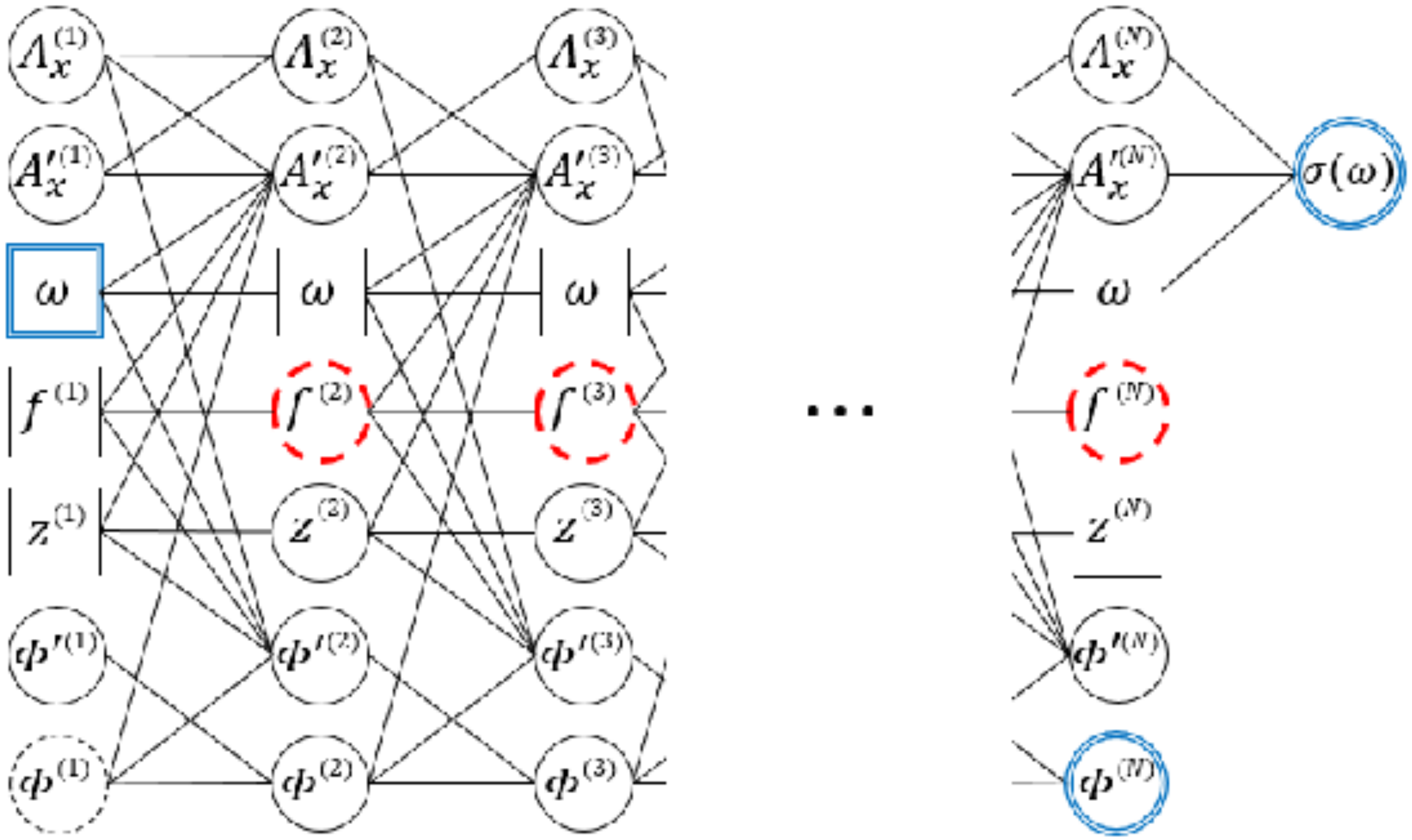
Loss

$\longrightarrow |\sigma\,(\omega) - \sigma_{true}(\omega)|$





AdS/Deep learning: optical conductivity



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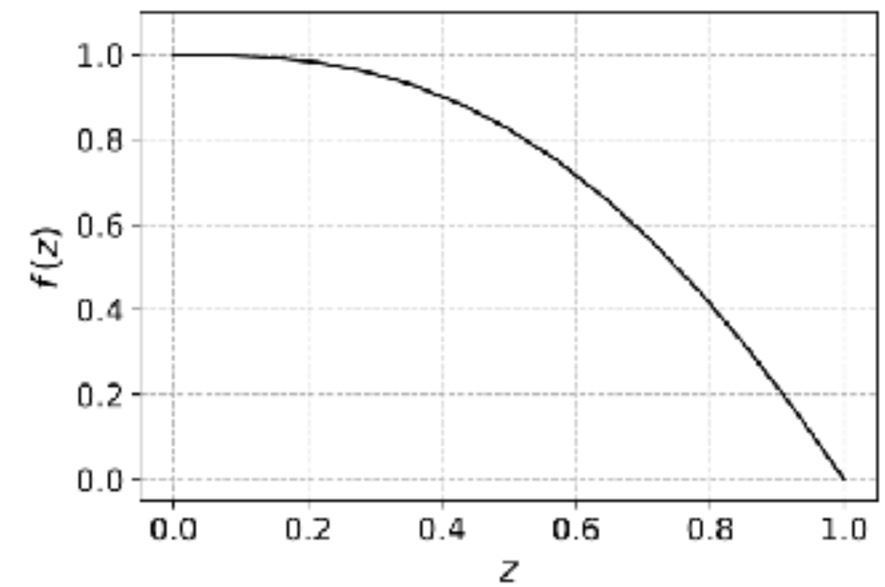

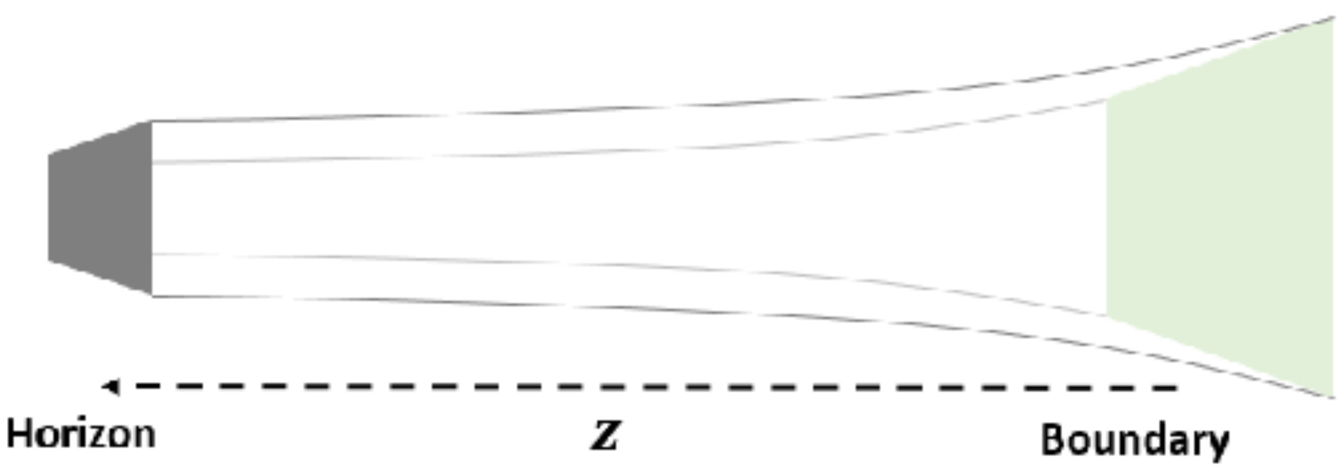

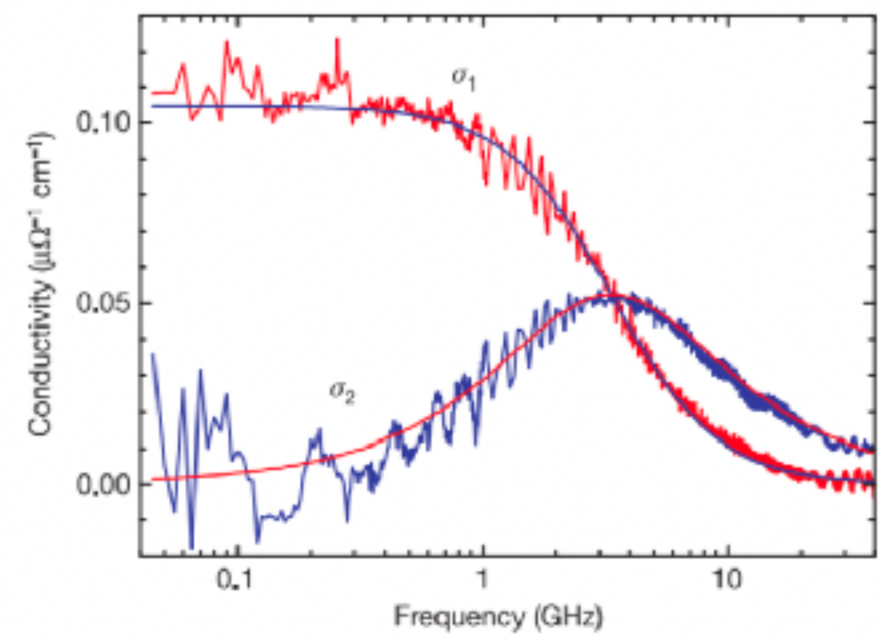

Figure 2 | Conductivity spectrum of UPd_2Al_3 at temperature 2.75 K

Holography

Deep Learning Bulk Spacetime from Boundary data

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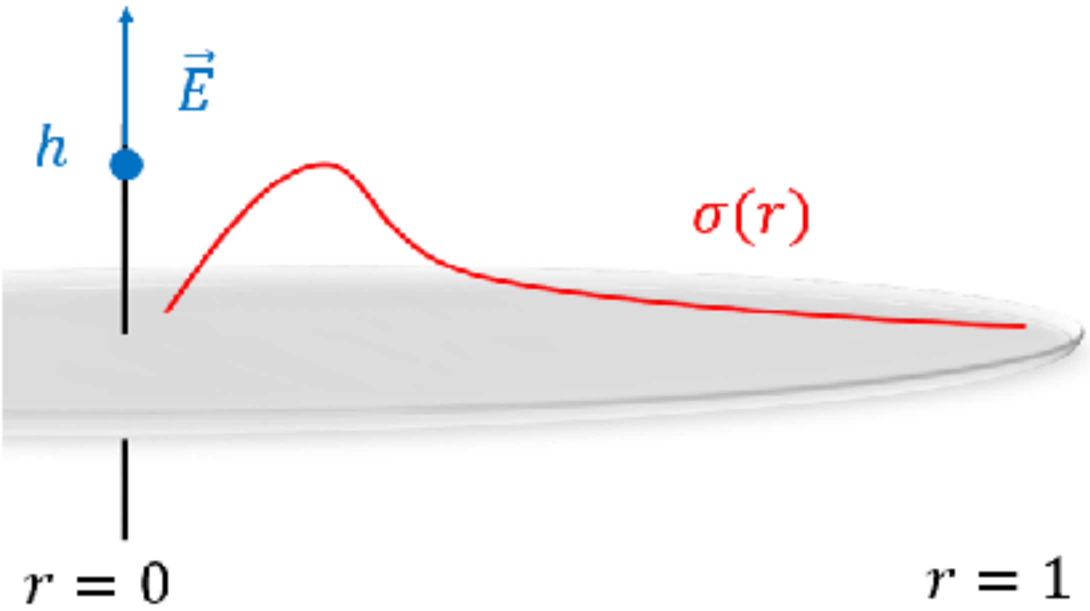
  - AdS/Deep Learning: Entanglement entropy

Deep Learning for integral: electrostatics

$$E(z) = \int_0^1 dr\; 2\pi r \frac{z\,\sigma(r)}{\sqrt{r^2+z^2}^3}$$



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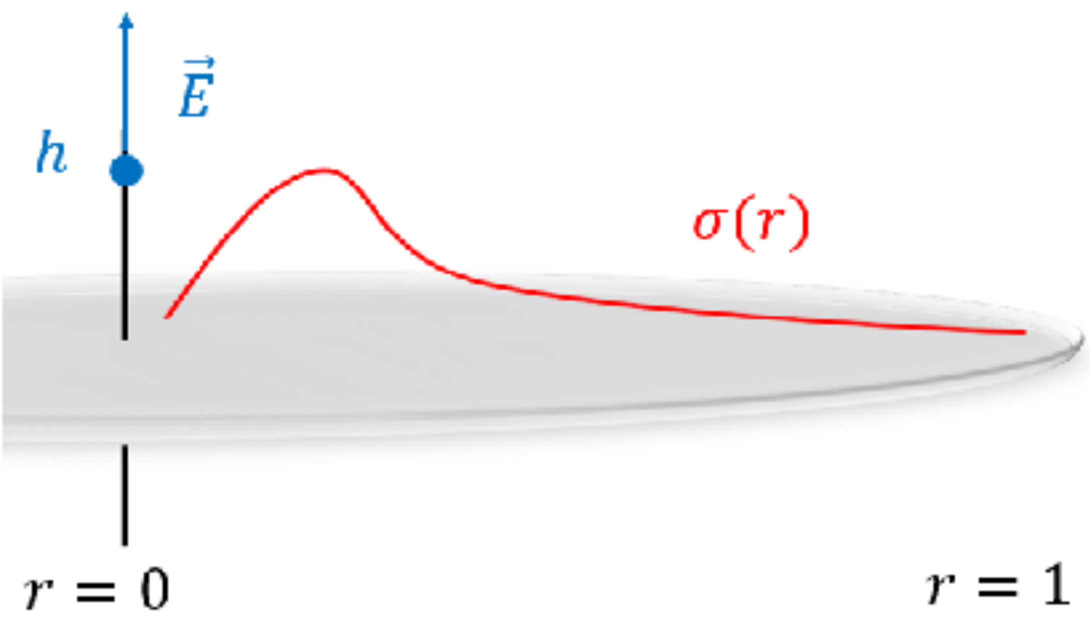
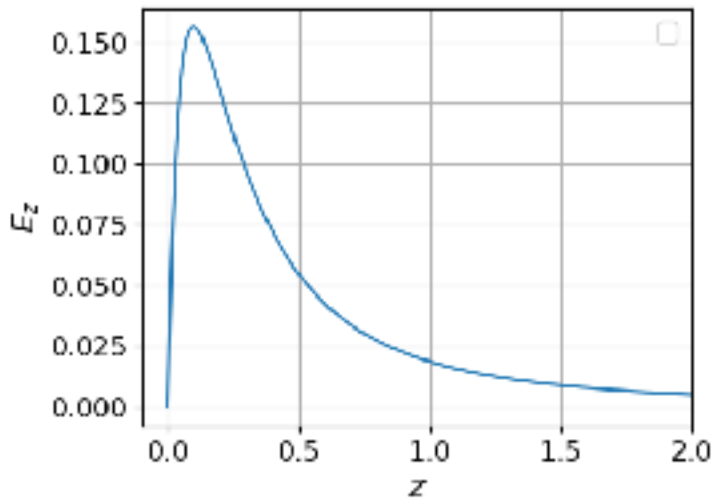
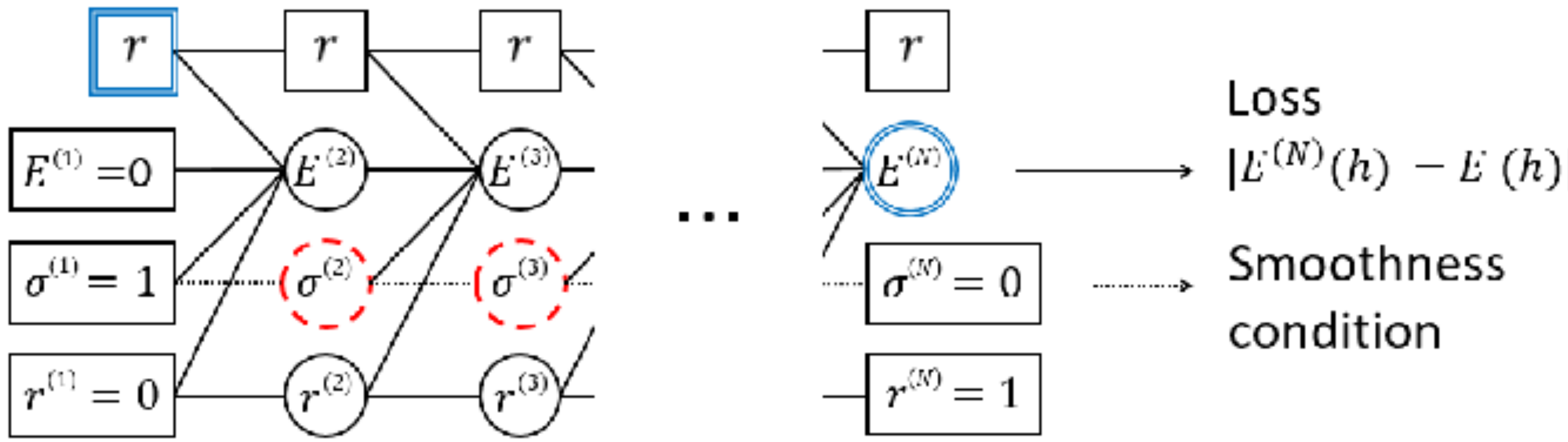




Deep Learning for integral: electrostatics

$$E^{(n+1)} = E^{(n)} + \Delta r \cdot 2\pi r \frac{z\,\sigma(r)}{\sqrt{r^2+z^2}^3},$$



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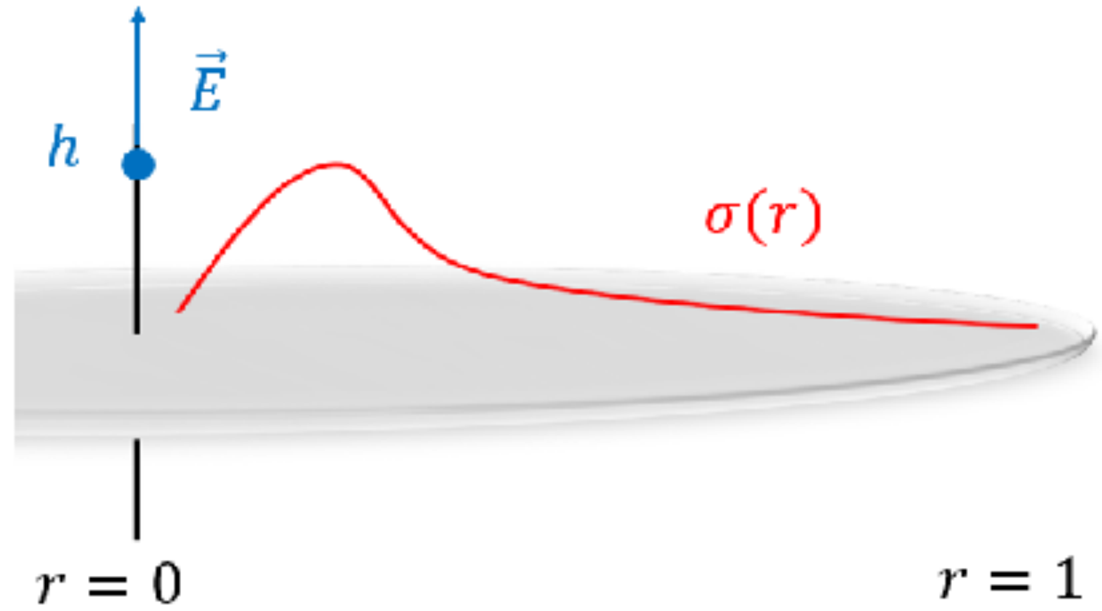

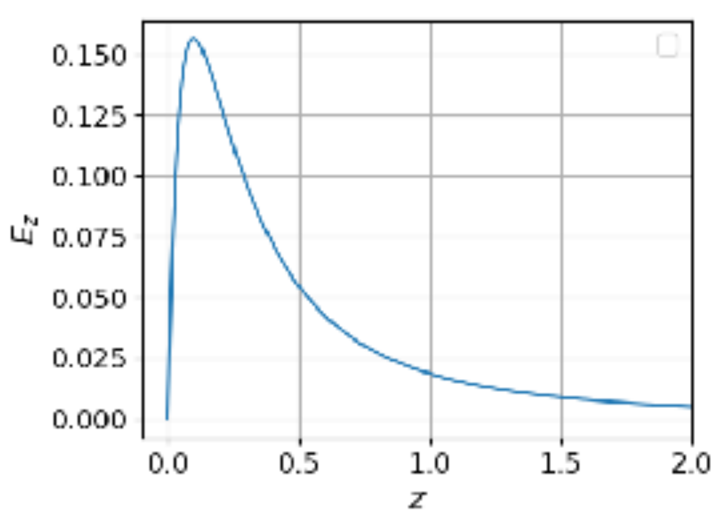

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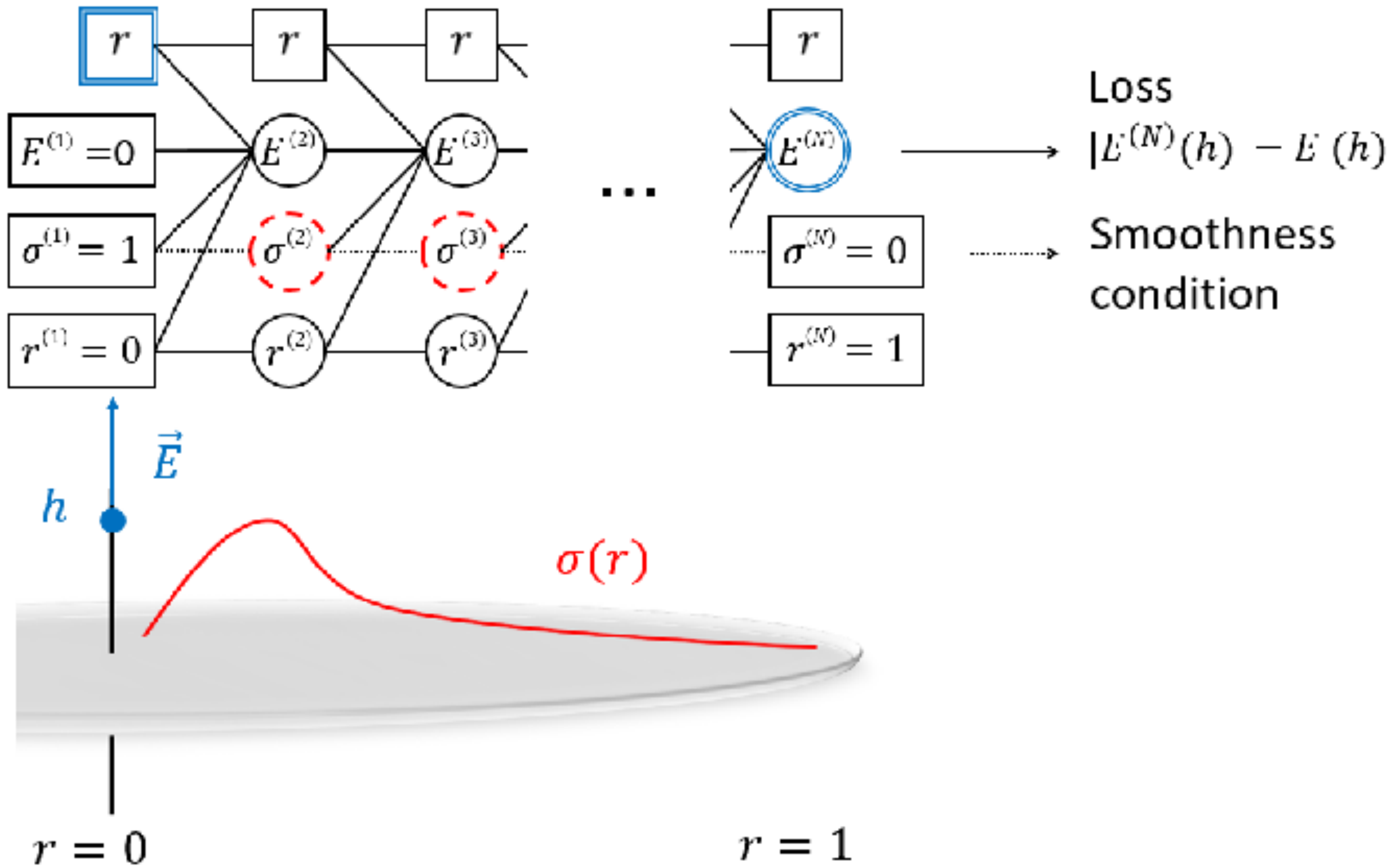

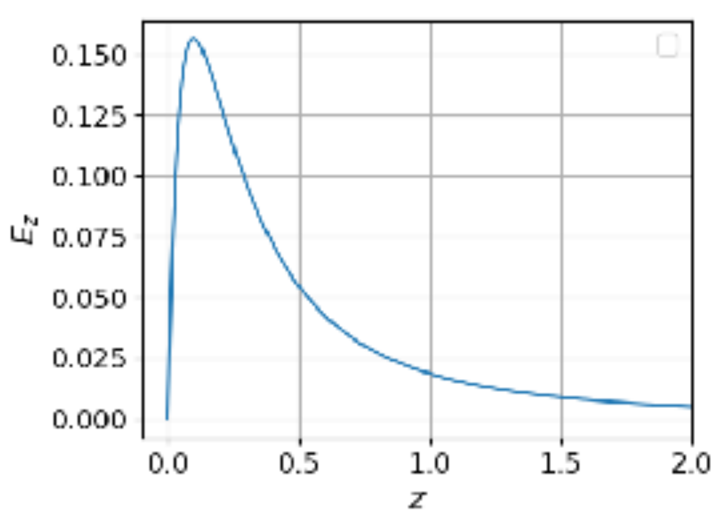

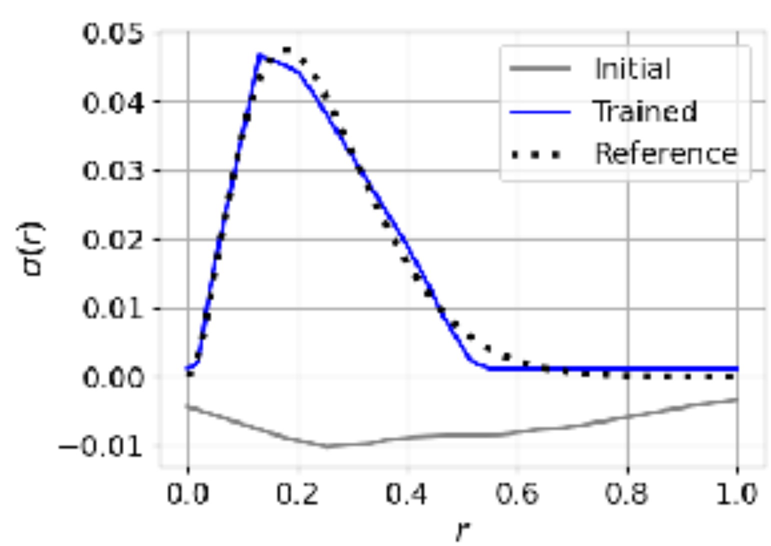

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AdS/Deep learning: entanglement entropy

$$E(z) = \int_0^1 dr\, 2\pi r \frac{z\, \sigma(r)}{\sqrt{r^2+z^2}^3}$$

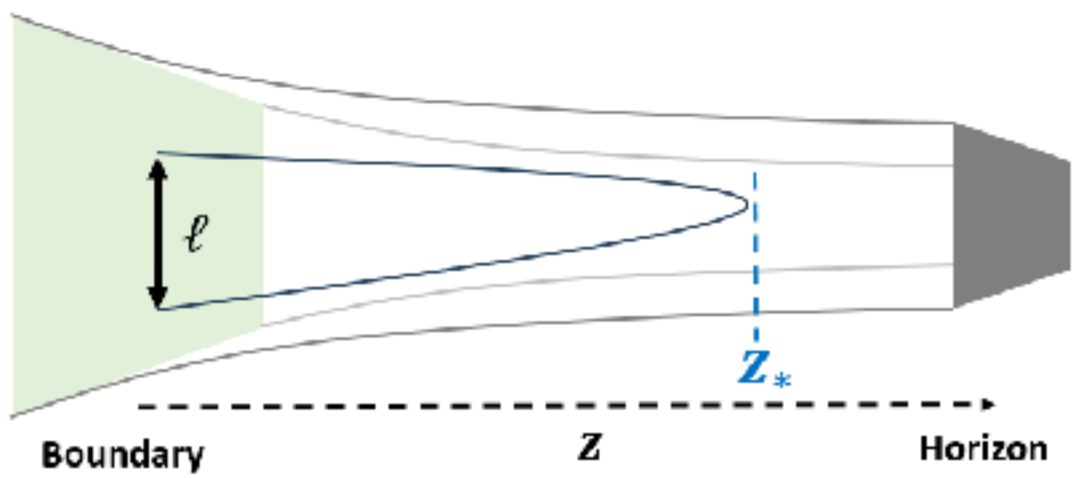
$$A(\alpha) = \int F[\, f_1(r;\alpha), f_2(r;\alpha), \cdots]\, dr$$

$$B(\alpha) = \int G[\, f_1(r;\alpha), f_2(r;\alpha), \cdots]\, dr$$

$$\ell(z_*) = \int_0^{z_*} dz \frac{2z^2}{\sqrt{z_*^4 - z^4}} \frac{1}{\sqrt{f(z)}}$$

$$C(z_*) := -1 + \int_0^{z_*} dz \cdot \frac{z_*}{z^2} \left(\sqrt{1 - \frac{z^2}{z_*^2}} \sqrt{\frac{1}{f(z)}} - 1 \right)$$

$$\bar{\sigma} := \frac{\sigma(\ell(z_*))}{s} = \frac{1}{z_*^2} + \frac{C(z_*)}{z_*} \frac{2}{\ell(z_*)} + \frac{4\pi}{\ell(z_*)^2} \left(\frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})} \right)^2$$



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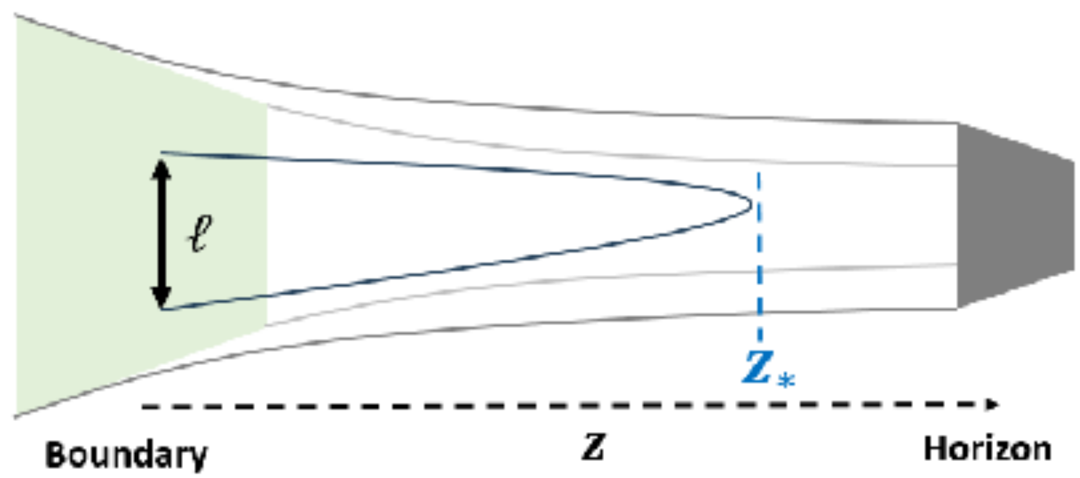

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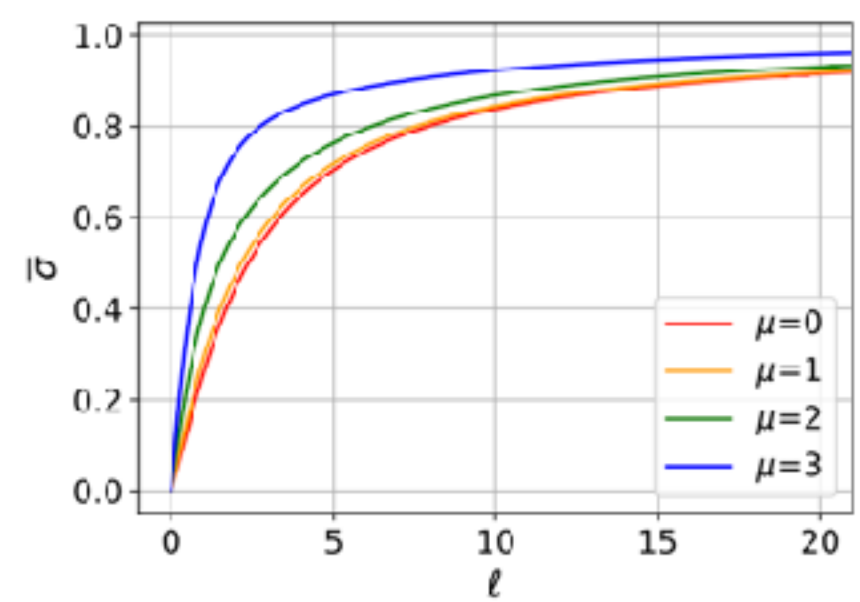
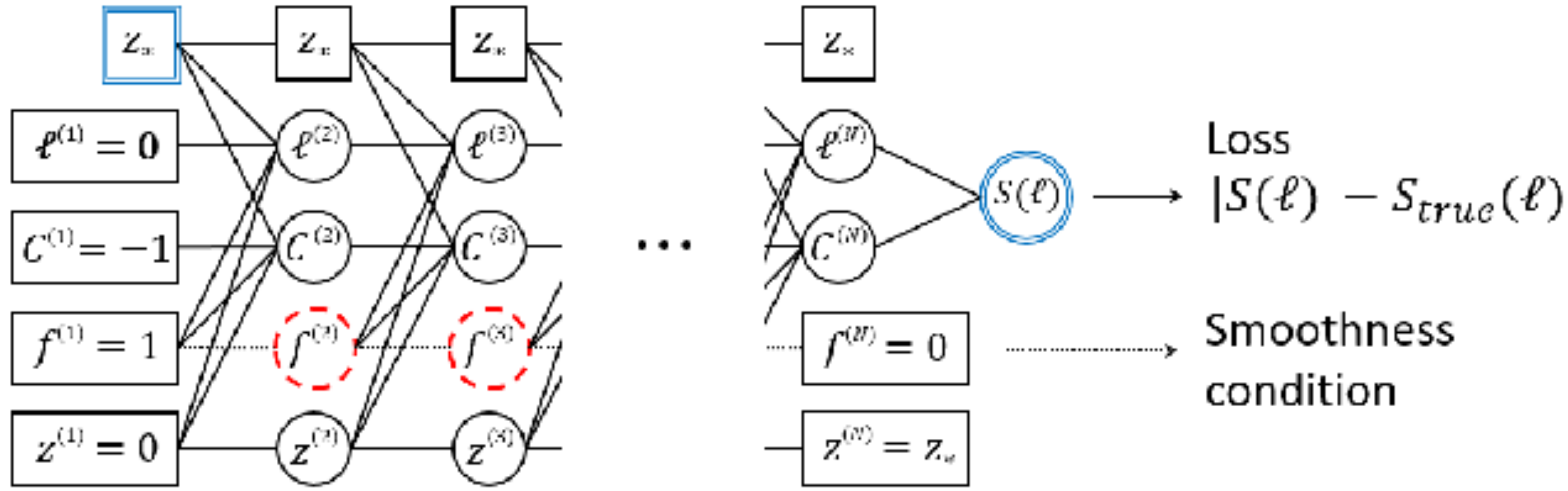
RN Black hole

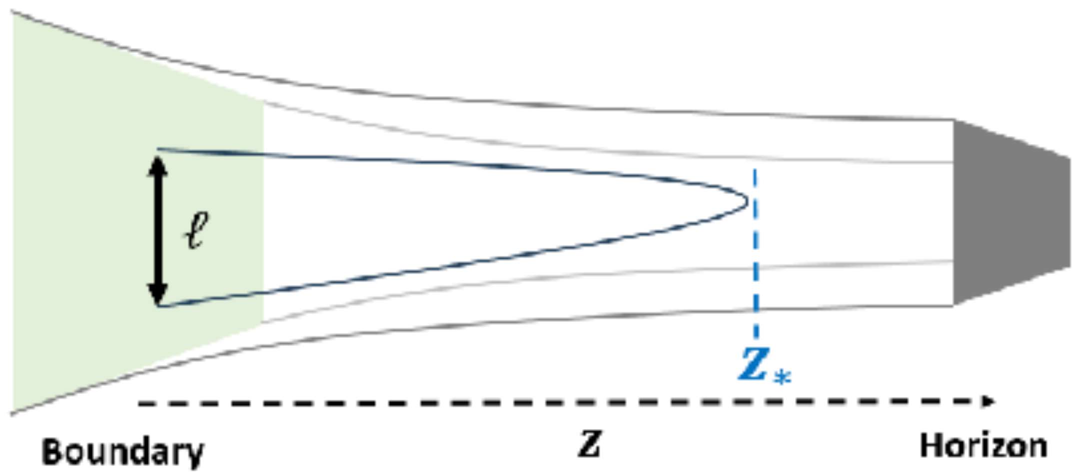


AdS/Deep learning: entanglement entropy

$$\ell^{(n+1)} = \ell^{(n)} + \Delta z \cdot \frac{2z^2}{\sqrt{(z_*^4 - z^4)\, f(z)}} \,,$$

$$C^{(n+1)} = C^{(n)} + \Delta z \cdot \frac{z_*}{z^2}\left(\sqrt{\left(1-\frac{z^2}{z_*^2}\right)\frac{1}{f(z)}} - 1\right)$$

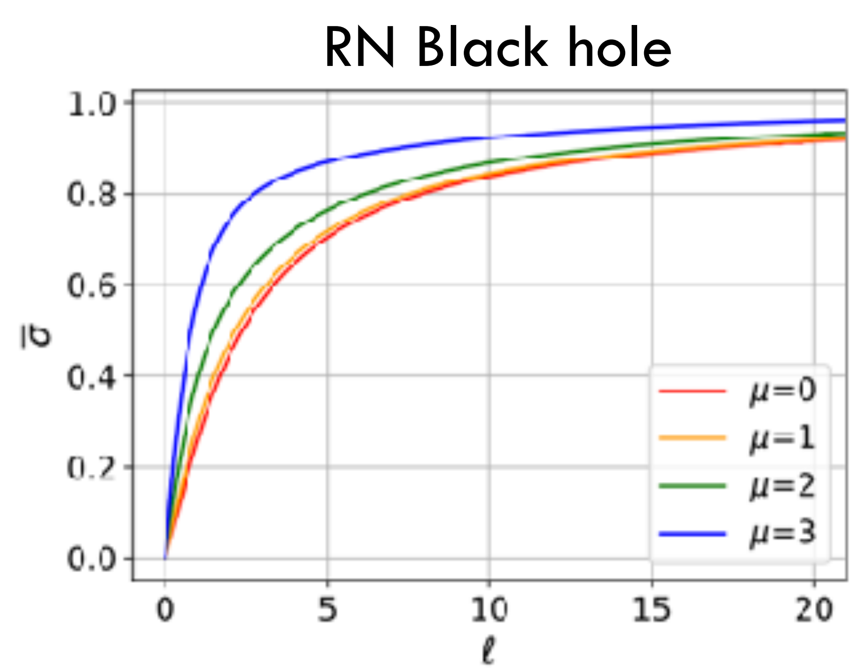




$$\ell(z_*) = \int_0^{z_*} dz \frac{2z^2}{\sqrt{z_*^4 - z^4}} \frac{1}{\sqrt{f(z)}}$$

$$C(z_*) := -1 + \int_0^{z_*} dz \cdot \frac{z_*}{z^2}\left(\sqrt{1-\frac{z^2}{z_*^2}}\sqrt{\frac{1}{f(z)}} - 1\right)$$

$$\bar{\sigma} := \frac{\sigma(\ell(z_*))}{s} = \frac{1}{z_*^2} + \frac{C(z_*)}{z_*}\frac{2}{\ell(z_*)} + \frac{4\pi}{\ell(z_*)^2}\left(\frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})}\right)^2$$



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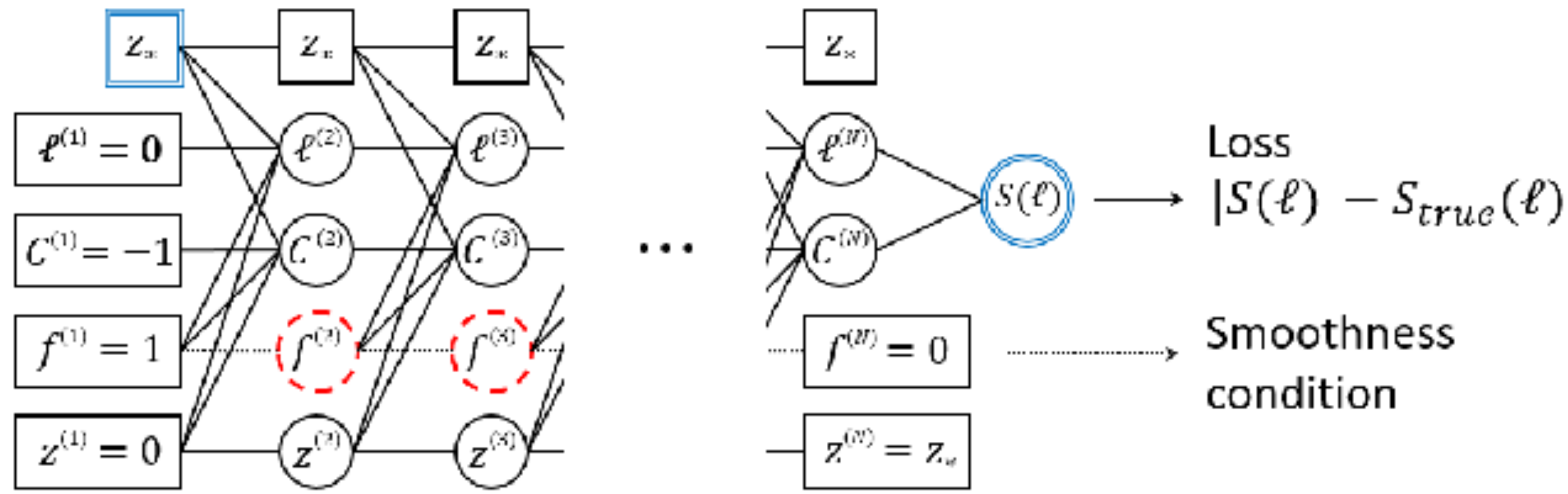

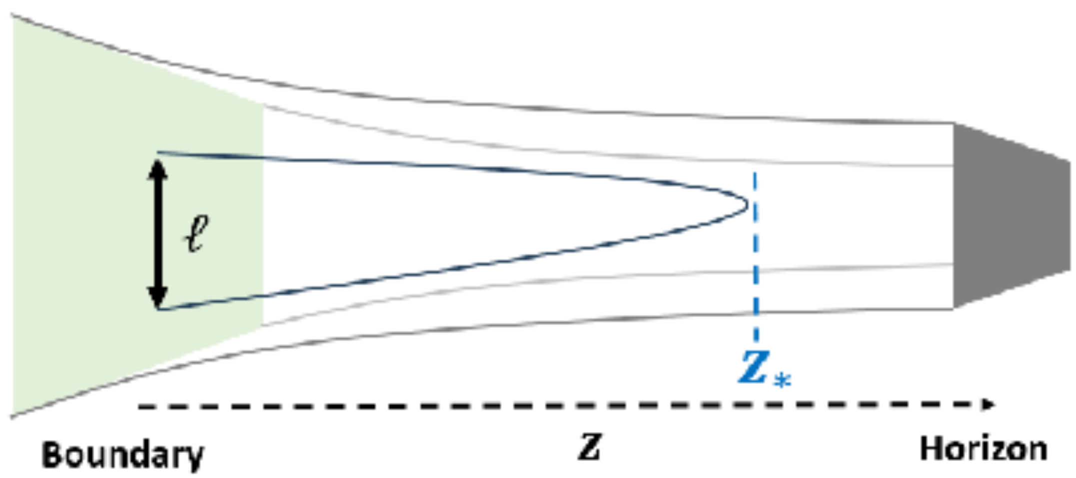

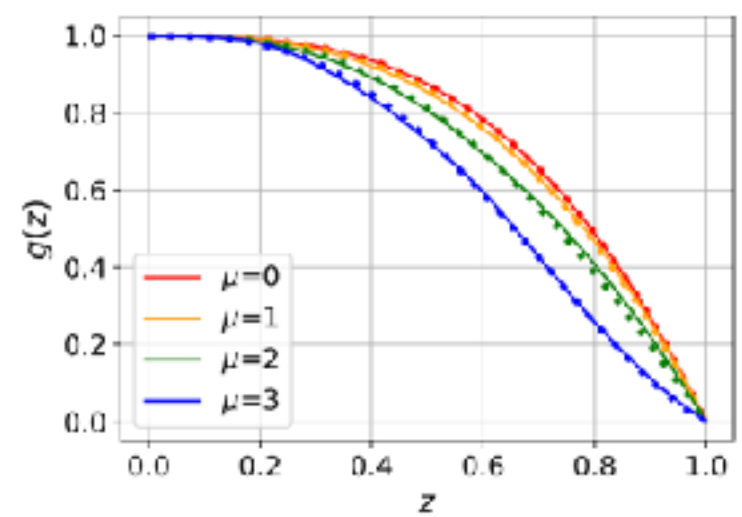

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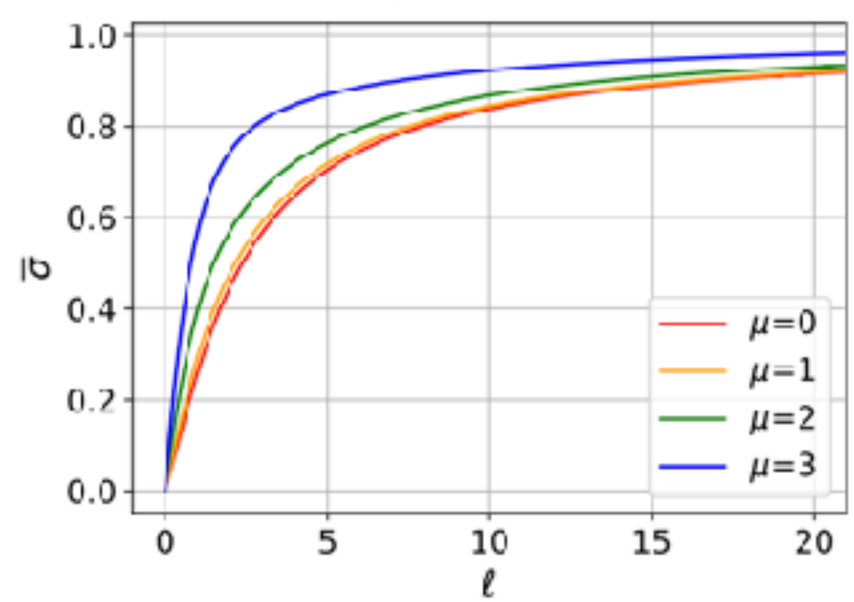
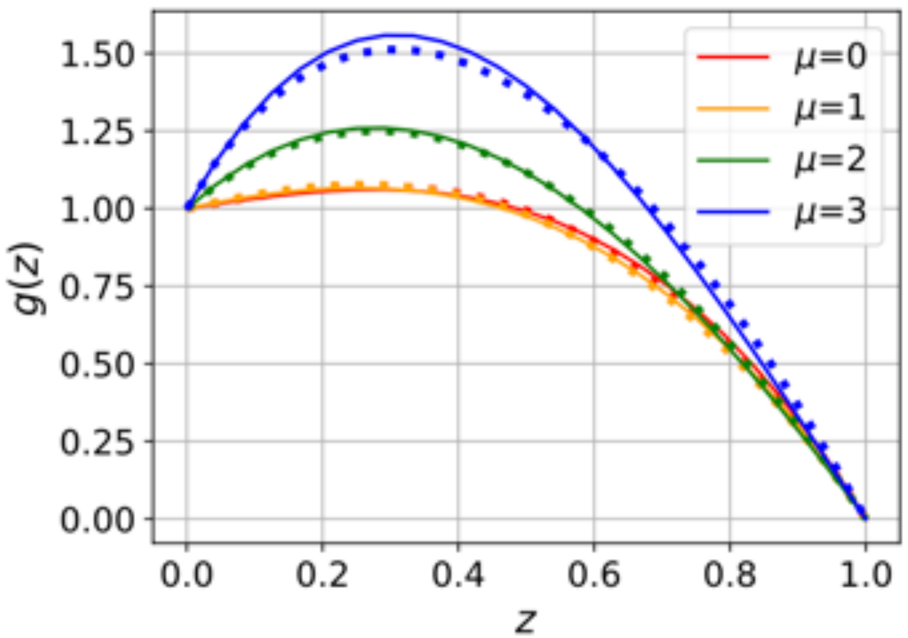
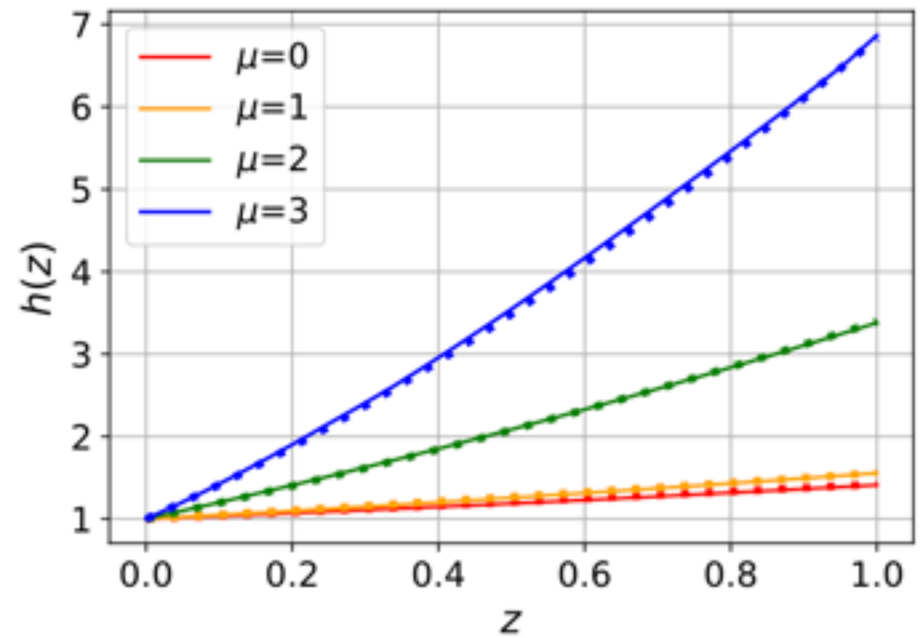
RN Black hole

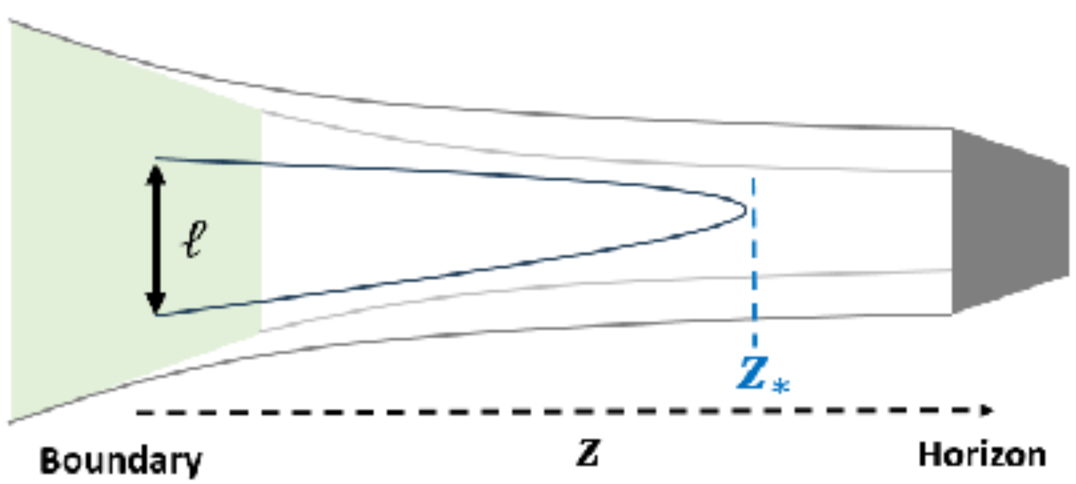


Gubser-Rocca case

$$ds^2 = \frac{L^2}{z^2}\left[-f(z)dt^2 + \frac{dz^2}{g(z)} + h(z)(dx^2 + dy^2)\right]$$







AdS/Deep learning: entanglement entropy

arXiv > hep-th > arXiv:2205.04445

High Energy Physics – Theory

[Submitted on 9 May 2022 (v1), last revised 11 Sep 2022 (this version, v3)]

Dual Geometry of Entanglement Entropy via Deep Learning

Chanyong Park, Chi-Ok Hwang, Kyungchan Cho, Se-Jin Kim

arXiv > hep-th > arXiv:2311.01724

High Energy Physics – Theory

[Submitted on 3 Nov 2023 (v1), last revised 15 Jan 2024 (this version, v3)]

Holography Transformer

Chanyong Park, Sejin Kim, Jung Hun Lee

Holography

Deep Learning Bulk Spacetime from Boundary data

- What is "Machine Learning" & "Deep Learning"?
  - Deep Learning 101: standard story
- Physics equation related Deep Learning I
  - Deep Learning for ODE: classical mechanics
  - AdS/Deep Learning: optical conductivity
- Physics equation related Deep Learning II
  - Deep Learning for Integral: classical electrostatics
  - AdS/Deep Learning: Entanglement entropy

Ongoing and future work

- Methodology development

  - Neural ODE

  - PDE

  - PINN (Physics Informed Neural Network)

  - Applications to other physics problems (ODE, PDE, Integral)

- Other physical quantities

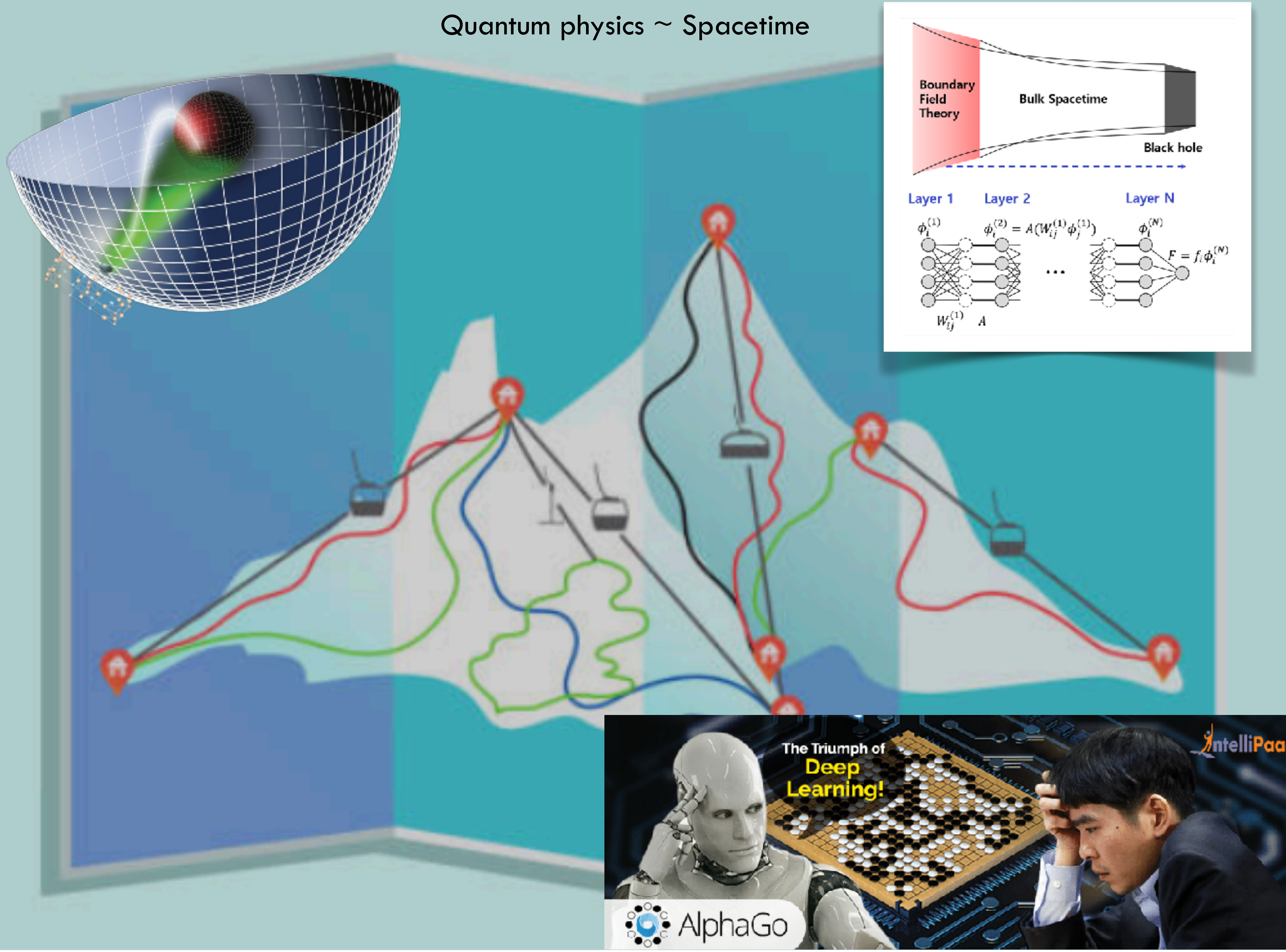
  - ARPES: Fermionic spectral function

  - Quantum info: complexity, entanglement entropy, etc

- Figuring out action itself

  - Linear T resistivity + T^2 Hall angle together

Quantum physics ~ Spacetime



Thank you