7th International Conference on Holography, String Theory in Da Nang

# Exploring hot and dense QCD by holography

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**Da Nang, Vietnam**

**22 August 2024**

ĐẠI HỌC DUY TÂN

# References：

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Probing QCD critical point and induced gravitational wave by black hole physics,
Phys. Rev. D 106 no.12, L121902 (2022)

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Holographic study of higher-order baryon number susceptibilities at finite temperature and density,
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Song He, **Li Li**, Sai Wang and Shao-Jiang Wang,
Constraints on holographic QCD phase transitions from PTA observations,
[arXiv:2308.07257 [hep-ph]]. (to appear in Sci.China Phys.Mech.Astron.)

Yan-Qing Zhao, Song He, Defu Hou, **Li Li** and Zhibin Li,
Phase structure and critical phenomena in 2-flavor QCD by holography,
Phys.Rev.D 109 (2024) 8, 086015

Rong-Gen Cai, Song He, **Li Li** and Hong-An Zeng,
QCD Phase Diagram at finite Magnetic Field and Chemical Potential: A Holographic Approach Using Machine Learning, [arXiv:2406.12772 [hep-th]].

# Outline

1. Introduction

2. Holographic QCD model

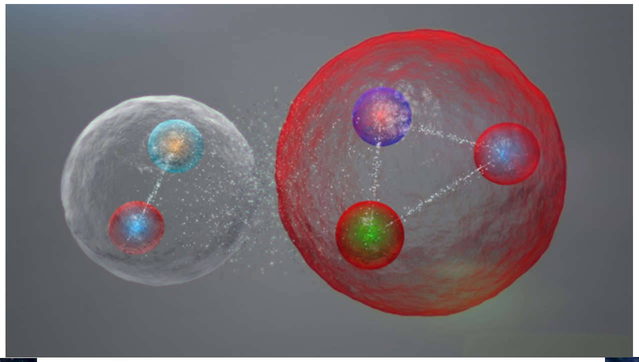
3. Phase diagram and GWs

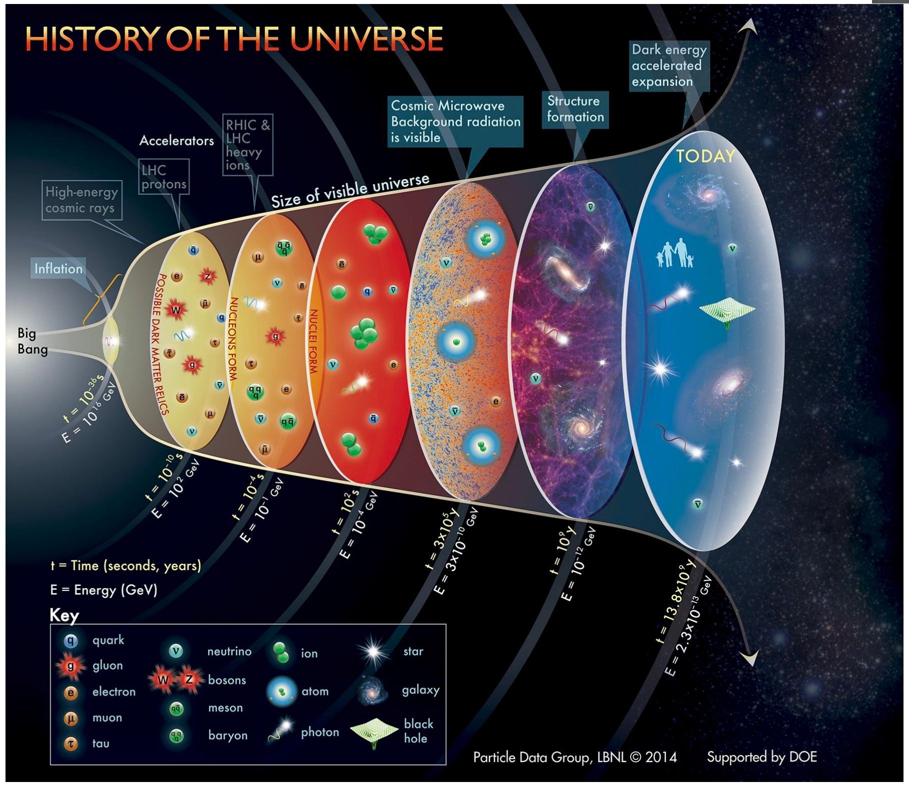
4. Summary and discussion

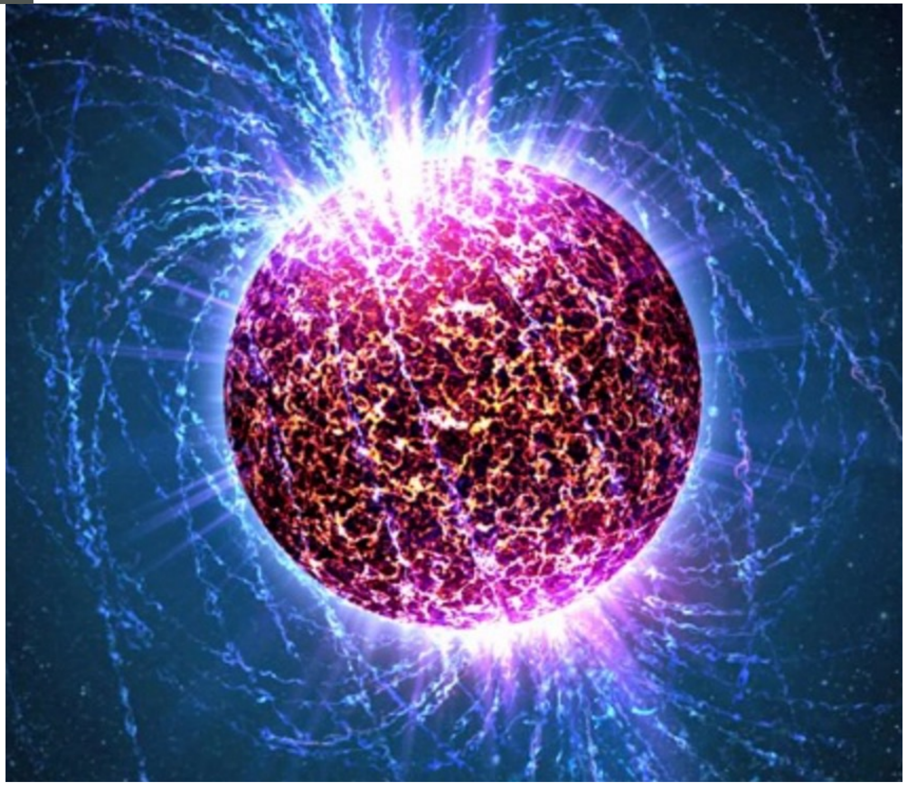
# 1. Introduction

## QCD phase diagram:

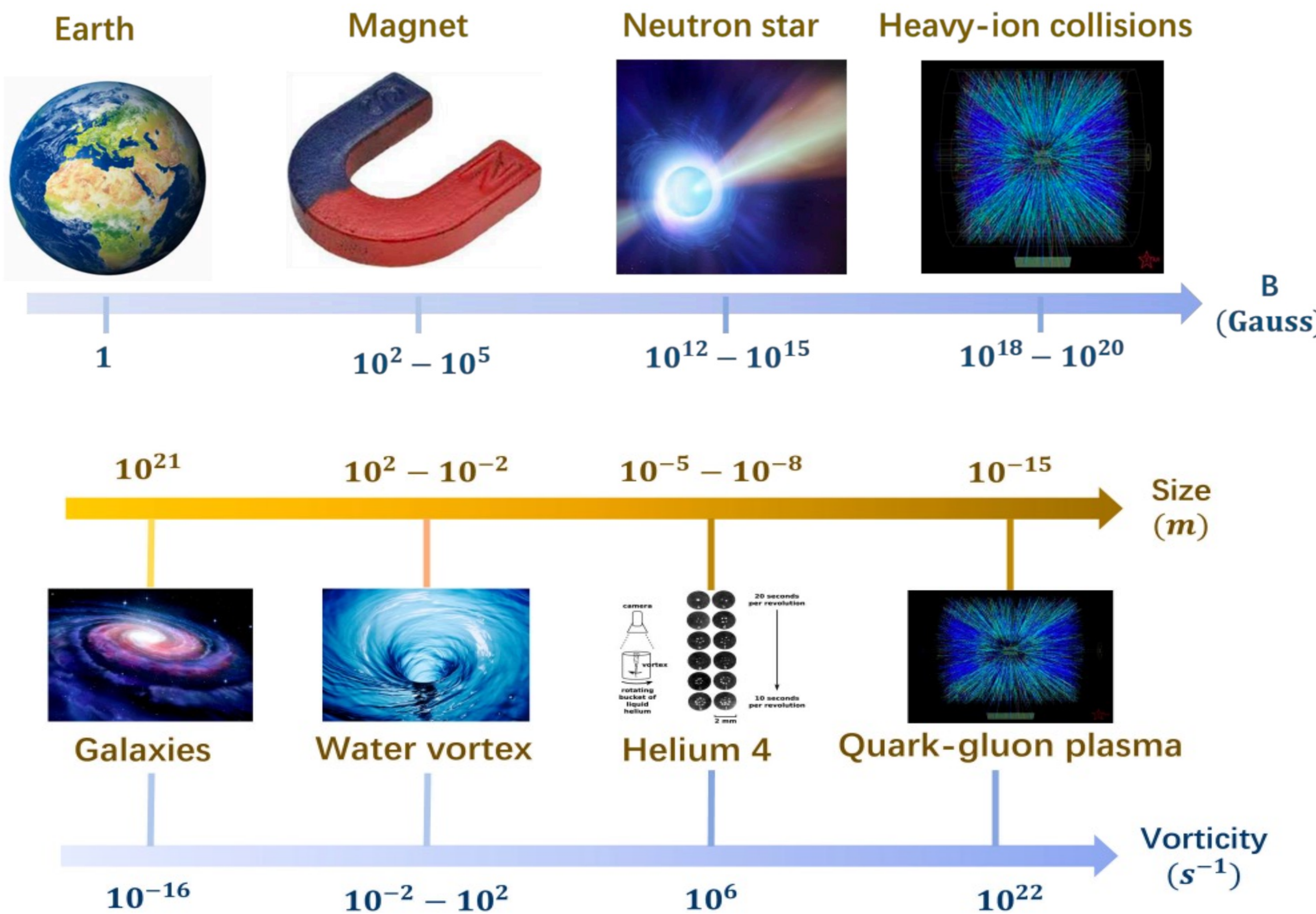
One of the most interesting and fundamental challenges of high energy physics!





It involves strongly interacting matter under extreme conditions.




Xu-Guang Huang

# 1. Introduction

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One of the most interesting and fundamental challenges of high energy physics!

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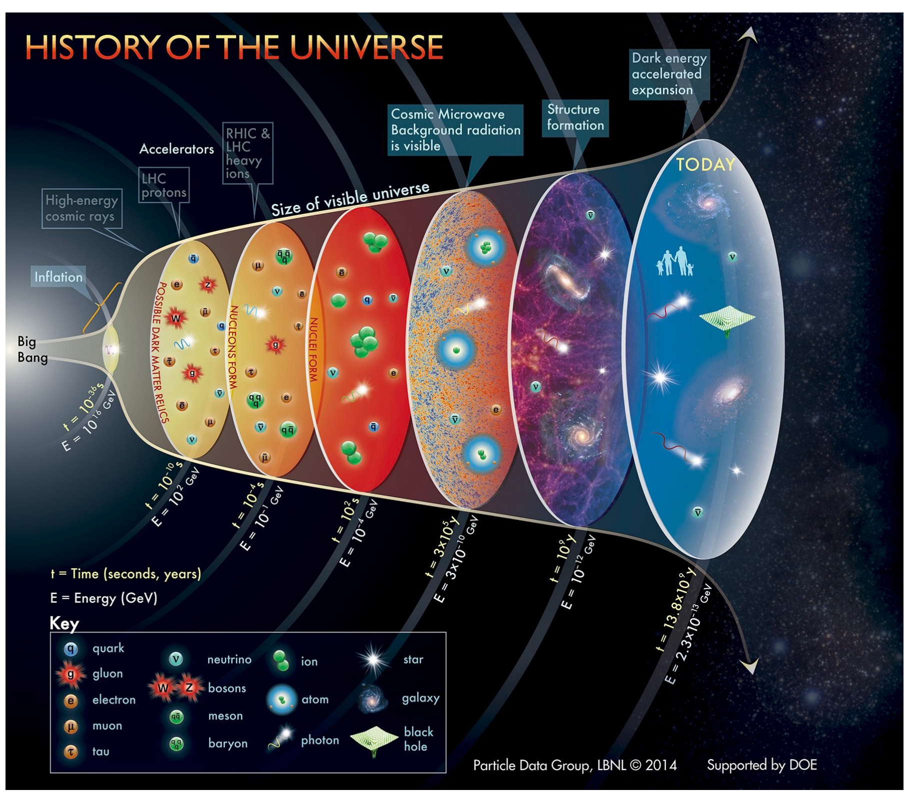
Only little is known about the phase structures in low-T and high-$\mu_B$ regions.



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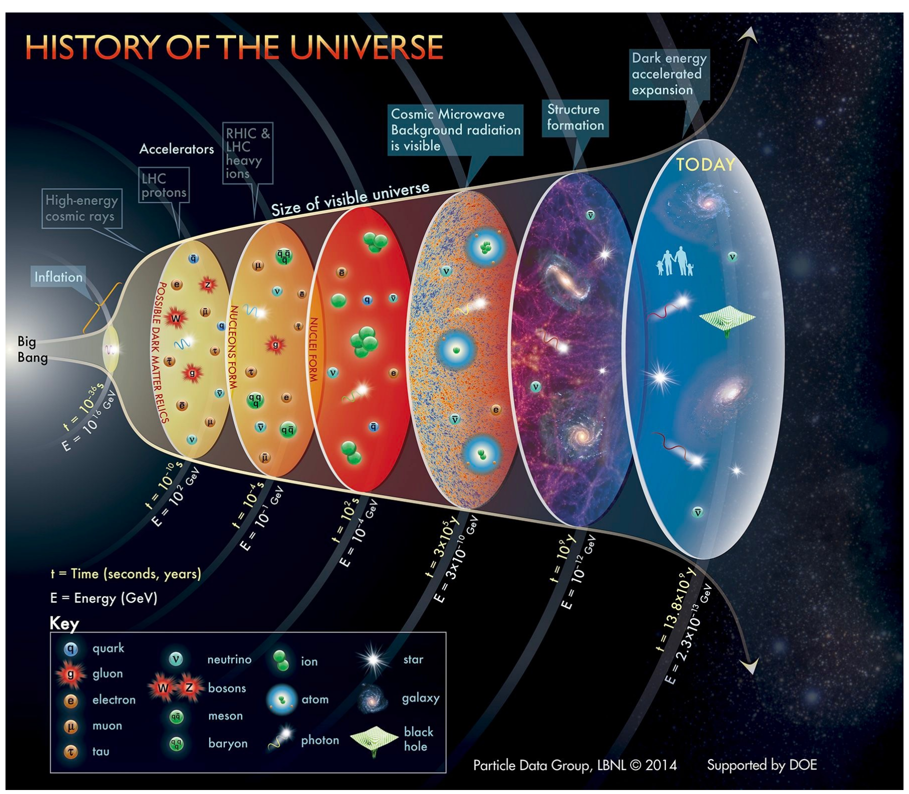
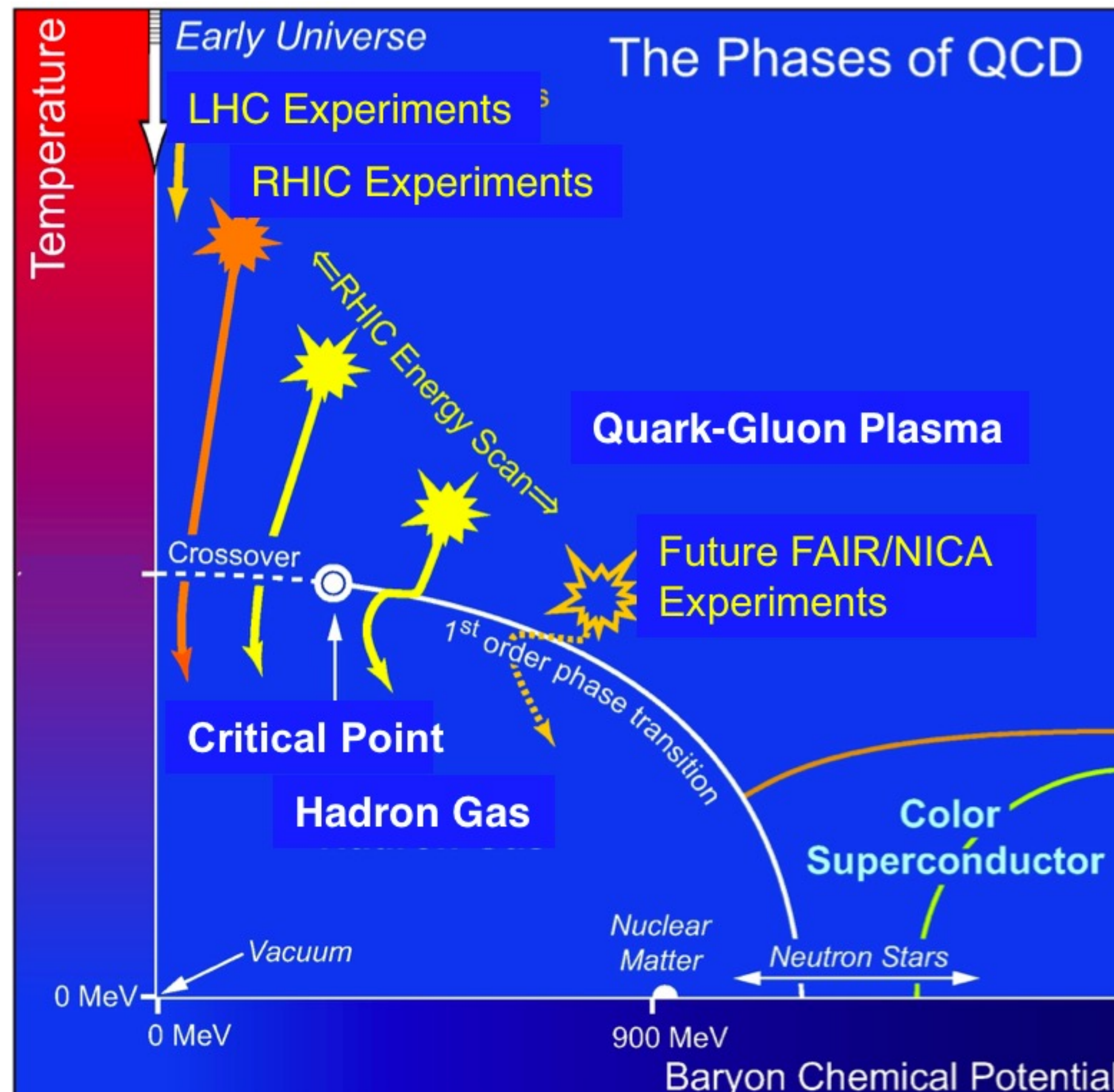
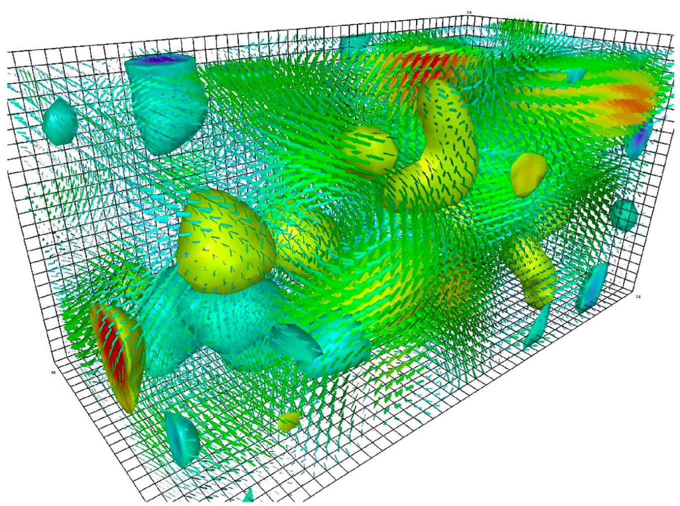


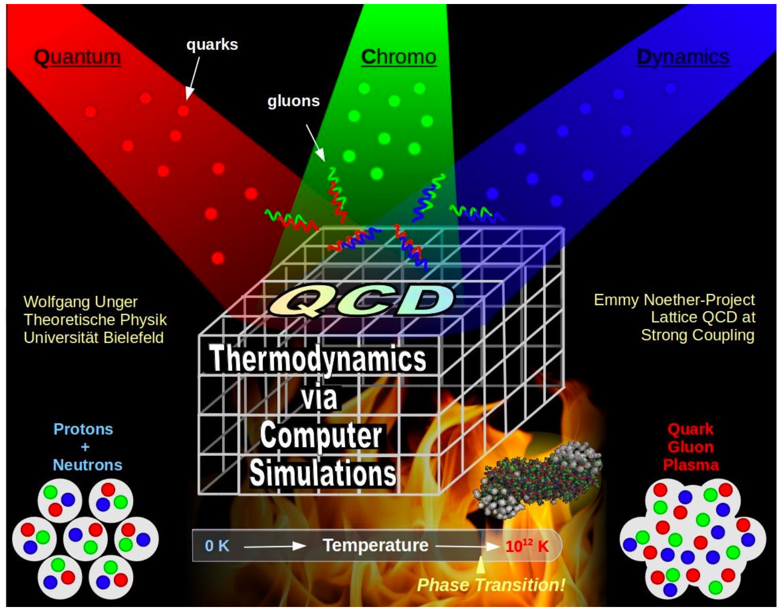



**It has not been possible to obtain QCD phase diagram directly from QCD!**

**Lattice QCD**: the update algorithm solves the path integral of discretized QCD.

It can give reliable information from the first principle at **zero density**,
but fails at finite density due to the famous **sign problem**





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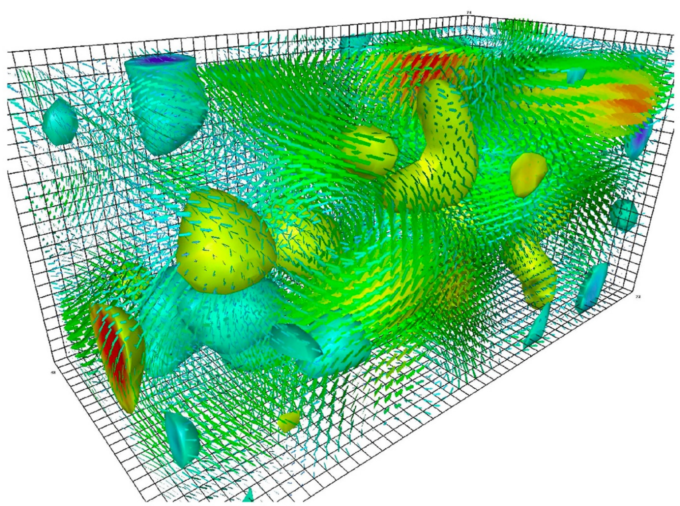

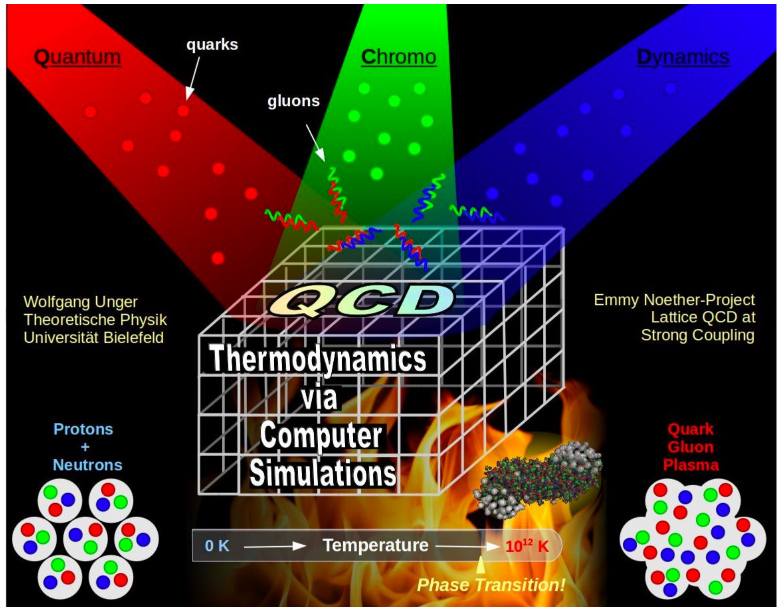


**Challenges of lattice QCD – costs**

*Today, 1000 configurations on a* $64^3 \times 16$ *lattice cost about 1 million core hours.*

Traditional supercomputer: 1 million core hours = cca 30 k€, 15 tons $CO_2$

GPU based supercomputer: 1 million core hours = cca 10 k€, 6 tons $CO_2$

[S. Borsanyi, 2018]

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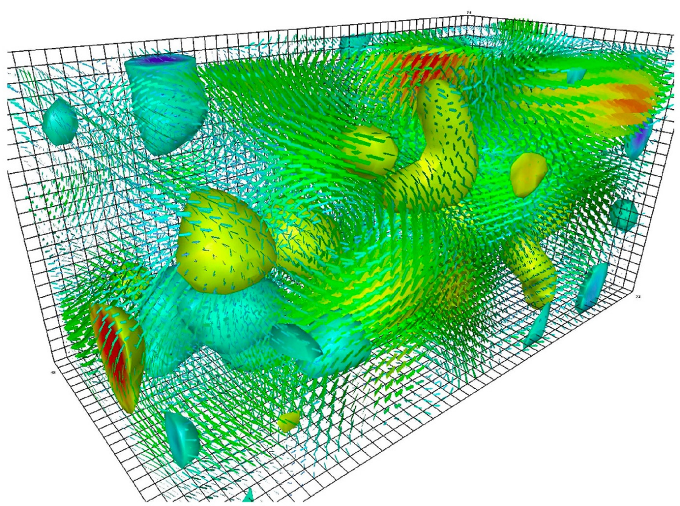
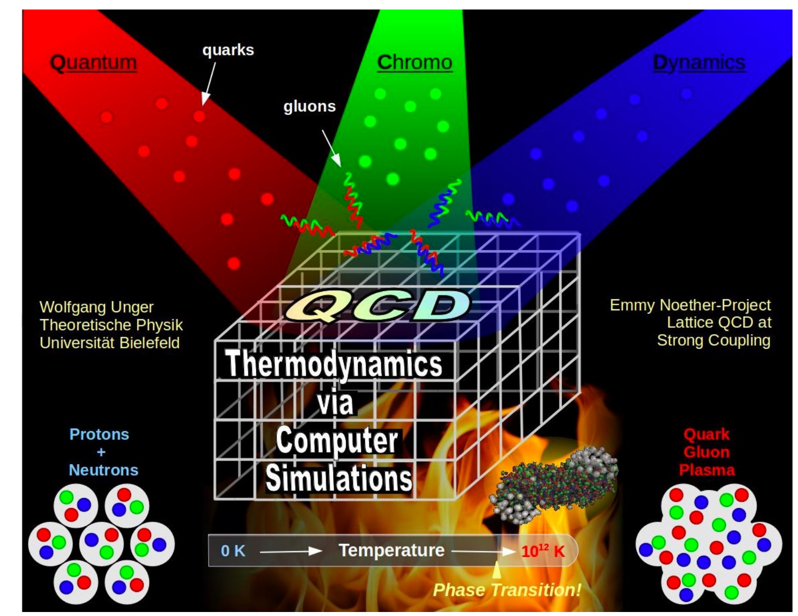
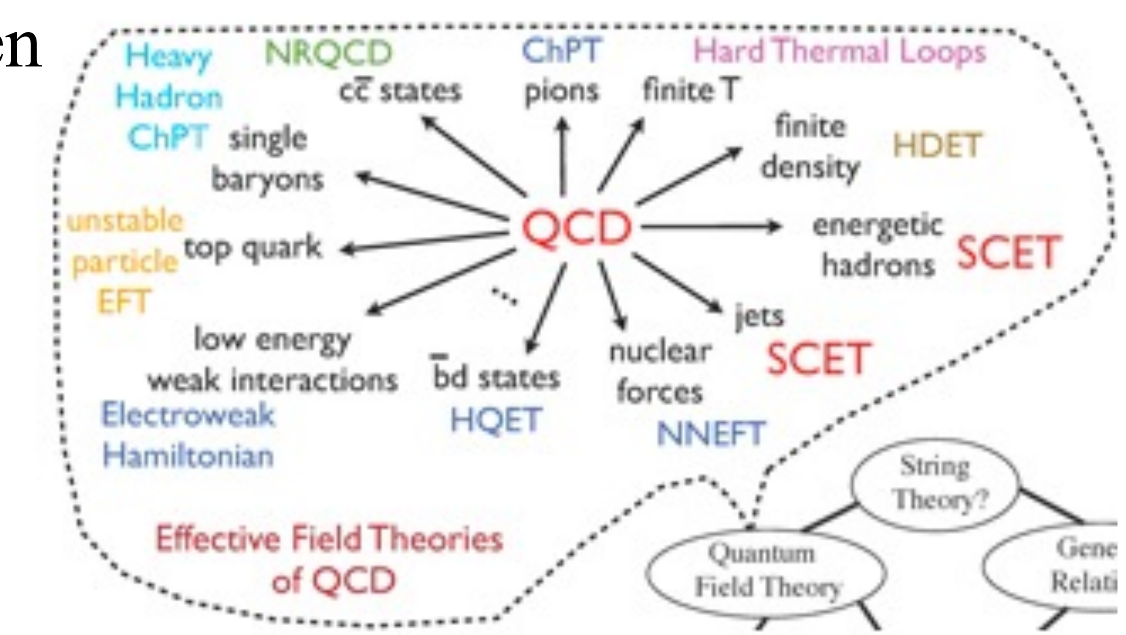




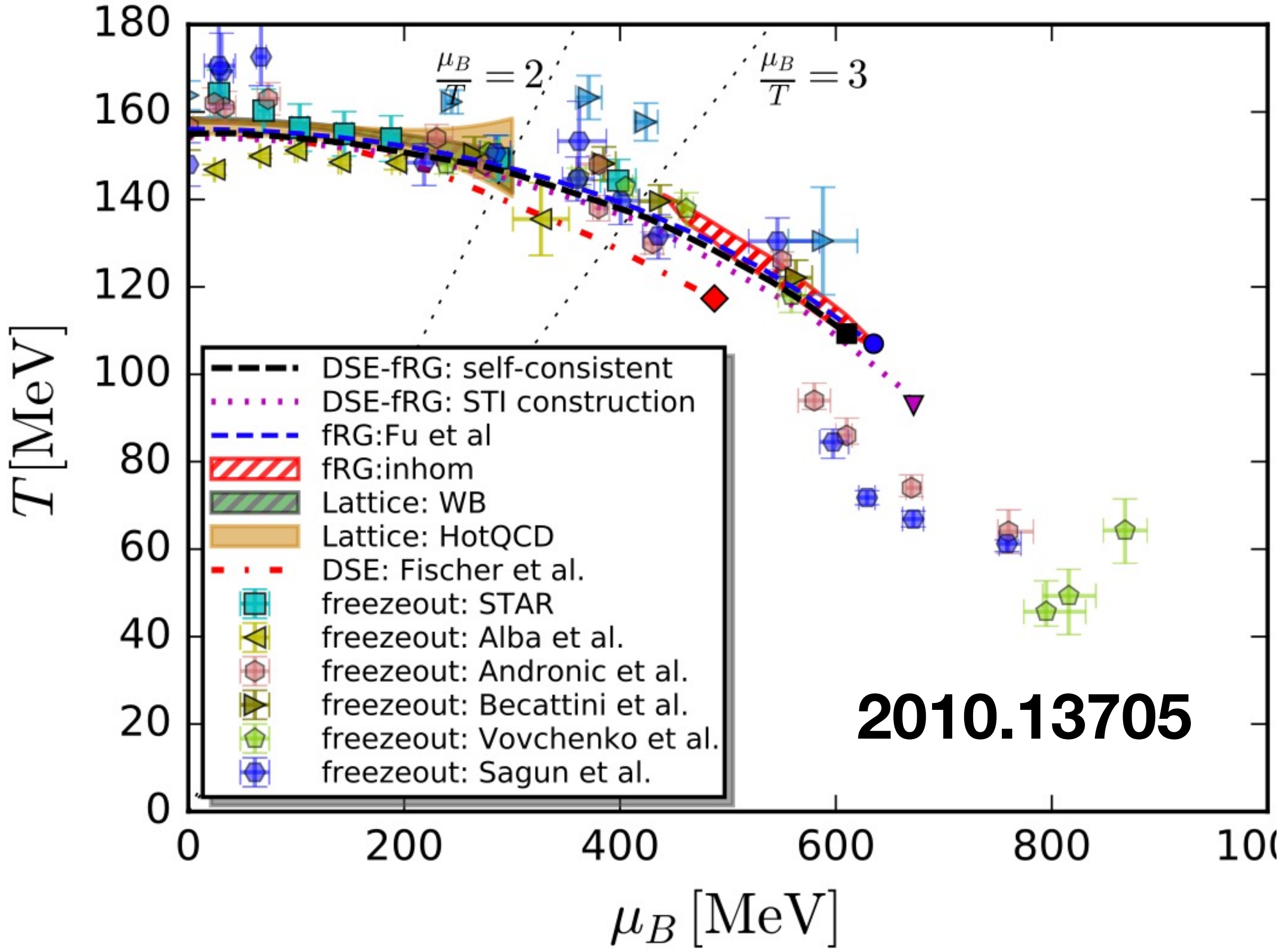

**EFT**: many low energy **effective models** have been proposed to study QCD in certain conditions.

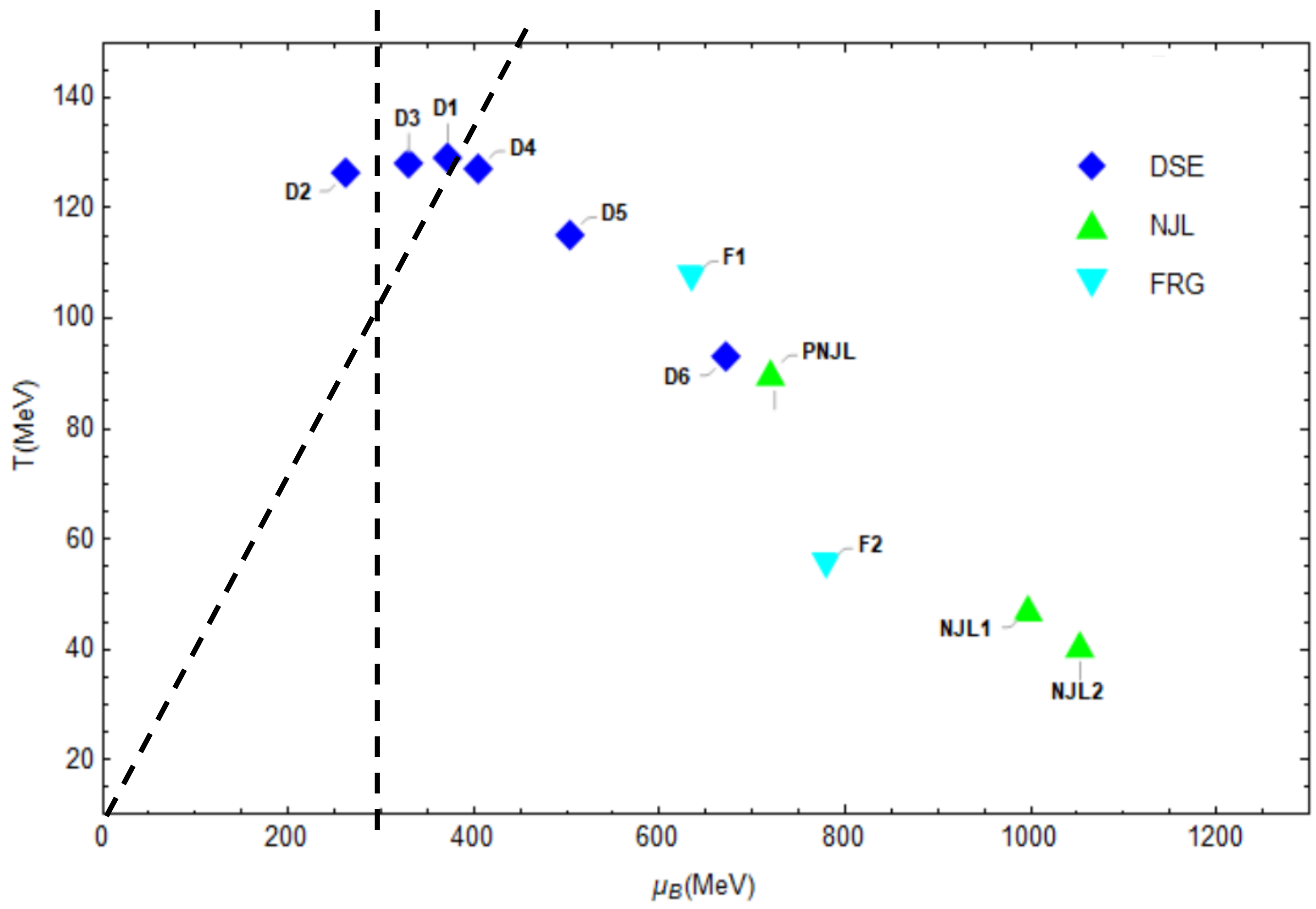



**Match lattice QCD data quantitatively?**

# Searching CEP from EFT

The exact location of CEP is still under dispute, and the lattice-QCD results disfavors the existence of the QCD Critical Point for $\mu_B/T \leq 3, \mu_B < 300$MeV.



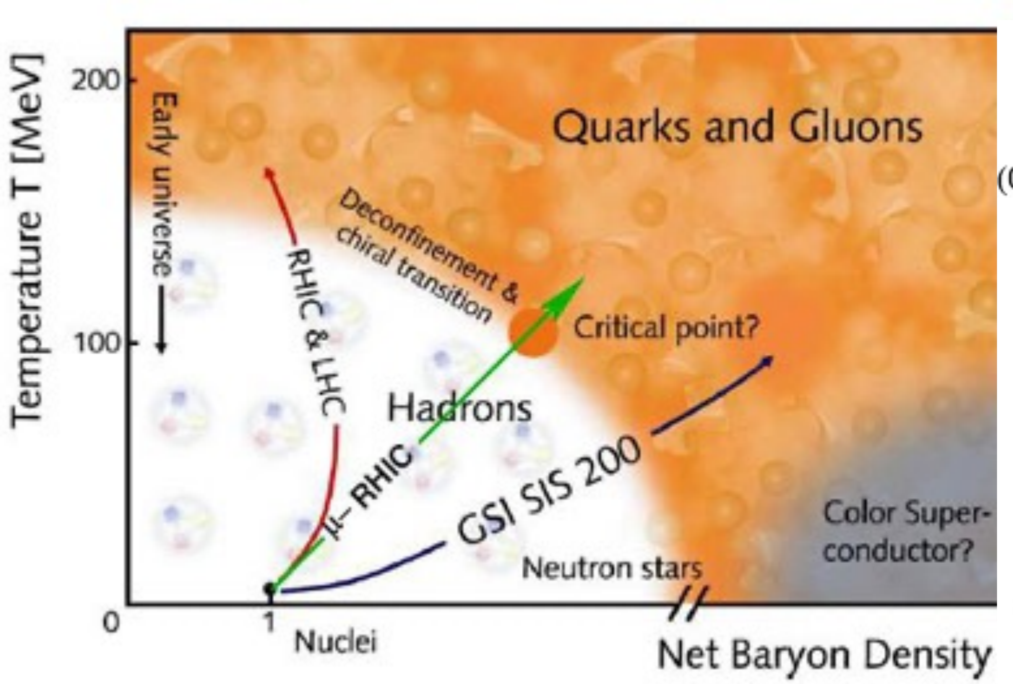

**Schwinger–Dyson equation (DSE):** 2109.09935, 1607.01675, 1405.4762, 2002.07500.
**Nambu–Jona-Lasinio models (NJL, PNJL):** 1801.09215, Nucl. Phys. A 504 (1989), 668-684
**Functional renormalization group (FRG):** 1909.02991, 1709.05654…

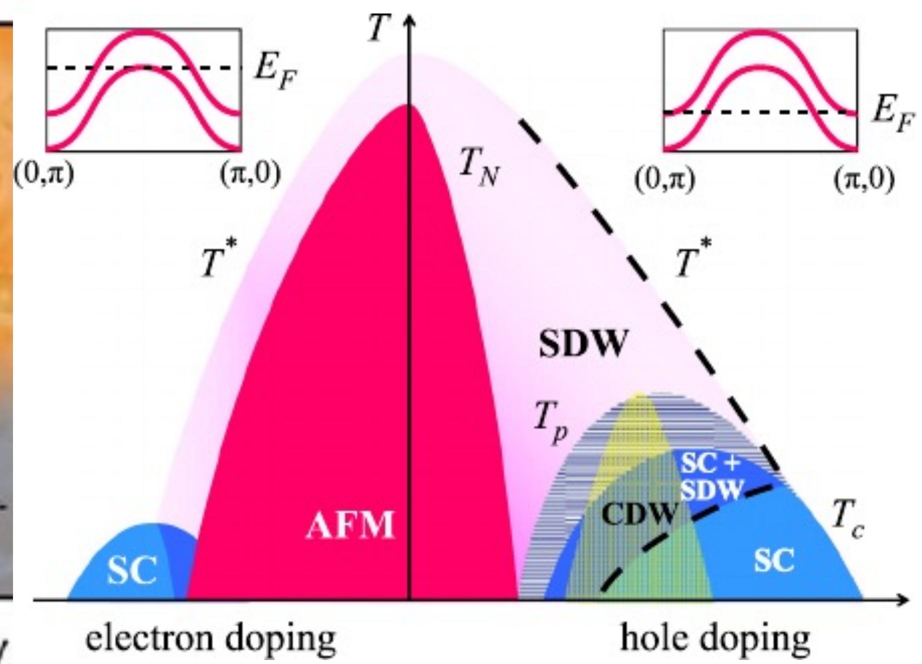
# Holography as a Theoretical Laboratory

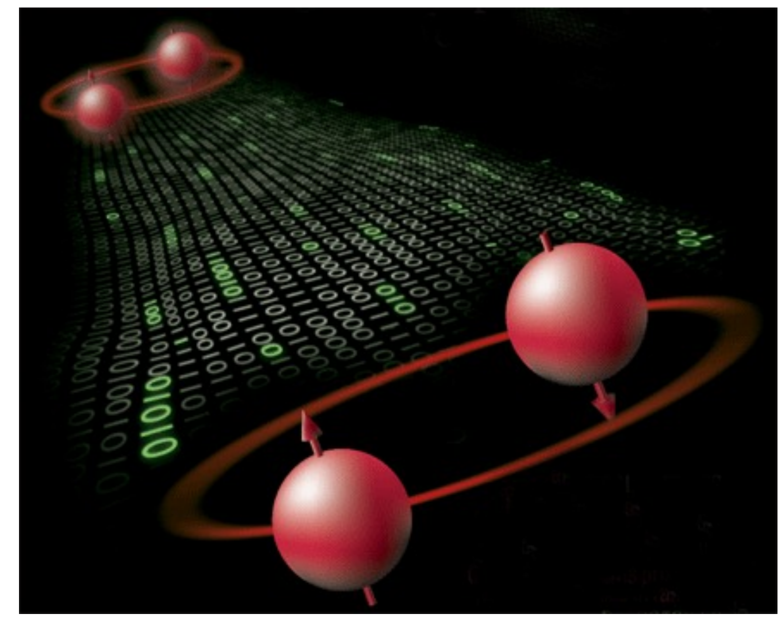
**Non gravitational system**
at strong coupling

↔

**Gravitational theory**
at weak coupling



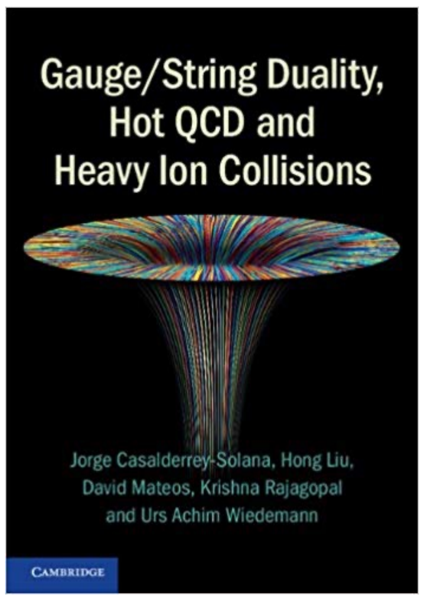



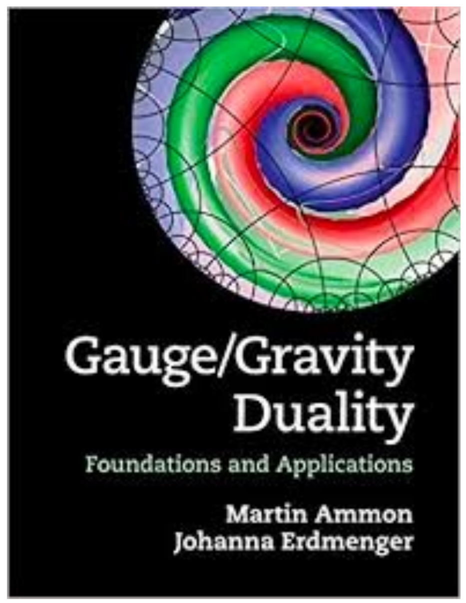



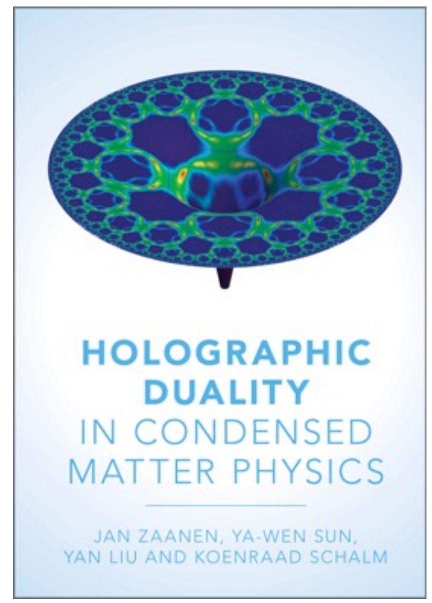
## Applied holography:

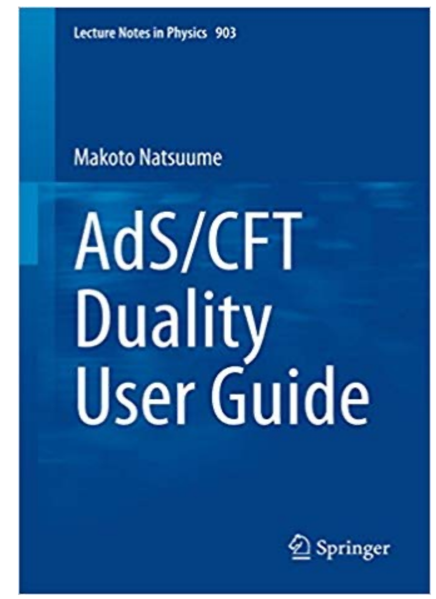
QGP and QCD (drag force, jet quenching, confinement/deconfinement,… ),
Condensed matter (quantum criticality, strange metal, superconductivity,…),
Quantum Entanglement, Non-equilibrium dynamics…

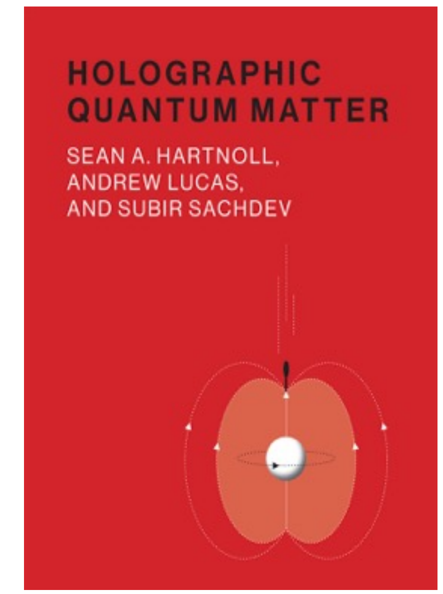
# References



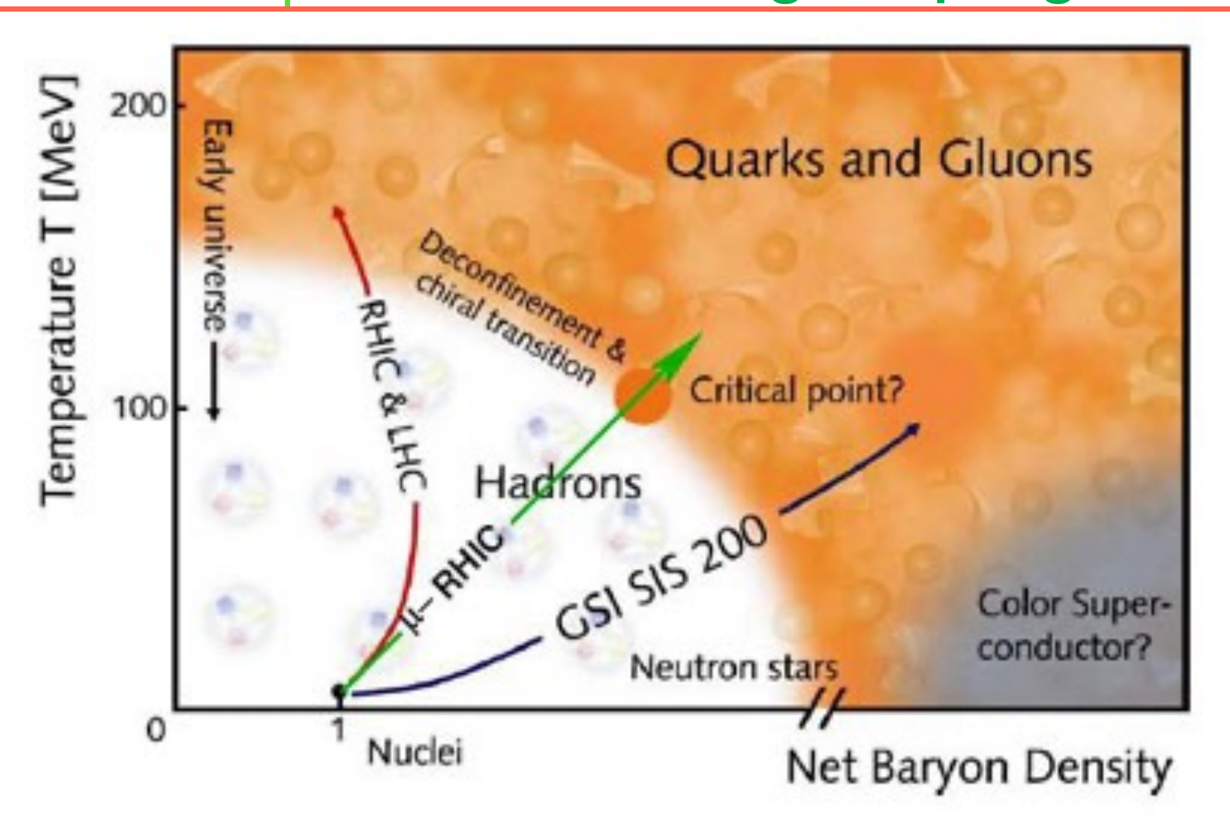



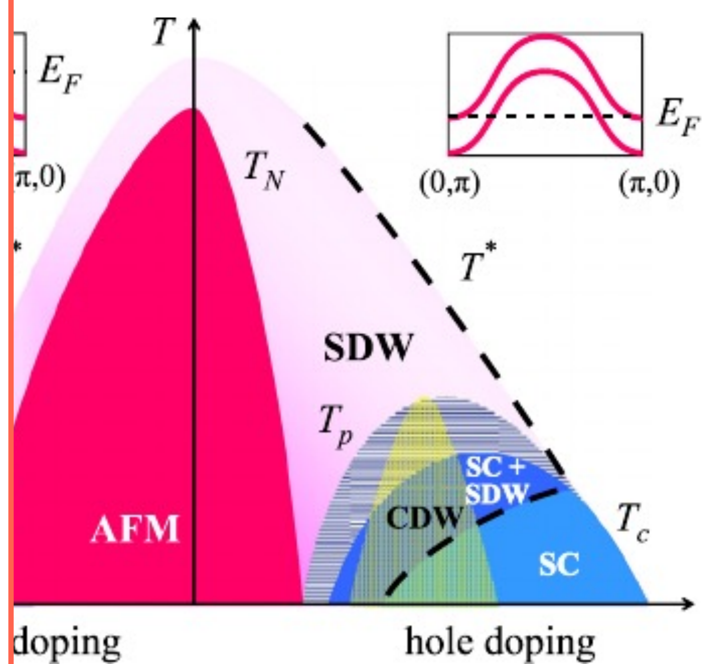

**Hartnoll, 0903.3246; Herzog, 0904.1975; Mcgreevy, 0909.0518; Hubeny et al, 1006.3675; Takayanagi, 1204.2450; Cai et al, 1502.00437; Landsteiner et al, 1911.07978; Baggioli et al, 2101.01892……**

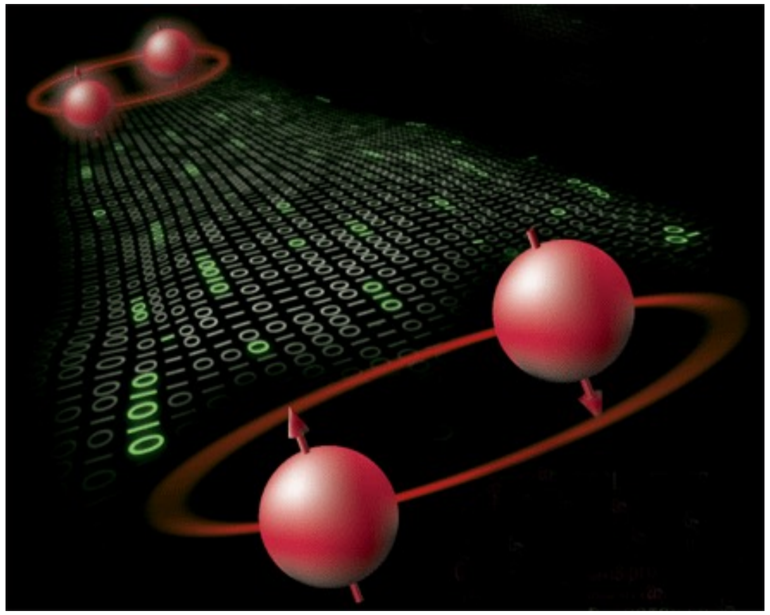
# Holography as a Theoretical Laboratory

**Non gravitational system**
at strong coupling

**Gravitational theory**
at weak coupling



**Holographic QCD:**

**Using holography to tackle non-perturbative QCD problems**

## ❖Top-Down approach: D-brane construction, from superstring

Witten-Sakai Sugimoto model [hep-th/0412141, hep-th/0507073]

D3-D7 model [hep-th/0306018]

D4-D6 model [hep-th/0311270]

Limited ability to characterize QCD properties as its rigidity

Far from real QCD

## ❖Top-Down approach: D-brane construction, from superstring

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Limited ability to characterize QCD properties as its rigidity

Far from real QCD

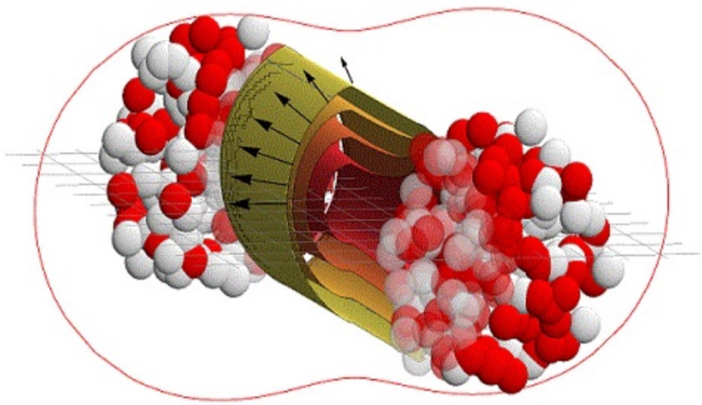
## ❖Bottom-up approach: phenomenology

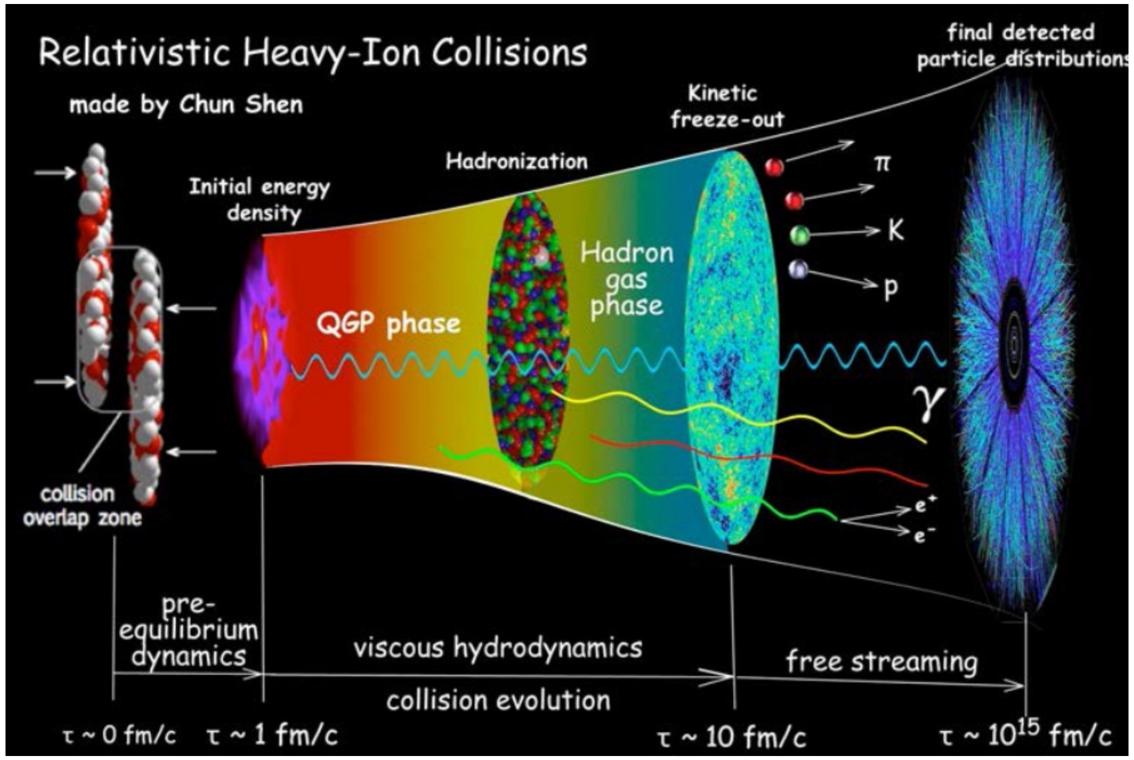
Effective models (more freedom), no clear string origin

However, low energy QCD properties are easier to incorporate

Remarkably, these models yield consistent results of low energy hadron physics with experiments.

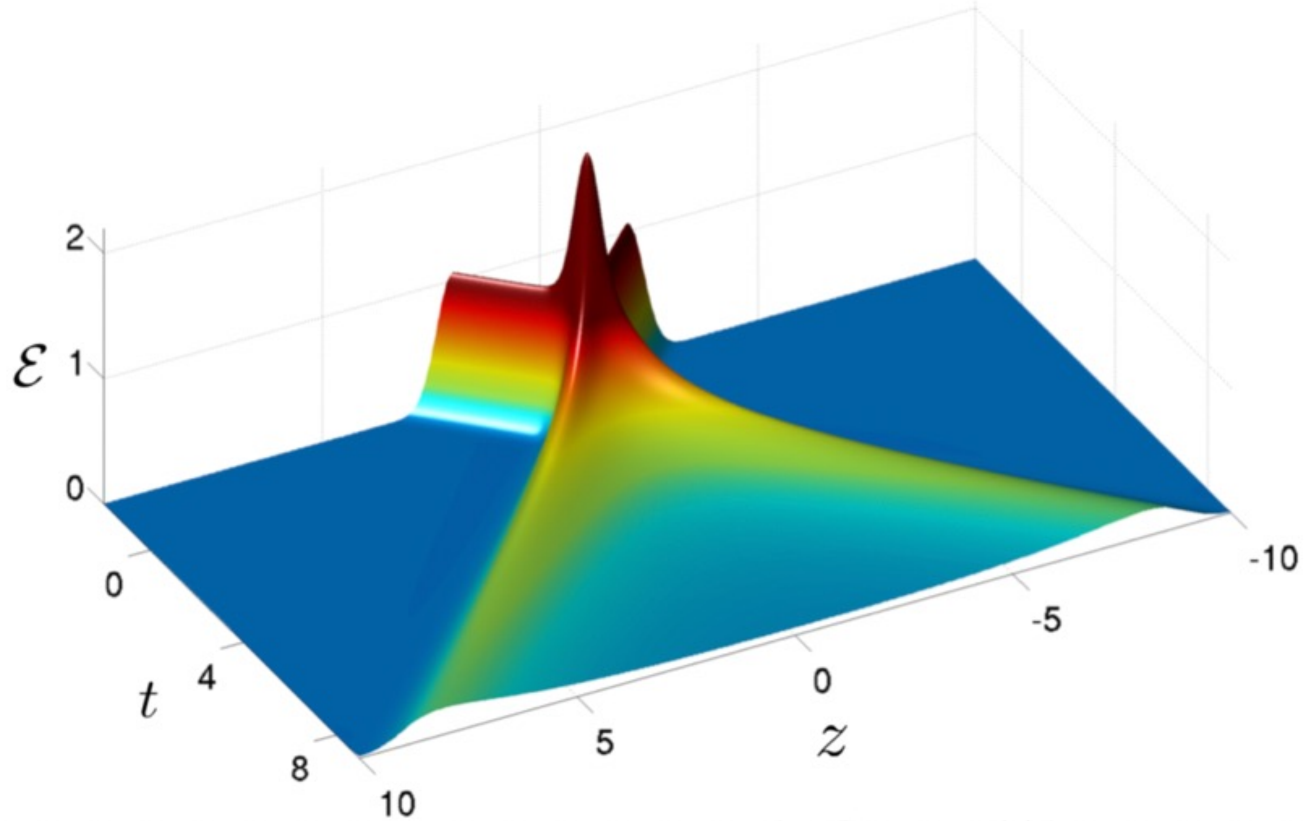
## Holographic QGP






**A fully dynamical simulation:**

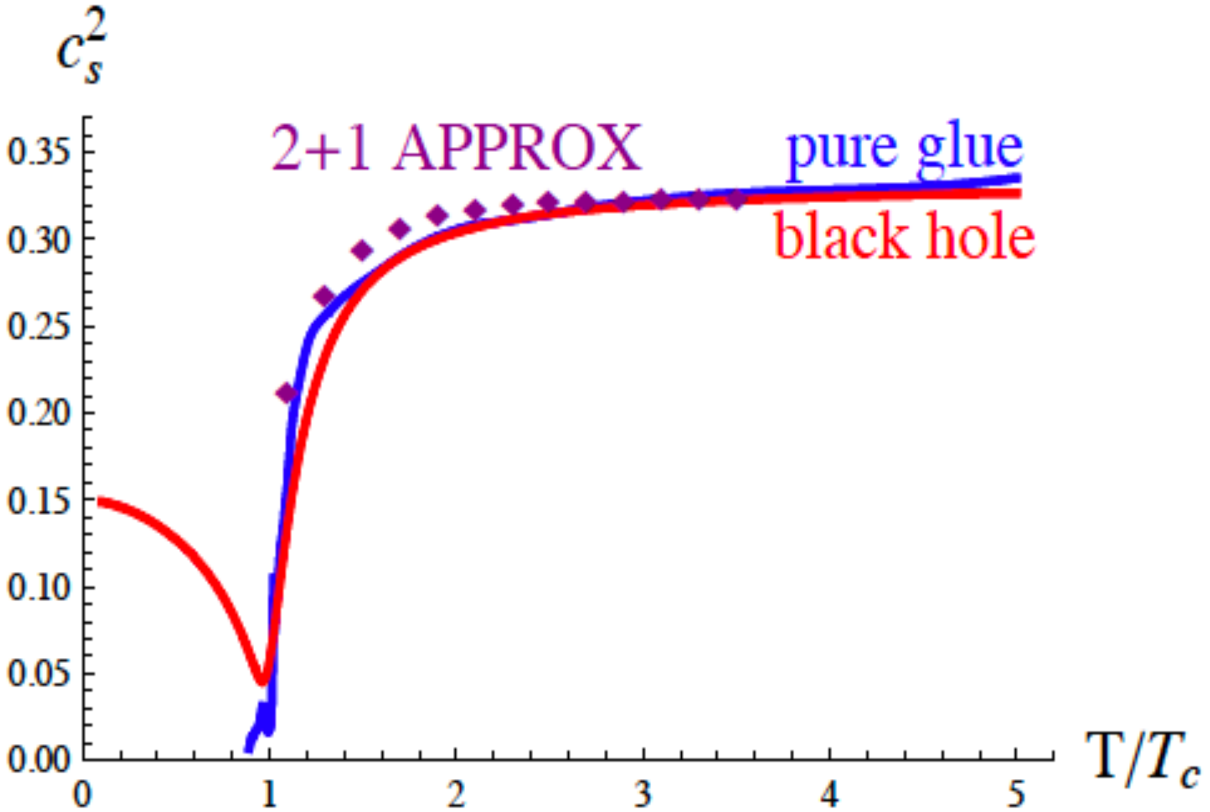
from far-from-equilibrium to viscous hydrodynamics, to a hadronic gas cascade, to the final (measured) particle spectra.



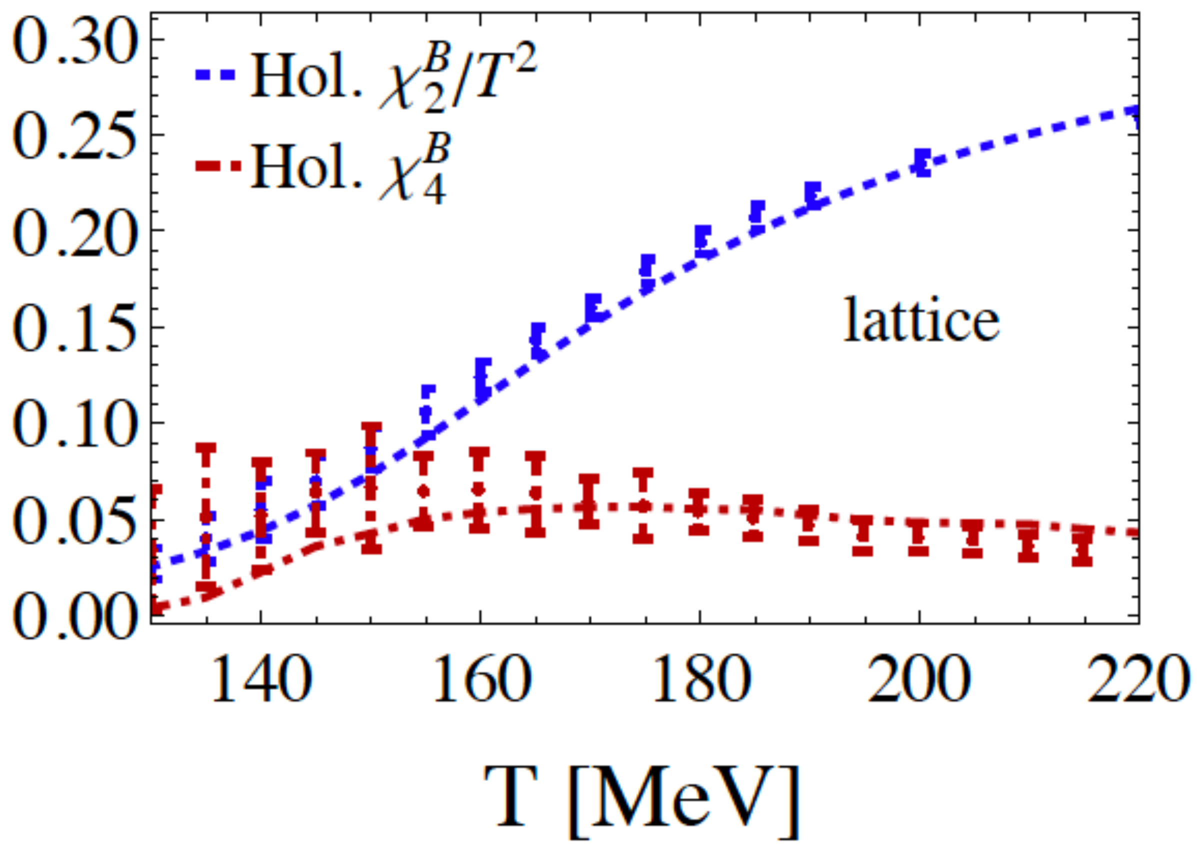

good agreement with head-on collisions performed at the LHC accelerator.

Wilke van der Schee, 1407.1849

## QCD equation of state at zero $\mu_B$




**Speed of sound,**
*Phys.Rev.Lett.* 101 (2008) 131601




**Baryon susceptibility,**
*Phys.Rev.Lett.* 115 (2015) 20, 202301

A strong indication that holography can make **quantitative predictions** for the properties of QCD in **non-perturbative** regime.

## Two recent reviews:

**Bottom-up approach**

### The dynamical holographic QCD method for hadron physics and QCD matter

Yidian Chen (Beijing, GUCAS), Danning Li (Jinan U.), Mei Huang (Beijing, GUCAS) (Jun 2, 2022)

Published in: *Commun.Theor.Phys.* 74 (2022) 9, 097201 • e-Print: 2206.00917 [hep-ph]

pdf DOI cite claim reference search

### Hot QCD Phase Diagram From Holographic Einstein-Maxwell-Dilaton Models

Romulo Rougemont (Goias U.), Joaquin Grefa (Houston U.), Mauricio Hippert (Illinois U., Urbana), Jorge Noronha (Illinois U., Urbana), Jacquelyn Noronha-Hostler (Illinois U., Urbana) et al. (Jul 7, 2023)

e-Print: 2307.03885 [nucl-th]

pdf cite claim reference search

# Outline

1. Introduction

**2. Holographic QCD model**

3. Phase diagram and GWs

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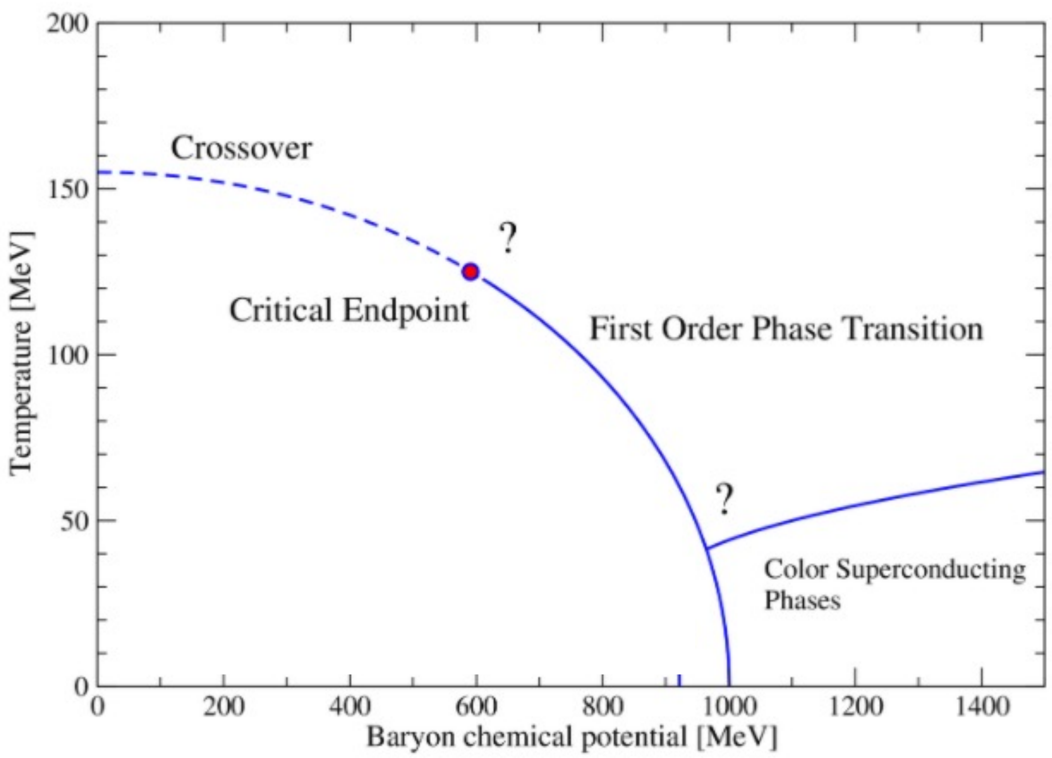
# 2. Holographic QCD Model

**Einstein-Maxwell-Dilaton theory:**

Rong-Gen Cai, Song He, **Li Li**, Yuan-Xu Wang,
Phys. Rev. D 106 no.12, L121902 (2022)

$$S = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\Big[\mathcal{R} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)\Big]\,,$$

To match the degrees of freedom in QCD phase Diagram: $\phi,\ \ A_\mu$



# 2. Holographic QCD Model

**Einstein-Maxwell-Dilaton theory:**

Rong-Gen Cai, Song He, **Li Li**, Yuan-Xu Wang,
Phys. Rev. D 106 no.12, L121902 (2022)

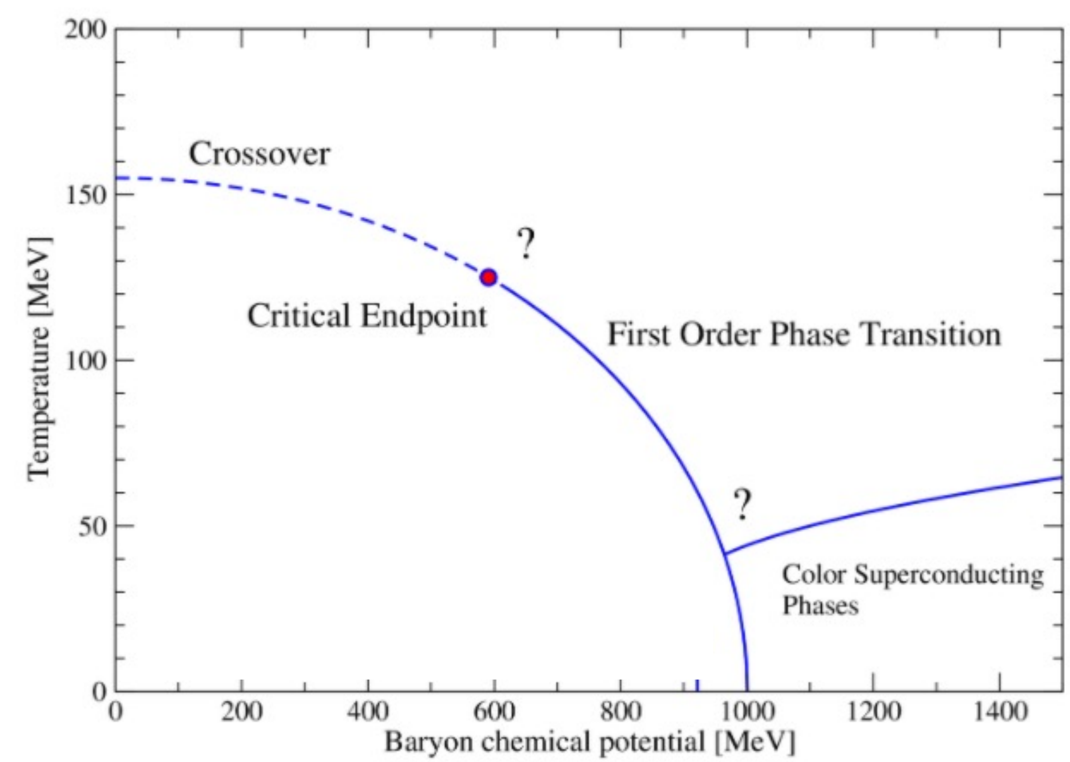
Break conformal symmetry

$$S = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\Big[\mathcal{R} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)\Big]\,,$$

Baryon chemical potential

To match the degrees of freedom in QCD phase Diagram: $\phi,\ A_\mu$

To match lattice QCD simulation at $\mu_B = 0$: $Z(\phi),\ V(\phi)$



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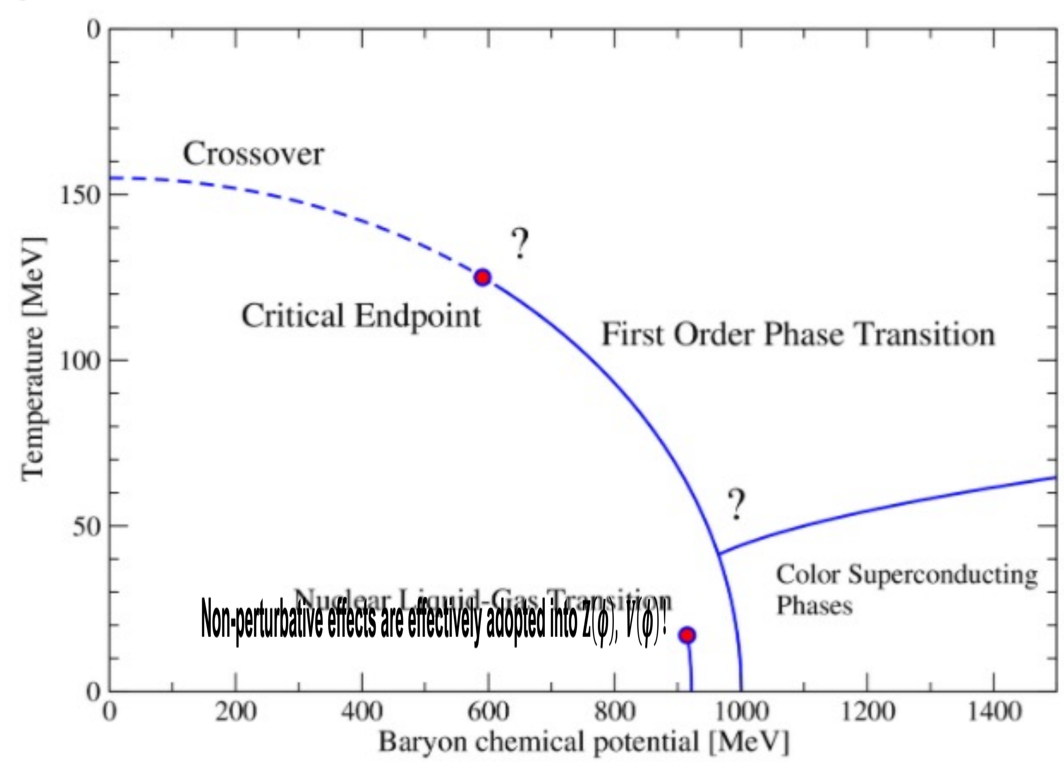



**Non-perturbative effects are effectively adopted into $\mathrm{Z}(\phi),\ V(\phi)$!**

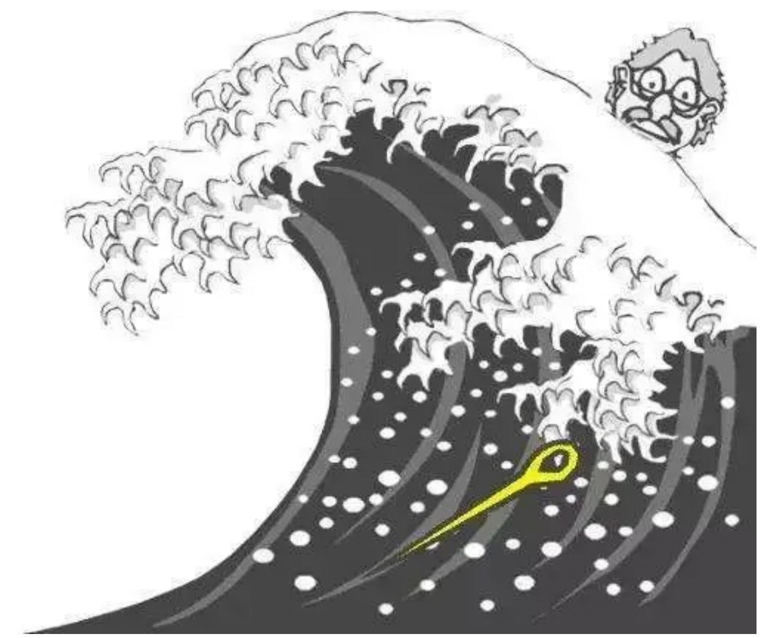
# 2. Holographic QCD Model

$$S = \frac{1}{2\kappa_N^2} \int d^5x \sqrt{-g} \left[ \mathcal{R} - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - \frac{Z(\phi)}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi) \right],$$

To fix $Z(\phi)$ and $V(\phi)$ is very challenging !

$$V(\phi) = -12 \cosh[c_1 \phi] + (6c_1^2 - \frac{3}{2})\phi^2 + c_2 \phi^6 ,$$

$$Z(\phi) = \frac{1}{1+c_3} \operatorname{sech}[c_4 \phi^3] + \frac{c_3}{1+c_3} e^{-c_5 \phi} ,$$



Free model parameters

$c_1, c_2, c_3, c_4, c_5$ $\kappa_N^2$

All parameters are fixed using state-of-the-art lattice QCD data

Equation of state in $(2+1)$-flavor QCD

A. Bazavov, Tanmoy Bhattacharya, C. DeTar, H.-T. Ding, Steven Gottlieb, Rajan Gupta, P. Hegde, U. M. Heller, F. Karsch, E. Laermann, L. Levkova, Swagato Mukherjee, P. Petreczky, C. Schmidt, C. Schroeder, R. A. Soltz, W. Soeldner, R. Sugar, M. Wagner, and P. Vranas (HotQCD Collaboration)

Phys. Rev. D **90**, 094503 – Published 4 November 2014

# Black hole thermodynamics

**Black hole ansatz:**

$$ds^2 = -f(r)e^{-\eta(r)}dt^2 + \frac{dr^2}{f(r)} + r^2 d\boldsymbol{x}_3^2$$

$$\phi = \phi(r), \quad A_t = A_t(r)$$

**To solve equations of motion:**

$$\phi'' + \left(\frac{f'}{f} - \frac{\eta'}{2} + \frac{3}{r}\right)\phi' + \frac{\partial_\phi Z}{2f}e^\eta A_t'^2 - \frac{1}{f}\partial_\phi V = 0$$

$$\partial_r(e^{\eta/2}\, r^3\, Z A_t') = 0$$

$$\frac{\eta'}{r} + \frac{1}{3}\phi'^2 = 0$$

$$\frac{2}{r}\frac{f'}{f} - \frac{\eta'}{r} + \frac{Z}{3f}e^\eta A_t'^2 + \frac{2}{3f}V + \frac{4}{r^2} = 0$$

$$V(\phi) = -12\cosh[c_1\phi] + (6c_1^2 - \frac{3}{2})\phi^2 + c_2\phi^6$$

$$Z(\phi) = \frac{1}{1+c_3}\mathrm{sech}[c_4\phi^3] + \frac{c_3}{1+c_3}e^{-c_5\phi},$$

asymptotic expansion at boundary:

$$\phi(r) = \frac{\phi_s}{r} + \frac{\phi_v}{r^3} - \frac{\ln(r)}{6r^3}(1-6c_1^4)\phi_s^3 + \mathcal{O}(\frac{\ln(r)}{r^5})\,.$$

$$\begin{aligned}A_t(r) = \mu_B &- \frac{2\kappa_N^2\rho_B}{2r^2} - \frac{2\kappa_N^2\rho_B\, c_3\, c_5\, \phi_s}{3(1+c_3)\, r^3} \\ &+ \frac{2\kappa_N^2\rho_B\, \phi_s^2\, ((1+c_3)^2 - 6(-1+c_3)\, c_3\, c_5^2)}{48(1+c_3)^2\, r^4} \\ &+ \frac{2\kappa_N^2\rho_B\, c_3\, c_5\, (-10\, c_5^2\, (1+(-4+c_3)\, c_3))\, \phi_s^3}{300(1+c_3)^3\, r^5} \\ &+ \frac{2\kappa_N^2\rho_B\, c_3\, c_5\, ((7-12\, c_1^4)\, \phi_s^3 - 60\, \phi_v)}{300(1+c_3)\, r^5} \\ &- \frac{2\kappa_N^2\rho_B\, c_3\, c_5\, \phi_s^3(-1+6\, c_1^4)\ln(r)}{30(1+c_3)\, r^5} + \mathcal{O}(\frac{\ln(r)}{r^6})\,.\end{aligned}$$

$$\begin{aligned}\eta(r) = 0 &+ \frac{\phi_s^2}{6r^2} + \frac{(1-6c_1^4)\phi_s^4 + 72\phi_s\phi_v}{144r^4} \\ &- \frac{\ln(r)}{12r^4}(1-6c_1^4)\phi_s^4 + \mathcal{O}(\frac{\ln(r)^2}{r^6})\,.\end{aligned}$$

$$\begin{aligned}f(r) = r^2\Big[1 &+ \frac{\phi_s^2}{6r^2} + \frac{f_v}{r^4} - \frac{\ln(r)}{12r^4}(1-6c_1^4)\phi_s^4 \\ &+ \mathcal{O}\Big(\frac{\ln(r)^2}{r^6}\Big)\Big]\,.\end{aligned}$$

# Black hole thermodynamics

**Black hole ansatz:**

$$ds^2 = -f(r)e^{-\eta(r)}dt^2 + \frac{dr^2}{f(r)} + r^2 d\boldsymbol{x}_3^2$$

$$\phi = \phi(r), \quad A_t = A_t(r)$$

Temperature and entropy

$$T = \frac{1}{4\pi} f'(r_h) e^{-\eta(r_h)/2}, \qquad s = \frac{2\pi}{\kappa_N^2} r_h^3$$

Free energy

$$\Omega = \frac{1}{2\kappa_N^2}\left(f_v - \phi_s\phi_v - \frac{3 - 48b - 8c_1^4}{48}\phi_s^4\right)$$

Energy and pressure

$$\epsilon := T_{tt} = \frac{1}{2\kappa_N^2}\left(-3f_v + \phi_s\phi_v + \frac{1+48b}{48}\phi_s^4\right),$$

$$P := T_{xx} = T_{yy} = T_{zz}$$

$$= \frac{1}{2\kappa_N^2}\left(-f_v + \phi_s\phi_v + \frac{3 - 48b - 8c_1^4}{48}\phi_s^4\right),$$

asymptotic expansion at boundary:

$$\phi(r) = \frac{\phi_s}{r} + \frac{\phi_v}{r^3} - \frac{\ln(r)}{6r^3}(1 - 6c_1^4)\phi_s^3 + \mathcal{O}(\frac{\ln(r)}{r^5}).$$

$$\begin{aligned} A_t(r) = \mu_B &- \frac{2\kappa_N^2\rho_B}{2r^2} - \frac{2\kappa_N^2\rho_B\, c_3\, c_5\, \phi_s}{3(1+c_3)\, r^3} \\ &+ \frac{2\kappa_N^2\rho_B\, \phi_s^2\, ((1+c_3)^2 - 6(-1+c_3)\, c_3\, c_5^2)}{48(1+c_3)^2\, r^4} \\ &+ \frac{2\kappa_N^2\rho_B\, c_3\, c_5\, (-10\, c_5^2\, (1 + (-4+c_3)\, c_3))\, \phi_s^3}{300(1+c_3)^3\, r^5} \\ &+ \frac{2\kappa_N^2\rho_B\, c_3\, c_5\, ((7 - 12\, c_1^4)\, \phi_s^3 - 60\, \phi_v)}{300(1+c_3)\, r^5} \\ &- \frac{2\kappa_N^2\rho_B\, c_3\, c_5\, \phi_s^3(-1 + 6\, c_1^4)\ln(r)}{30(1+c_3)\, r^5} + \mathcal{O}(\frac{\ln(r)}{r^6}). \end{aligned}$$

$$\begin{aligned} \eta(r) = 0 &+ \frac{\phi_s^2}{6r^2} + \frac{(1-6c_1^4)\phi_s^4 + 72\phi_s\phi_v}{144r^4} \\ &- \frac{\ln(r)}{12r^4}(1-6c_1^4)\phi_s^4 + \mathcal{O}(\frac{\ln(r)^2}{r^6}). \end{aligned}$$

$$\begin{aligned} f(r) = r^2\Big[1 &+ \frac{\phi_s^2}{6r^2} + \frac{f_v}{r^4} - \frac{\ln(r)}{12r^4}(1-6c_1^4)\phi_s^4 \\ &+ \mathcal{O}\Big(\frac{\ln(r)^2}{r^6}\Big)\Big]. \end{aligned}$$

# Black hole thermodynamics

**Black hole ansatz:**

$$ds^2 = -f(r)e^{-\eta(r)}dt^2 + \frac{dr^2}{f(r)} + r^2 d\boldsymbol{x}_{\boldsymbol{3}}^{\boldsymbol{2}}$$

$$\phi = \phi(r), \quad A_t = A_t(r)$$

Temperature and entropy

$$T = \frac{1}{4\pi} f'(r_h) e^{-\eta(r_h)/2}, \quad s = \frac{2\pi}{\kappa_N^2} r_h^3$$

Free energy

$$\Omega = \frac{1}{2\kappa_N^2}\left(f_v - \phi_s\phi_v - \frac{3-48b-8c_1^4}{48}\phi_s^4\right)$$

Energy and pressure

$$\epsilon := T_{tt} = \frac{1}{2\kappa_N^2}\left(-3f_v + \phi_s\phi_v + \frac{1+48b}{48}\phi_s^4\right),$$

$$P := T_{xx} = T_{yy} = T_{zz}$$

$$= \frac{1}{2\kappa_N^2}\left(-f_v + \phi_s\phi_v + \frac{3-48b-8c_1^4}{48}\phi_s^4\right),$$

◆Thermodynamic relation

$$\boldsymbol{\Omega = \epsilon - Ts - \mu_B\rho_B = -P}$$

$$\mathcal{Q} = \frac{1}{2\kappa_N^2} r^3 e^{\eta/2}\left[r^2\left(\frac{f}{r^2}e^{-\eta}\right)' - ZA_tA_t'\right]$$

◆First law of thermodynamics

$$\boldsymbol{d\epsilon = Tds + \mu_B d\rho_B - O d\phi_s}$$

Scalar source $\boldsymbol{\phi_s}$ will be fixed

## **Thermodynamics** of AdS Black Holes with Scalar Hair

$$S = \frac{1}{2\kappa_N^2}\int d^{d+1}x\sqrt{-g}\left[\mathcal{R} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right]$$

$$ds^2 = -f(r)e^{-\eta(r)}dt^2 + \frac{dr^2}{f(r)} + r^2 d\vec{x}^2_{d-1}, \quad \phi = \phi(r)$$

Naively-expected 1[st] law of thermodynamics does not hold [1408.0010,1408.1514]

$$dE = TdS + XdY = TdS - (c_1\phi_v d\phi_s - c_2\phi_s d\phi_v)$$

## Thermodynamics of AdS Black Holes with Scalar Hair

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Naively-expected 1st law of thermodynamics does not hold [1408.0010,1408.1514]

$$dE = TdS + XdY = TdS - (c_1\phi_v d\phi_s - c_2\phi_s d\phi_v)$$

**Wald formula**

$$\delta H = -\frac{\Sigma}{2\kappa_N^2} r^{d-1} e^{-\eta/2}\left[\frac{d-1}{r}\delta f + f\phi'\delta\phi\right]$$

**holographic renormalization**

$$\langle O\rangle = \frac{\delta}{\delta\phi_s}(S + S_\partial)_{on-shell}$$

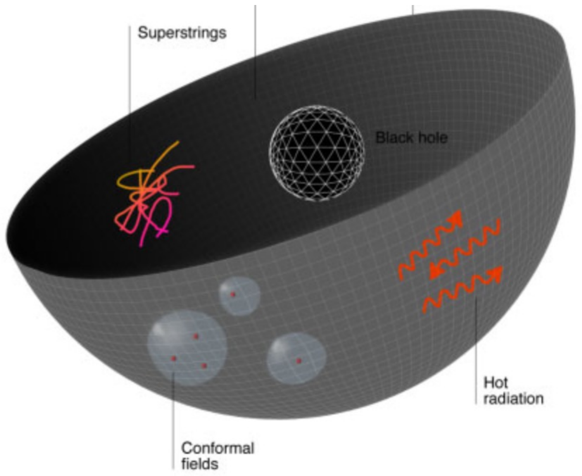
$$\frac{\delta H_h}{\Sigma} = \frac{1}{2\kappa_N^2} e^{-\eta(r_h)/2} f'(r_h)(d-1) r_h^{d-2}\delta r_h = T\delta s$$

$$\frac{\delta H_\infty}{\Sigma} = \delta\mathcal{E} + \langle O\rangle\,\delta\phi_s$$

$$d\mathcal{E} = Tds - \langle O\rangle\, d\phi_s$$

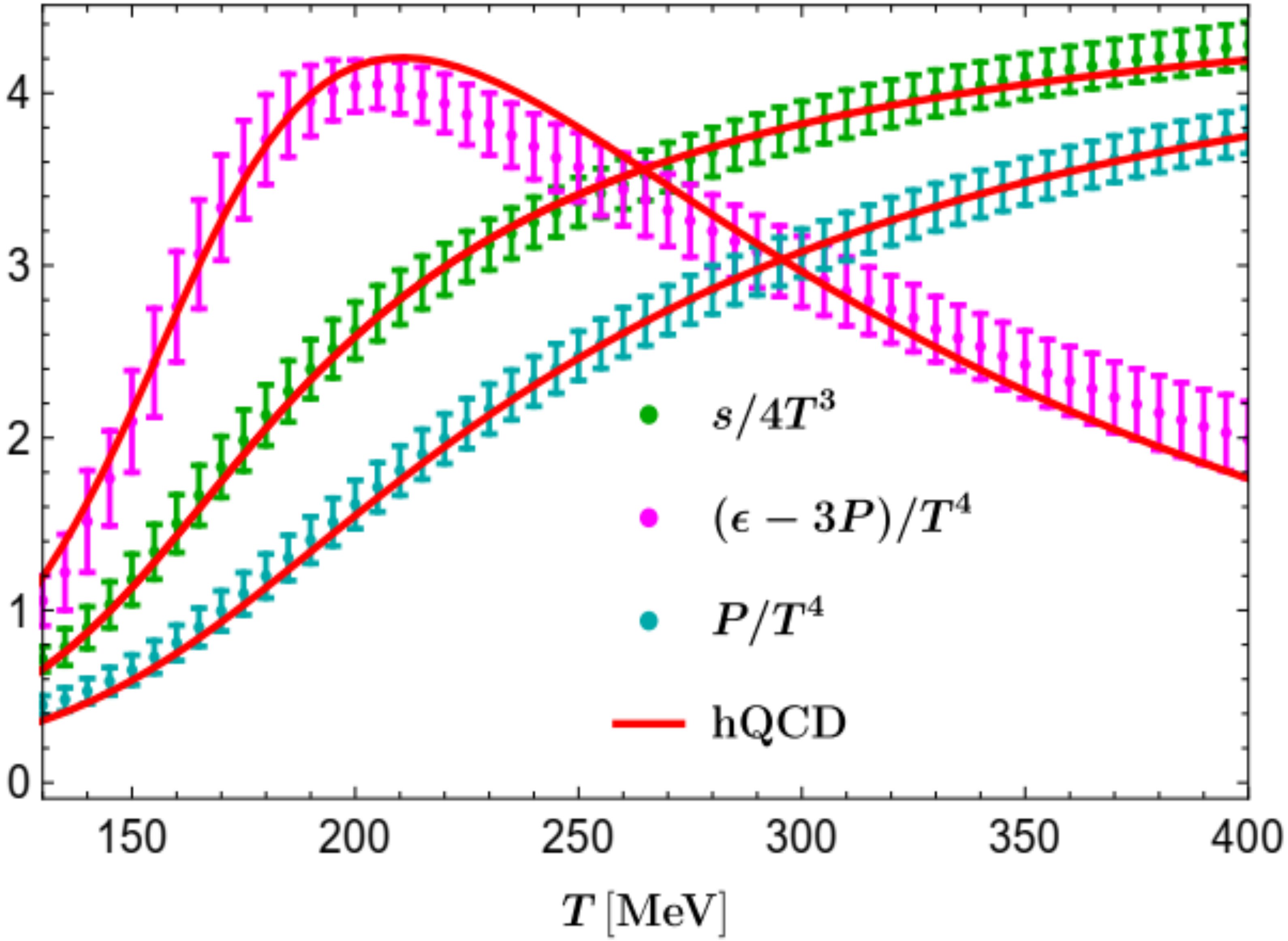
**Li Li**, Phys. Lett. B 815 (2021), 136123

# Fix model parameters by thermodynamics




**Equations of state at $\mu_B = 0$ :**

entropy, trace anomaly, pressure




$$T = \frac{1}{4\pi} f'(r_h) e^{-\eta(r_h)/2}$$

$$s = \frac{2\pi}{\kappa_N^2} r_h^3$$

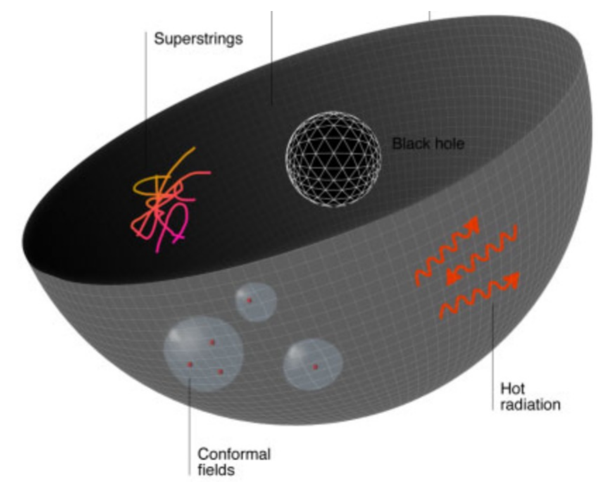
$$\epsilon := T_{tt}$$

$$P := T_{xx}$$

Lattice data for (2+1)-flavor QCD

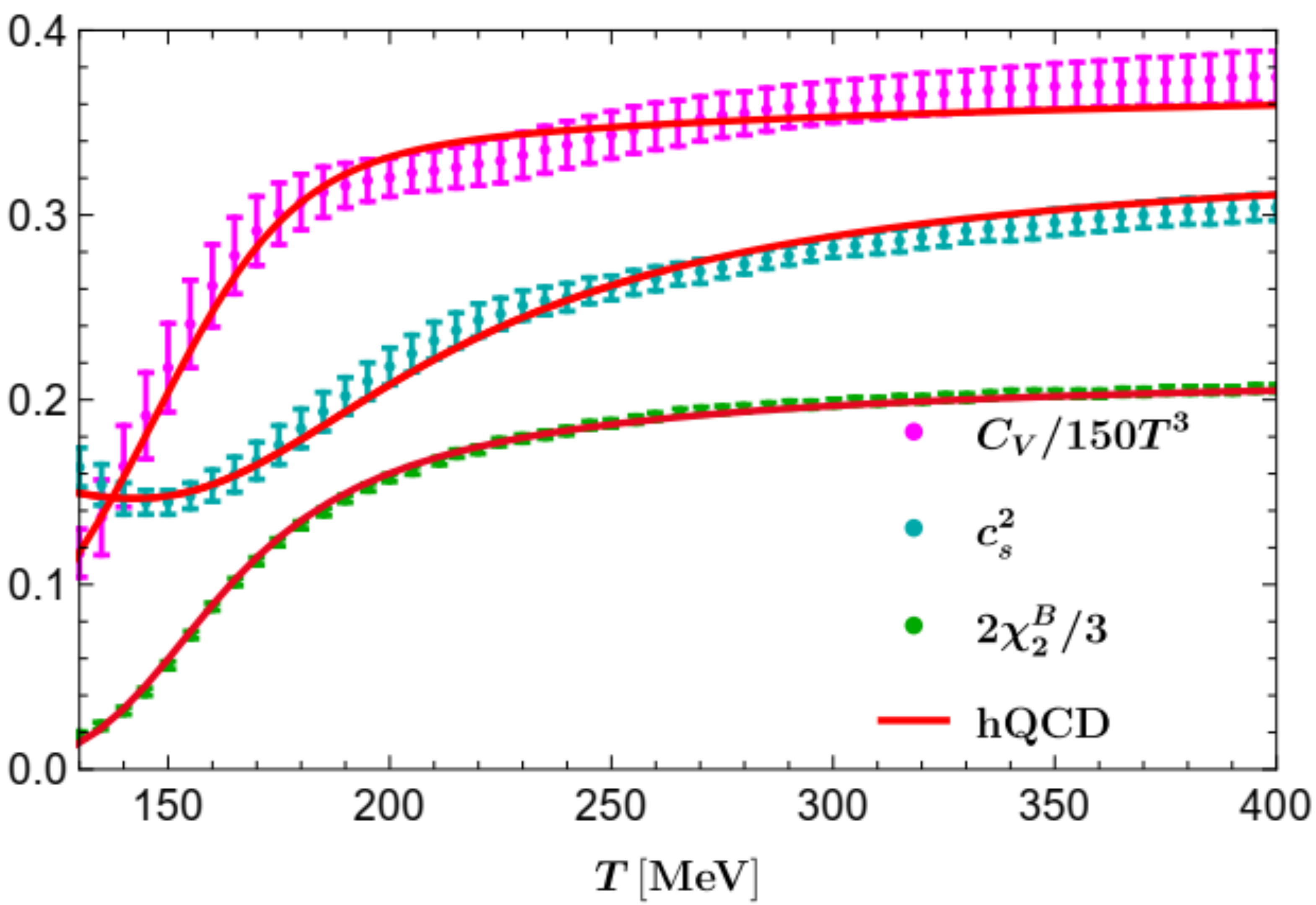
A. Bazavov *et al.* [HotQCD], Phys. Rev. D 90 (2014), 094503

# Fix model parameters by thermodynamics




**Transport coefficients at $\mu_B = 0$ :**

specific heat, sound speed, baryon susceptibility




$$c_s = \sqrt{(dP/d\epsilon)_{\mu_B}}$$

$$C_V = (d\epsilon/dT)_{\mu_B}$$

$$\chi_2^B = (d\rho_B/d\mu_B)_T$$

Lattice data for (2+1)-flavor QCD

**A. Bazavov *et al.* [HotQCD], Phys. Rev. D 90 (2014), 094503**

# Outline

1. Introduction

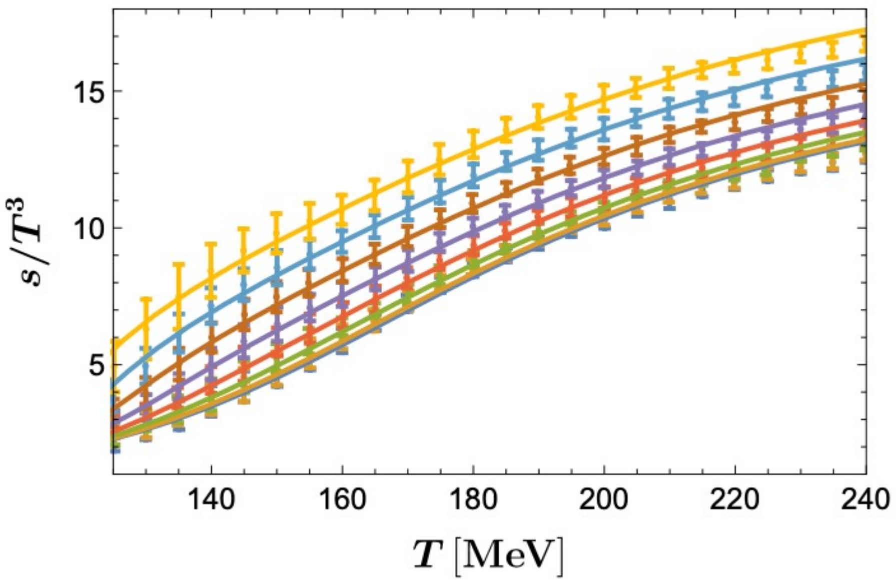
2. Holographic QCD model

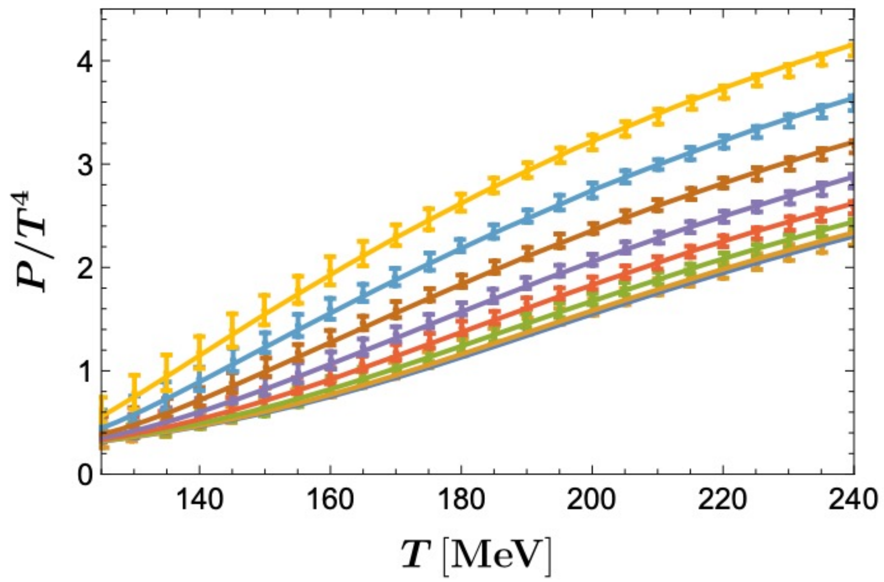
**3. Phase diagram and GWs**

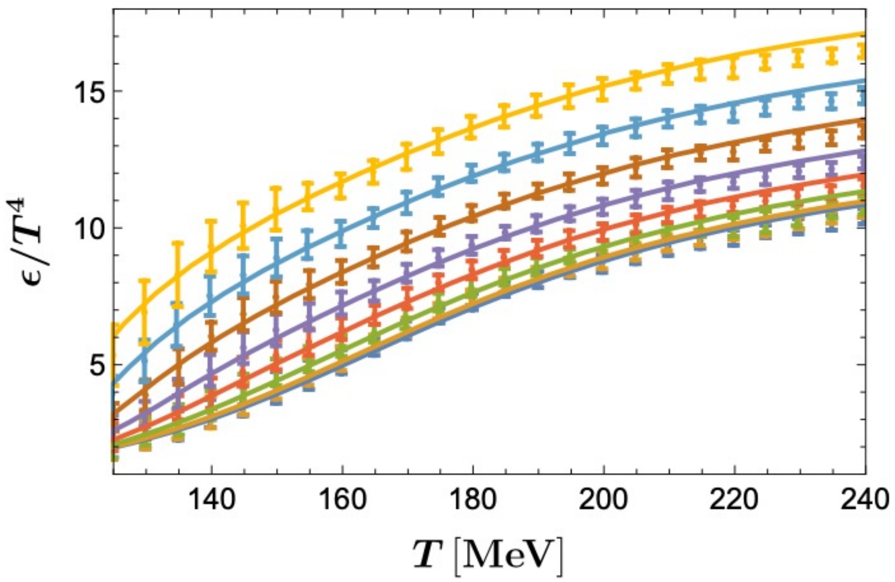
4. Summary and discussion

# 3. QCD Phase diagram and GWs

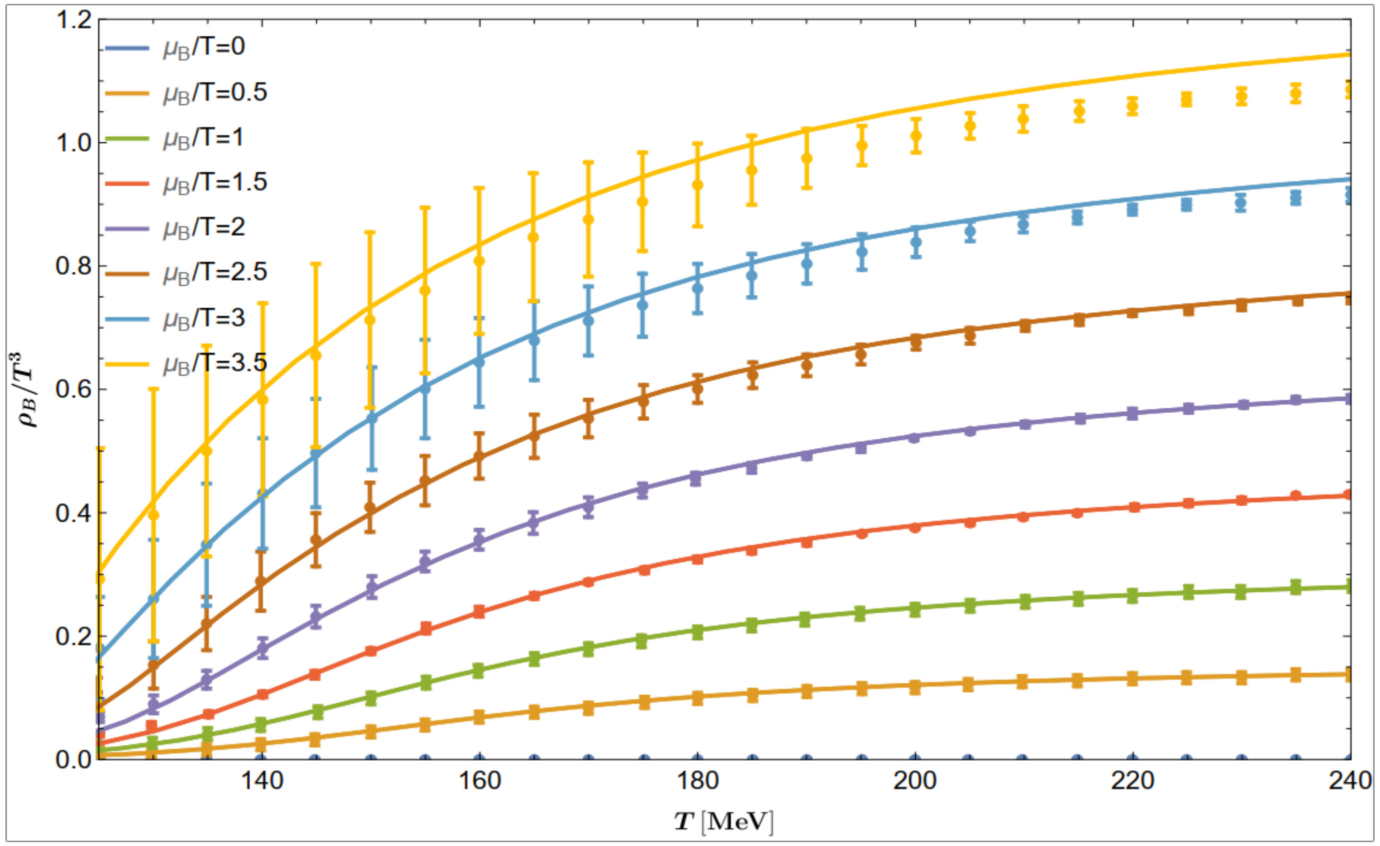
## Predictions at finite chemical potential




More challenging to fit $\rho_B$

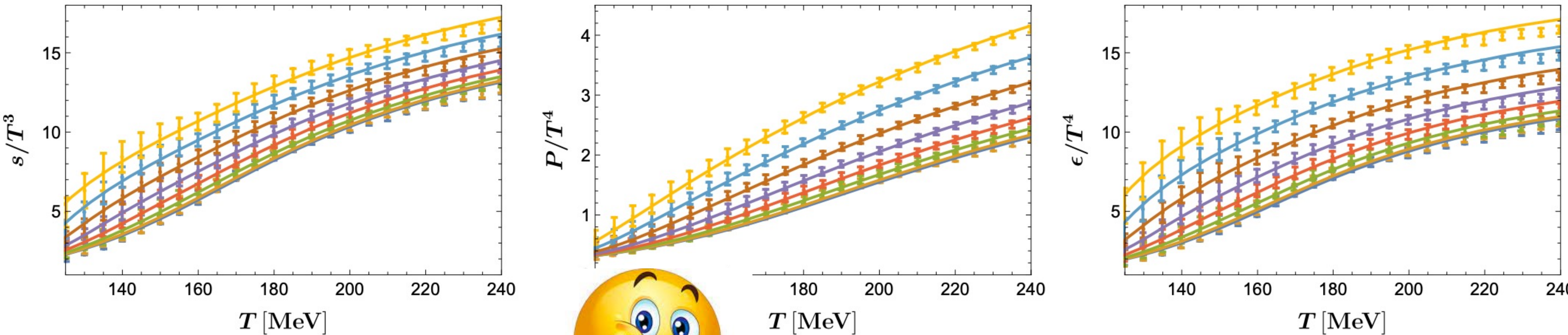



**S. Borsanyi, et al., Phys. Rev. Lett. 126 (2021) no.23, 232001**

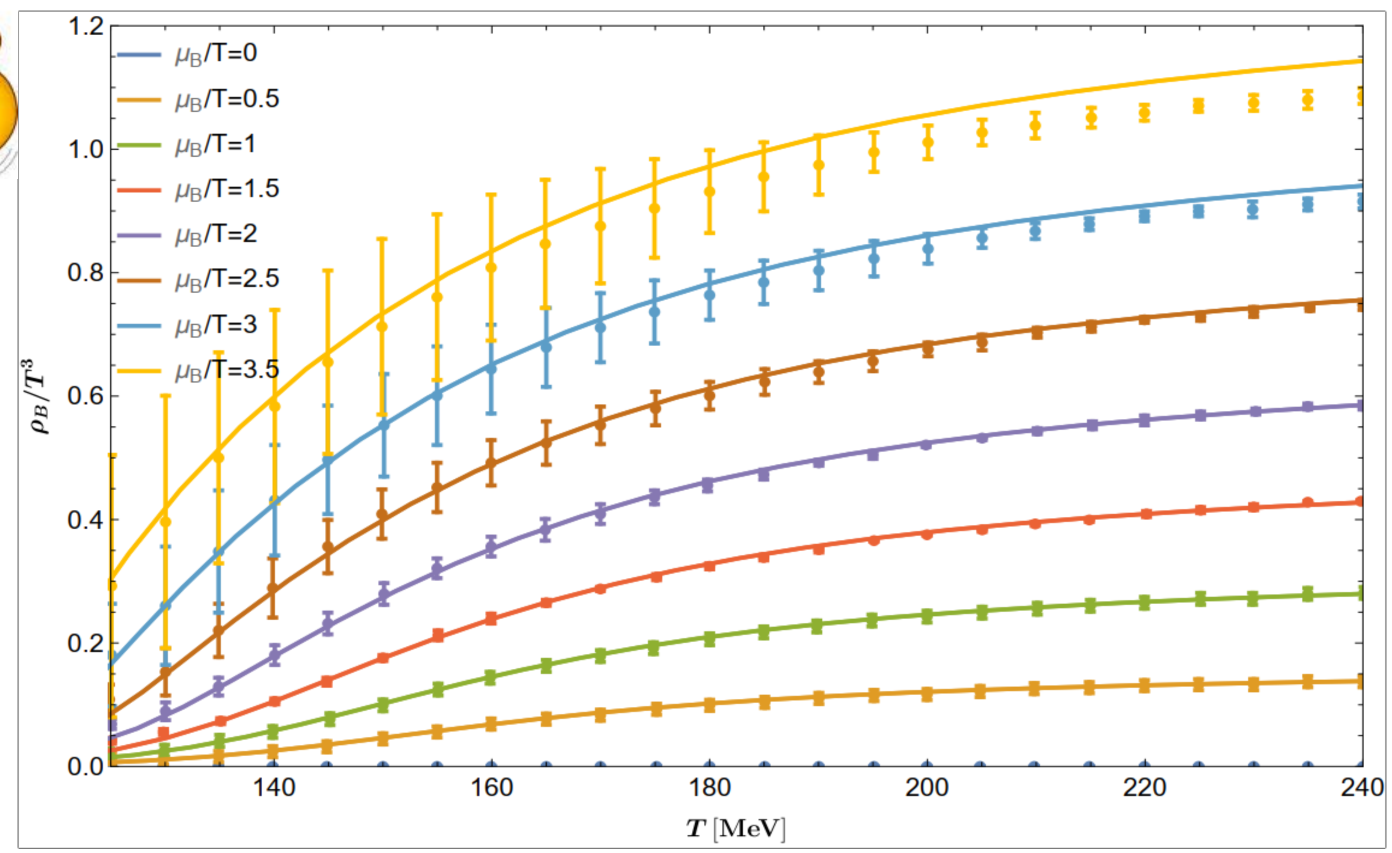
extrapolate lattice QCD data to finite $\mu_B$

# 3. QCD Phase diagram and GWs

## Predictions at finite chemical potential



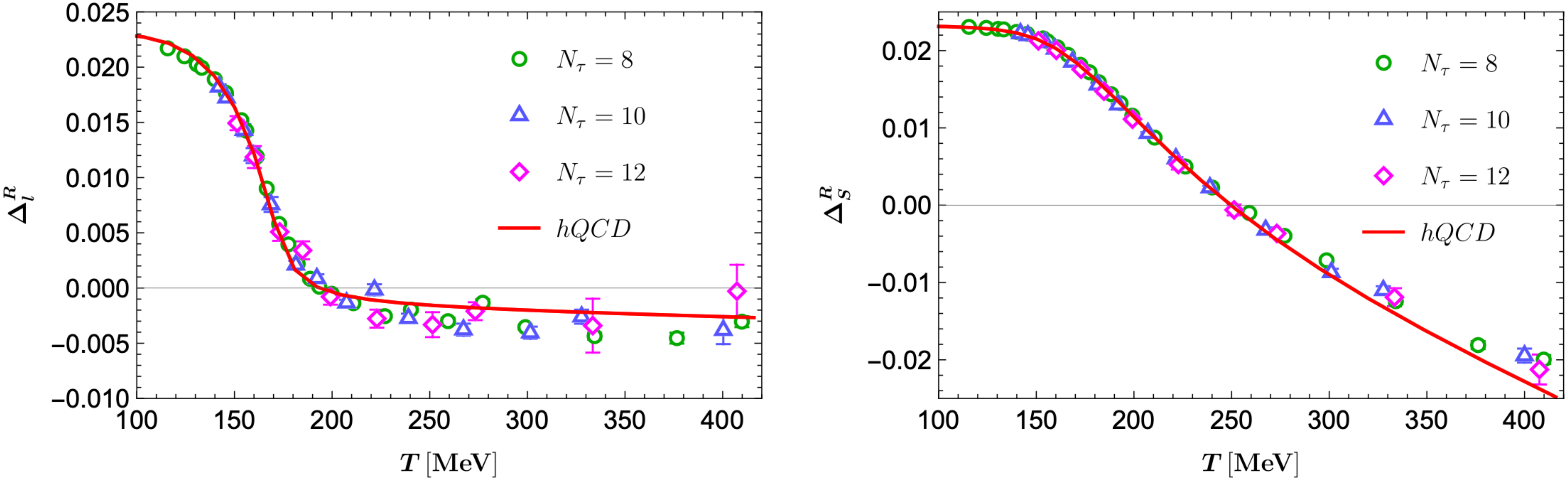

More challenging to fit $\rho_B$




S. Borsanyi, et al., Phys. Rev. Lett. 126 (2021) no.23, 232001

extrapolate lattice QCD data to finite $\mu_B$

## The chiral condensation:




$$\Delta_q^R = \hat{d} + 2\, m_s r_1^4 \left[ \langle \bar{\psi}\psi \rangle_{q,T} - \langle \bar{\psi}\psi \rangle_{q,0} \right], \quad q = l, s$$

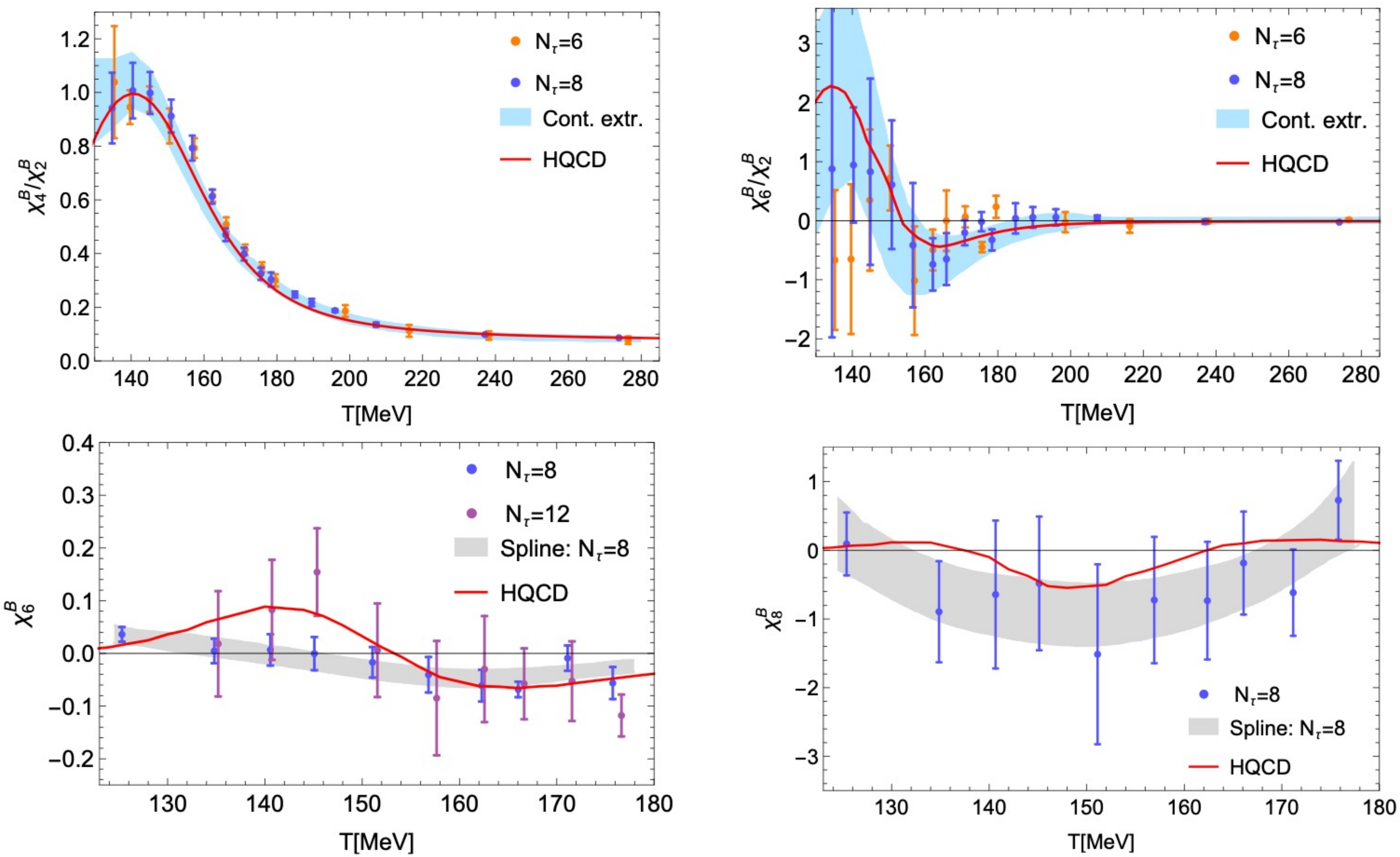
Left panel: u, d quarks
Right panel: s quark
$N_\tau$: the number of lattice sites in the imaginary time direction

A. Bazavov *et al.* [HotQCD], Phys. Rev. D 90 (2014), 094503

**Generalized susceptibilities**

$$\chi_n^B(T,\mu_B) = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{P}{T^4}$$

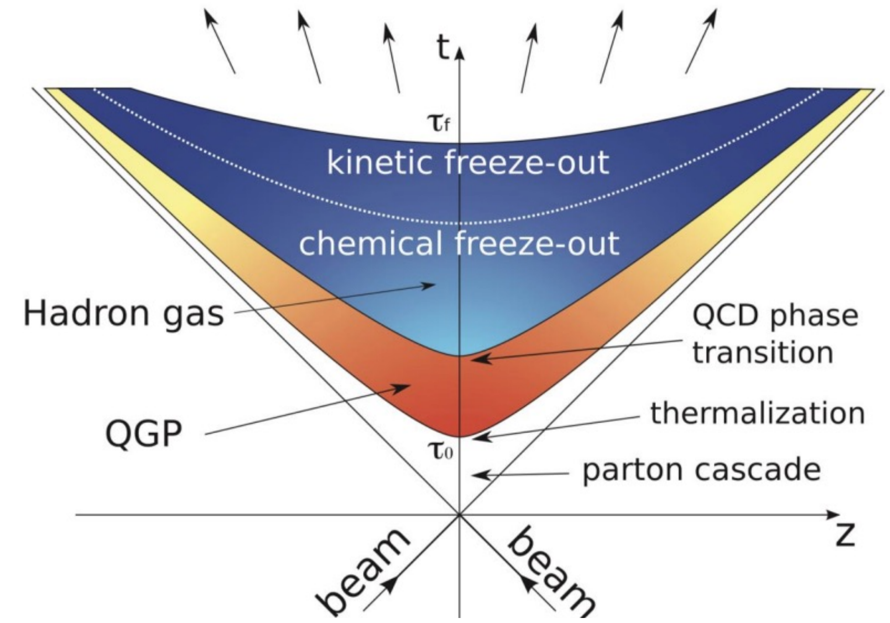
closely related to various cumulants of the baryon number distribution measured in **heavy-ion collision experiments**

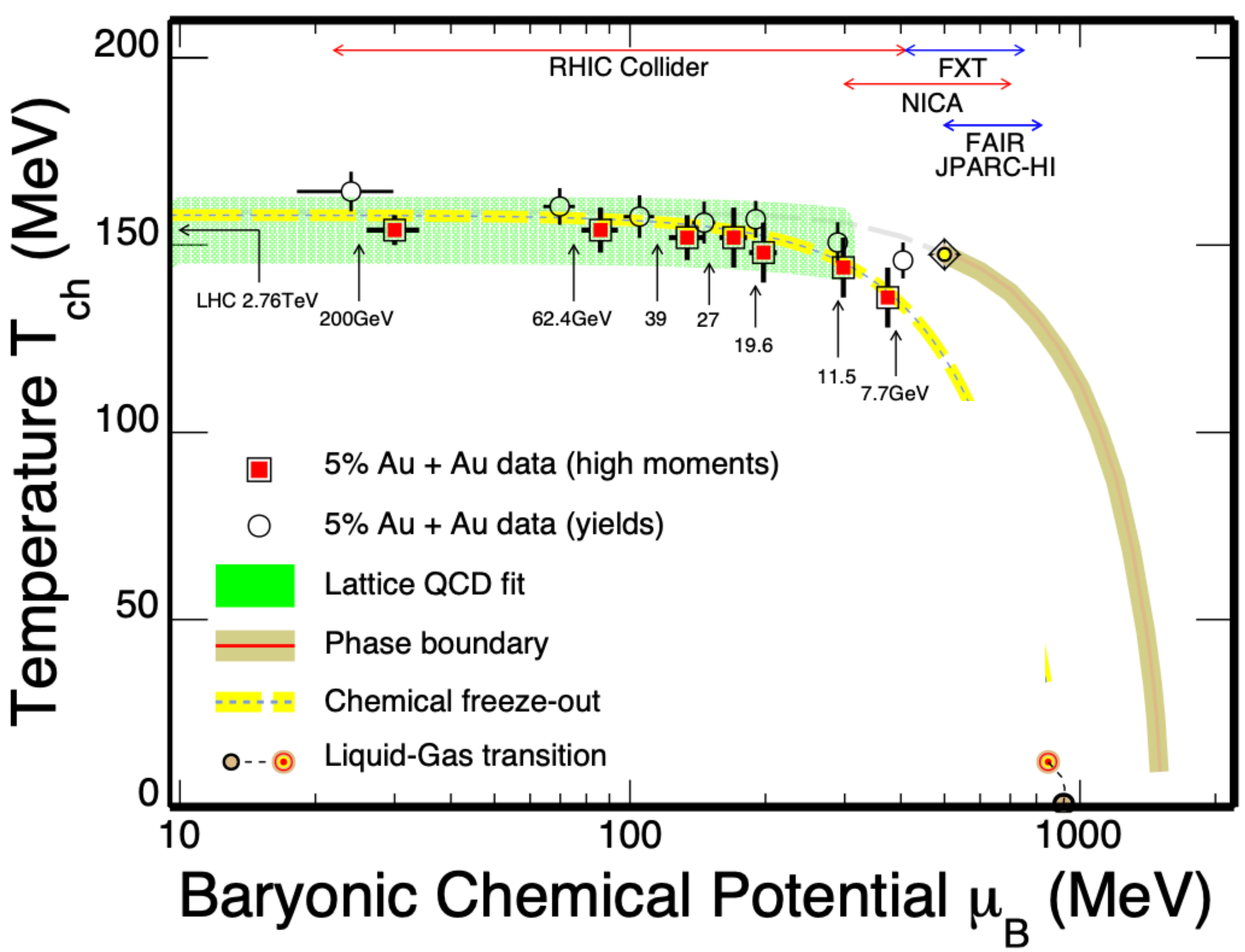


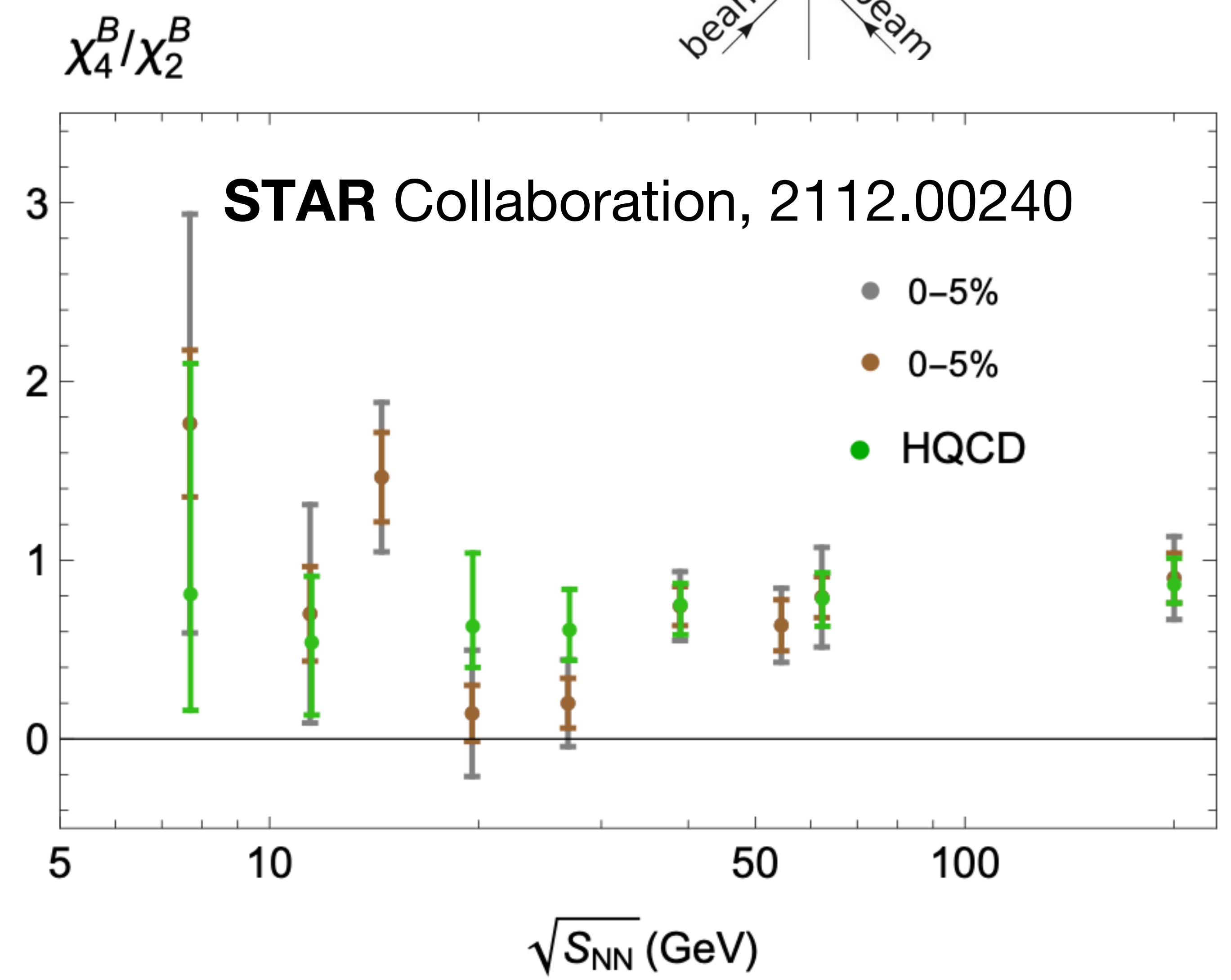

A. Bazavov *et al.* [HotQCD], Phys. Rev. D 95 (2017) 5, 05404

Z. Li, J. Liang, S. He and Li Li, Phys. Rev. D 108, no.4, 046008 (2023)

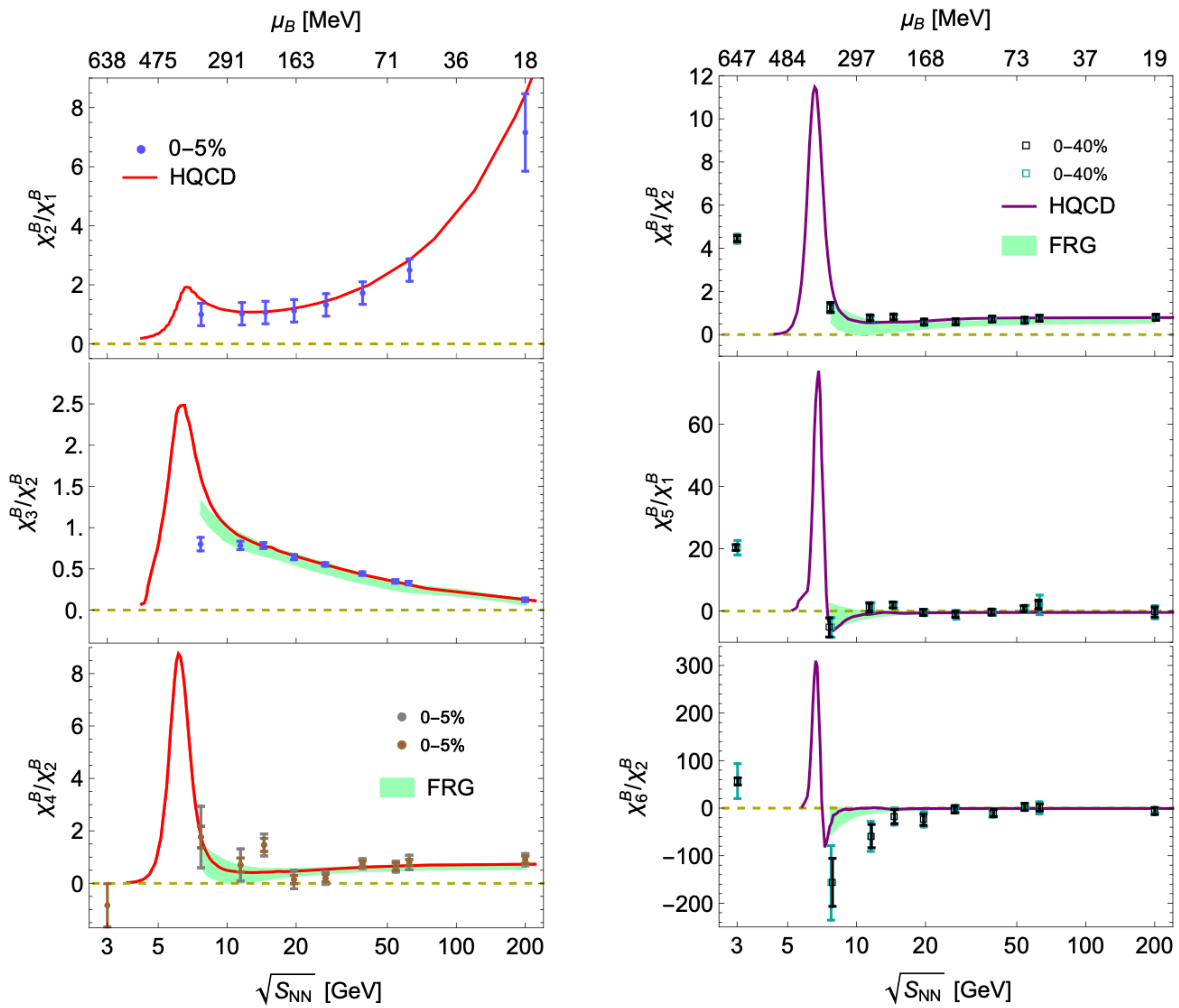
Compare with **STAR** data of net-proton distributions in 0 – 5% centrality (0% being head-on collisions) Au-Au collisions along the chemical freeze-out line



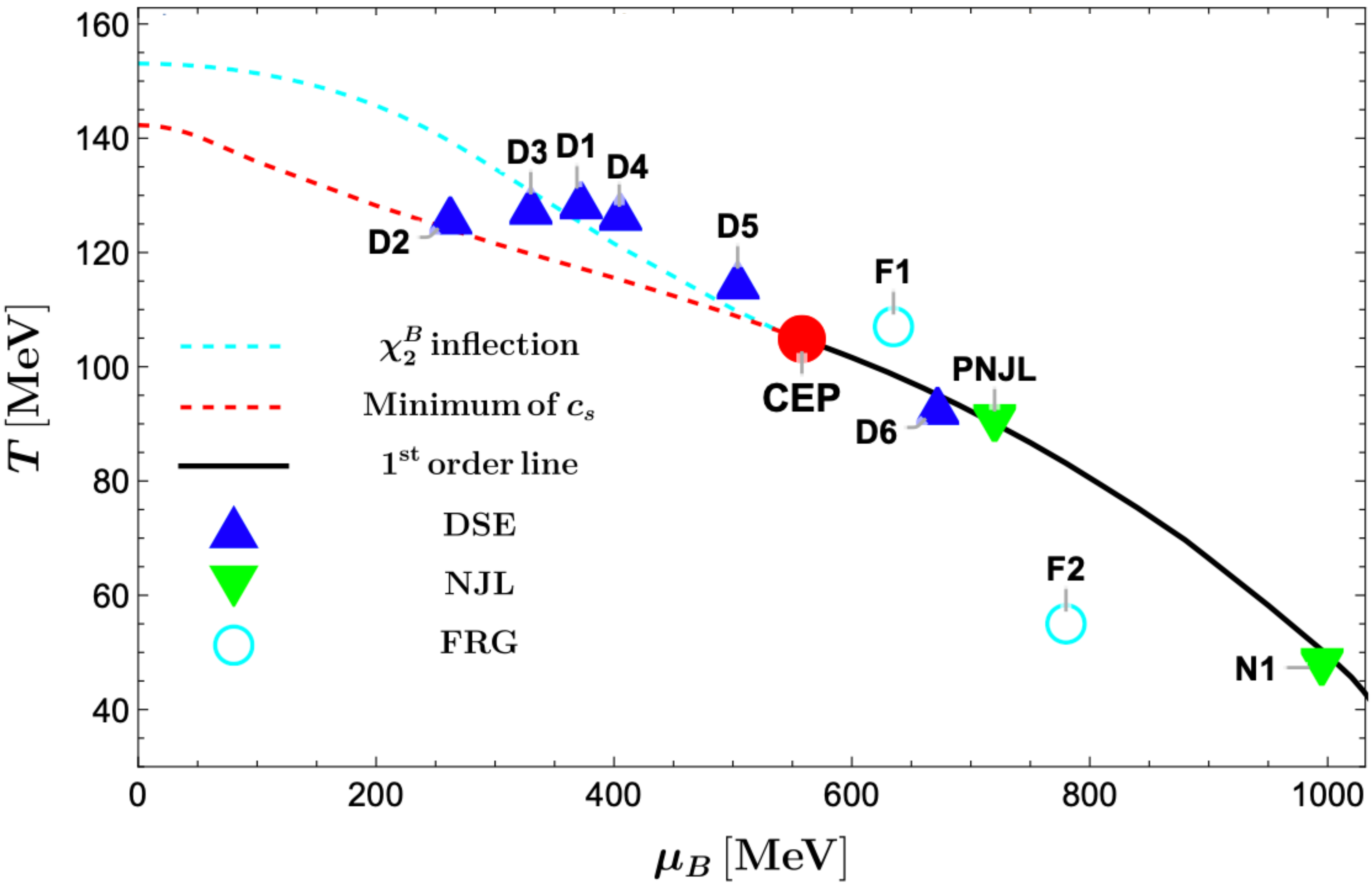




Z. Li, J. Liang, S. He and **Li Li**, Phys. Rev. D 108, no.4, 046008 (2023)




Z. Li, J. Liang, S. He and **Li Li**, Phys. Rev. D 108, no.4, 046008 (2023)

## QCD phase diagram from our holographic model




Rong-Gen Cai, Song He, **Li Li**, Yuan-Xu Wang,
Phys. Rev. D 106 (2022) no.12, L121902

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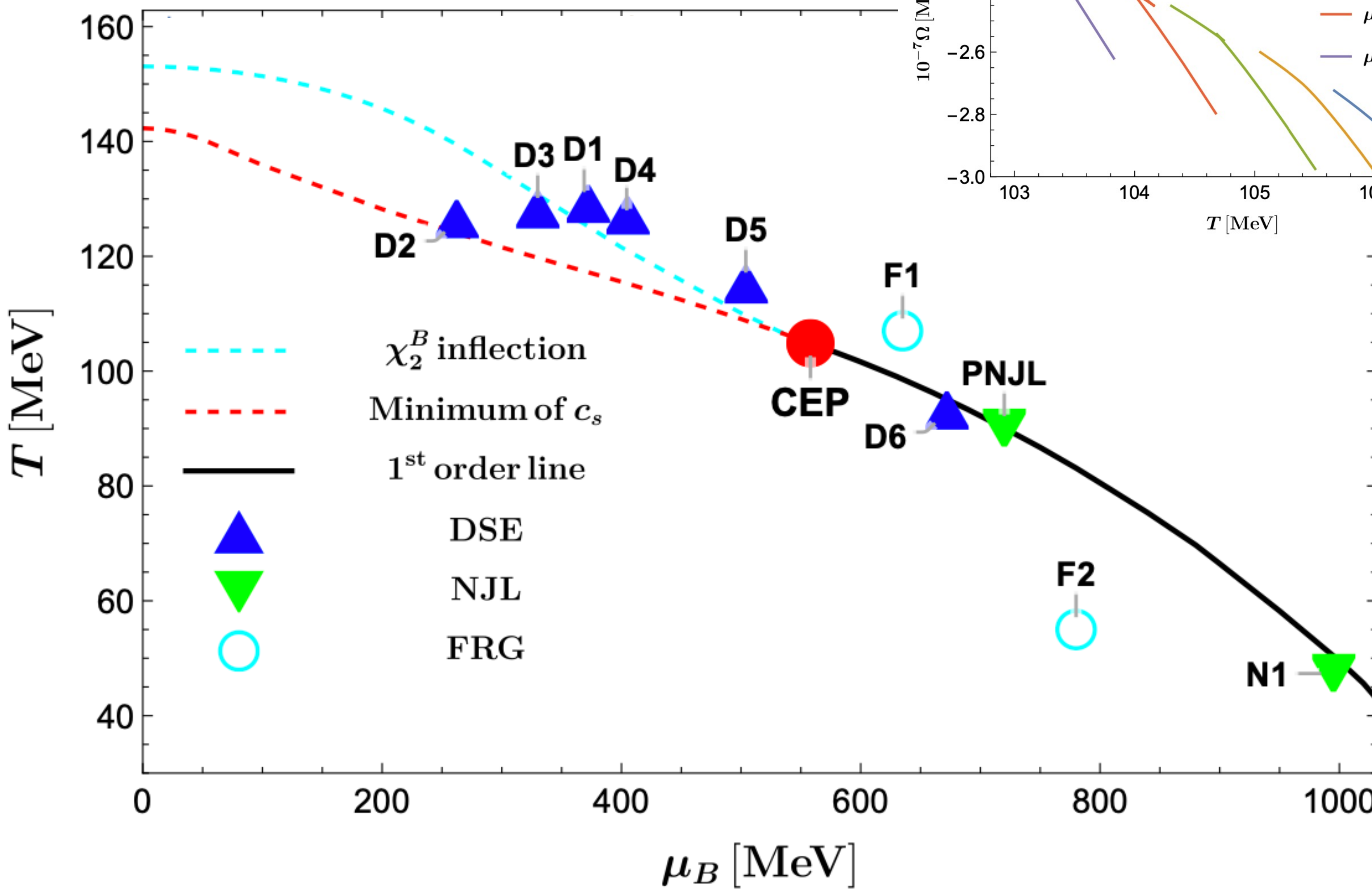


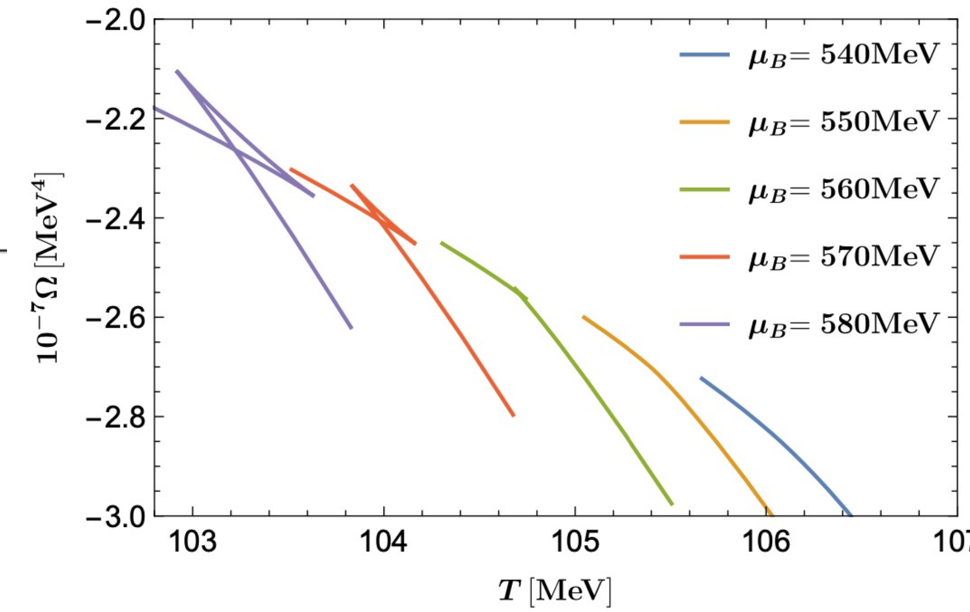


Rong-Gen Cai, Song He, **Li Li**, Yuan-Xu Wang,
Phys. Rev. D 106 (2022) no.12, L121902

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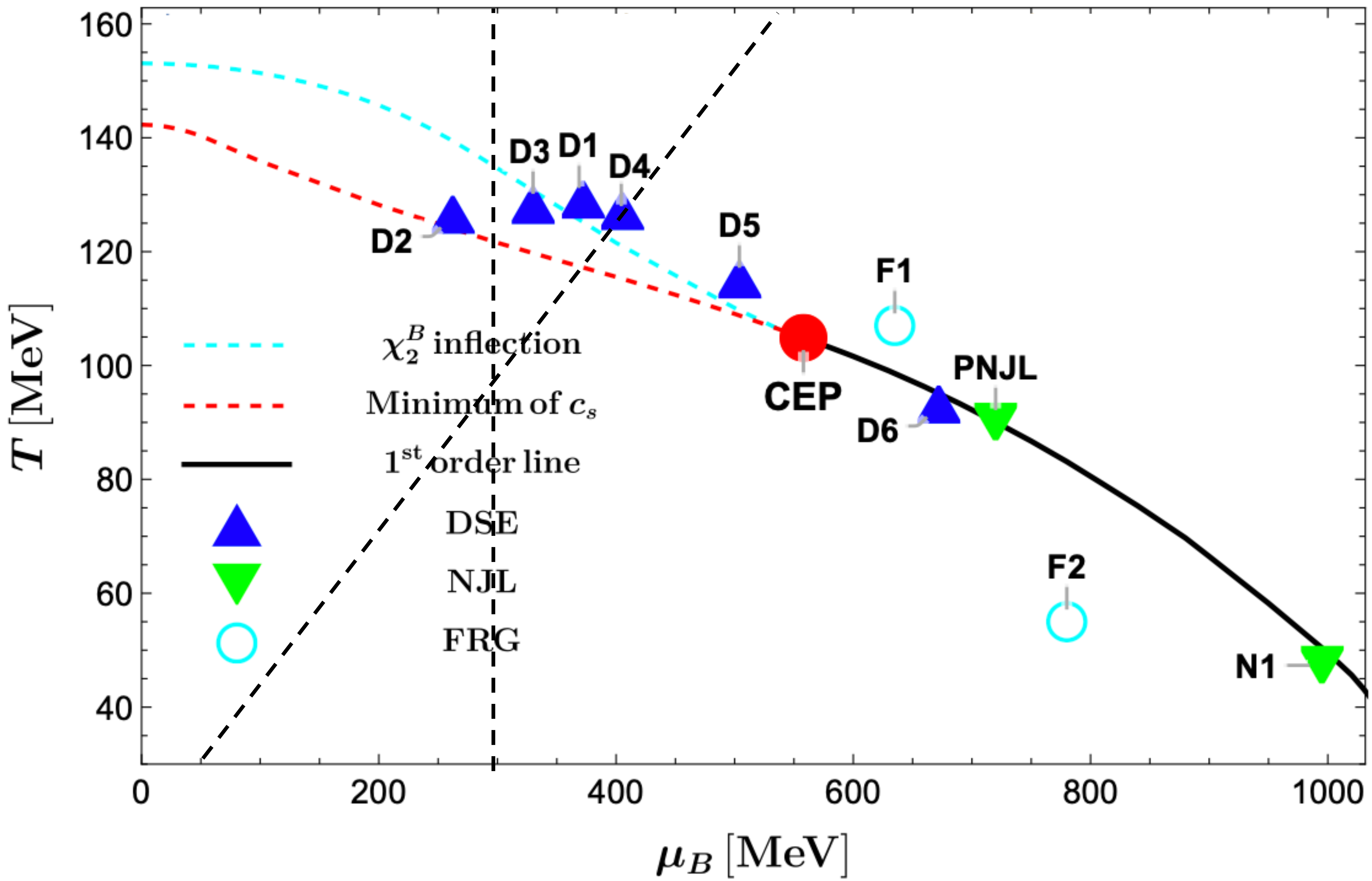
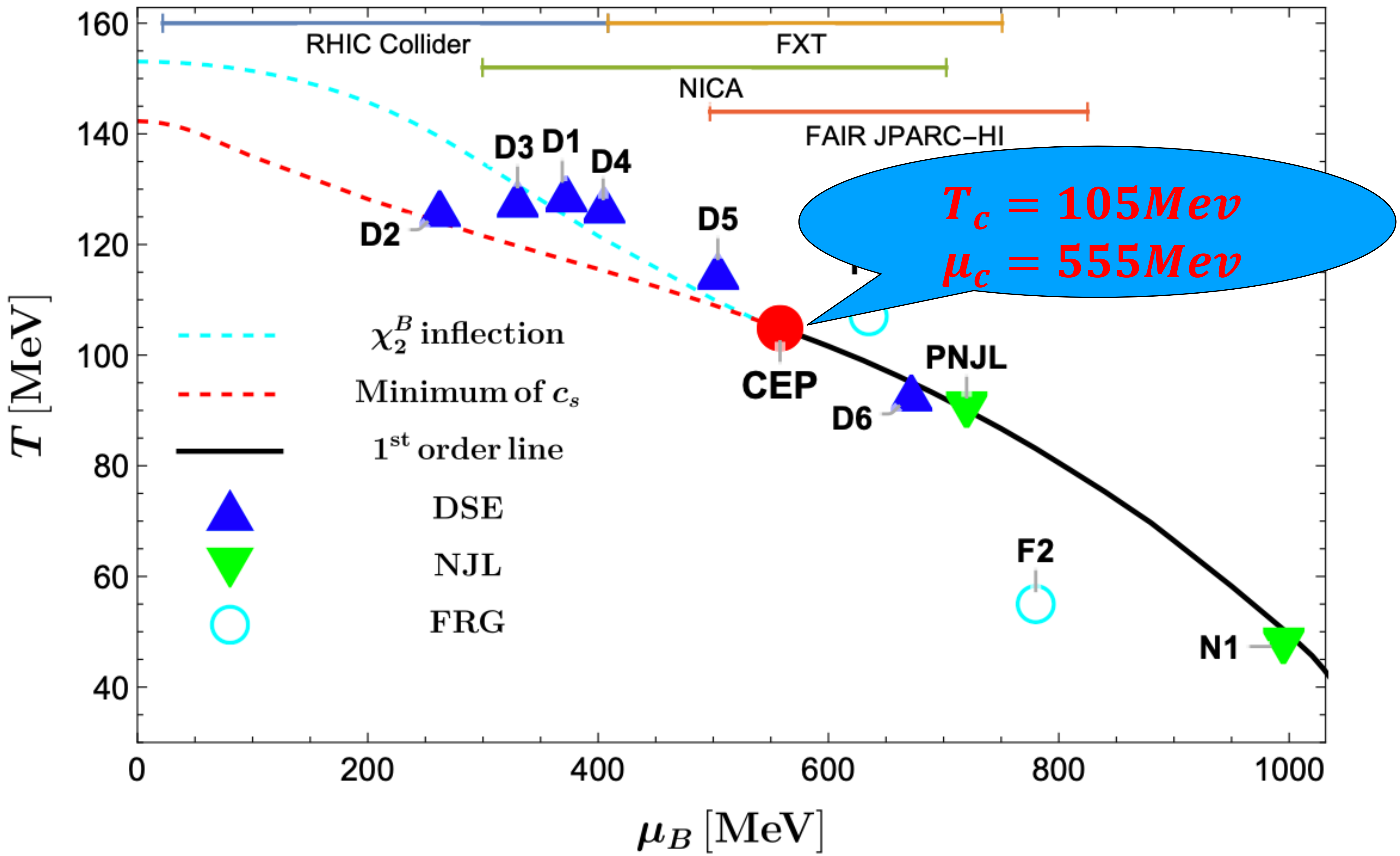



$\mu_B / T \leq 3, \mu_B < 300$MeV was excluded by lattice simulation

A. Bazavov, etc. Phys. Rev. D 95 (2017) no.5, 054504

## QCD phase diagram from our holographic model



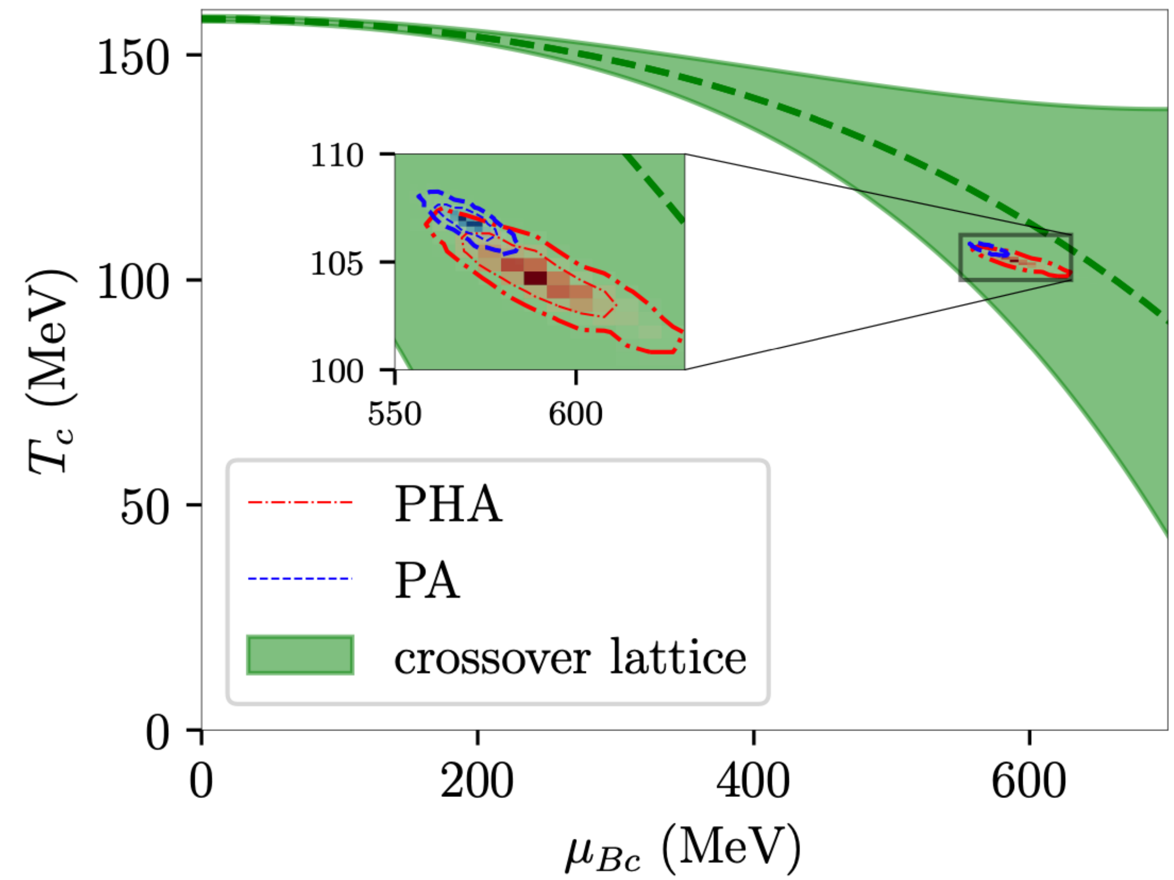

The predicted location of CEP is within the coverage of future (FAIR, JPARC-HI, and NICA) experimental facilities.

**Our location of CEP $(T_c, \mu_c) = (105, 555)$Mev has been supported by recent studies.**

M. Hippert, et al, **Bayesian location** of the QCD critical point from a holographic perspective, [arXiv:2309.00579 [nucl-th]].

$$(T_c,\ \mu_{Bc})_{PHA} = (104 \pm 3,\ 589^{+36}_{-26})\ \mathrm{MeV}$$

$$(T_c,\ \mu_{Bc})_{PA} = (107 \pm 1,\ 571 \pm 11)\ \mathrm{MeV}$$



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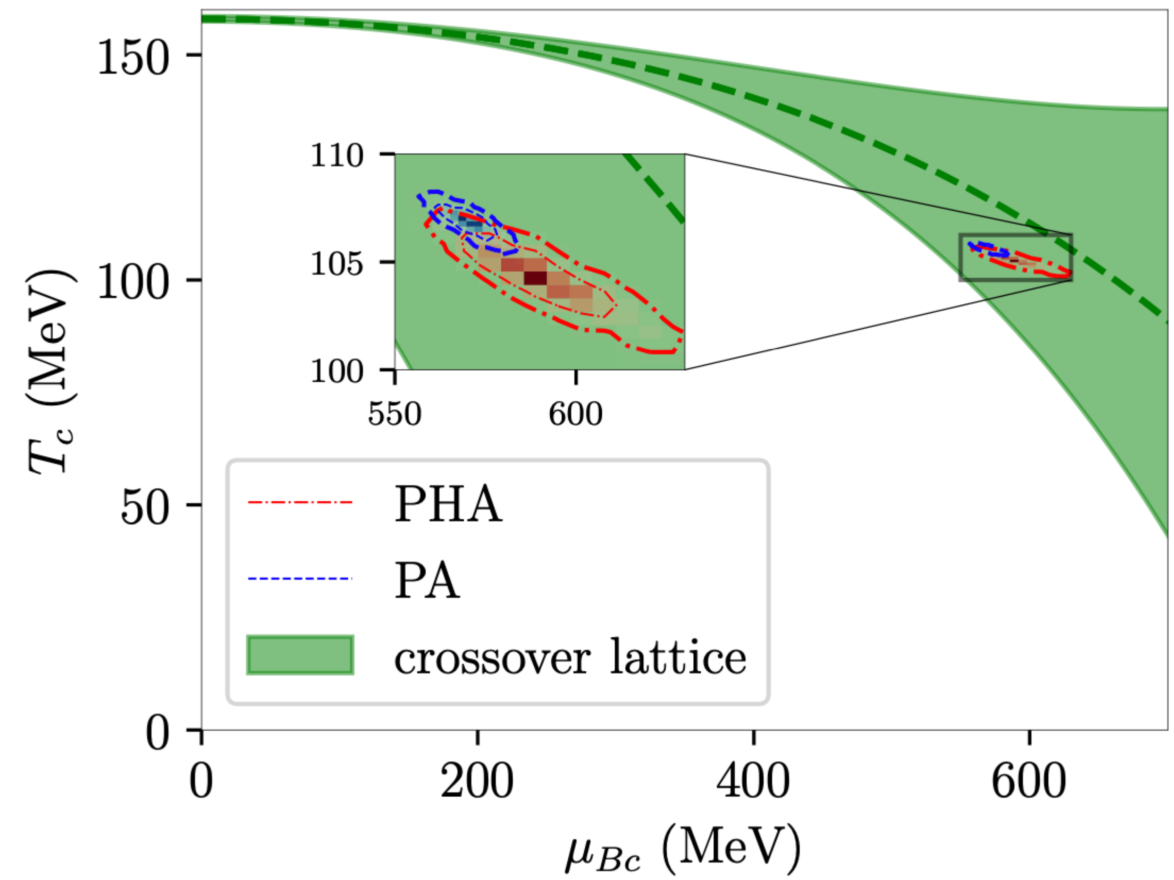
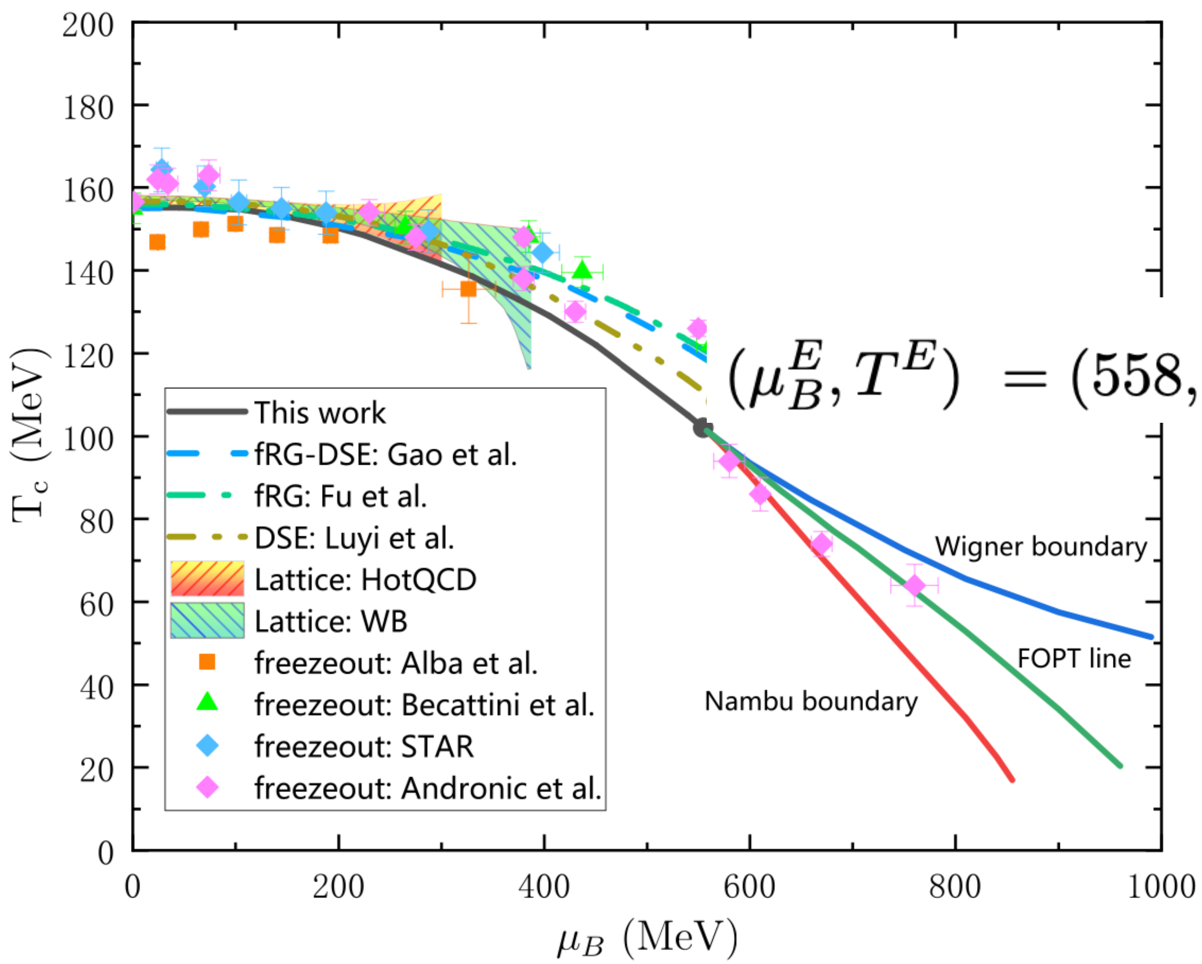


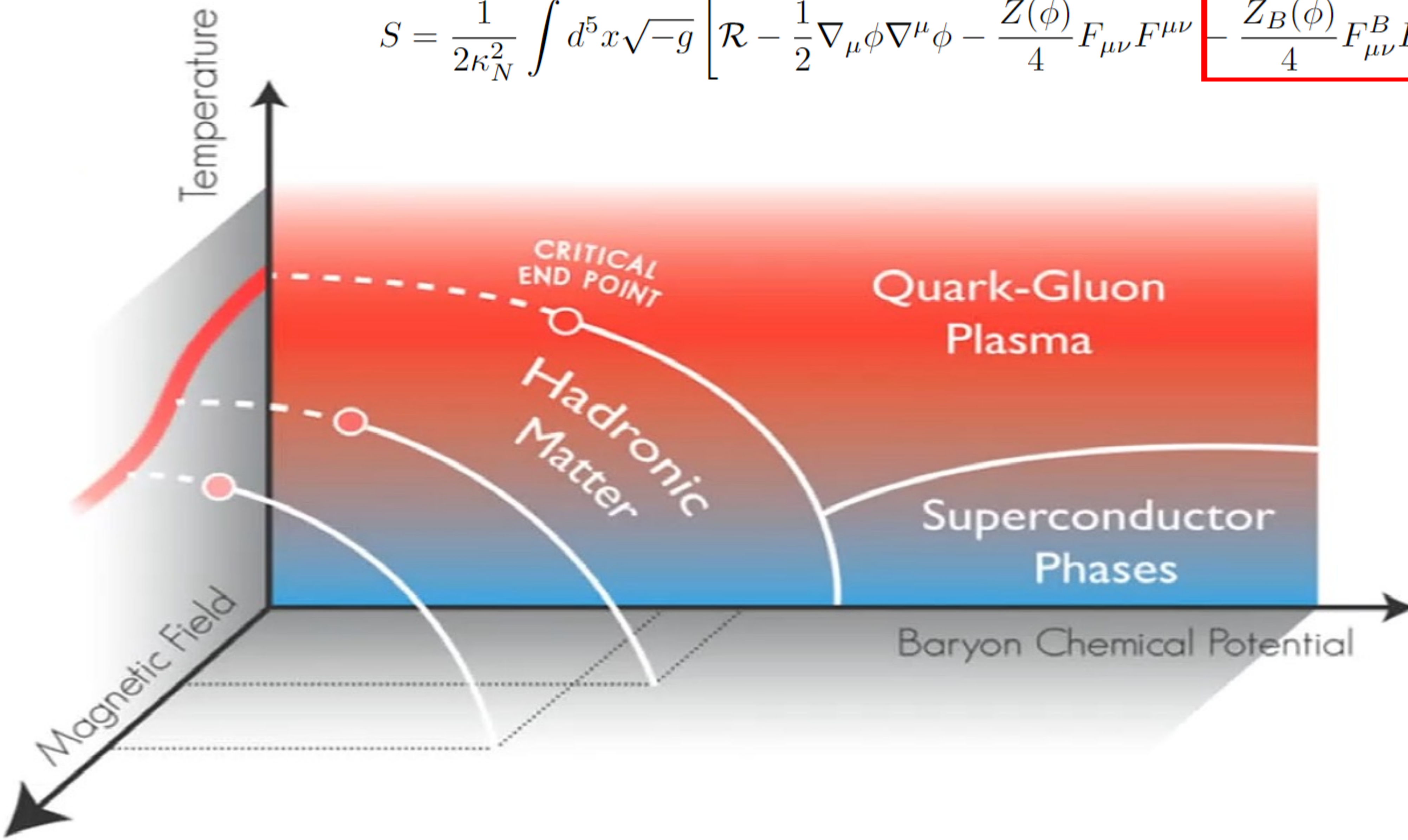

$$(\mu_B^E, T^E) = (558, 101.3)\ \text{MeV}.$$

H. w. Zheng, et al, The **effective potential of composite operator** in the first order region of QCD phase transition, [arXiv:2312.00382 [hep-ph]].

# hQCD model for 2+1 flavors with B field

Rong-Gen Cai, Song He, Li Li, Hong-An Zeng, 2406.12772

$$S=\frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\left[\mathcal{R}-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi-\frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu}-\frac{Z_B(\phi)}{4}F^B_{\mu\nu}F^{B\mu\nu}-V(\phi)\right]$$

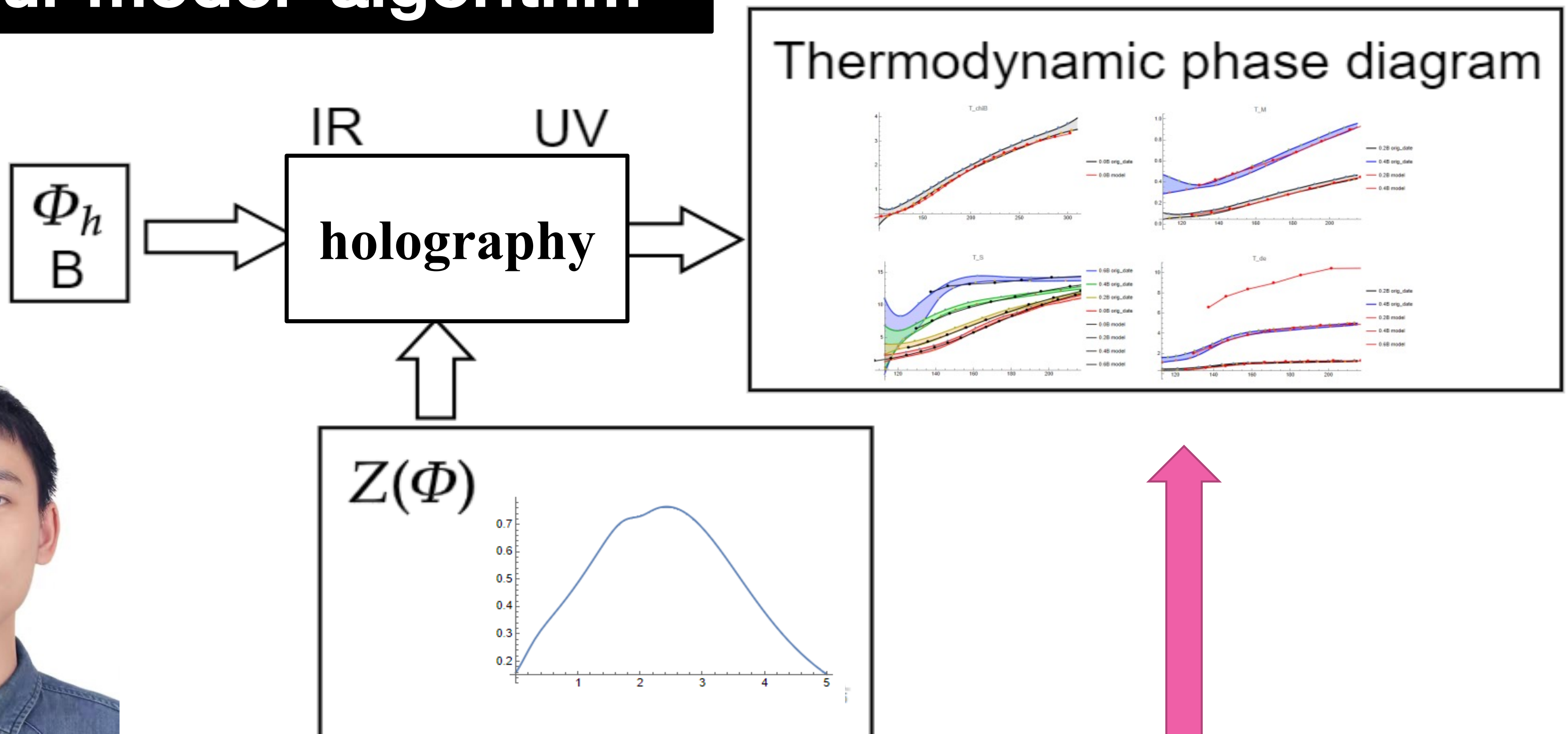



A. N. Tawfik and A. M. Diab, arXiv:2106.04576 [hep-ph]

# Neural Network

$$S = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\left[R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi) - \frac{Z_B(\phi)}{4}F^B_{\mu\nu}F^{B\mu\nu}\right]$$

## Our model algorithm



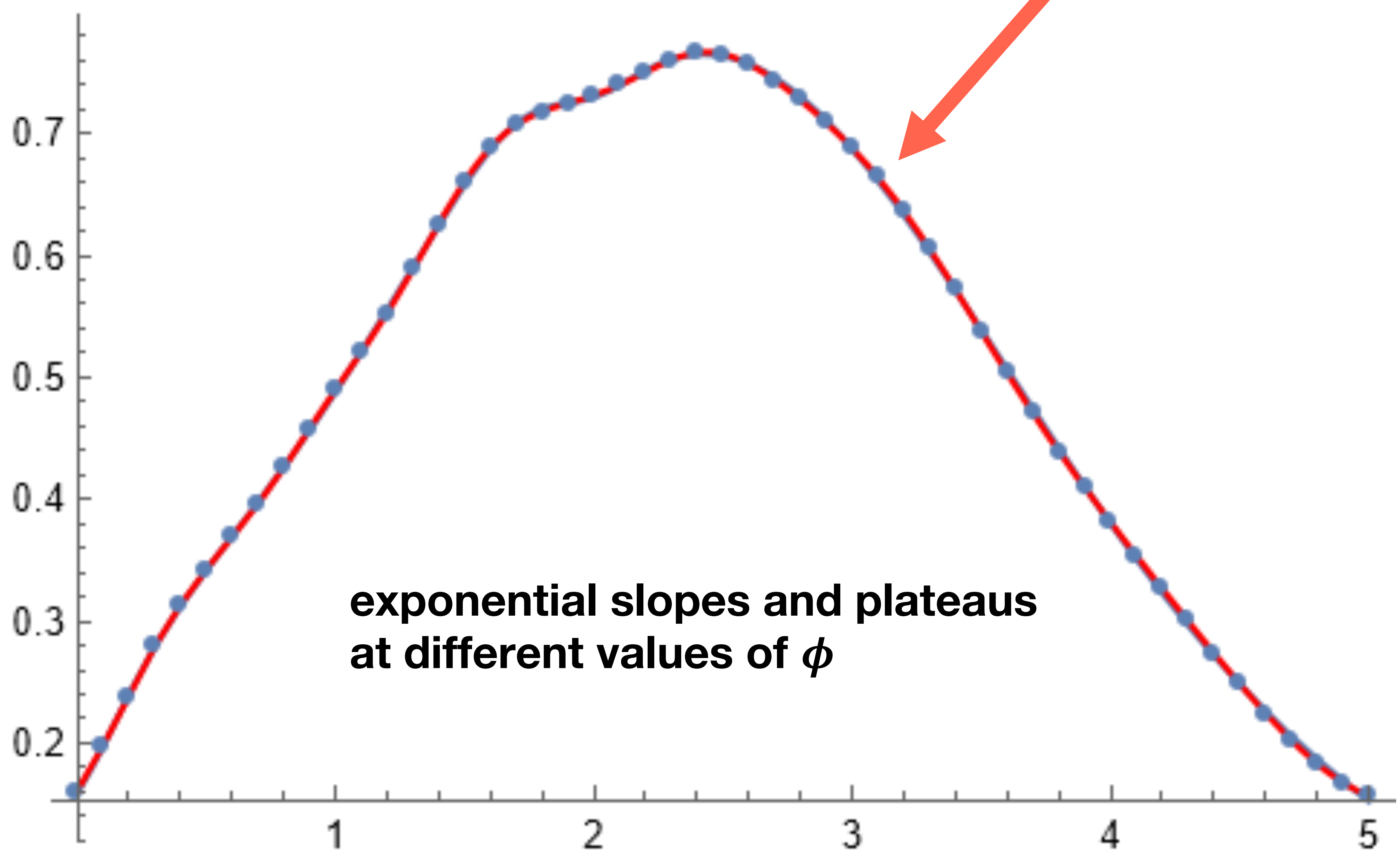

Hong-An Zeng

(See talk by Keun-Young Kim)

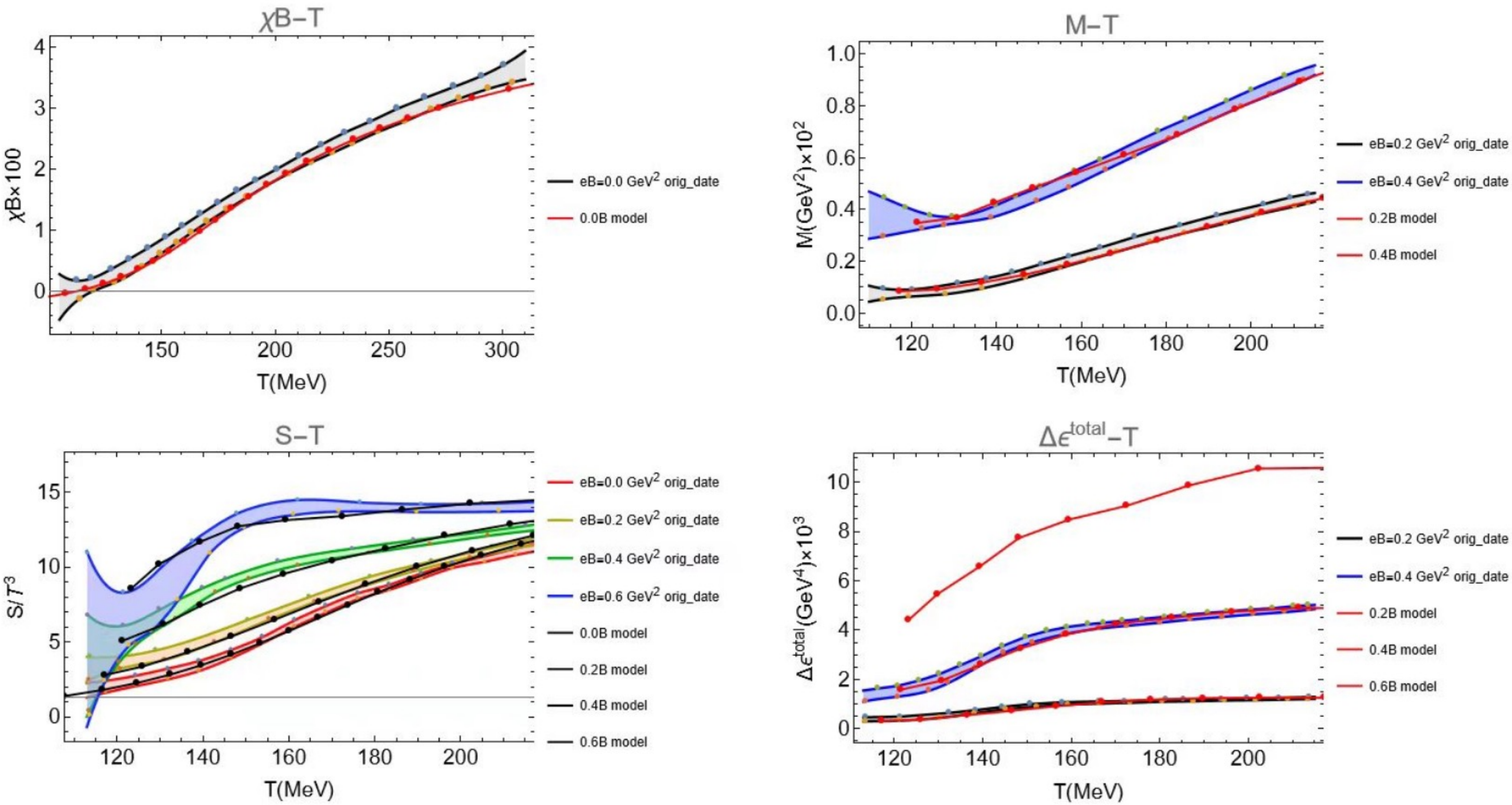
**Target: To find a $Z_B$-function that matches the thermodynamic quantities from Lattice QCD**

# Neural Network

$$S = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\left[R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi) - \frac{Z_B(\phi)}{4}F^B_{\mu\nu}F^{B\mu\nu}\right]$$



Rong-Gen Cai, Song He, **Li Li**, Hong-An Zeng, 2406.12772

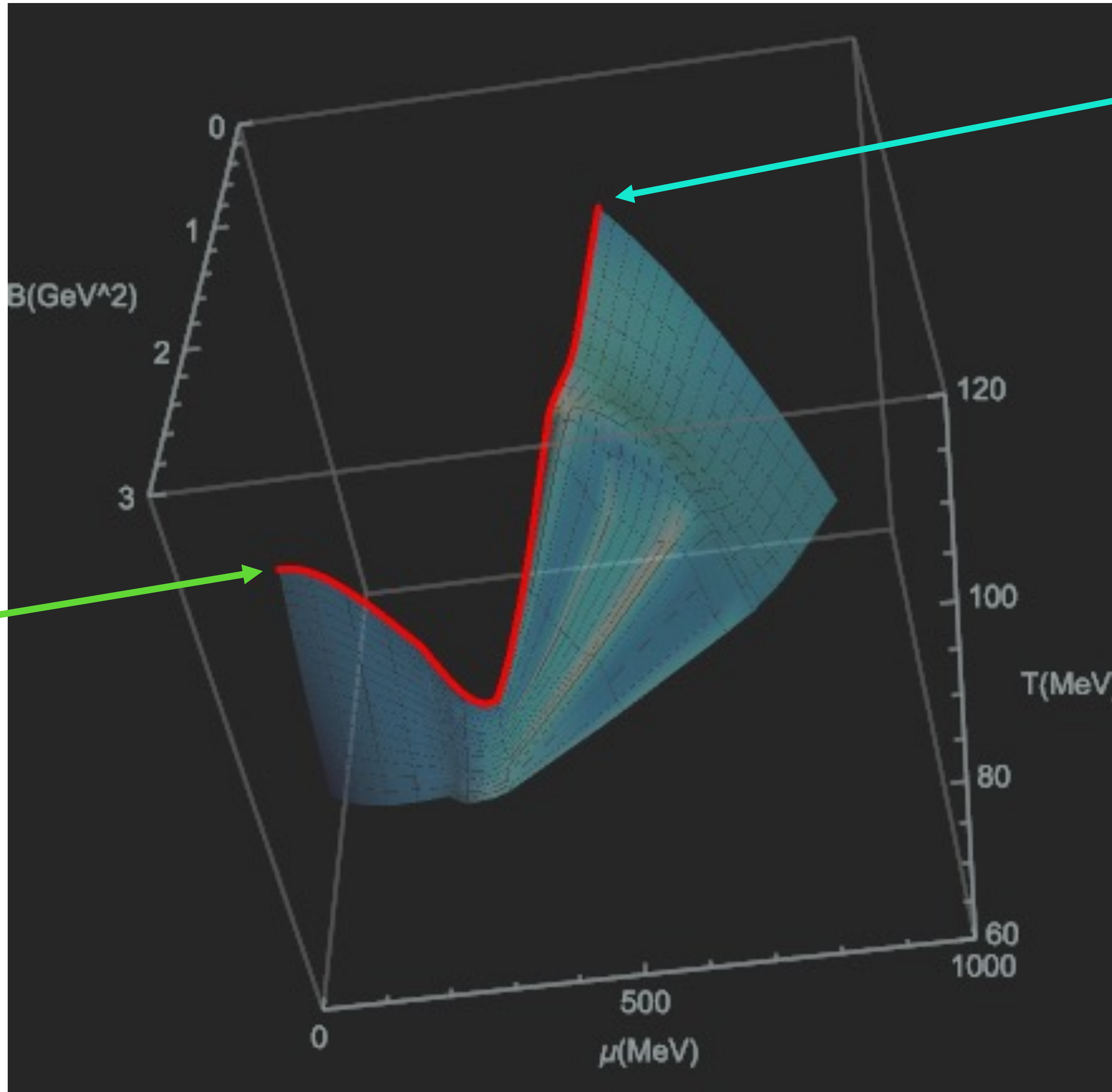



**G.S. Bali, et al, The QCD equation of state in background magnetic fields [arXiv:1406.0269 [hep-lat]]**

$eB = 1GeV^2 \leftrightarrow 1.602\times10^{19} Gauss$

# 3D Phase diagram

Rong-Gen Cai, Song He, **Li Li**, Hong-An Zeng, 2406.12772

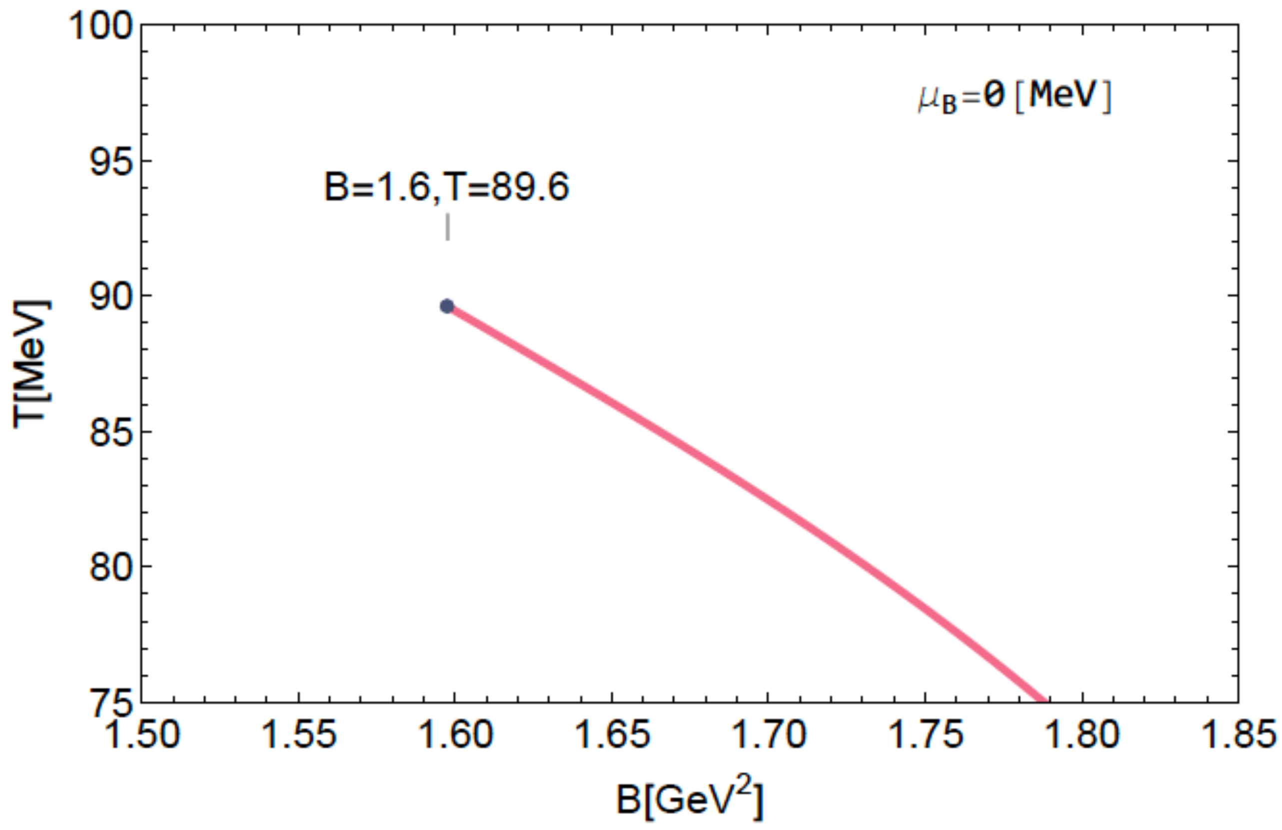


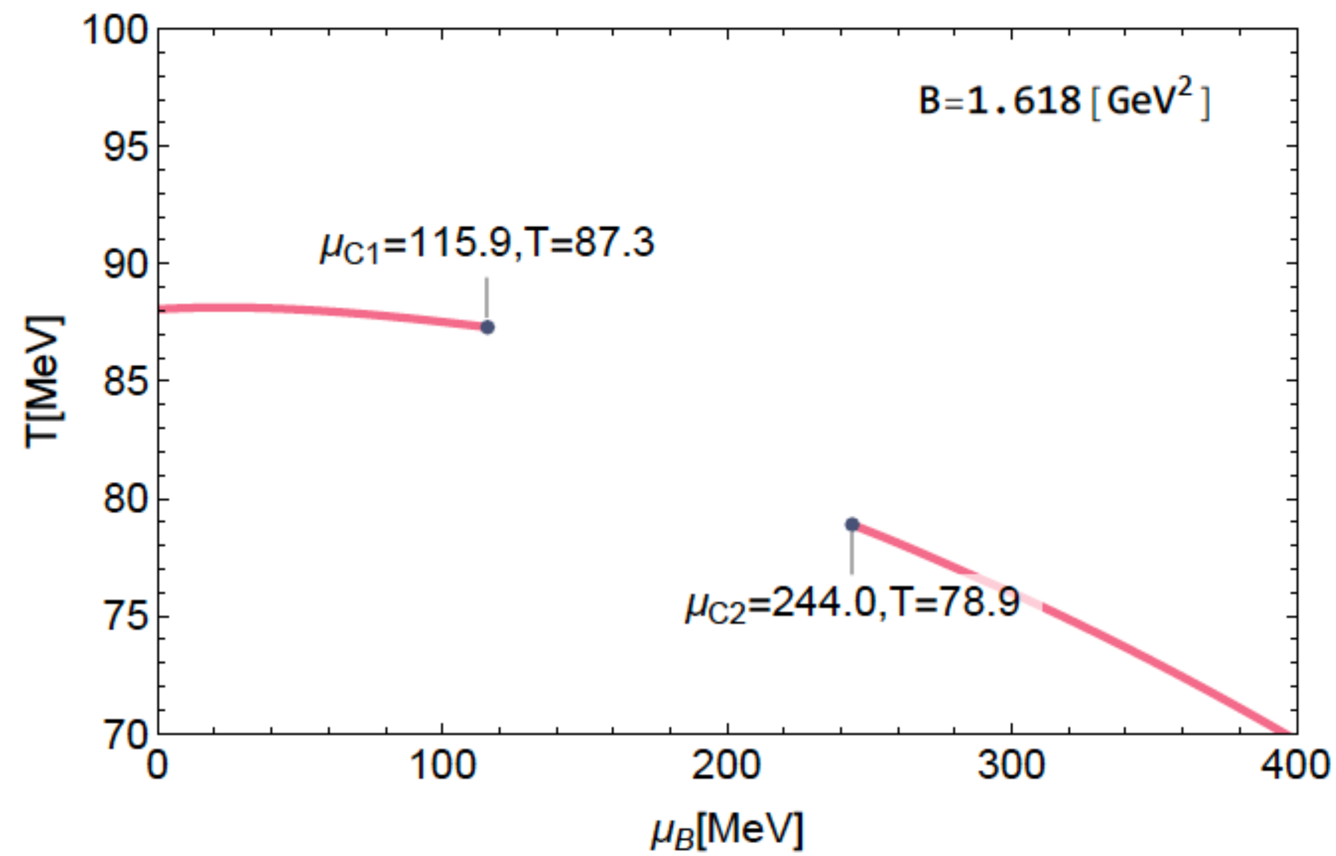

**B=0:**
$T_c = 105Mev,$
$\mu_c = 555Mev$

**$\mu$=0:**
$T_c = 89.6Mev,$
$B_c = 1.6Gev^2$

# Rich phase structure

Rong-Gen Cai, Song He, **Li Li**, Hong-An Zeng, 2406.12772




| | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ |
|---|---|---|---|---|
| Experiment | 0.110-0.116 | 0.316-0.327 | 1.23-1.25 | 4.6-4.9 |
| 3D Ising | 0.110(5) | 0.325±0.0015 | 1.2405±0.0015 | 4.82(4) |
| Mean field | 0 | 1/2 | 1 | 3 |
| DGR model | 0 | 0.482 | 0.942 | 3.035 |
| hQCD(I) | 0.0046233 | 0.532779 | 0.91629 | 3.00825 |
| hQCD(II) | 0.0233477 | 0.481381 | 1.05119487 | 3.238166 |
| hQCD(III) | 0.019778 | 0.46259 | 1.03499 | 3.48613 |

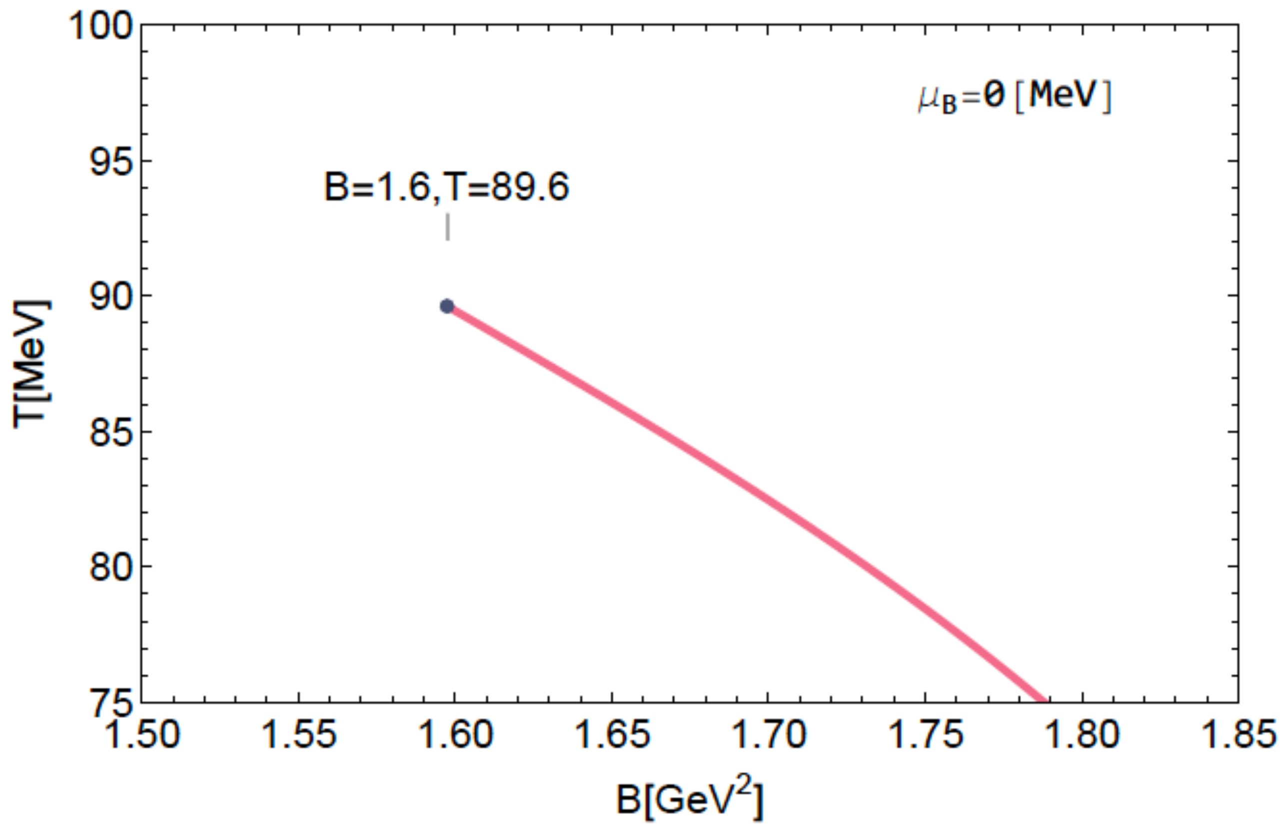
$\mu_B = 554.66Mev, B = 0Gev^2$

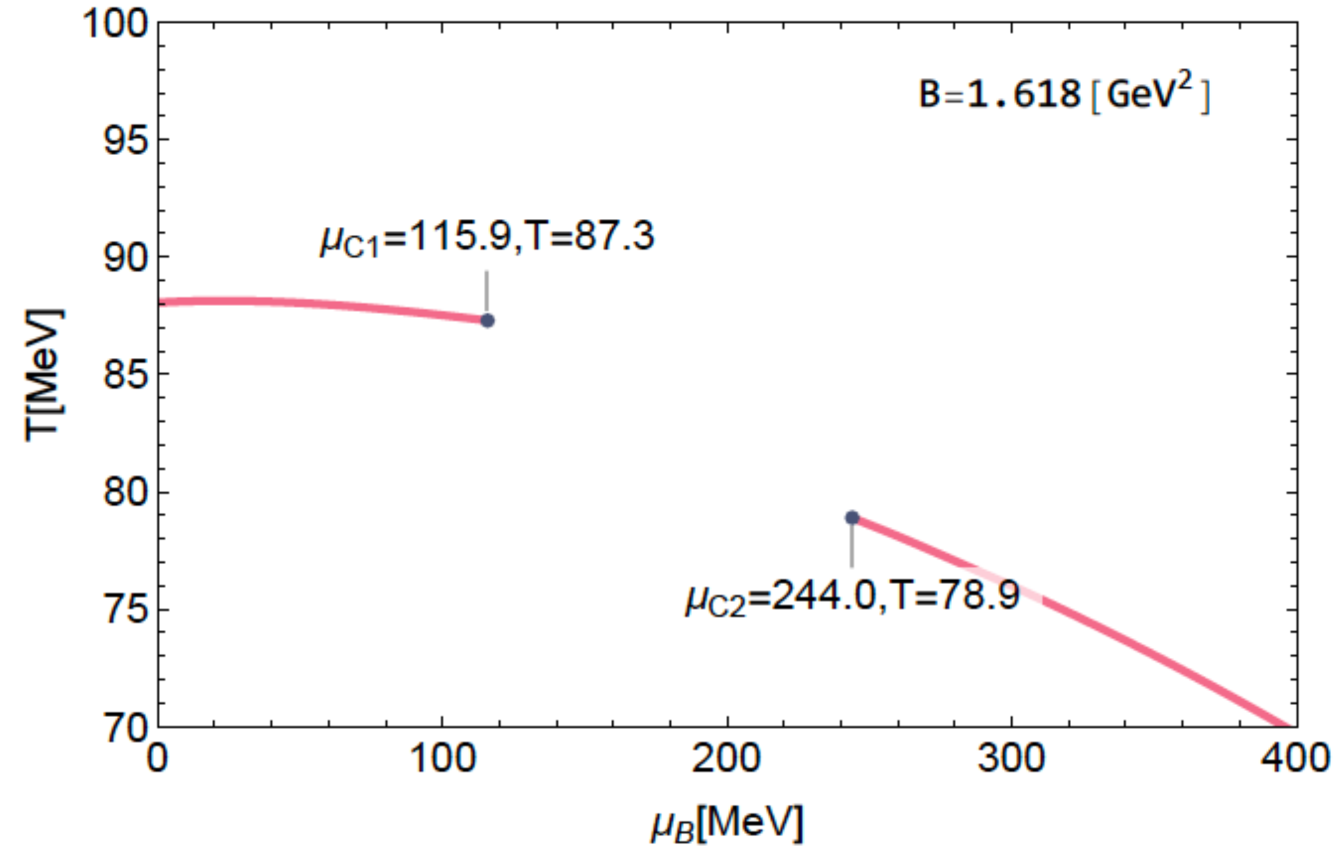
$\mu_B = 501.4Mev, B = 0.3Gev^2$

$\mu_B = 0Mev, B = 1.6Gev^2$

# Rich phase structure

Rong-Gen Cai, Song He, **Li Li**, Hong-An Zeng, 2406.12772




$$\alpha + 2\beta + \gamma = 2, \quad \alpha + \beta(1+\delta) = 2$$

| | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ |
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$\mu_B = 554.66 Mev, B = 0 Gev^2$

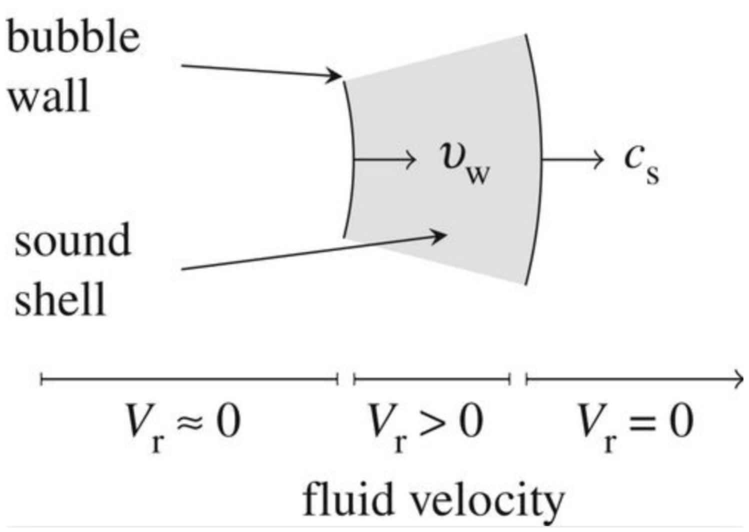
$\mu_B = 501.4 Mev, B = 0.3 Gev^2$

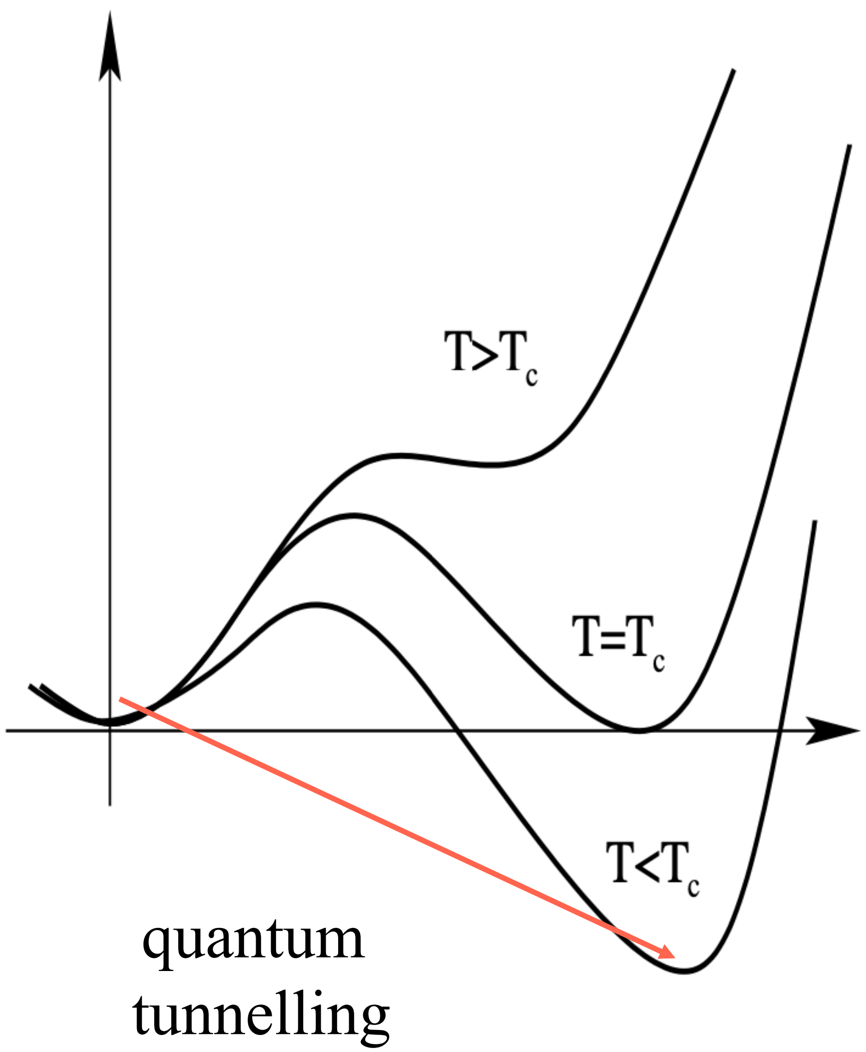
$\mu_B = 0 Mev, B = 1.6 Gev^2$

# Induced gravitational wave

Strong first order phase transition will result in the production of GWs:

**bubble collision + sound wave + MHD turbulence**



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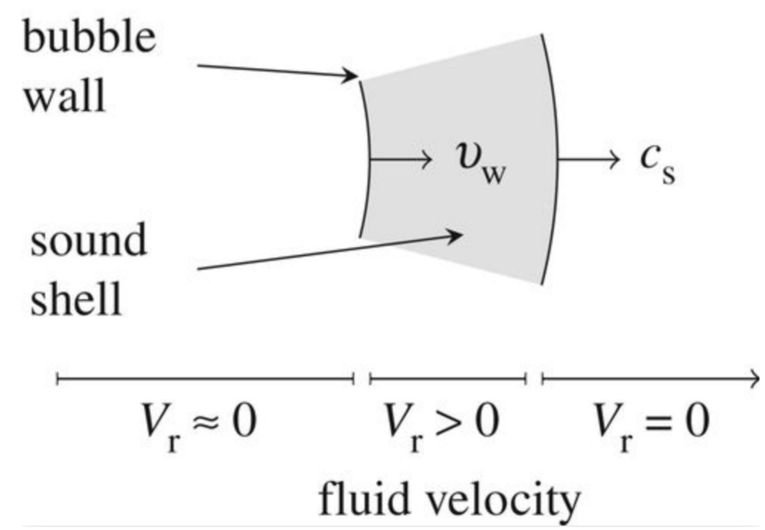


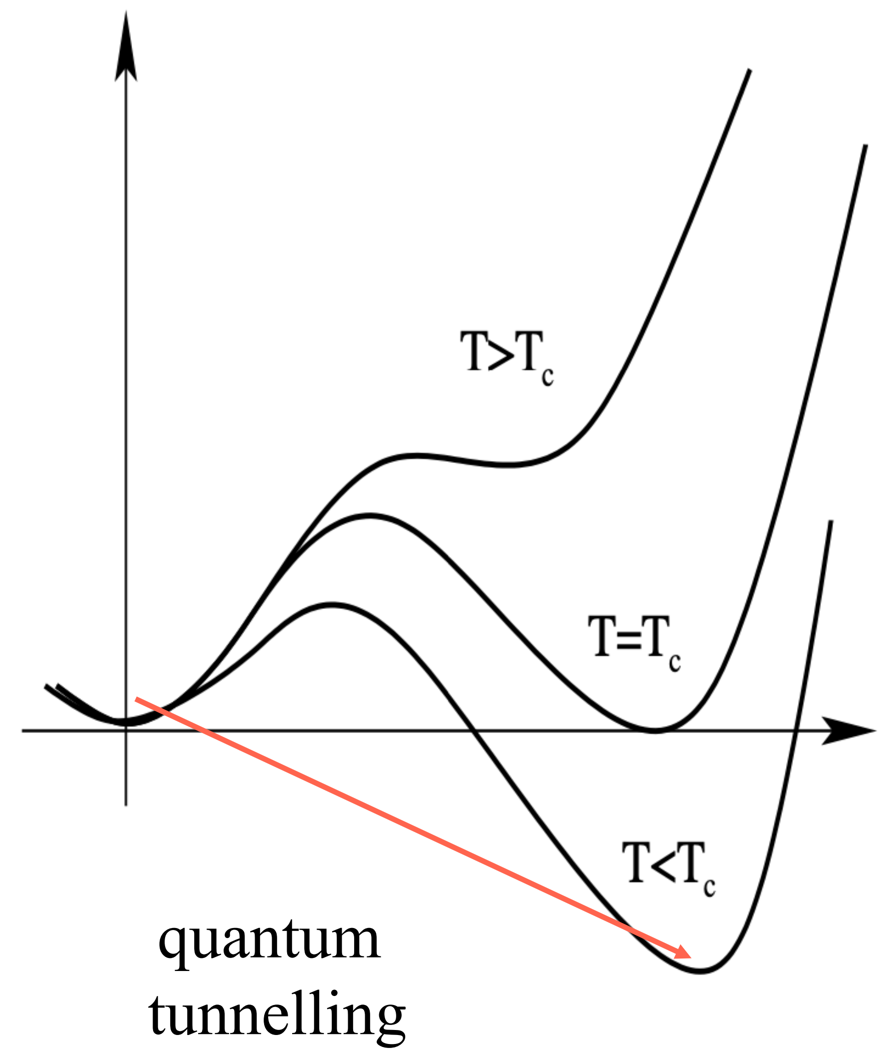

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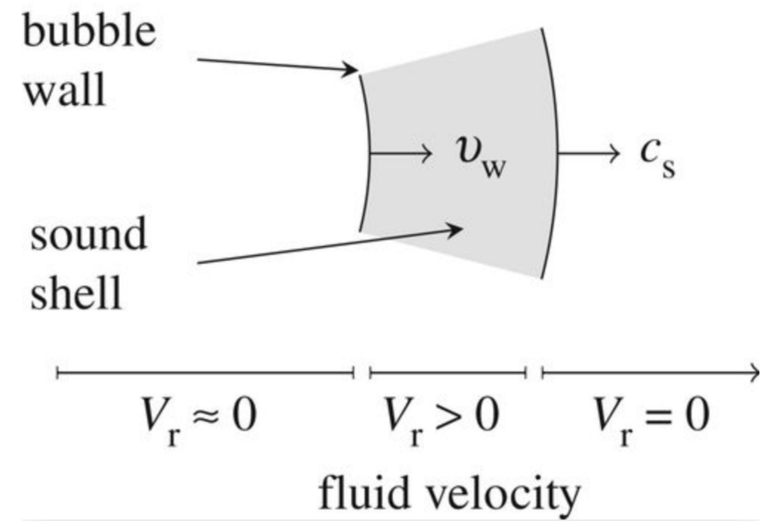
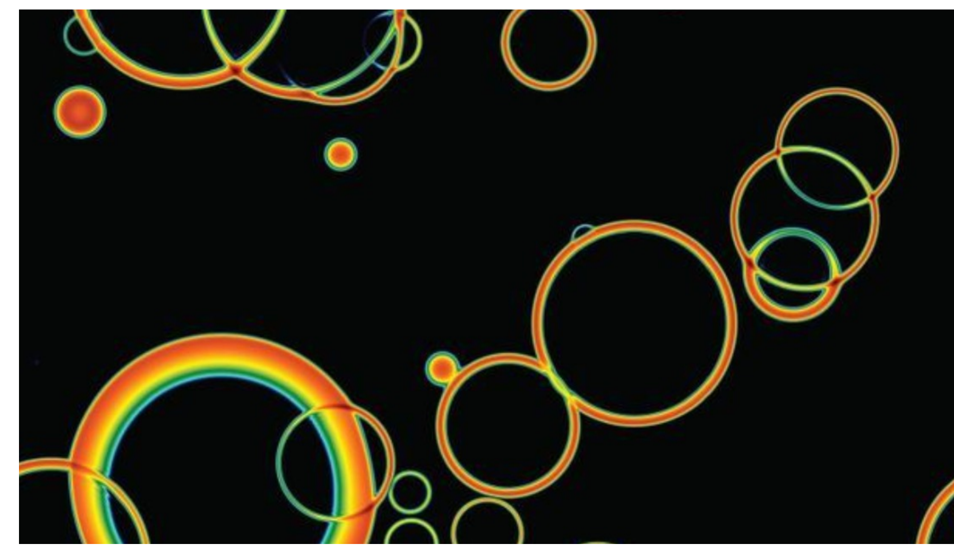
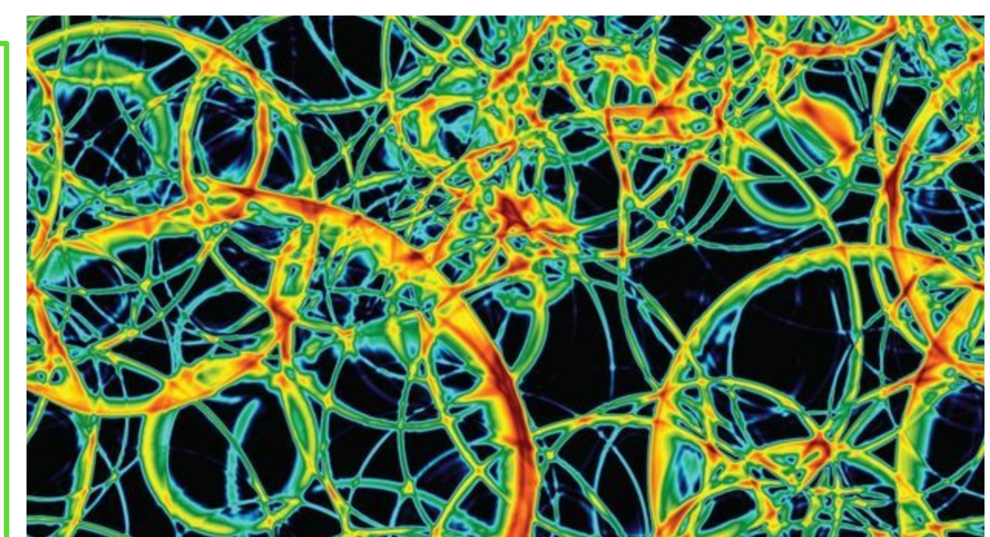



GWs are dominated by **sound waves** with the energy spectrum

$$h^2\Omega_{GW}(f) = 8.5 \times 10^{-6} \left(\frac{100}{g_n}\right)^{1/3} \left(\frac{\kappa\alpha}{1+\alpha}\right)^2 \times \left(\frac{H_n}{\beta}\right) v_w S_{SW}(f)\,.$$

$\alpha$: phase transition strength parameter
$v_w$: bubble wall terminal velocity
$\frac{\beta}{H_n}$: the inverse time duration of the phase transition
$H_n$: Hubble rate at the nucleation temperature $T_n$
$g_n$: the number of degrees of freedom
$\kappa$: the fraction of bulk kinetic energy relative to the available vacuum energy.





Weir David J. R. Soc. A.376: 20170126(2018)

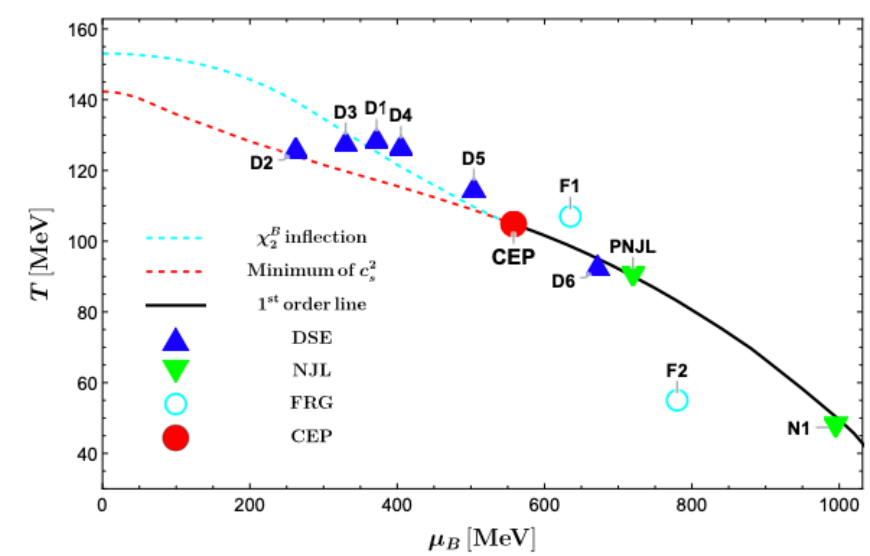
# Induced gravitational wave

$$h^2\Omega_{GW}(f) = 8.5\times 10^{-6}\left(\frac{100}{g_n}\right)^{1/3}\left(\frac{\kappa\alpha}{1+\alpha}\right)^2 \times\left(\frac{H_n}{\beta}\right) v_w S_{SW}(f)\,.$$

**Spectral shape and peak frequency**

$$S_{SW}(f) = \left(\frac{f}{f_{SW}}\right)^3\left[\frac{7}{4+3(f/f_{SW})^2}\right]^{7/2},$$

$$f_{SW} = 1.9\times 10^{-8}\left(\frac{\beta}{H_n}\right)\left(\frac{T_n}{100\,\mathrm{Mev}}\right)\left(\frac{g_n}{100}\right)^{1/6}\ \mathrm{Hz}$$




**Parameters:**

$$g_n = \frac{45 S_+}{2\pi^2 T_n^2} \qquad v_w = 0.95$$

$$\kappa = \frac{\alpha}{0.73+0.083\sqrt{\alpha}+\alpha}$$

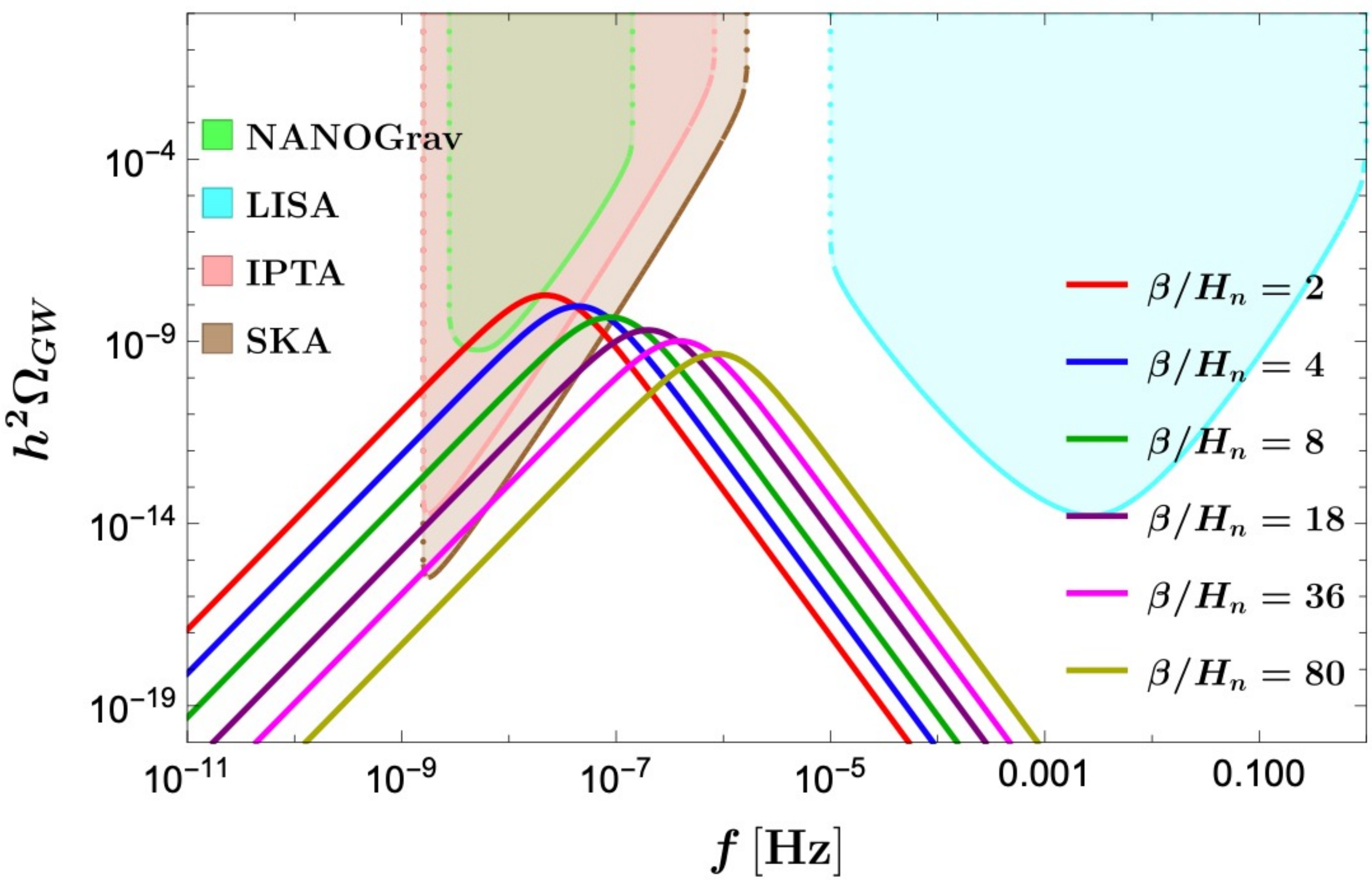
$$\alpha = \frac{4(\theta_+ - \theta_-)}{3 w_+} \qquad \theta = (\epsilon - 3P)/4 \qquad w = \epsilon + P$$

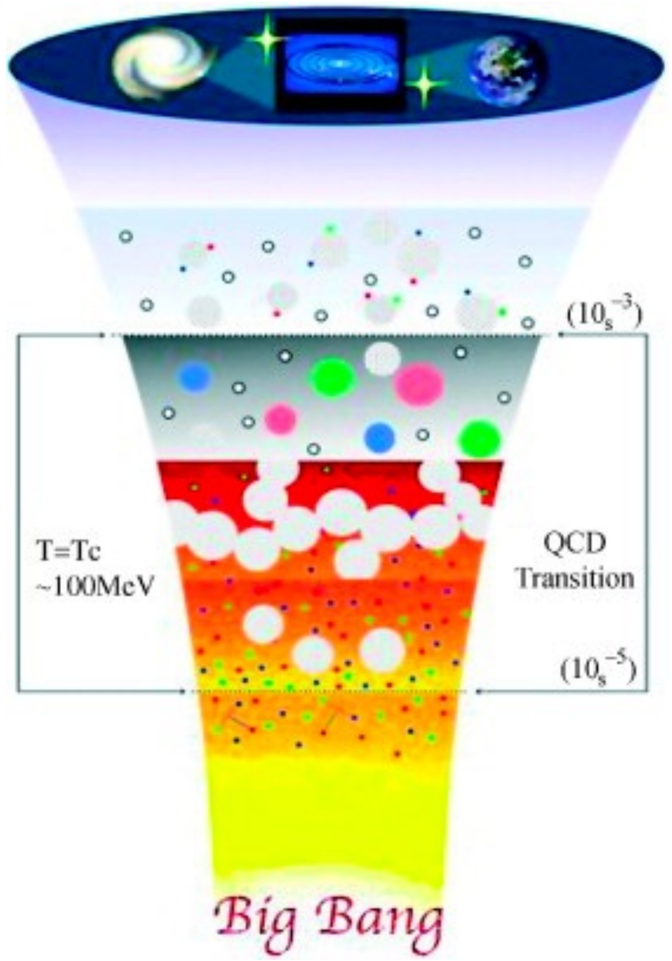
between the false (+) and true (-) vacuums

**Most parameters can be fixed.**

# Induced gravitational wave

*Kai Schmitz, JHEP* 01 (2021) 097




$(\mu, T, \alpha) =$ (1000 MeV, 49.53 MeV, 0.33).

The upper curve in each band is for $\beta/Hn = 2$ and the lower curve is for $\beta/Hn = 80$.

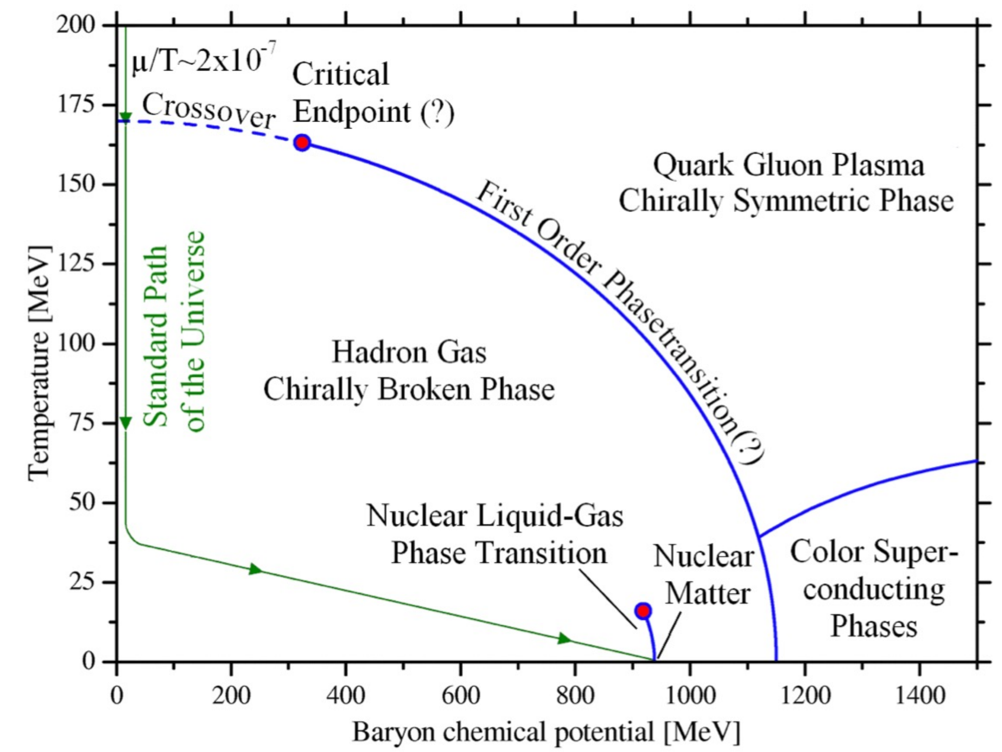
The GW energy spectrum is within the projected sensitivity of **IPTA** and **SKA**

**Standard cosmology:** from microwave background radiation (CMB) and big bang nucleosynthesis (BBN)

$$\eta_B = \frac{n_B}{s} \sim \frac{n_B}{n_\gamma} \sim \frac{\mu_B}{T} \sim 10^{-9}$$

Baryon number per entropy is conserved and early universe evolves along $\mu/T \sim 10^{-9}$

crossover, no cosmological signals




[Boeckel and Bielich, 2011]

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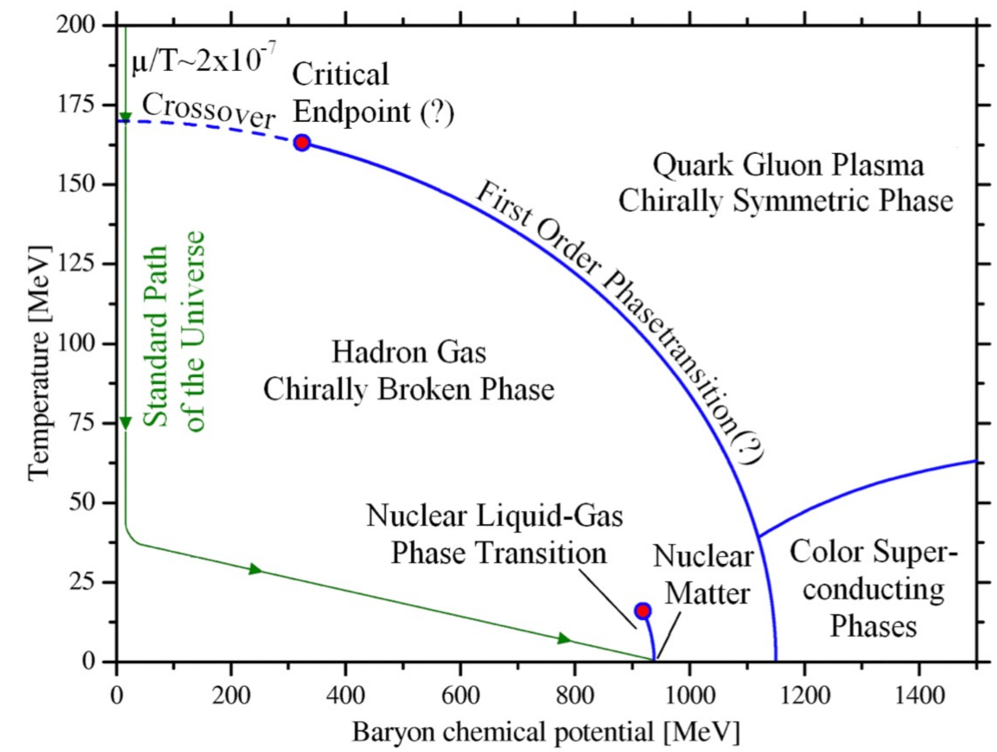


[Boeckel and Bielich, 2011]

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⇓

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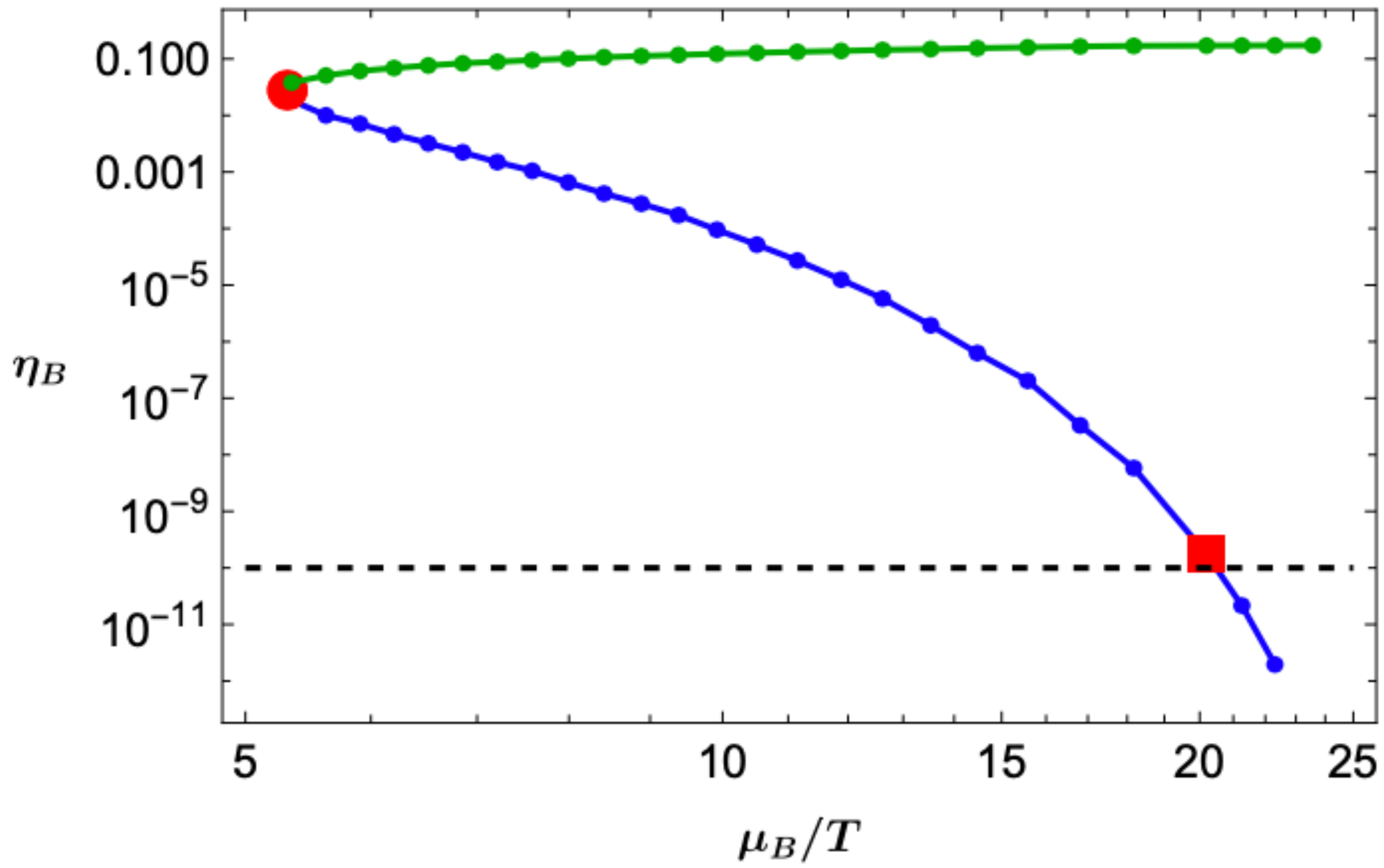
Is it possible to have a 1st phase transition without contradiction with present data?




Is it possible to have a 1st phase transition without contradiction with present data?

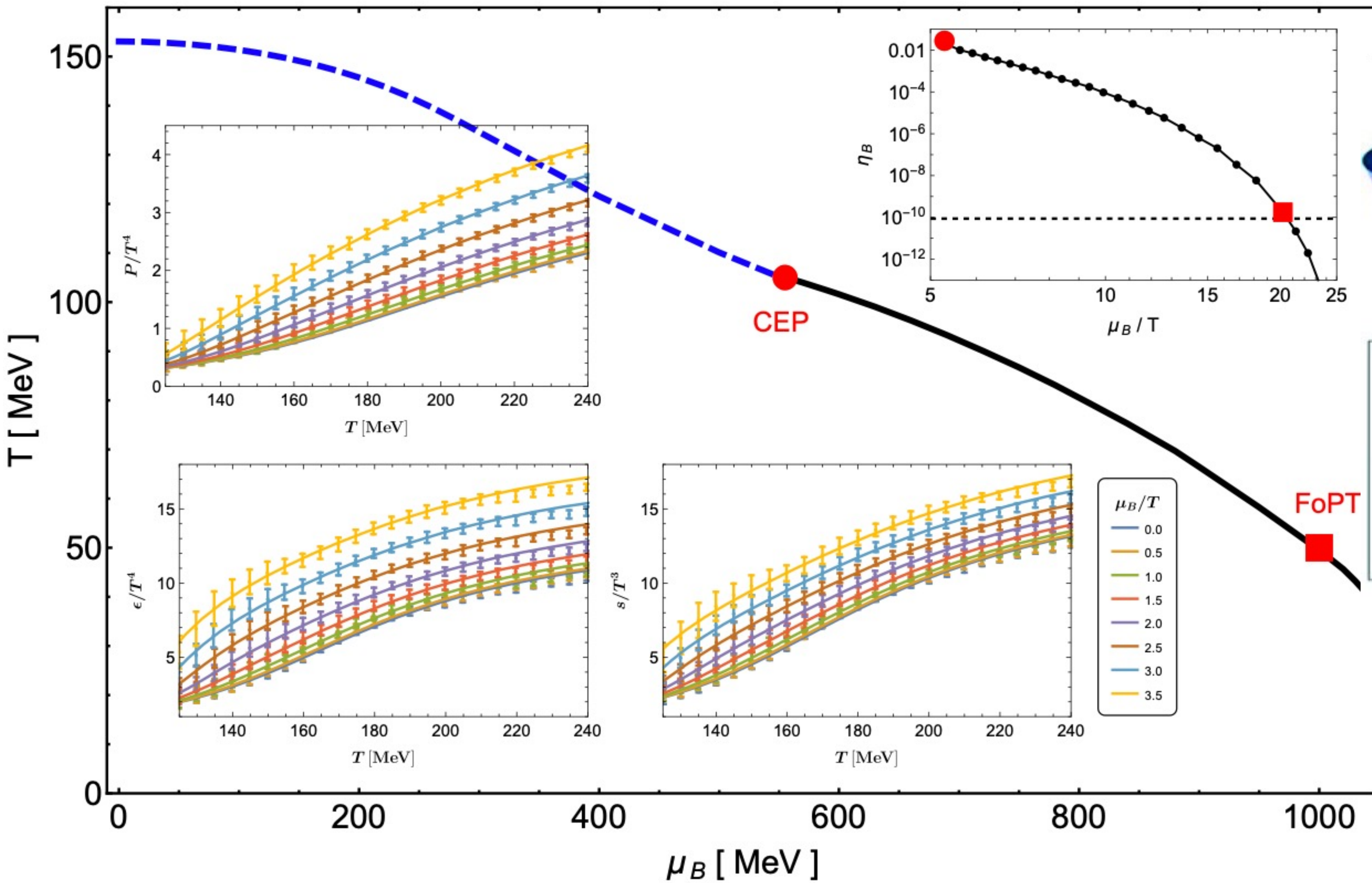
Start with $\eta_B \sim$ O(0.1) generated by Affleck-Dine baryogenesis

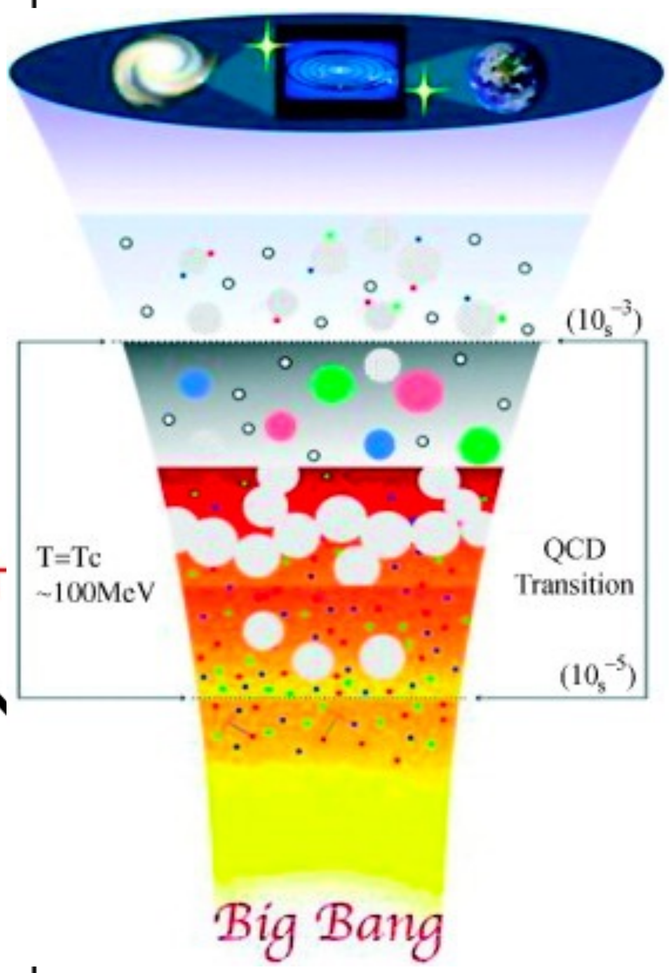
**[hep-ph/0303065]**

The upper limit for the Affleck-Dine baryogenesis: $\eta_B \sim$ O(1)
[Linde, PLB, 1985]

# Phase transition gravitational wave

$$\eta_B = \frac{\rho_B}{s}$$



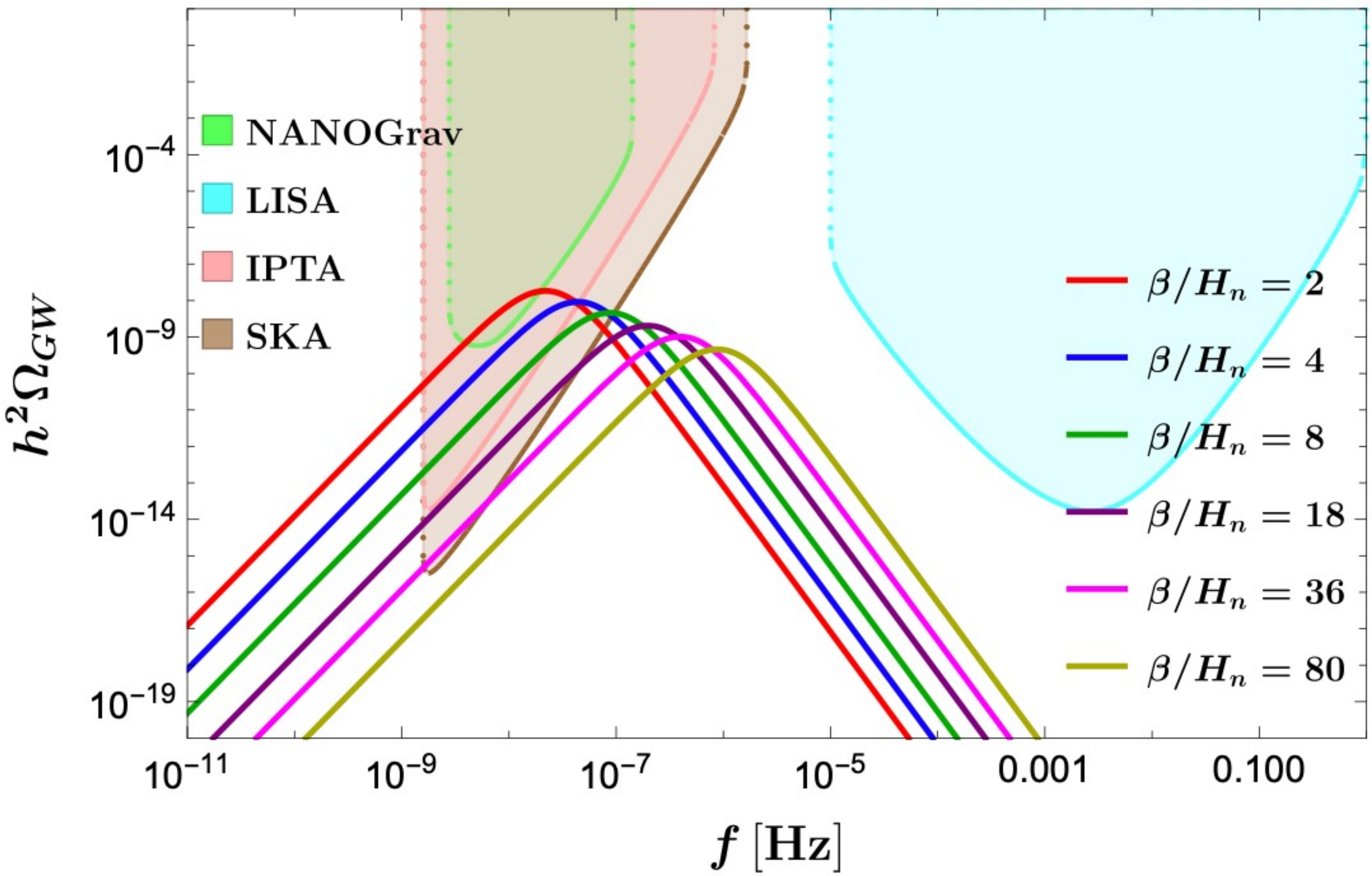



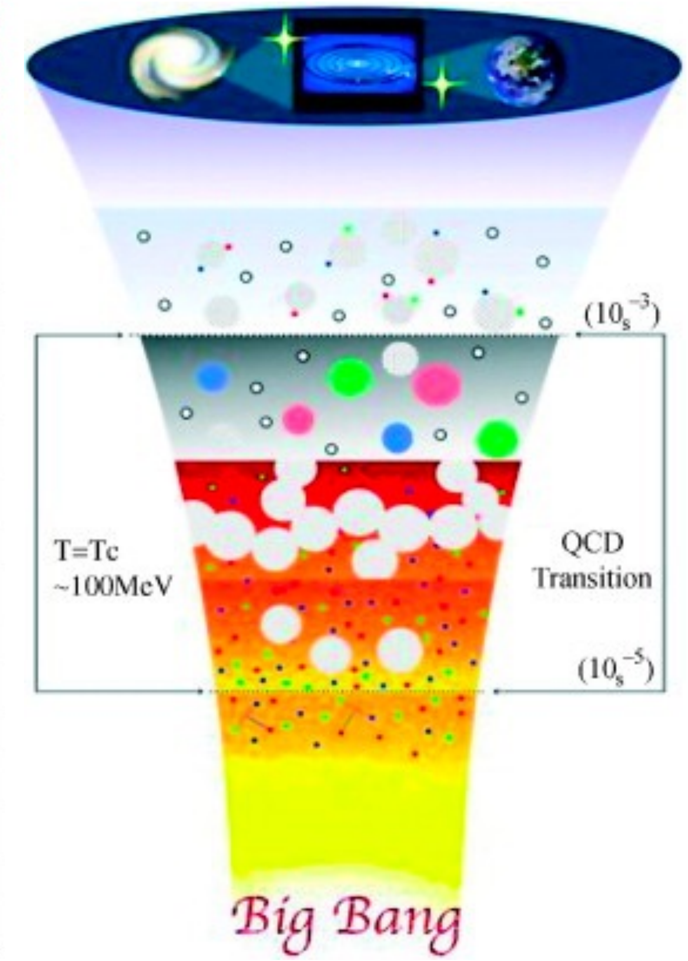

Our model provides a scenario for phase transition GWs within the Standard Model of particle physics.

$$\mu_B = 1000Mev, T_n = 49.53Mev$$

# GWs from sound wave

*Kai Schmitz, JHEP* 01 (2021) 097

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The upper curve in each band is for $\beta/Hn = 2$ and the lower curve is for $\beta/Hn = 80$.

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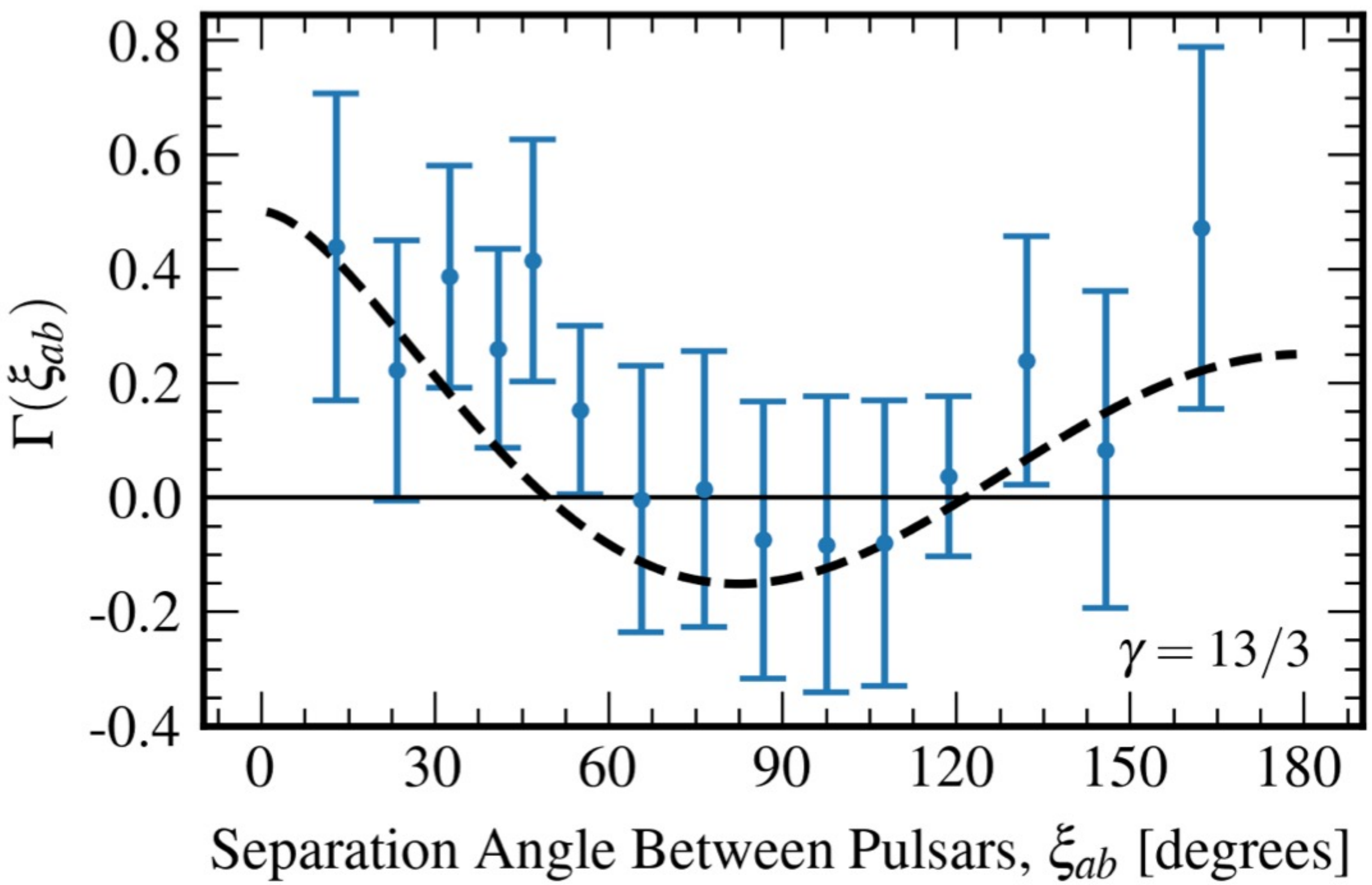
Recently, independent evidence for detecting a GW background around the nano-Hz band has been reported by different **PTA observations.**




NANOGrav 15yr data

[2306.16213]

The parameter space of the FoPT predicted by our QCD model can be constrained by NANOGrav data by assuming that it produces the dominant contribution to the signals.

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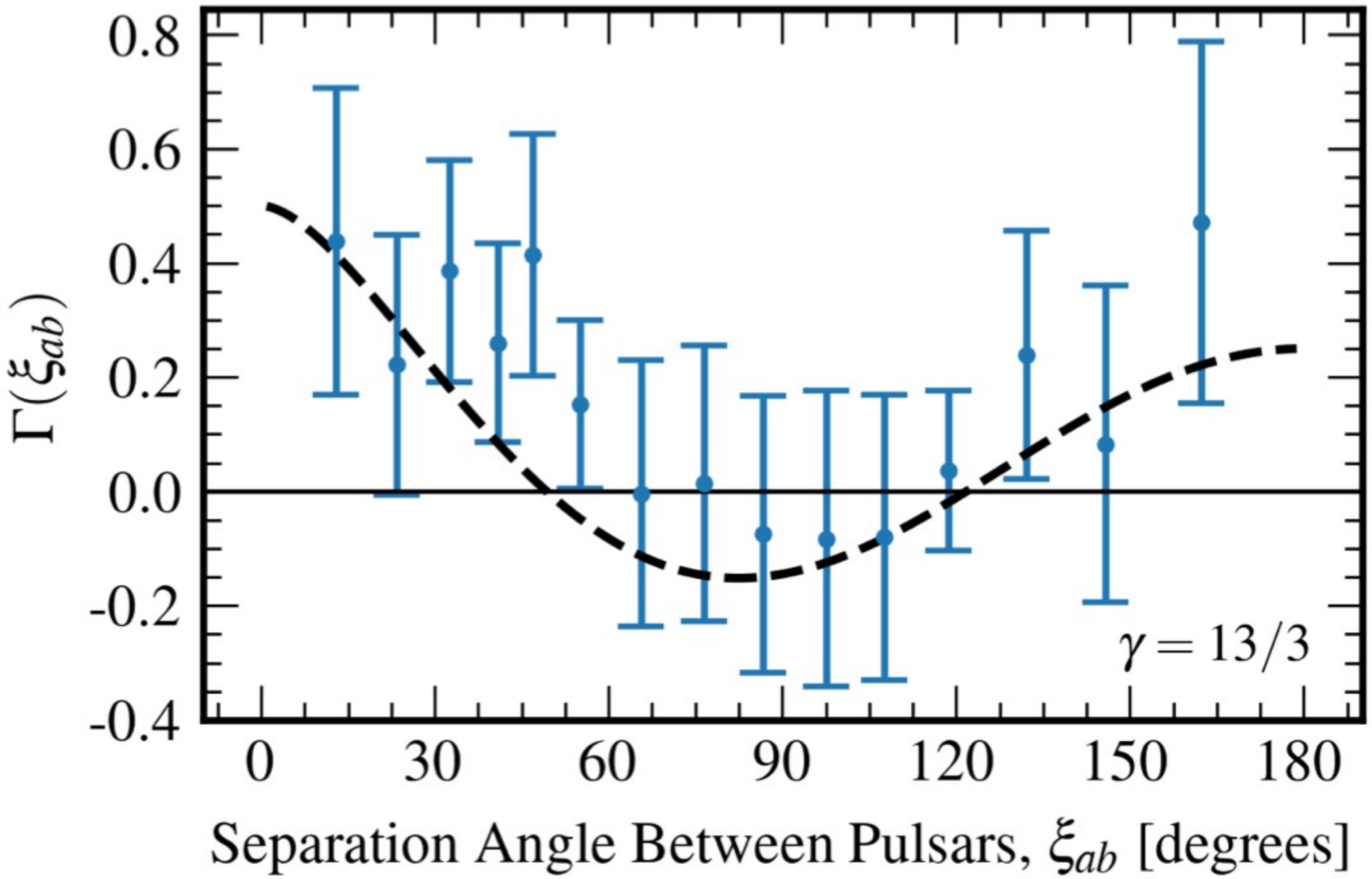


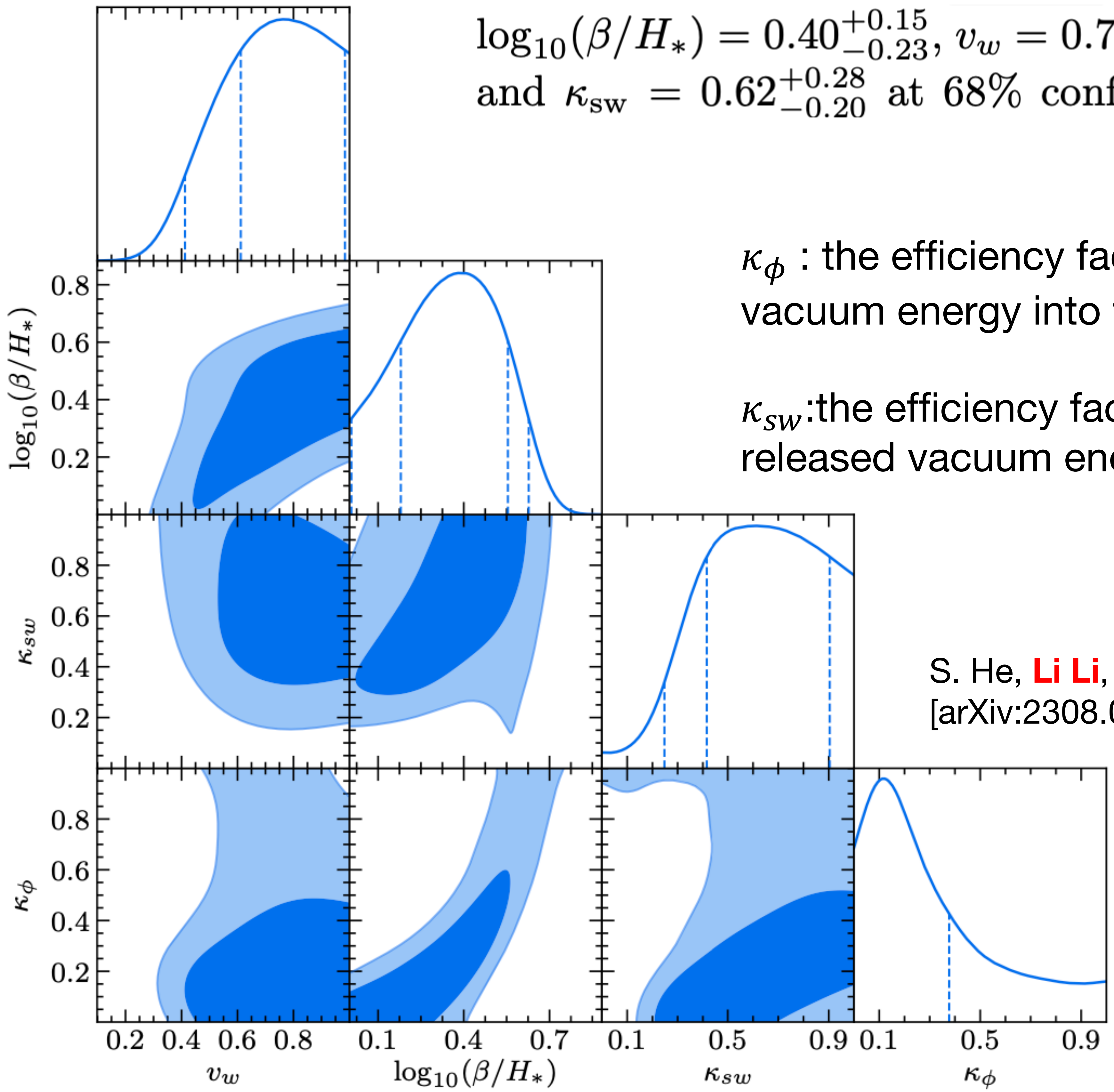

NANOGrav 15yr data

[2306.16213]

The parameter space of the FoPT predicted by our QCD model can be constrained by NANOGrav data by assuming that it produces the dominant contribution to the signals.

**bubble collision + sound wave + MHD turbulence**

**Posteriors of the four independent model parameters inferred from the NANOGrav 15-year data release**

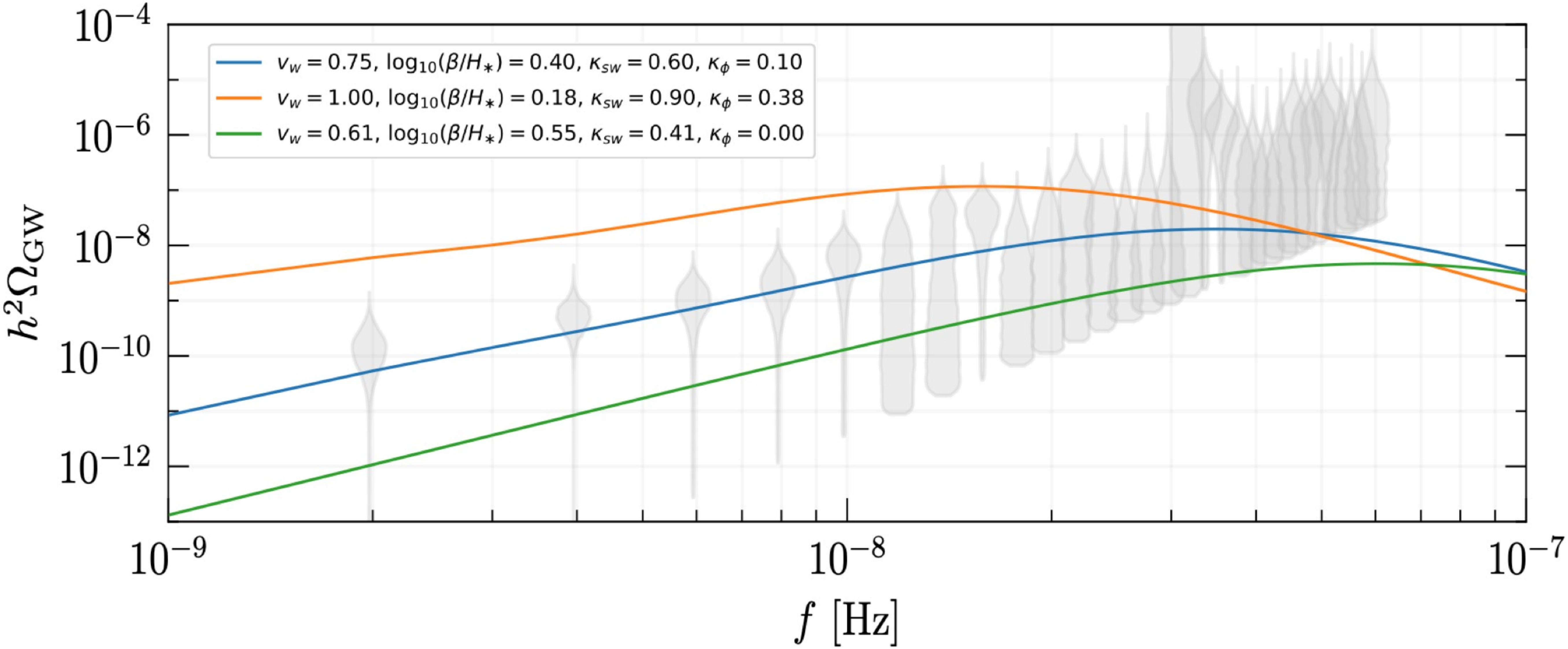



$\log_{10}(\beta/H_*) = 0.40^{+0.15}_{-0.23}$, $v_w = 0.76^{+0.22}_{-0.15}$, $\kappa_\phi = 0.12^{+0.26}_{-0.12}$, and $\kappa_{\rm sw} = 0.62^{+0.28}_{-0.20}$ at 68% confidence level.

$\kappa_\phi$ : the efficiency factor of converting the vacuum energy into the bubble wall motion,

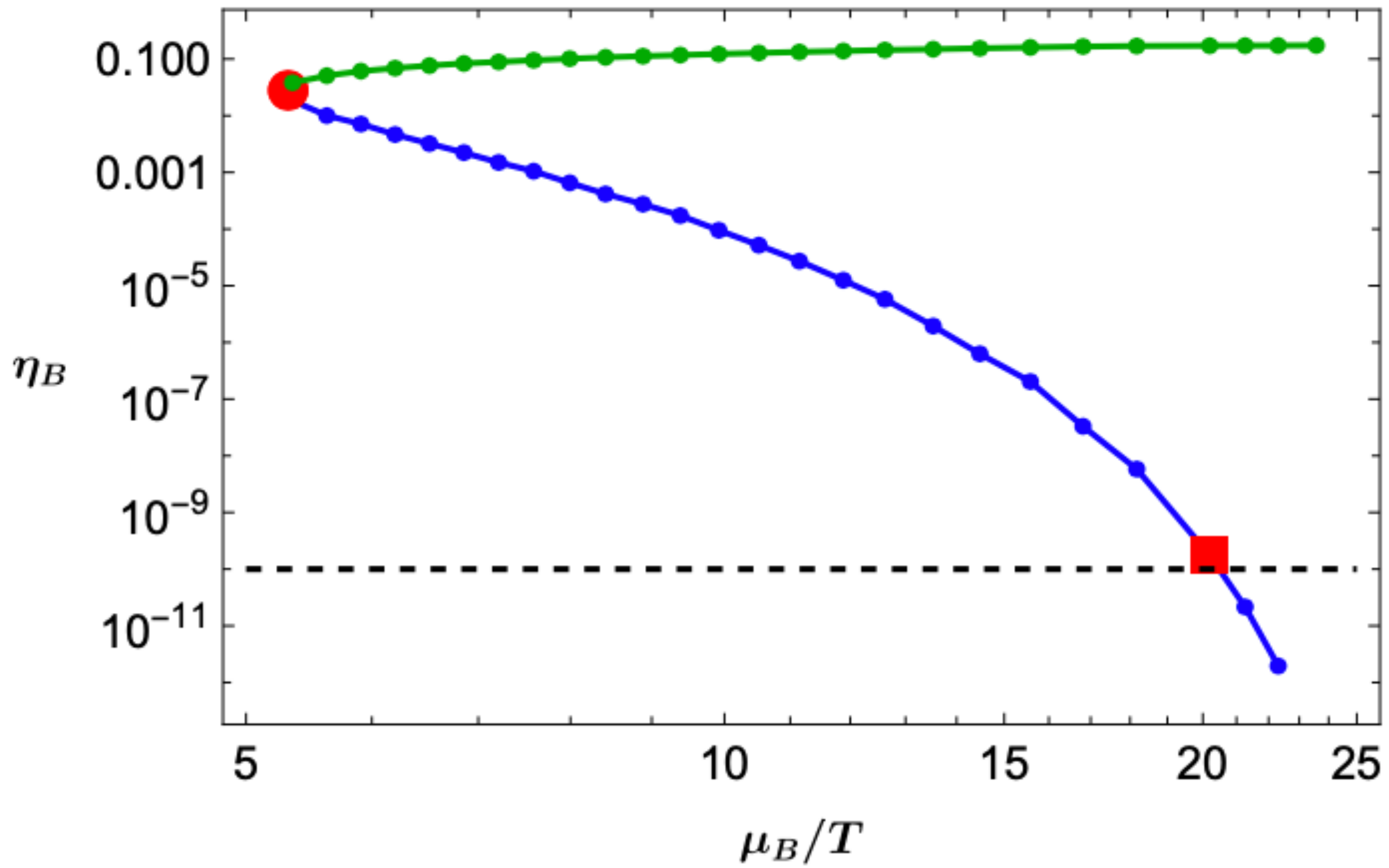
$\kappa_{sw}$:the efficiency factor of converting the released vacuum energy into the fluid motions

S. He, **Li Li**, S. Wang and S. J. Wang, [arXiv:2308.07257 [hep-ph]].




Energy-density spectra with three different sets of values for the four independent model parameters. Violin data points stand for the NANOGrav 15yr observations.

S. He, **Li Li**, S. Wang and S. J. Wang, [arXiv:2308.07257 [hep-ph]].




Is it possible to have a 1st phase transition without contradiction with present data?

**Little inflation scenario**
**at cosmological QCD phase transition**

A strong mechanism for baryogenesis + A quasistable QCD-medium state that triggers a short inflationary period of inflation diluting the baryon asymmetry to the value observed today.

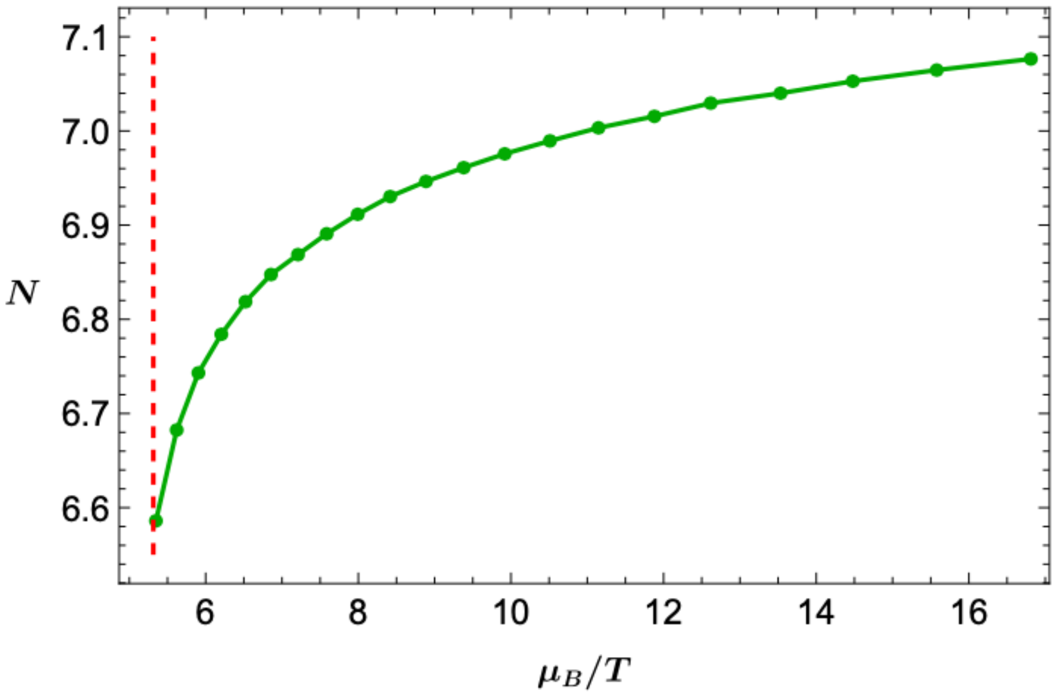
[Linde, 1985; Kampfer et al., 1986, Borghini et al., 2000]

## A little inflation in QCD phase diagram

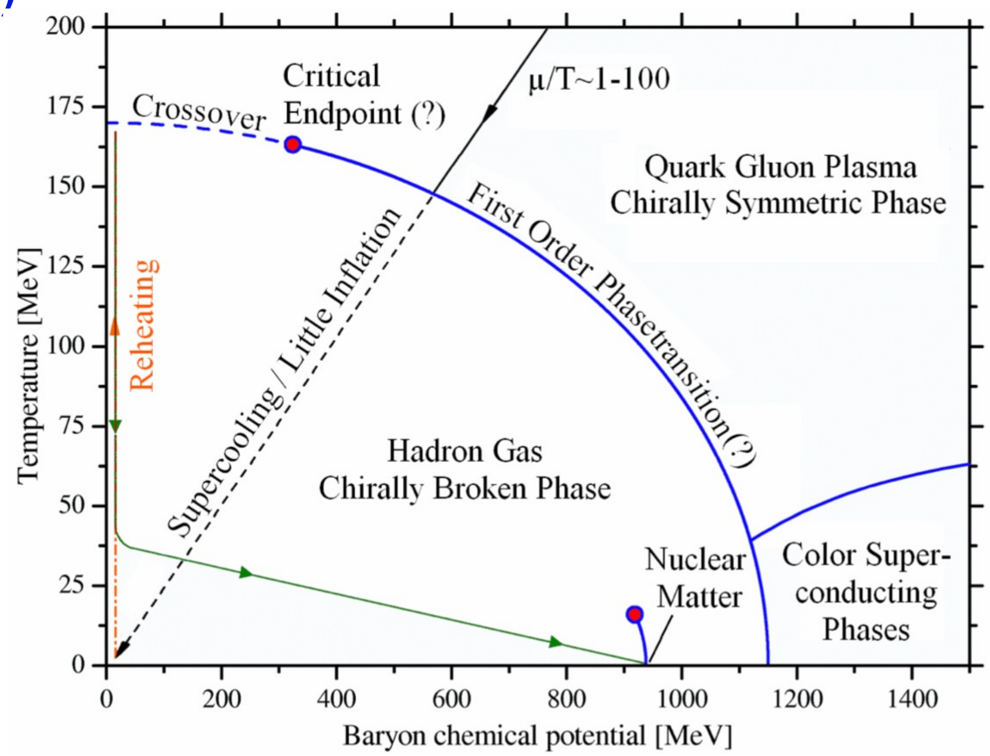
a few e-folds are enough (standard inflation needs N ~ 50)

$$\frac{a_f}{a_i} = \left(\frac{\eta_{Bi}}{\eta_{Bf}}\right)^{1/3} = \left(\frac{\eta_{B+}}{\eta_B^{ob}}\right)^{1/3}$$

Hence $N = \ln\left(\frac{a_f}{a_i}\right) = \ln(10^3) \approx$ **7 e-folds** is enough




- ➢Start with $\mu/T$ ~ O(1) (e.g. Affleck-Dine baryogenesis)
- ➢ Universe trapped in false vacuum at the 1st transition line
- ➢ Supercooling and dilution with $\mu/T$ = const
- ➢Decay to the true vacuum state → reheating so that $\mu/T{\sim}10^{-9}$
- ➢Then standard cosmological evolution to BBN




[Boeckel and Bielich, 2011]

# Outline

1. Introduction

2. Holographic QCD model

3. Phase diagram and GWs

**4. Summary and discussion**

# 4. Summary

➢Build a hQCD model to describe real QCD dynamics.

➢ Find good quantitative agreement with lattice data at zero/non-zero chemical potential.

➢ Predict the QCD CEP that agrees qualitatively with effective field results.

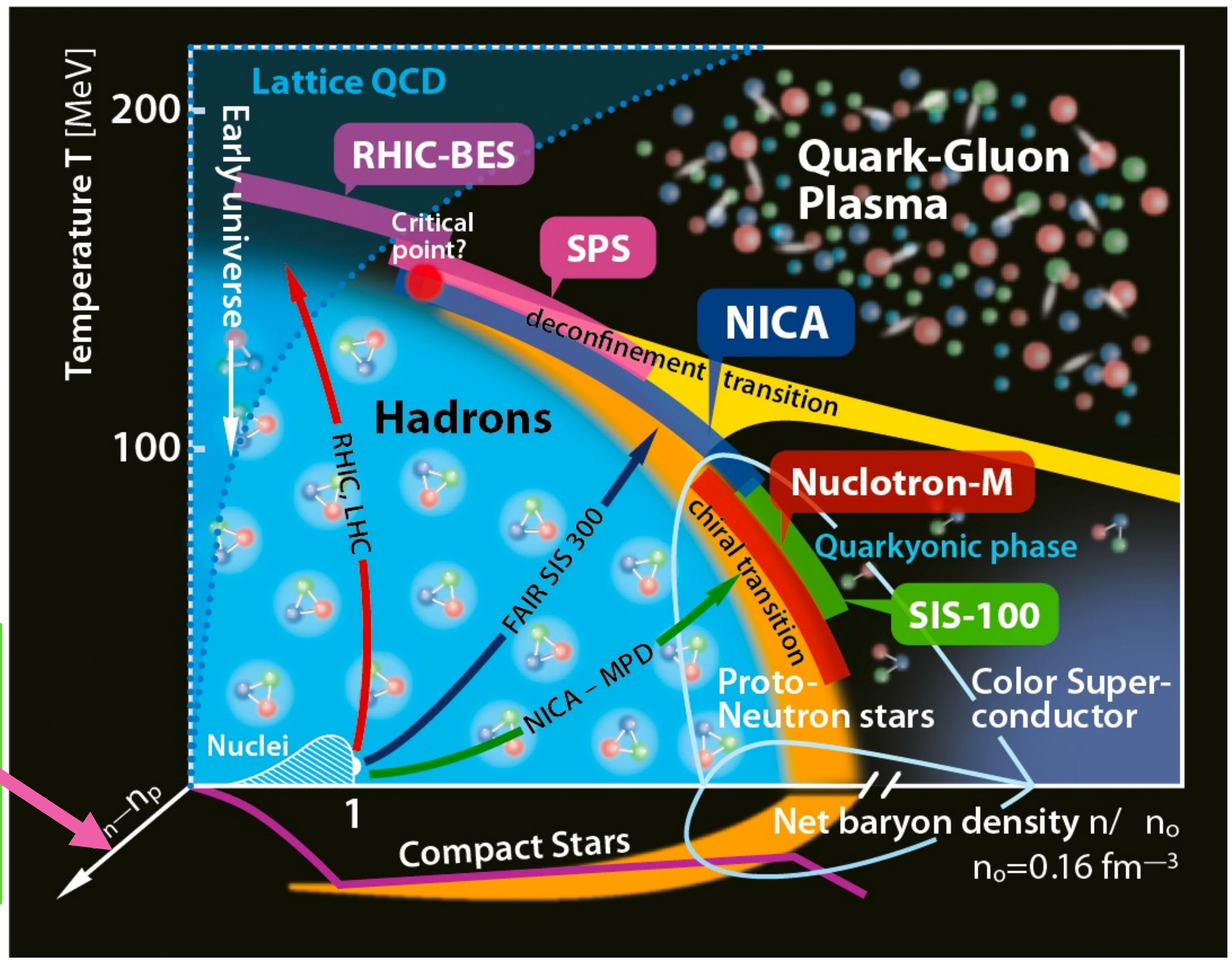
➢ The predicted location of CEP is within the coverage of future (FAIR, JPARC-HI, and NICA) experimental facilities.

➢ Compute the stochastic GW spectrum

**Holography is a useful approach to study real hot and dense QCD matter in non-perturbative regime.**

# 4. Discussion

It is desirable to further study the non-perturbative features of QCD dynamics using this holographic model.

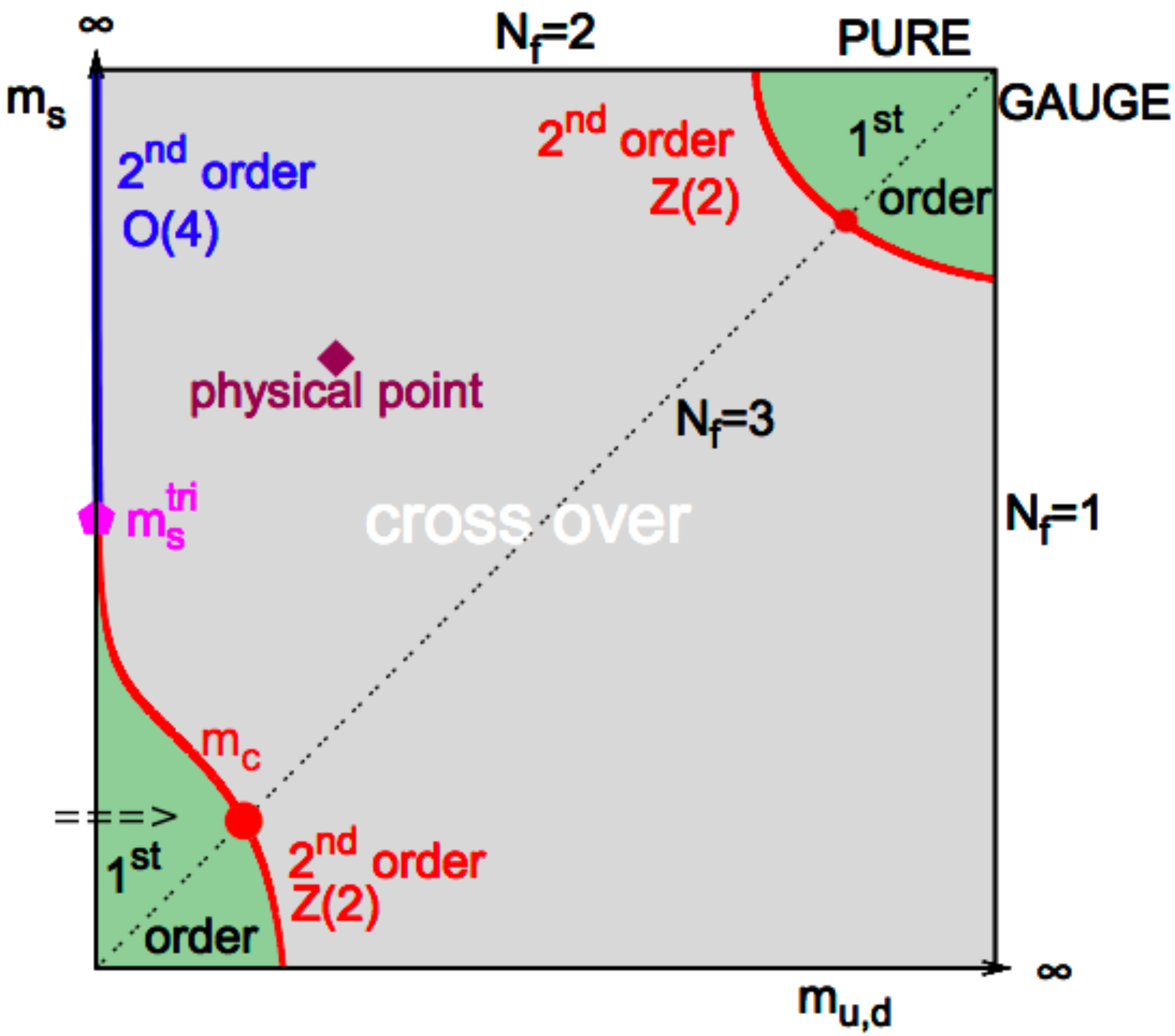



Magnetic field
**Isospin**
Rotation
…

GWs:
1st phase transition
bubble collisions
PBH
…

New method is necessary: Machine Learning…

# Columbia plot of QCD at $\mu_B = 0$




To build effective models to capture the main feature of QCD matter;

Non-perturbative effects are effectively adopted into the model parameters.

# QCD at zero chemical potential

**Einstein-Dilaton theory:**

$$S = \frac{1}{2\kappa_N^2}\int \mathrm{d}^5x\sqrt{-g}\left[\mathcal{R} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right]$$

**To match lattice QCD simulation**

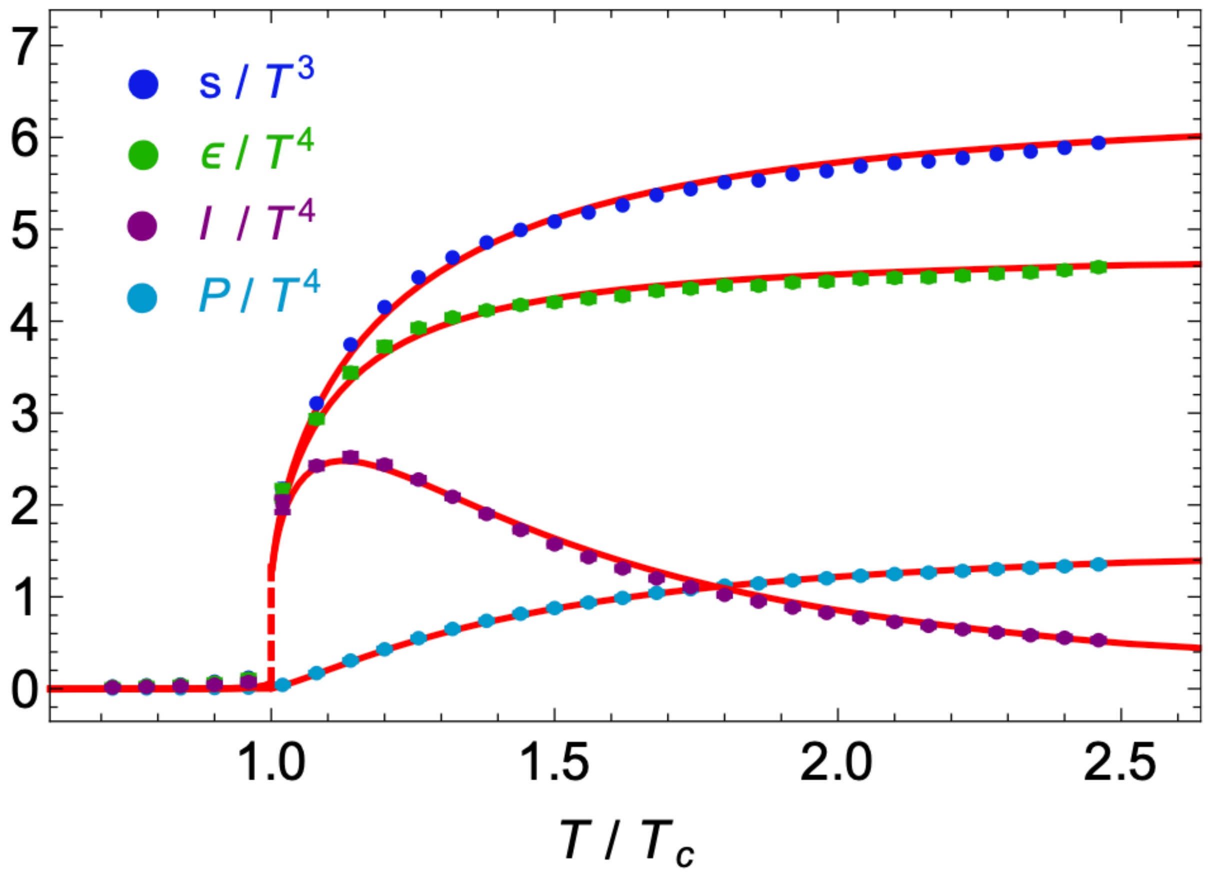
$$V(\phi) = -12\cosh[c_1\phi] + (6c_1^2 - \frac{3}{2})\phi^2 + c_2\phi^6$$

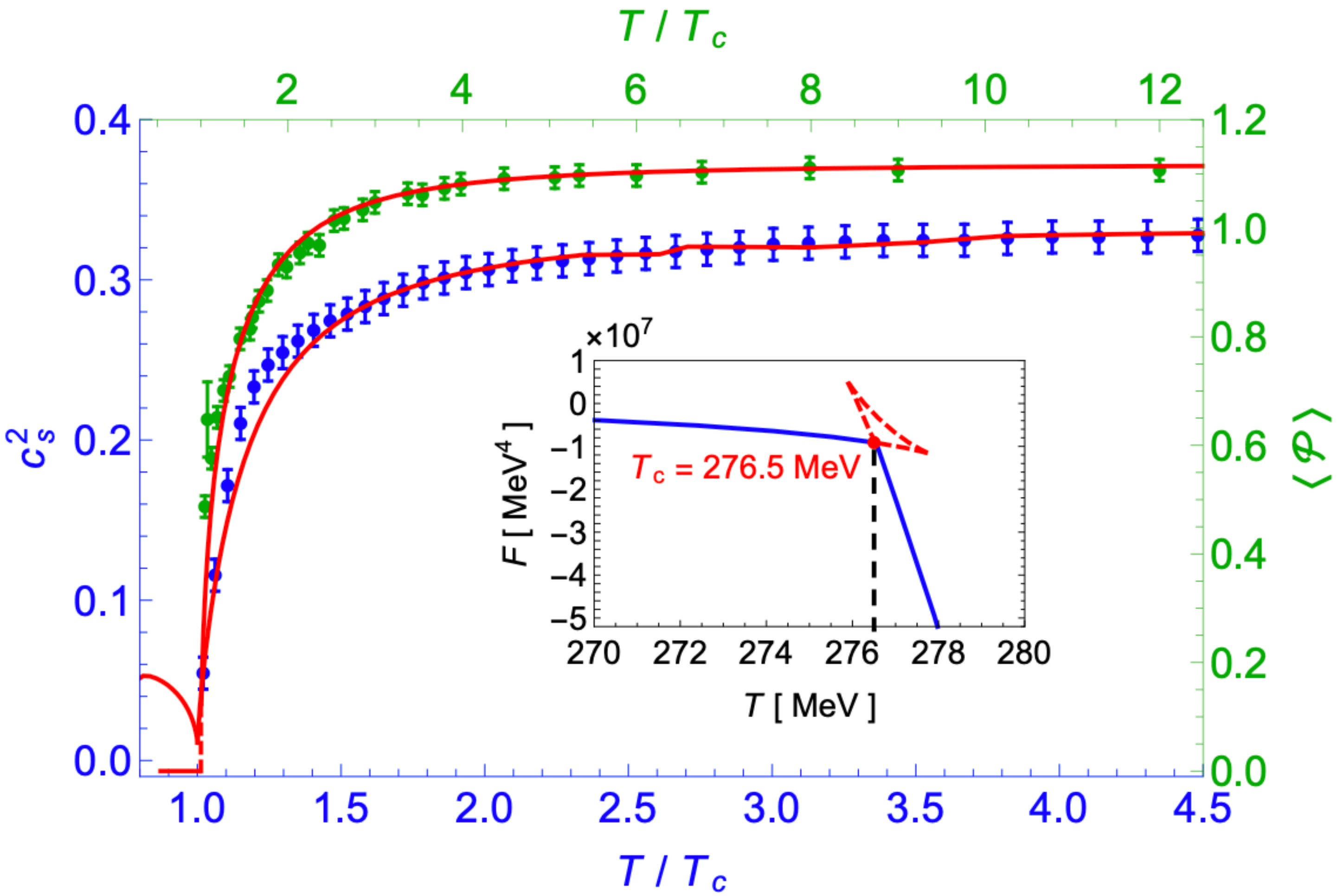
**Non-perturbative effects are effectively adopted into the model parameters by matching with up-to-date lattice QCD data.**

# Pure Gluon at zero chemical potential

**Much simpler than (2+1)-flavor QCD!**

$$c_1 = 0.735, c_2 = 0$$

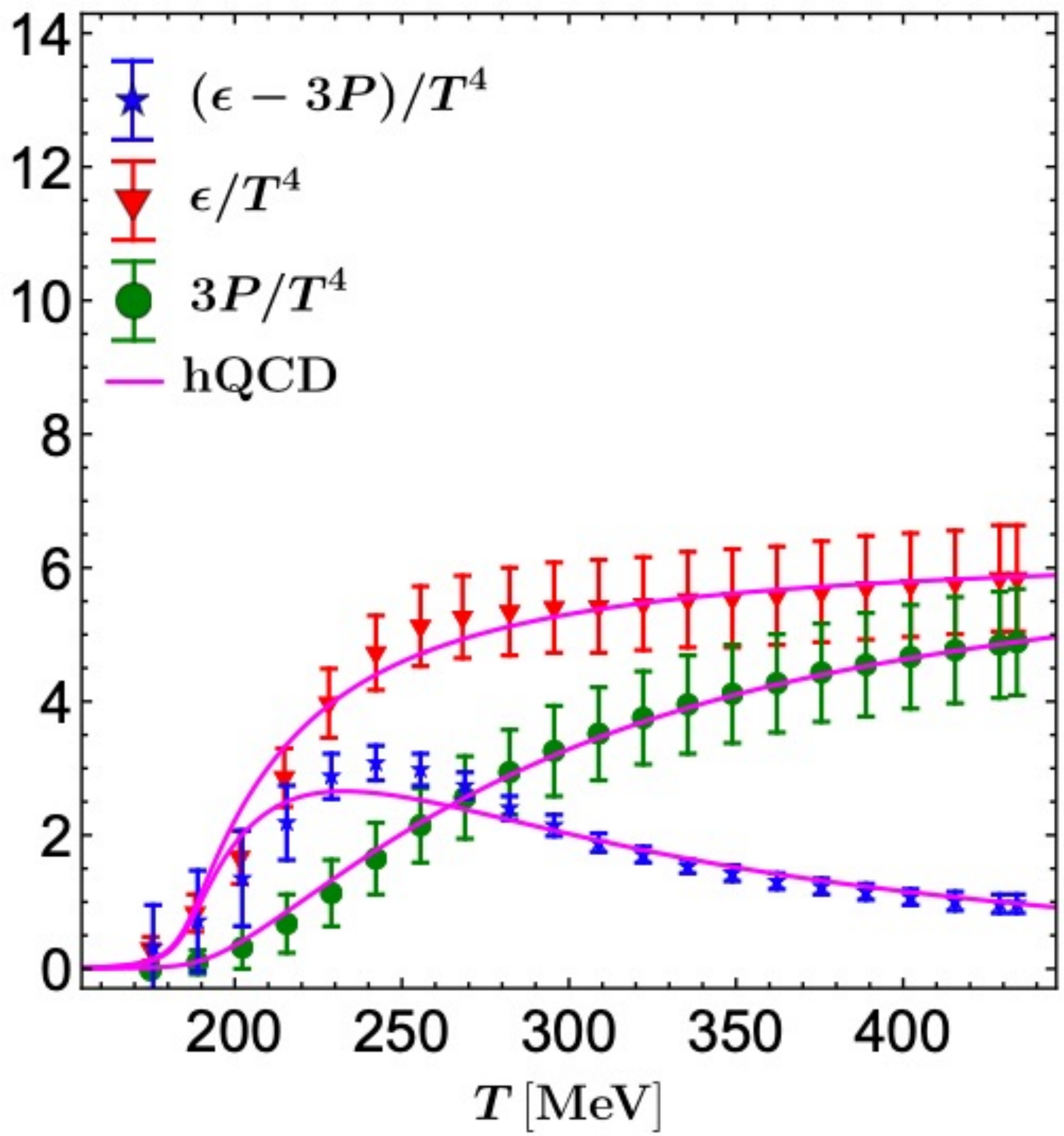



**There is a first-order confinement/deconfinement PT at Tc=276.5 Mev !**

Song He, **Li Li**, Zhibin Li, Shao-Jiang Wang,
Sci.China Phys.Mech.Astron. 67 (2024) 240411

# 2-favor QCD at zero chemical potential




$$V(\phi) = -12\cosh[c_1\phi] + (6c_1^2 - \frac{3}{2})\phi^2 + c_2\phi^6$$

| model | $c_1$ | $c_2$ |
|---|---|---|
| pure $SU(3)$ | 0.735 | 0 |
| 2 flavor | 0.710 | 0.0002 |
| 2+1 flavor | 0.710 | 0.0037 |

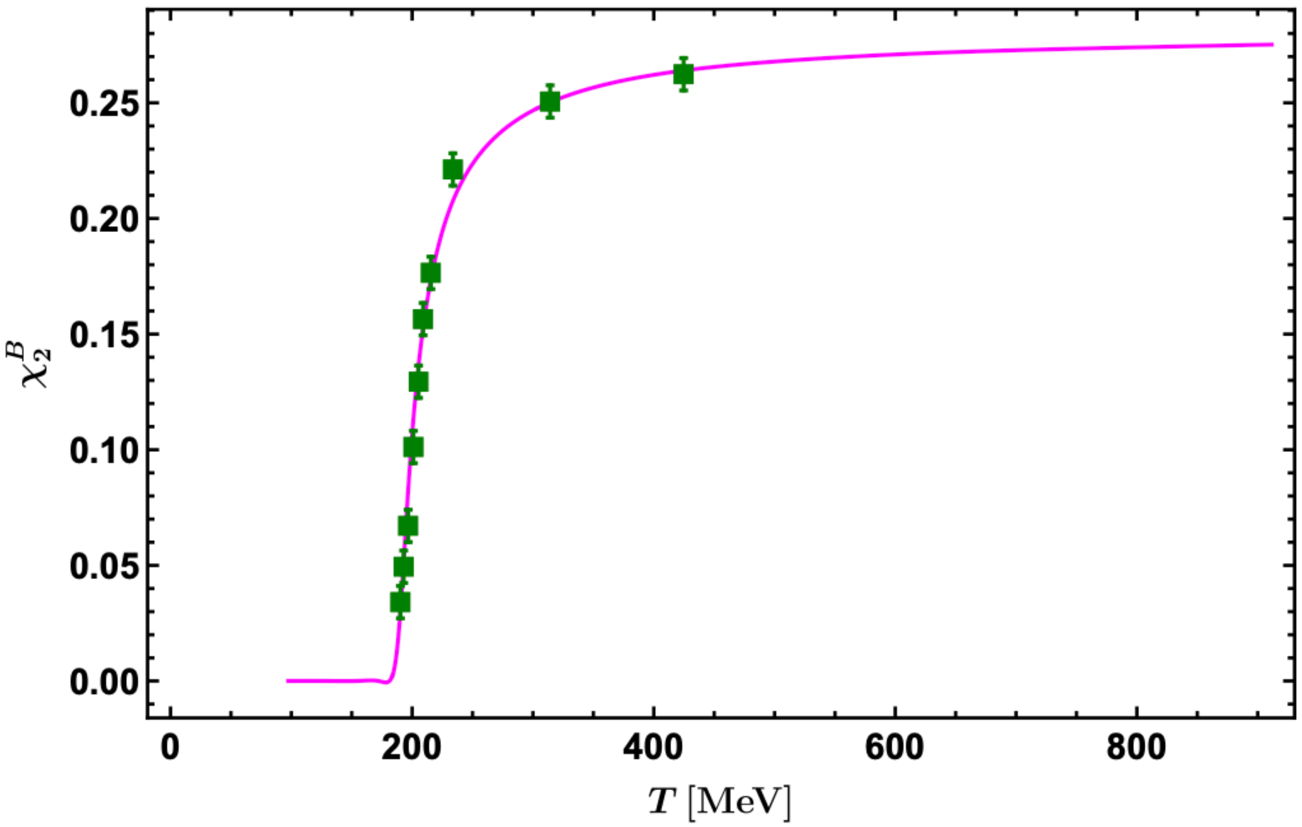
Yan-Qing Zhao, Song He, Defu Hou, **Li Li**, Zhibin Li, Phys.Rev.D 109 (2024) 8, 086015

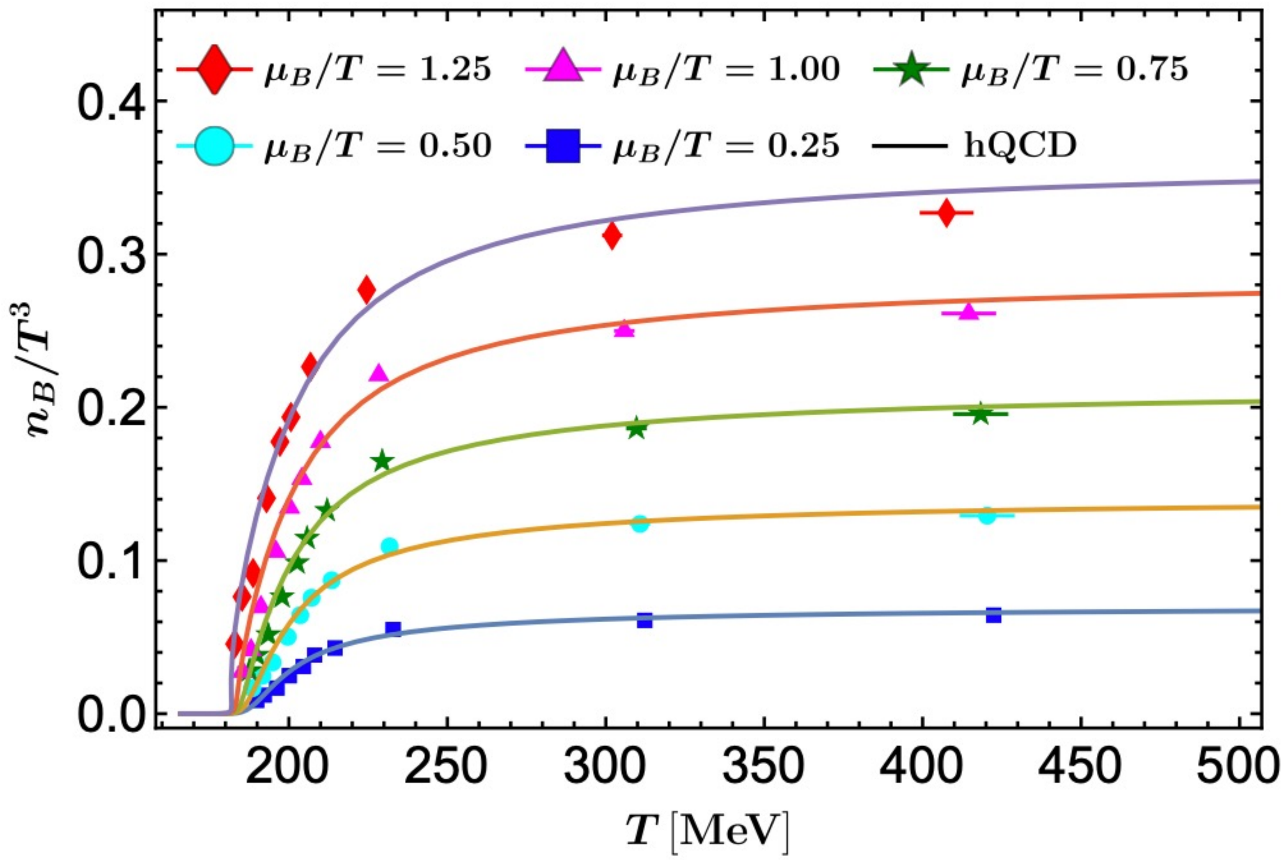
**tmfT Collaboration**, arXiv:1412.6748 [hep-lat]

# Baryon number density and second-order baryon susceptibility

$$Z(\phi) = \frac{1}{1+c_3}\mathrm{sech}[c_4\phi^3] + \frac{c_3}{1+c_3}e^{-c_5\phi}$$

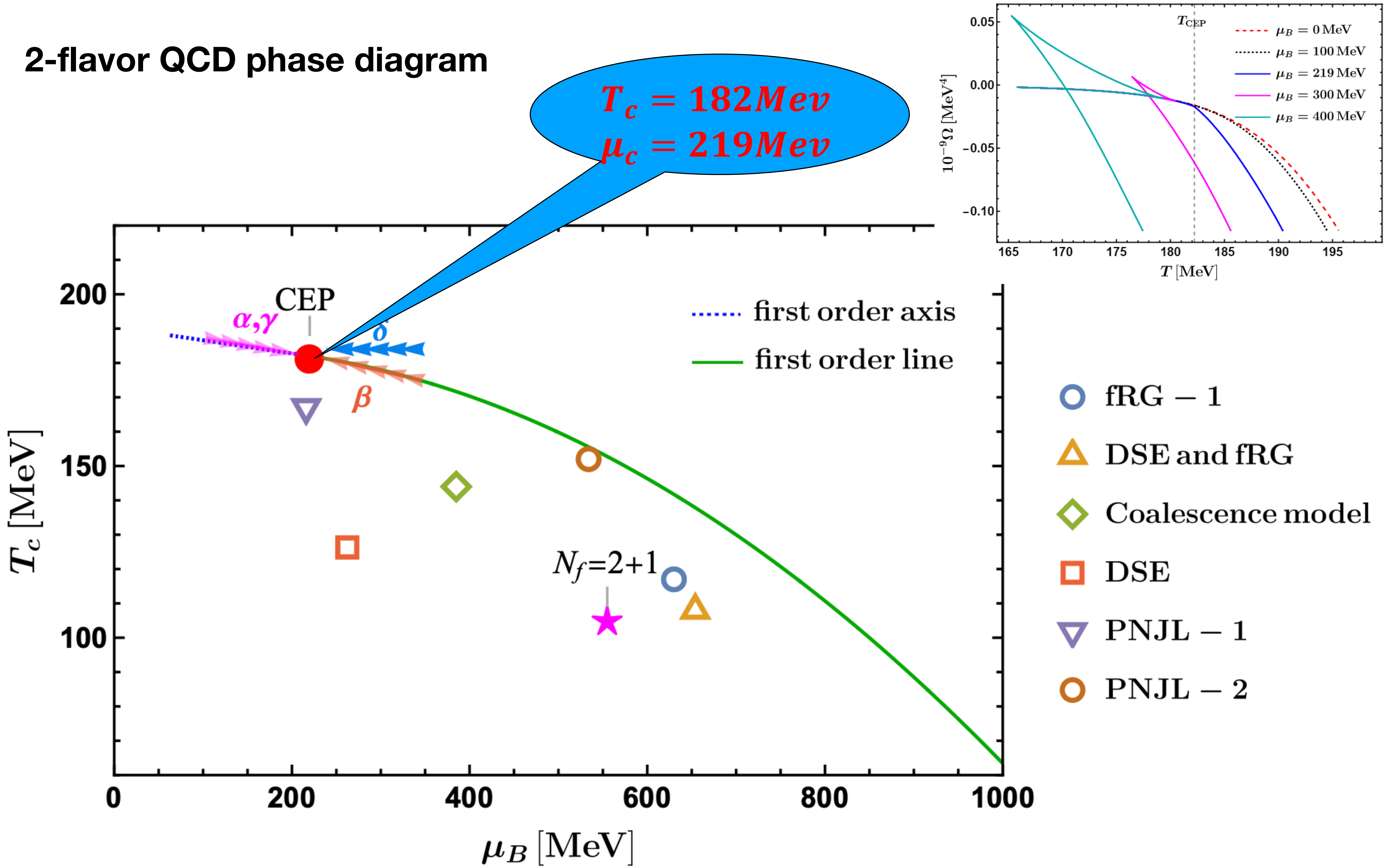
$$S = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\Big[\mathcal{R} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)\Big],$$




S. Datta, R. V. Gavai and S. Gupta, arXiv:1612.06673 [hep-lat].

## 2-flavor QCD phase diagram



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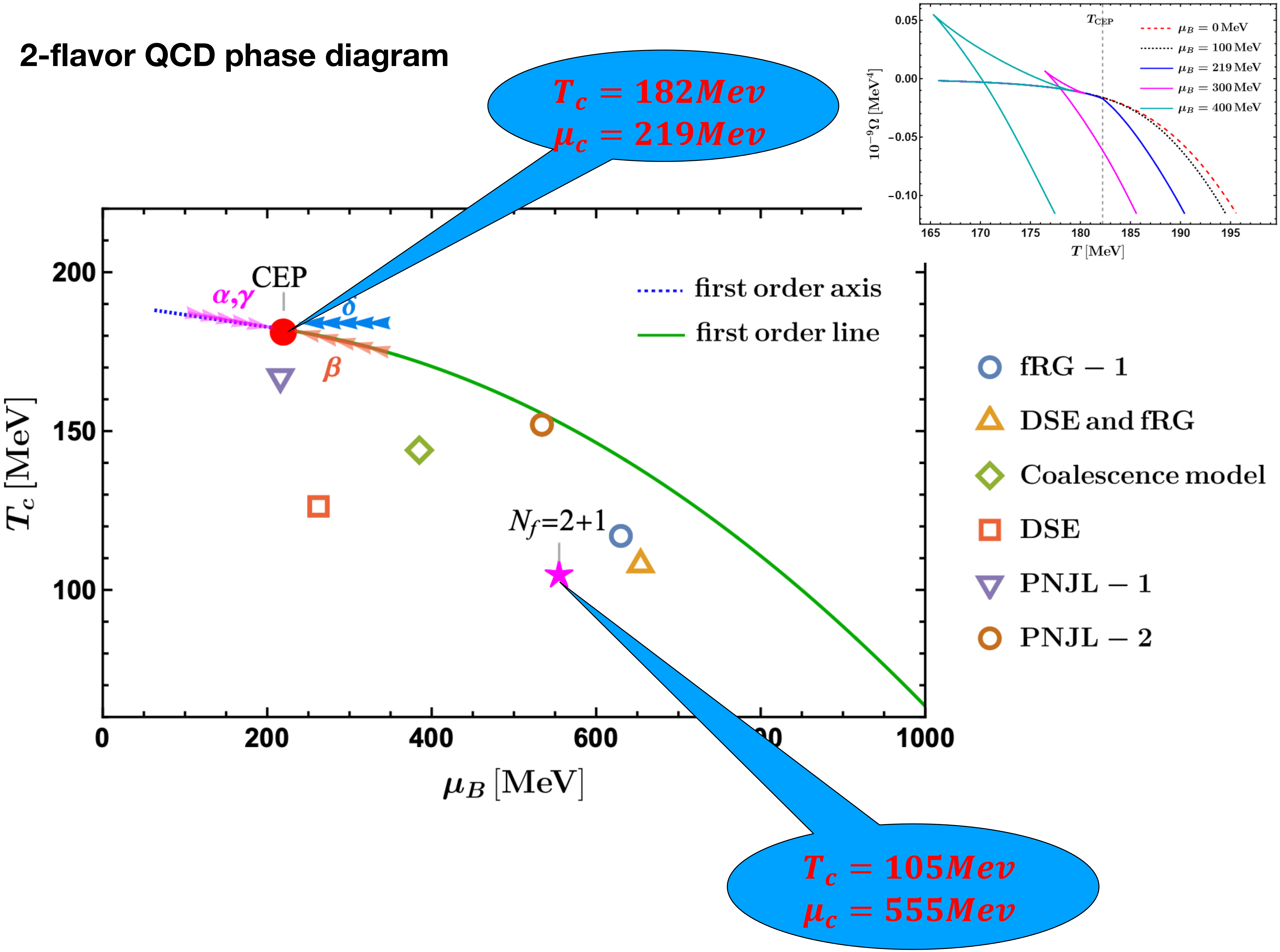
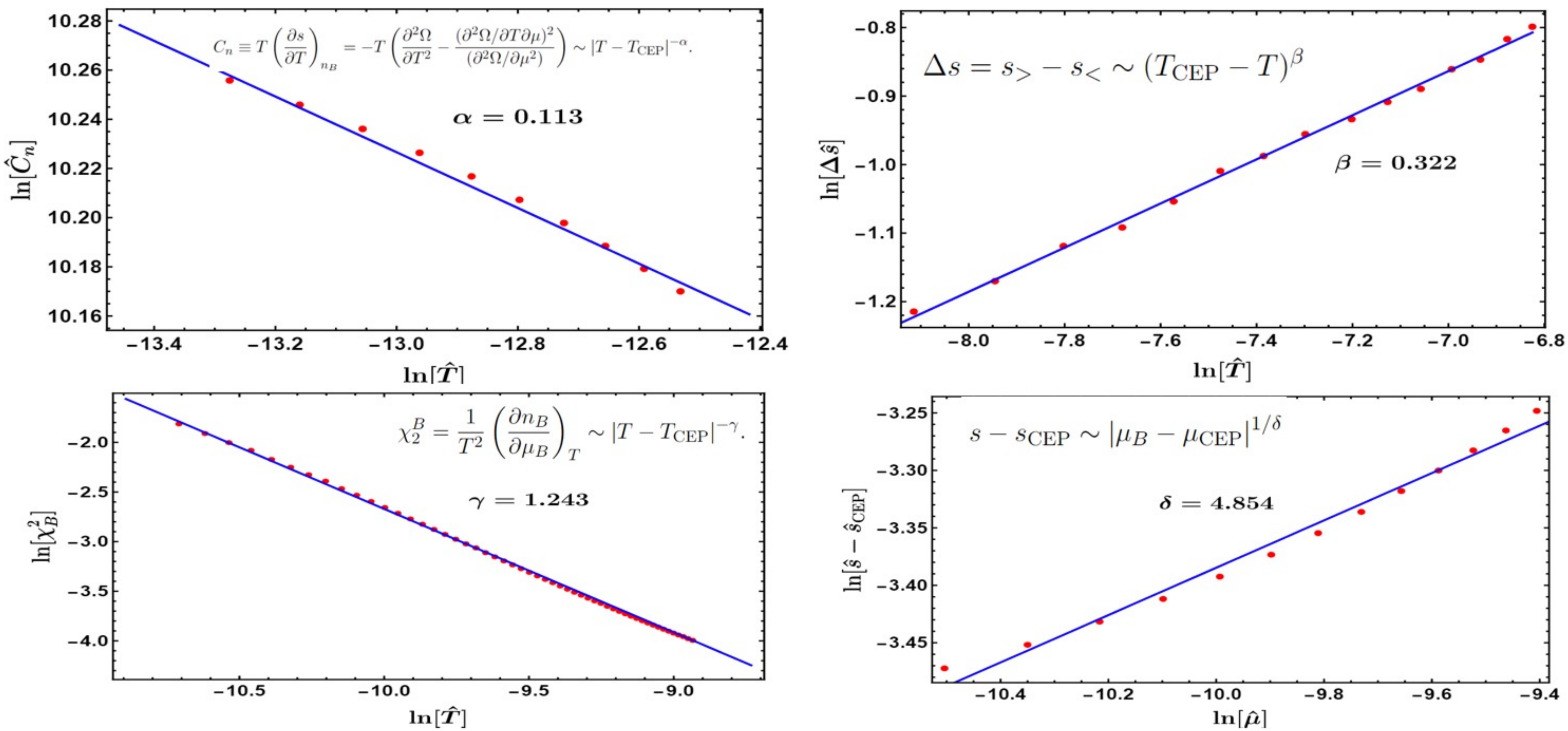


## 2-flavor QCD critical exponents




$$\alpha + 2\beta + \gamma = 2, \quad \alpha + \beta(1+\delta) = 2$$

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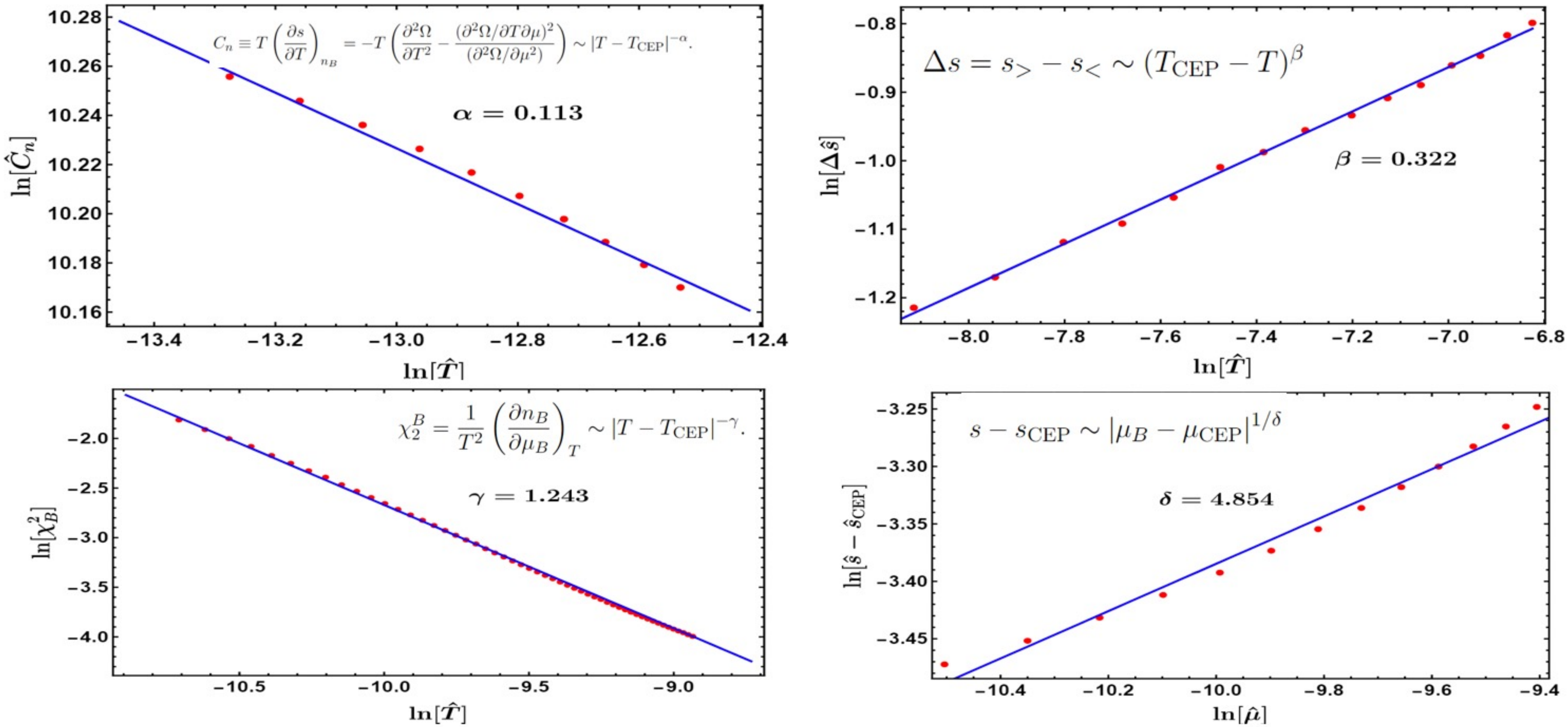


$$\alpha + 2\beta + \gamma = 2, \quad \alpha + \beta(1+\delta) = 2$$

| | Experiment | 3D Ising | Mean field | DGR model | Ours |
|---|---|---|---|---|---|
| $\alpha$ | 0.110-0.116 | 0.110(5) | 0 | 0 | 0.113 |
| $\beta$ | 0.316-0.327 | 0.325±0.0015 | 1/2 | 0.482 | 0.322 |
| $\gamma$ | 1.23-1.25 | 1.2405±0.0015 | 1 | 0.942 | 1.243 |
| $\delta$ | 4.6-4.9 | 4.82(4) | 3 | 3.035 | 4.854 |

# Thank you!