

**4th INTERNATIONAL CONFERENCE
on HOLOGRAPHY, STRING THEORY and DISCRETE APPROACH
in HANOI, VIETNAM**

NEW HINTS TOWARDS TRANSPORT UNIVERSALITY

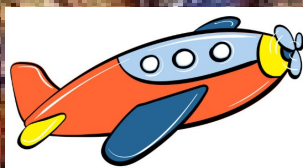
Matteo Baggioli

(<https://members.ift.uam-csic.es/matteo.baggioli/>)



Instituto de
Física
Teórica
UAM-CSIC

SOON



上海交通大学
SHANGHAI JIAO TONG UNIVERSITY

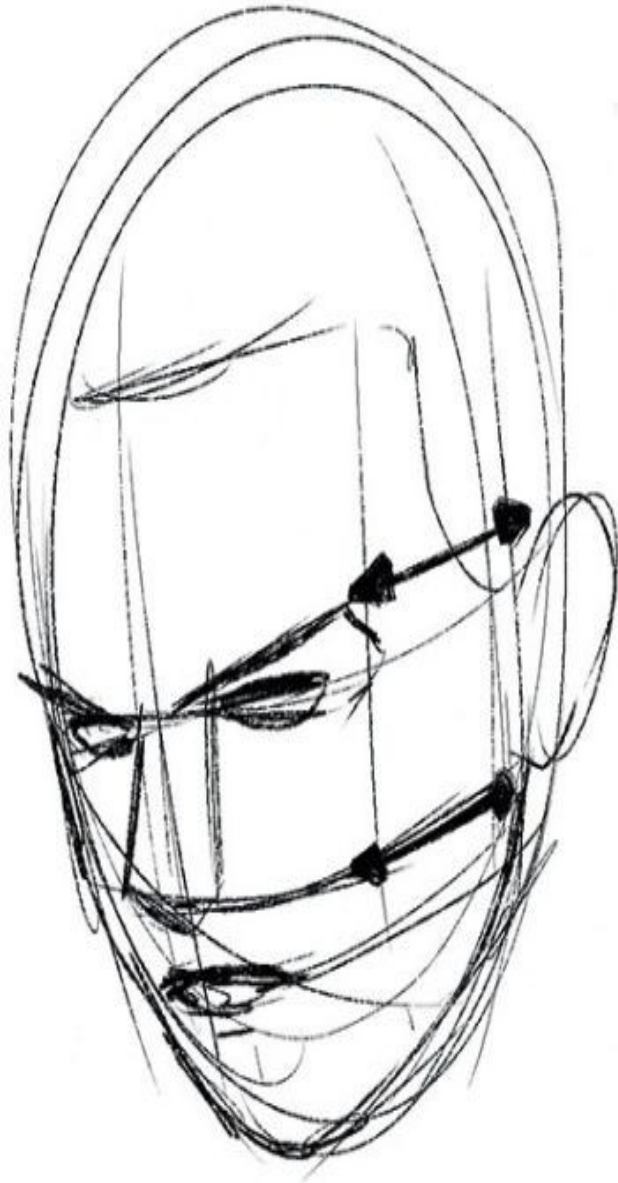
THANKS A LOT SHINGO SAN







UNIVERSALITY



References & collaborators

[arXiv:2005.06482](#) [pdf, other]

hep-th

cond-mat.dis-nn

cond-mat.str-el

quant-ph

Universal Bounds on Transport in Holographic Systems with Broken Translations

Authors: Matteo Baggioli, Wei-Jia Li

[arXiv:2003.13506](#) [pdf, other]

hep-th

cond-mat.soft

nucl-th

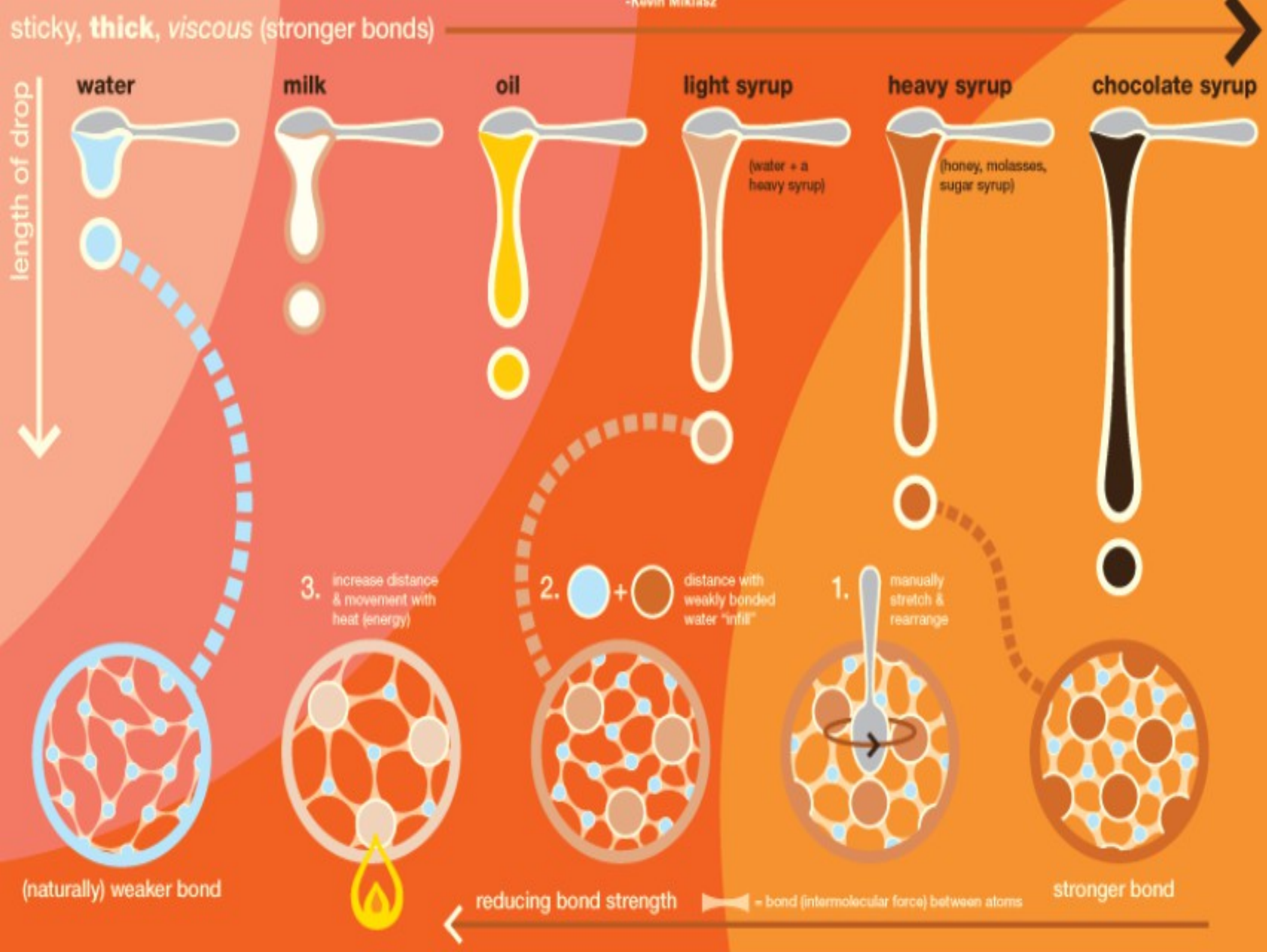
Similarity between the kinematic viscosity of quark-gluon plasma and liquids at the viscosity minimum

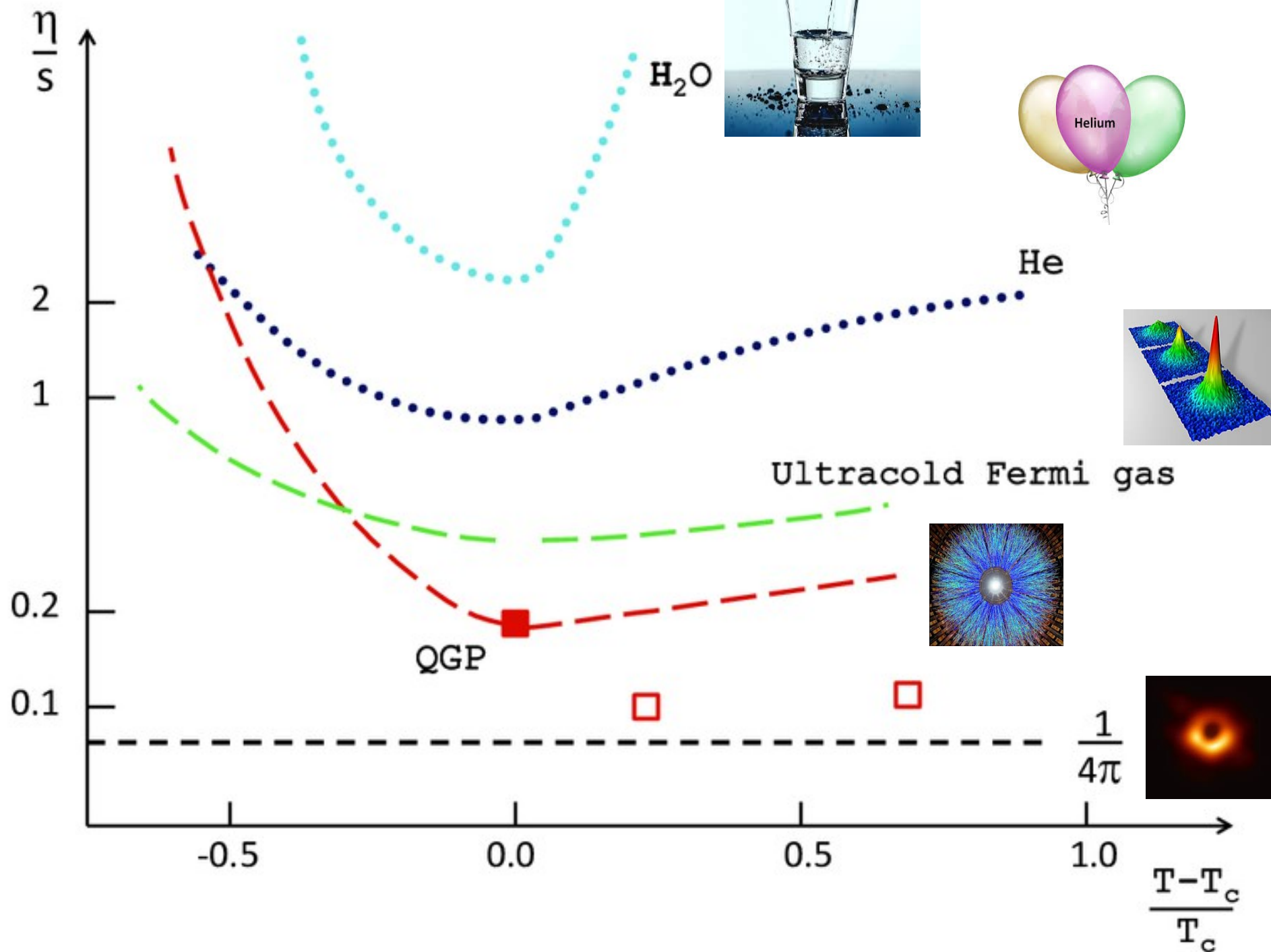
Authors: Matteo Baggioli, Vadim Brazhkin, Kostya Trachenko

"Universal lower bounds in energy and momentum diffusion in liquids"

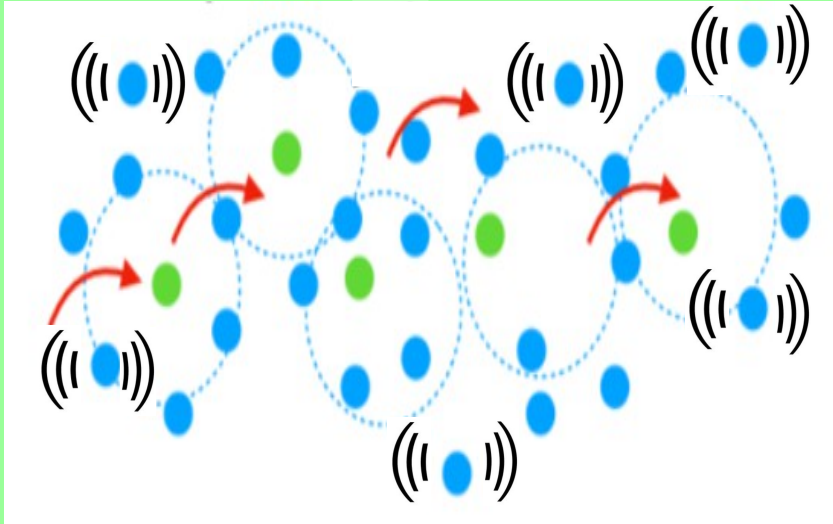
Coming soon! MB, Trachenko, Behnia, Brazhkin







WHY A MINIMUM?

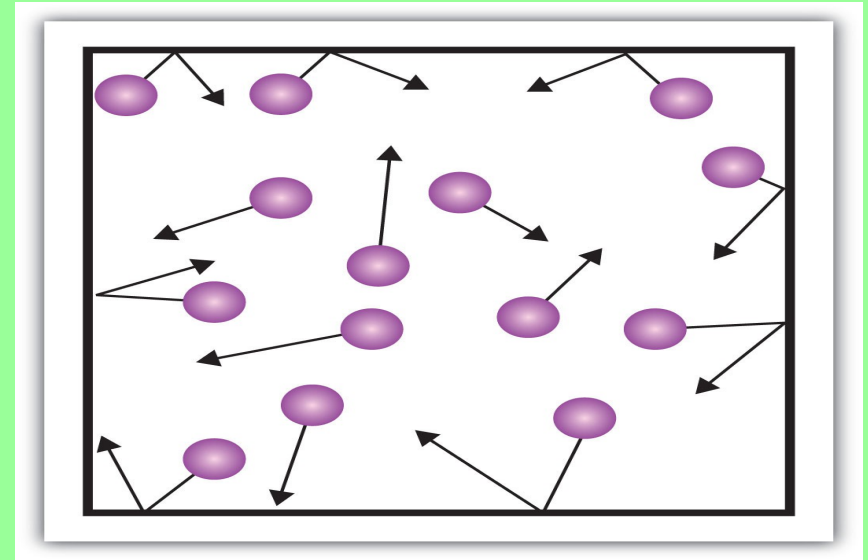


**Oscillatory motion
+
Diffusive jumps**

$$\eta = \eta_0 \exp \left(\frac{U}{T} \right)$$

$$D \sim \tau^{-1} \sim \eta^{-1}$$

LIQUID DYNAMICS



**Ballistic motion
Viscosity comes
from collisions**

$$\eta = \frac{1}{3} \rho v L$$

$$D \sim \eta \sim l_{mfp}$$

GAS DYNAMICS

T_c

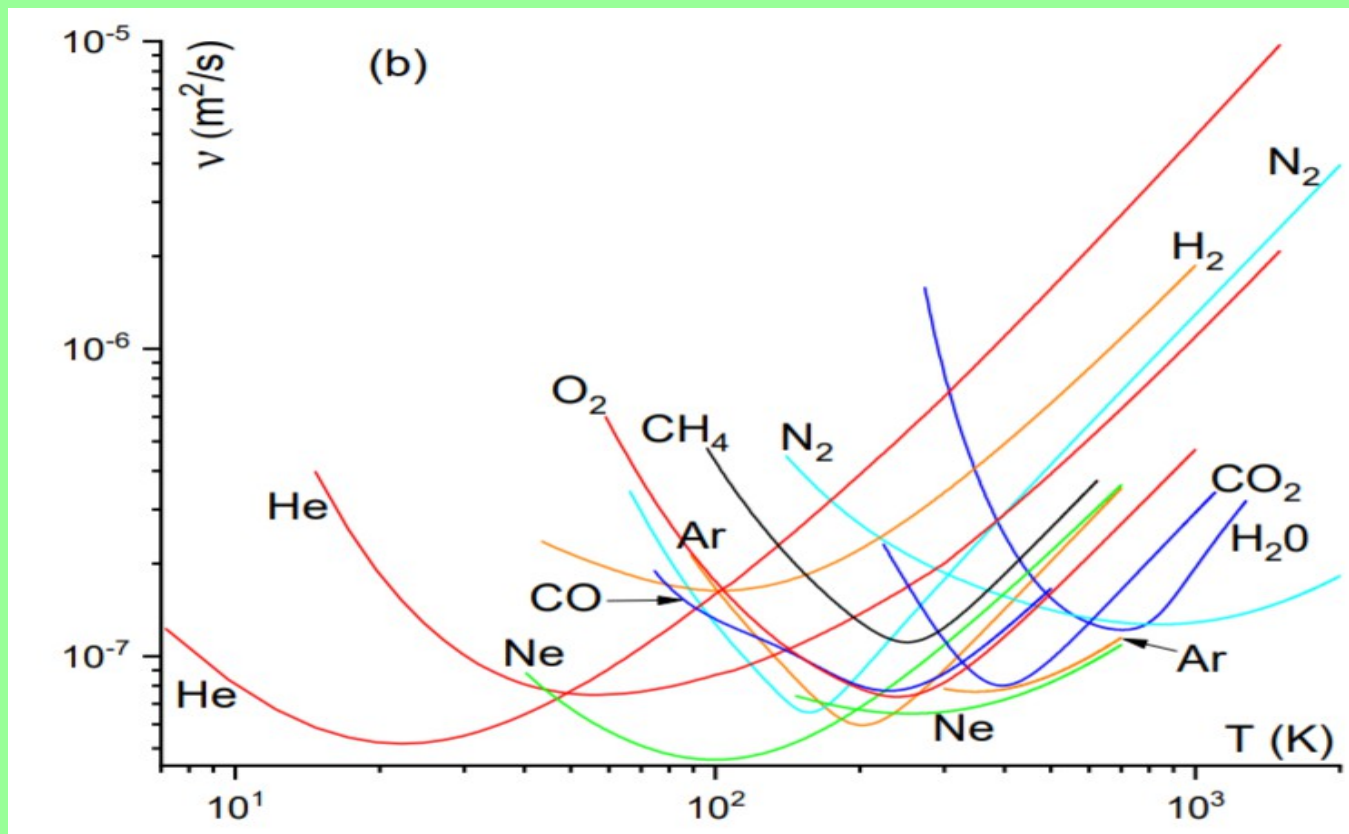
A UNIVERSAL MINIMUM

[Trachenko, Brazhkin, Sci.Advances 2020]

$$\nu_m = \frac{1}{4\pi} \frac{\hbar}{\sqrt{m_e m}}$$

$$\nu_m^{exp} = (0.5 - 2) \cdot 10^{-7} \frac{\text{m}^2}{\text{s}}$$

[See derivation therein]



UNIVERSAL

WHAT ABOUT QGP ?

E/V	1 GeV/fm ³ [23]
η	$5 \cdot 10^{11}$ Pa·s [7]
m_p	$1.67 \cdot 10^{-27}$ kg
a_p	$0.84 \cdot 10^{-15}$ m
a	$0.5 \cdot 10^{-15}$ m [24]
T_{QGP}	$2 \cdot 10^{12}$ K [7]

THE SHEAR VISCOSITY IS HUGE
(COMPARABLE TO LIQUIDS
AT THE GLASS TRANSITION)

But the density is also huge !

$$D = \frac{a_p^2}{\hbar} k_B T_{\text{QGP}}$$

Compatible with the standard liquid
formula and using the Planckian
relaxation time !

It can be derived in several ways :
(check the paper 😊)

$$\nu_{\text{QGP}}^{\text{exp}} \approx 10^{-7} \frac{\text{m}^2}{\text{s}}$$

CONSEQUENCES

we found that (a) the calculated ν_{QGP}^{th} in (11) and D_s in (13) are close to the experimental value of ν_{QGP}^{exp} in (10), and (b) these values of kinematic viscosity of QGP are close to both experimental and theoretical values of kinematic viscosity in liquids at the minimum ν_m .

Given that the dynamic viscosity η and the density of QGP are about 16 orders of magnitude larger than those values in liquids, the similarity of ν is striking.

FIRST CONCLUSION

 $\frac{\eta}{s}$

Is very well defined in relativistic systems but
away from that it does not govern anything

 $\frac{\eta}{s}$

[Hartnoll 2015, Nature]

Momentum
Diffusion
Constant



THERMAL DIFFUSIVITY

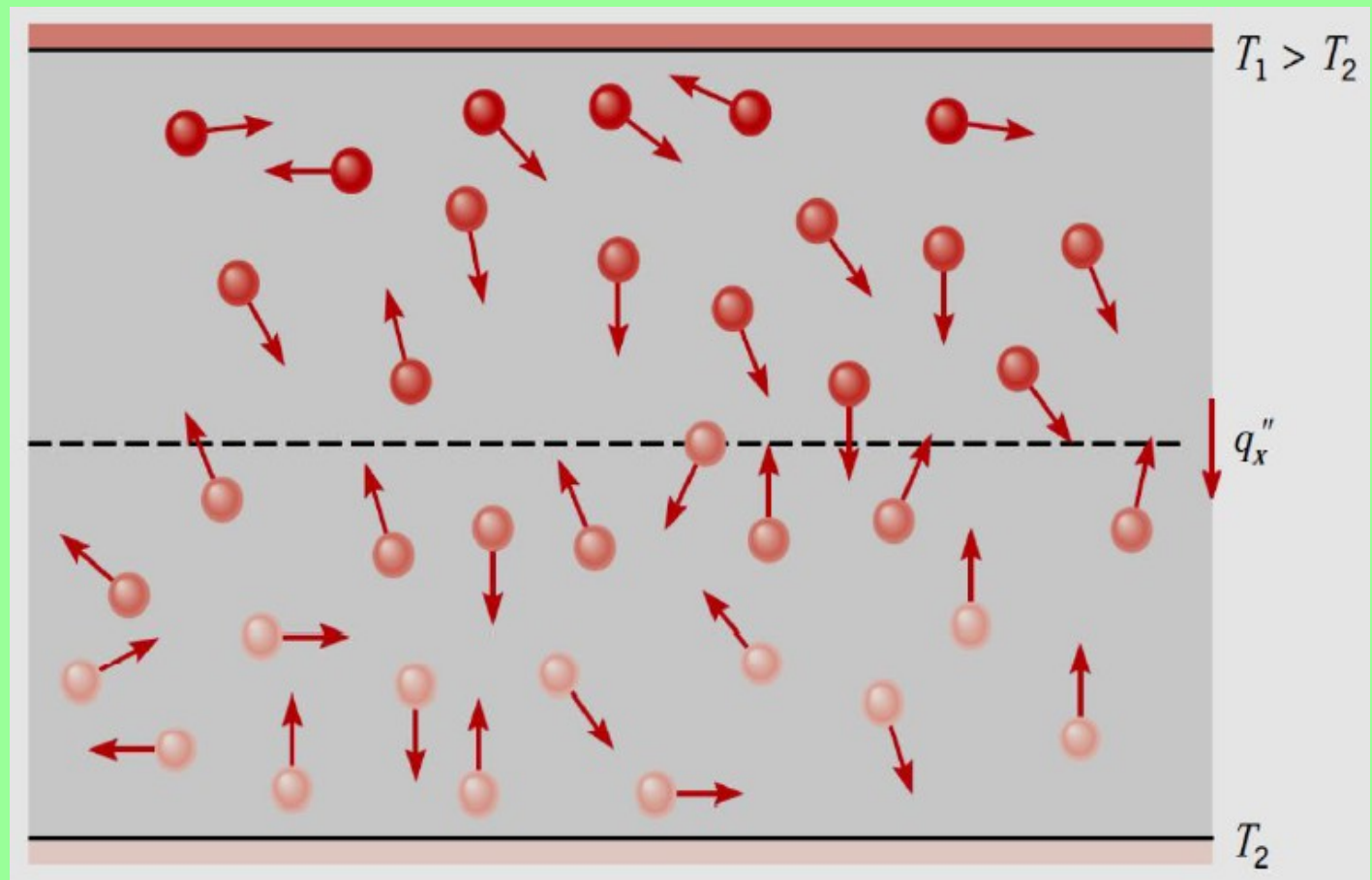
$$J_Q = \kappa \frac{\partial T}{\partial x},$$

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\alpha = \frac{\hat{k}}{c_p}$$

$$\kappa \approx c v l$$

$$\alpha = v l$$



THE MINIMUM

$$\text{Using } l = a \text{ and } v = \frac{a}{\tau_D} = \frac{1}{2\pi} a \omega_D$$

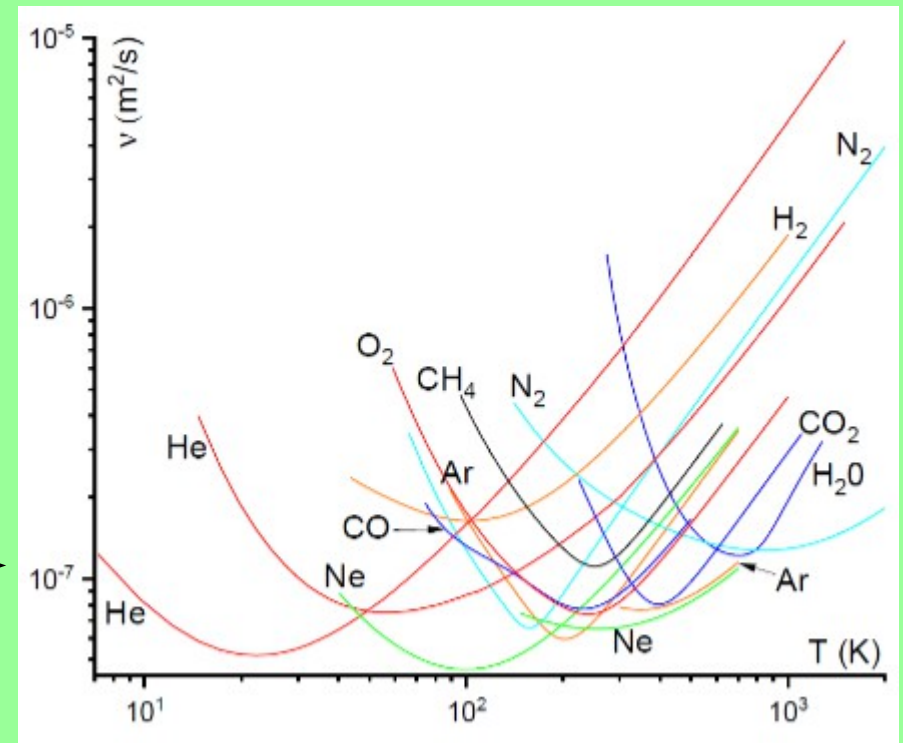
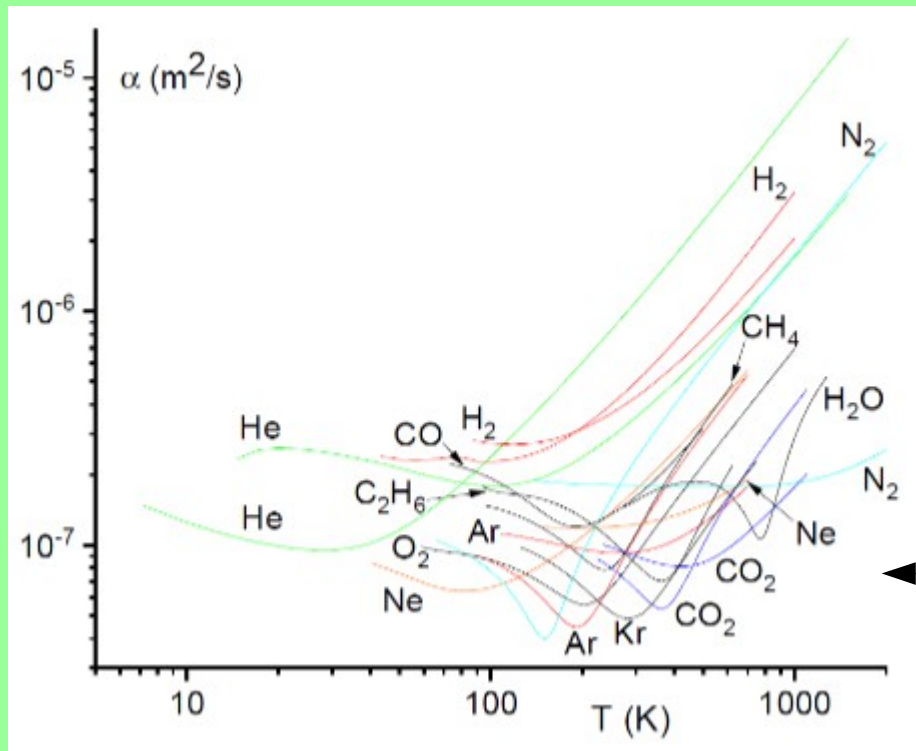
UV CUTOFFS IN CONDENSED MATTER

Bohr radius, a_B ,

Rydberg energy, E_R

$$\alpha_m = \frac{1}{4\pi} \frac{\hbar}{\sqrt{m_e m}}$$

EXPERIMENTS



National Institute of Standards and Technology database,
see <https://webbook.nist.gov/chemistry/fluid>.

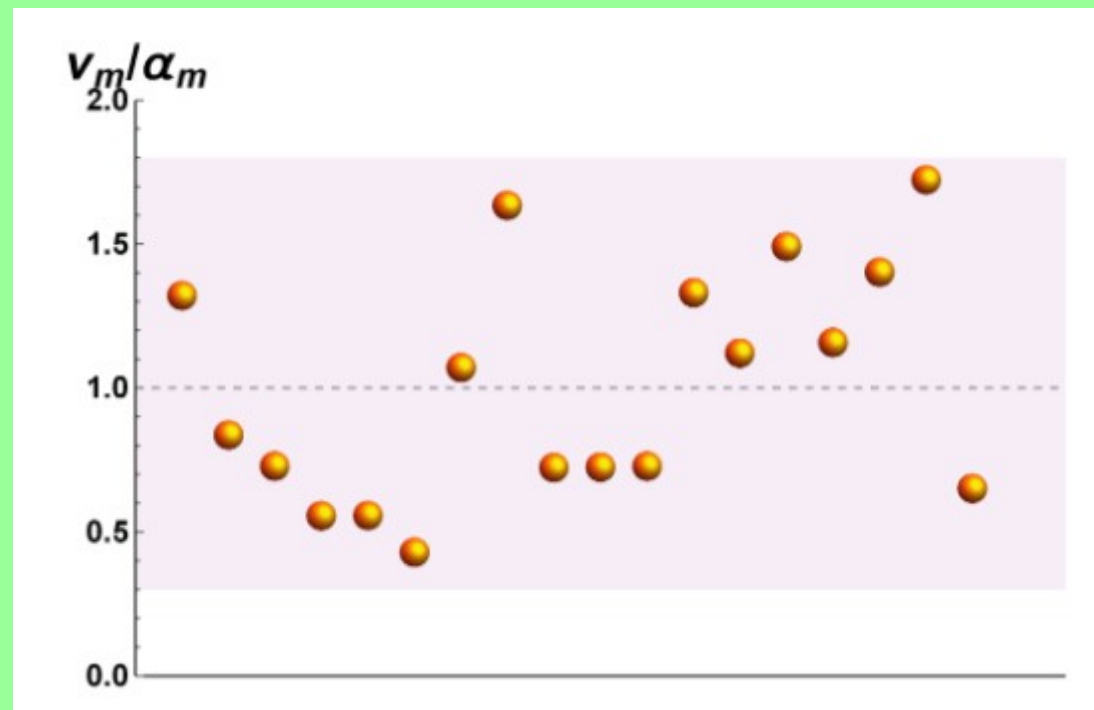
Minima at different temperatures but

$$\alpha_m = \mathcal{C} \nu_m, \quad \mathcal{C} \in [0.4, 1.7],$$

$$\frac{1}{4\pi} \frac{\hbar}{\sqrt{m_e m_p}} \approx 10^{-7} \frac{\text{m}^2}{\text{s}}.$$

DATA RECAP

	$\alpha_m^{th} = \nu_m^{th}$	α_m^{exp}	ν_m^{exp}	ν_m/α_m
Ar (20 MPa)	3.4	4.5	5.9	1.3
Ar (100 MPa)	3.4	9.3	7.7	0.8
Ne (50 MPa)	4.8	6.4	4.6	0.7
Ne (300 MPa)	4.8	11.9	6.5	0.6
He (20 MPa)	10.7	9.5	5.2	0.6
He (100 MPa)	10.7	17.9	7.5	0.4
Kr (30 MPa)	2.3	4.9	5.2	1.1
N ₂ (10 MPa)	4.1	4.0	6.5	1.6
N ₂ (500 MPa)	4.1	17.8	12.7	0.7
H ₂ (50 MPa)	15.2	22.8	16.3	0.7
H ₂ (100 MPa)	15.2	27.0	19.4	0.7
O ₂ (30 MPa)	3.8	5.6	7.4	1.3
H ₂ O (70 MPa)	5.1	10.7	11.9	1.1
CO ₂ (30 MPa)	3.2	5.4	8.0	1.5
CO ₂ (90 MPa)	3.2	8.1	9.3	1.2
CH ₄ (20 MPa)	5.4	7.9	11.0	1.4
C ₂ H ₆ (20 MPa)	3.9	7.0	12.0	1.7
CO (20 MPa)	4.1	12.0	7.7	0.6



**1) PREDICTIONS OF THE MINIMA
VERY ACCURATE**

**2) VALUES AT THE MINIMA VERY CLOSE
TO EACH OTHER**

SECOND CONCLUSION

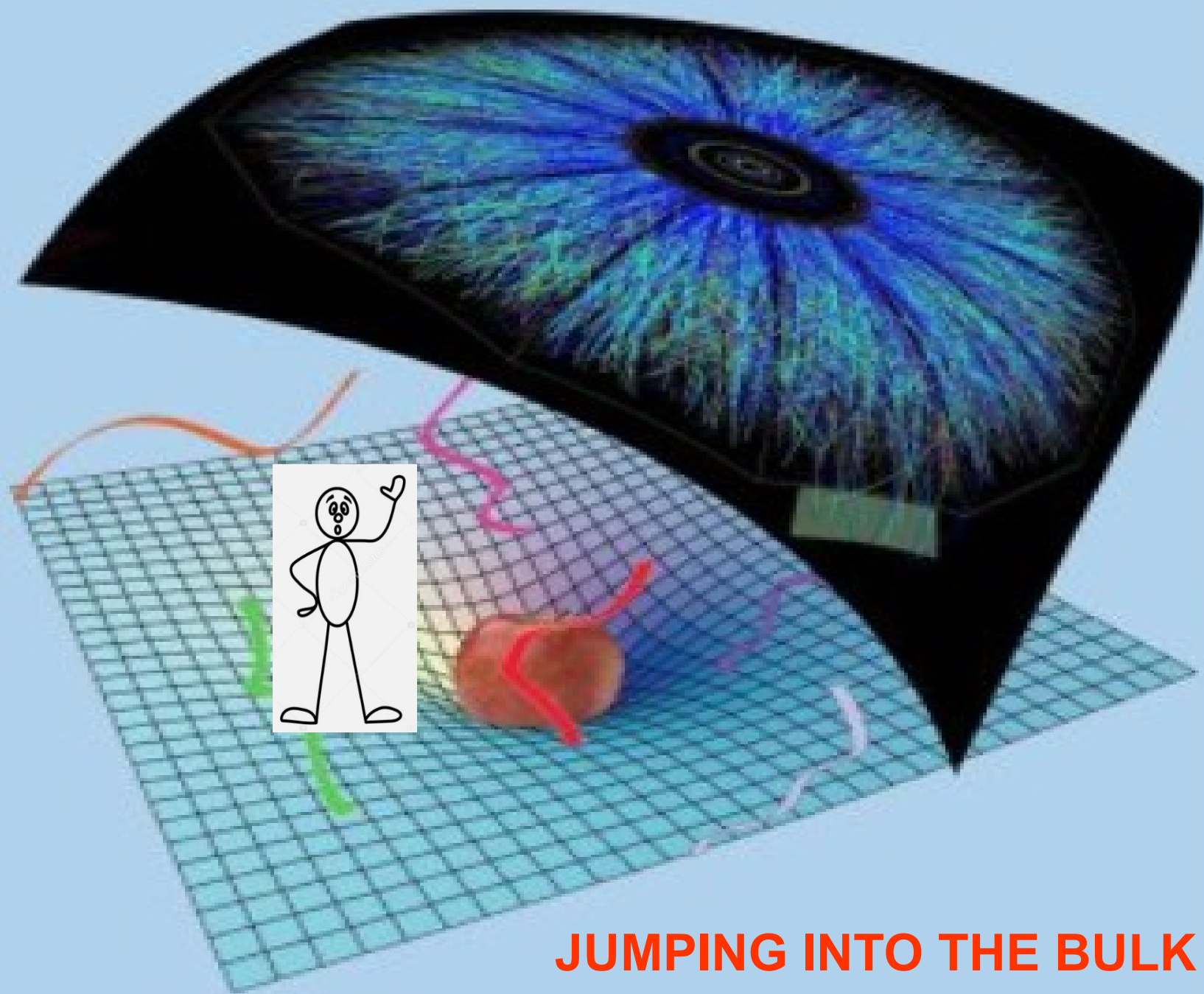


STRONG UNIVERSALITY AT THE MINIMUM

**No need of weird/fancy materials
to see quantum bounds!**

Physics is unavoidably quantum in cond-mat





JUMPING INTO THE BULK

From AdS / CFT correspondence to hydrodynamics

Giuseppe Policastro (Pisa, Scuola Normale Superiore), Dam T. Son, Andrei O. Starinets (Washington U., Seattle). May 2002. 18 pp.

Published in **JHEP 0209 (2002) 043**

INT-PUB-02-32

DOI: [10.1088/1126-6708/2002/09/043](https://doi.org/10.1088/1126-6708/2002/09/043)

e-Print: [hep-th/0205052](https://arxiv.org/abs/hep-th/0205052) | [PDF](#)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[ADS Abstract Service](#); [AMS MathSciNet](#)

[Detailed record](#) - [Cited by 684 records](#) 500+



Dynamics of black hole horizon

Fluid-dynamics

**Relativistic
hydrodynamics**

(fluid-gravity, QNMs, transport)

Shear diffusion

Sound mode

A HOLOGRAPHIC LIE

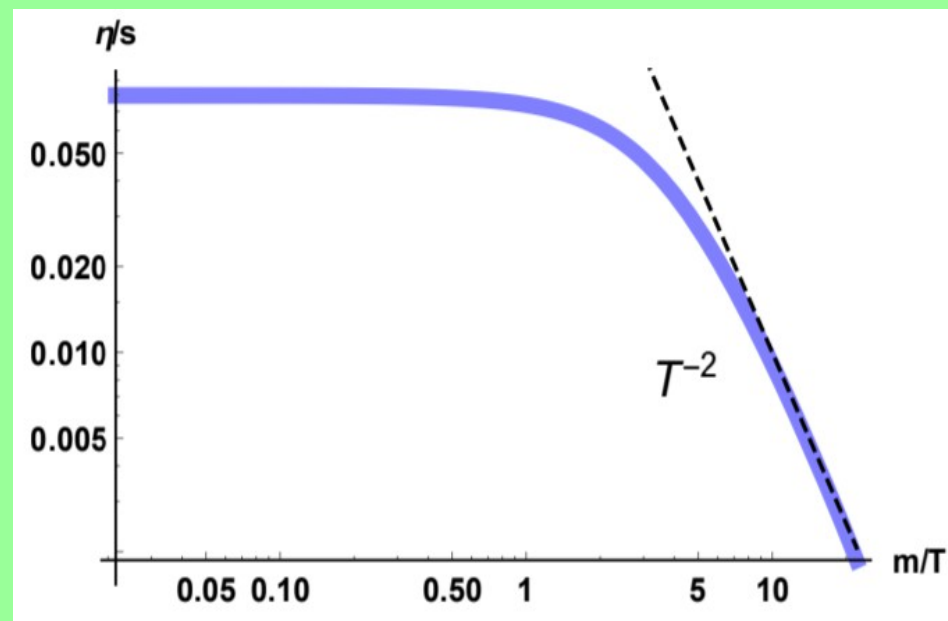
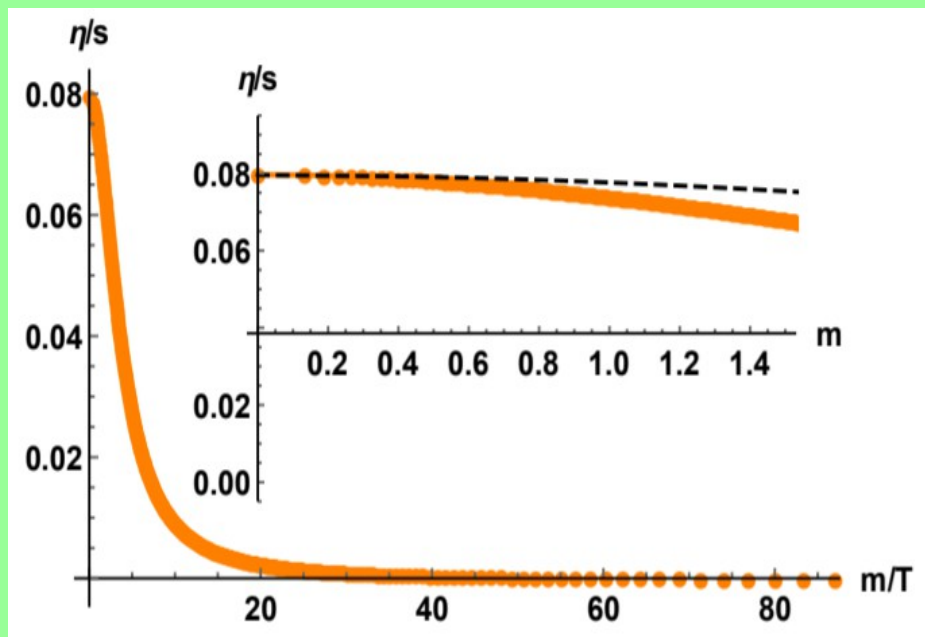


A SPECIFIC CASE

$$\phi^I = \alpha x^I$$

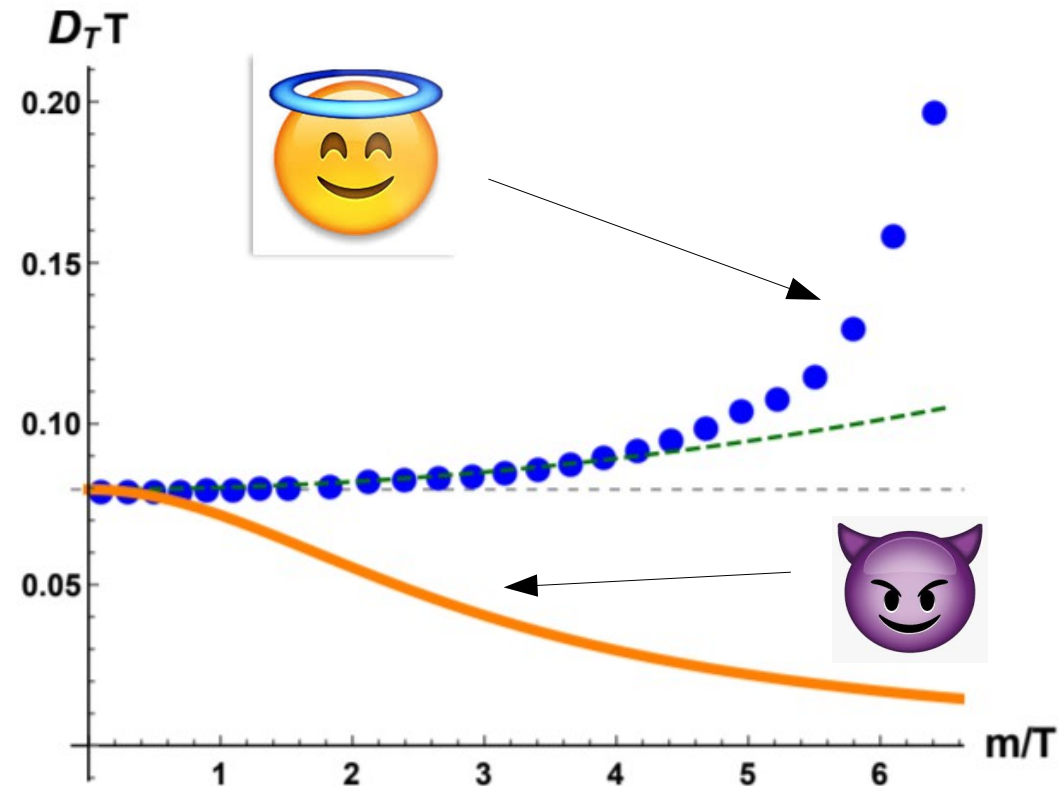
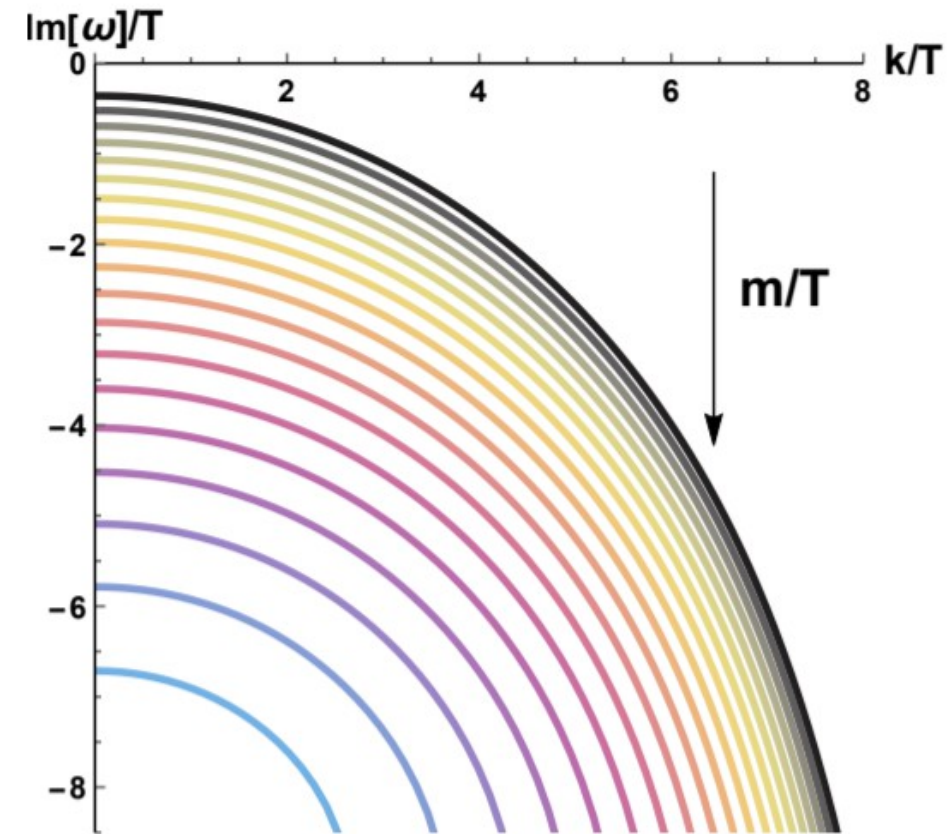
[Hartnoll, Ramirez, Santos 2016]

[MB, Albete, Pujolas 2016 +1 day]



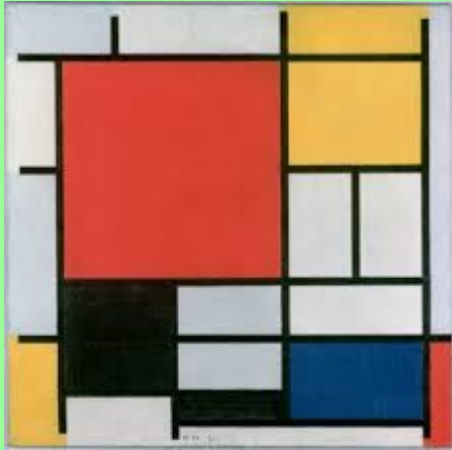
$$\frac{\eta}{s} = \frac{1}{4\pi} \left(1 - \frac{4}{3} m^2 u_h^{2N} \frac{\mathcal{H}_{\frac{2N}{3}-1}}{2N-3} \right) + \mathcal{O}(m^4),$$

A SIMPLE SOLUTION



$$D_T T \geq \frac{1}{4\pi},$$

OUTLOOK



$$\frac{\eta}{s}$$

loses its universality !

But the diffusion
constant not !

Same happens in systems with broken rotations!



[Blake, 2016, PRL]

[See Dimitrios' next talk]

PROVOCATIVE STATEMENT

**JUST CONSIDER THE
DIFFUSION CONSTANTS**



HINTS FROM EXPERIMENTS

arXiv:2001.03805 [pdf, other] cond-mat.str-el hep-th doi 10.1073/pnas.1910131116

Thermalization and Possible Signatures of Quantum Chaos in Complex Crystalline Materials

Authors: Jiecheng Zhang, Erik D. Kountz, Kamran Behnia, Aharon Kapitulnik

arXiv:1905.03551 [pdf, other] cond-mat.mtrl-sci cond-mat.stat-mech

doi 10.1088/1361-648X/ab2db6

A lower bound to the thermal diffusivity of insulators

Authors: Kamran Behnia, Aharon Kapitulnik

arXiv:1908.04792 [pdf, other] cond-mat.mtrl-sci cond-mat.stat-mech

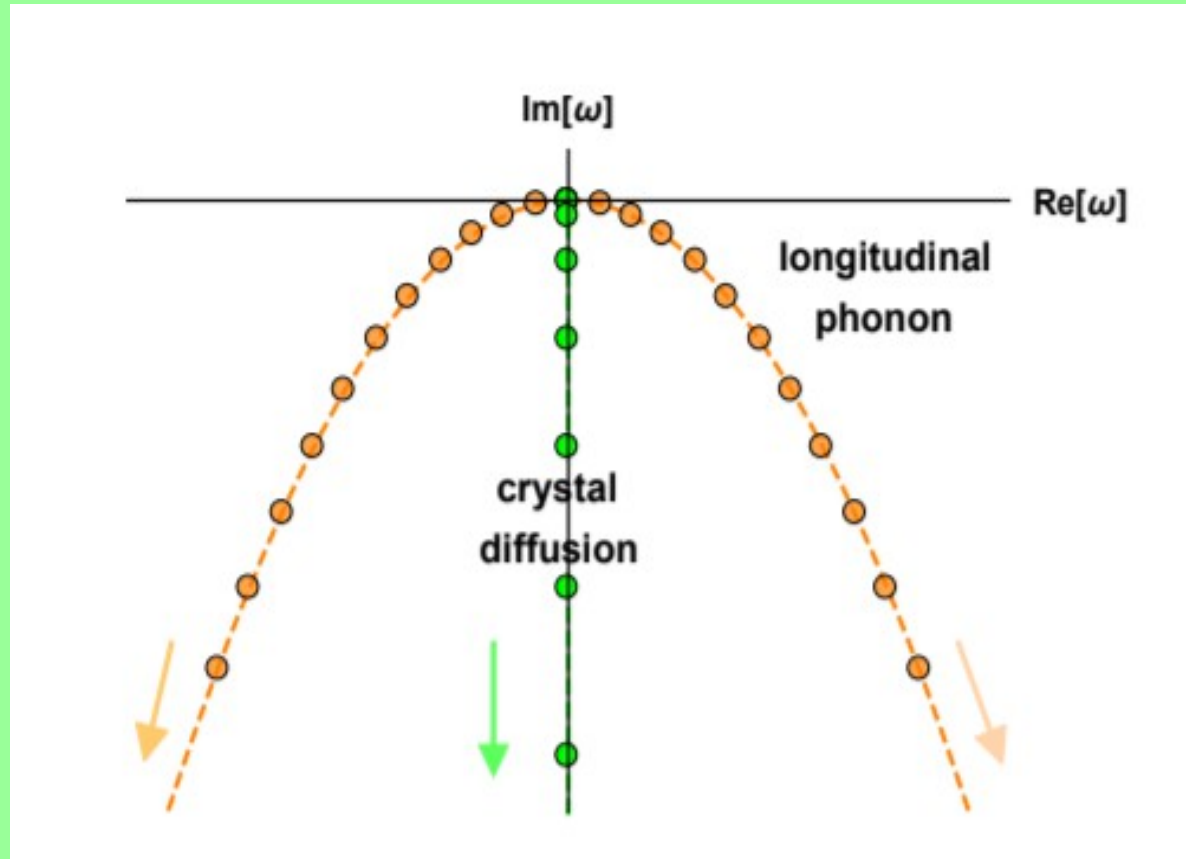
On the Planckian bound for heat diffusion in insulators

Authors: Connie H. Mousatov, Sean A. Hartnoll



SSB OF TRANSLATIONS

[MB et al
, 2018, JHEP]

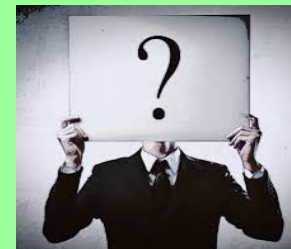


$$\omega = \pm c_L k - i D_p k^2 + \dots ,$$

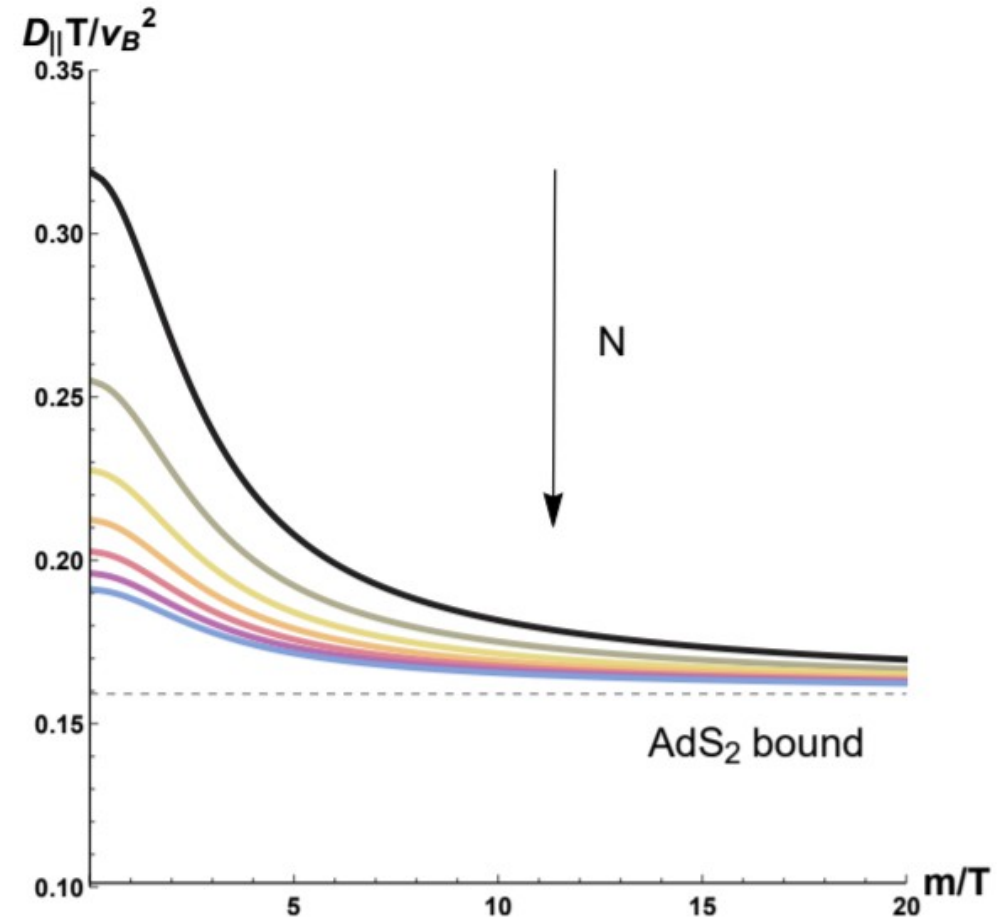
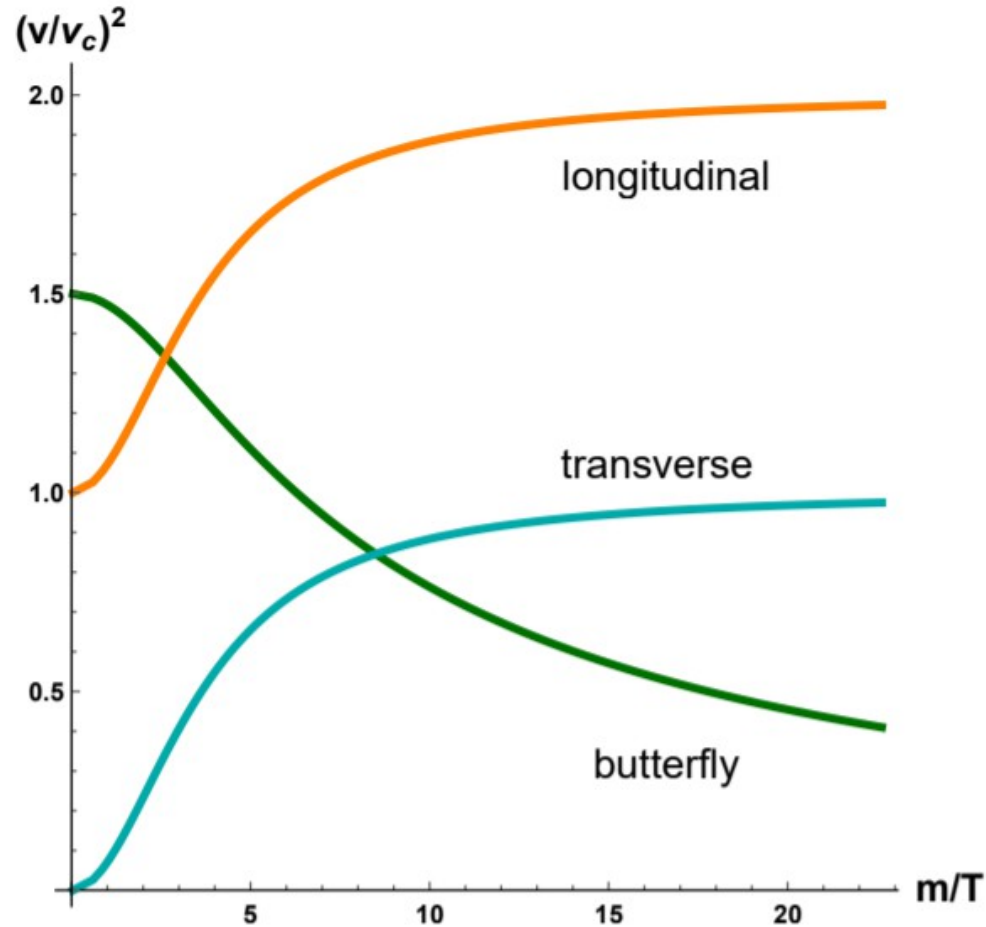


Longitudinal sound

$$\omega = -i D_\Phi k^2 + \dots ,$$



CRYSTAL DIFFUSION



$$\frac{D_{||} T}{v_B^2} \geq \frac{1}{2\pi}$$

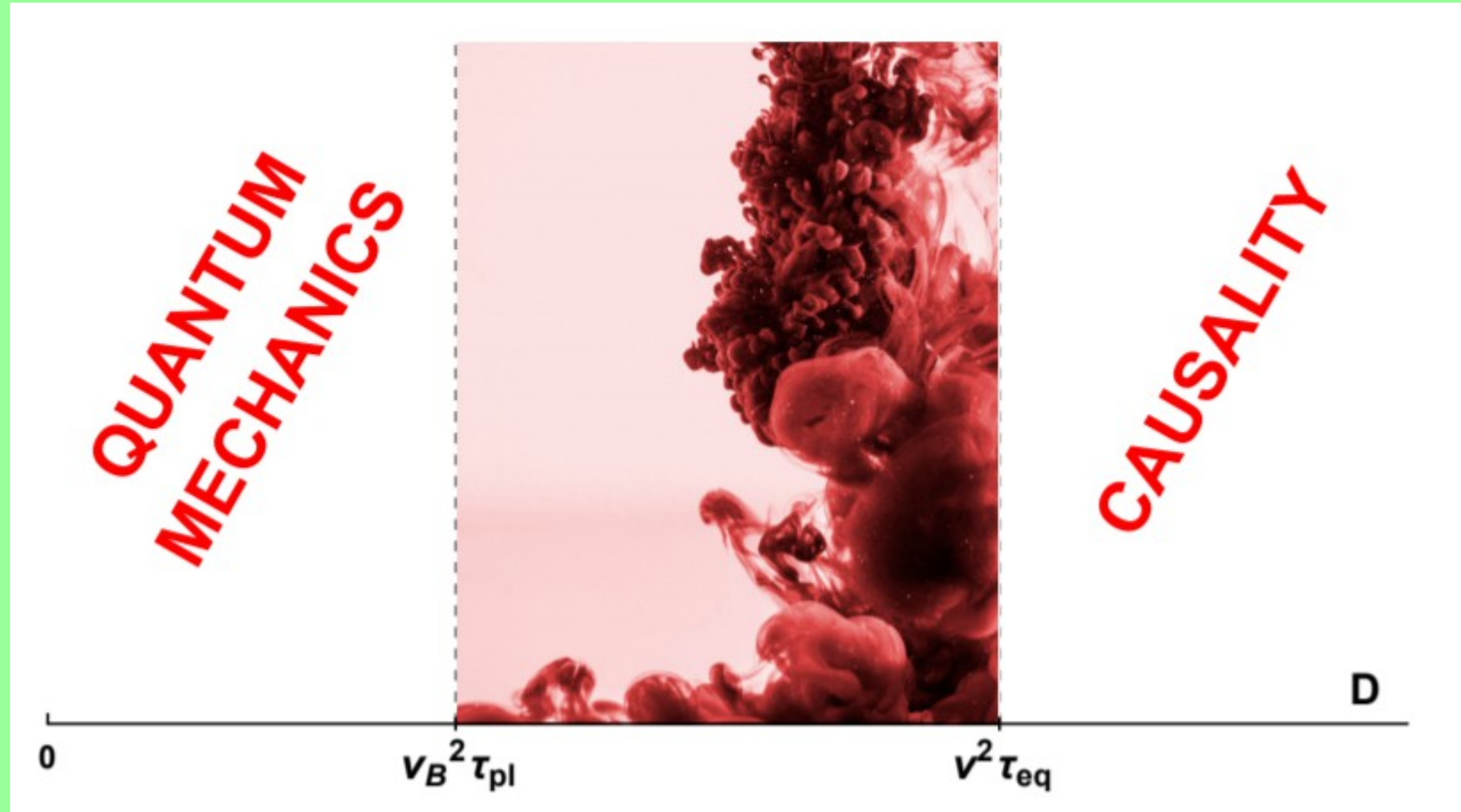
$\equiv$ AdS₂ bound for energy diffusion

NOT SO TRIVIAL ...

$$D_{\parallel} = \xi \frac{(B + G - \mathcal{P}) \chi_{\pi\pi}}{s' T^2 v_L^2},$$

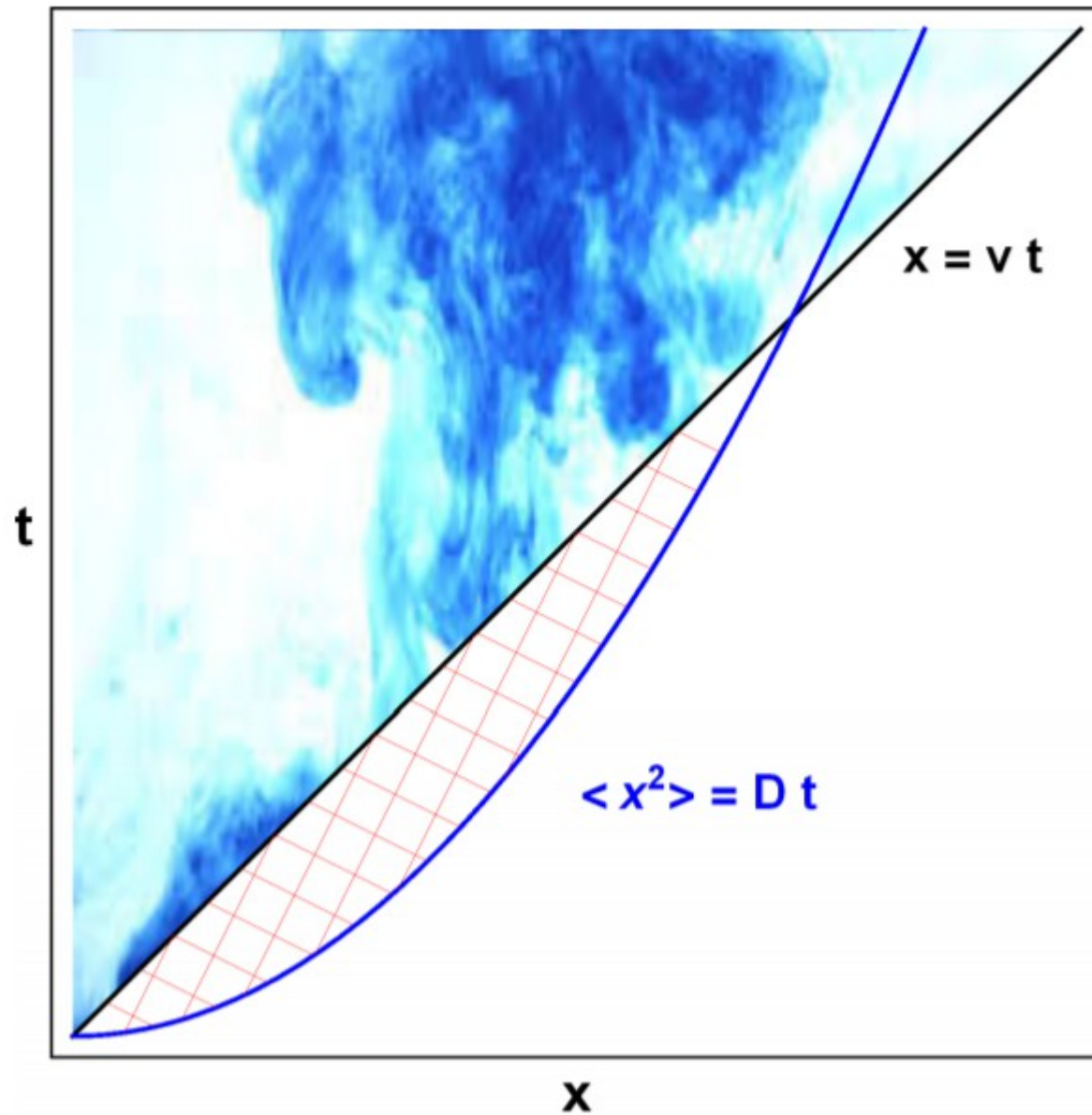
Coefficient	$m/T \rightarrow 0$	$m/T \rightarrow \infty$ ($\delta_m \rightarrow 0$)
T	$3/4\pi$	$\delta_m/4\pi$
χ	$3/2$	3
G	m^2	$3/2$
v_T^2	0	$1/2$
v_L^2	$1/2$	1
s'	$16\pi^2/3$	$8\pi^2/9$
$\mathcal{P}$	m^2	3
$\mathcal{B}$	$3m^2$	$9/2$
ξ	$1/(3m^2)$	$\delta_m^2/324$

AN UPPER BOUND



$$v_B^2 \tau_{pl} \lesssim D \lesssim v_{lighthouse}^2 \tau_{eq} ,$$

FROM CAUSALITY



[Hartman, Hartnoll, Mahajan,
PRL 2017]

$$\langle x^2 \rangle = D t$$

$$\sqrt{D t} \leq v_{\text{lightcone}} t$$

$$D \leq v_{\text{lightcone}}^2 \tau_{eq}$$

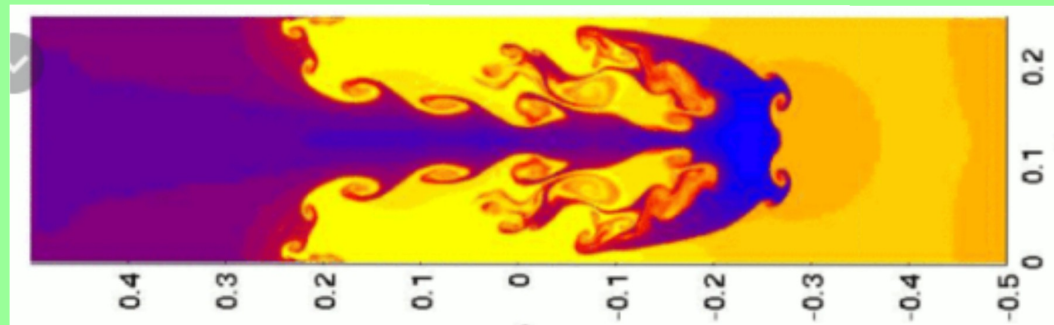
A BEAUTIFUL EXAMPLE

$$\omega = -i \frac{\eta}{\epsilon + p} k^2$$

$$|v| = \left| \frac{\partial \omega}{\partial k} \right| \sim k > c.$$

ISRAEL-STEWART FORMALISM

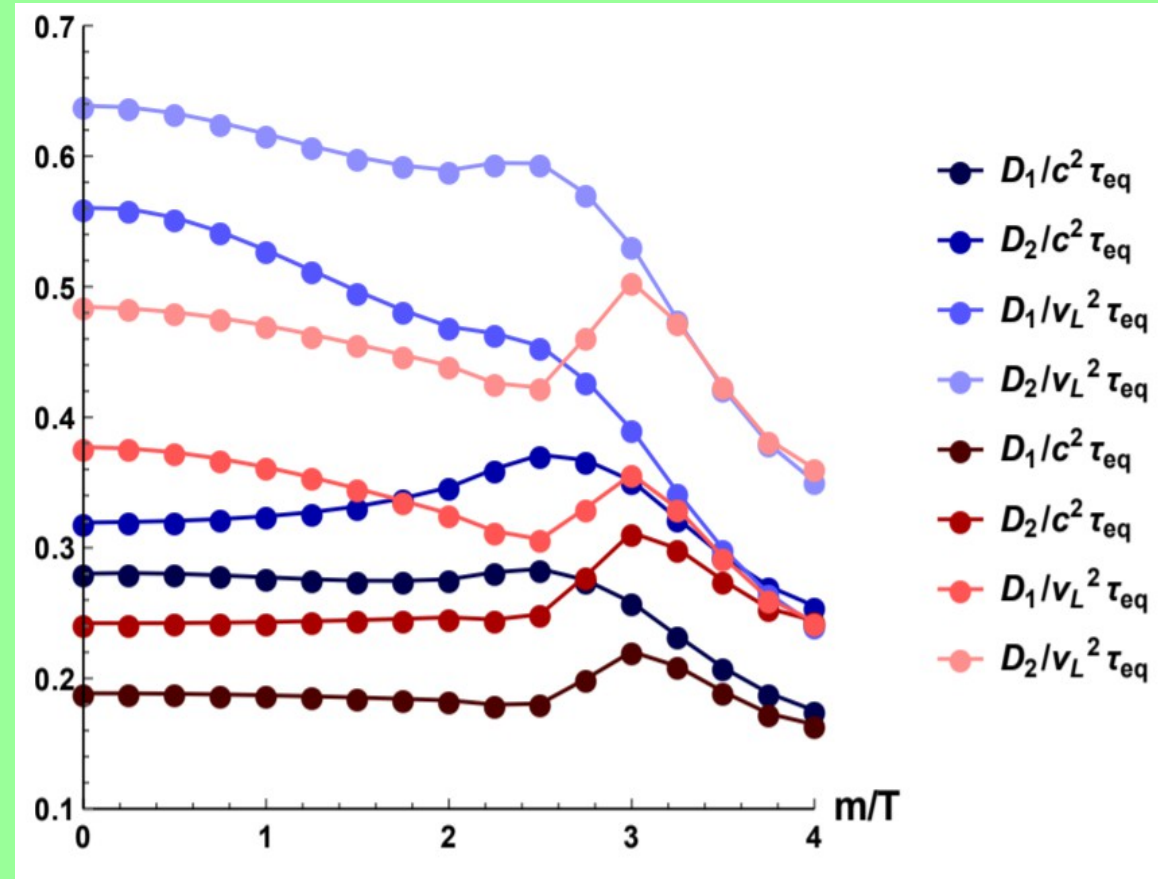
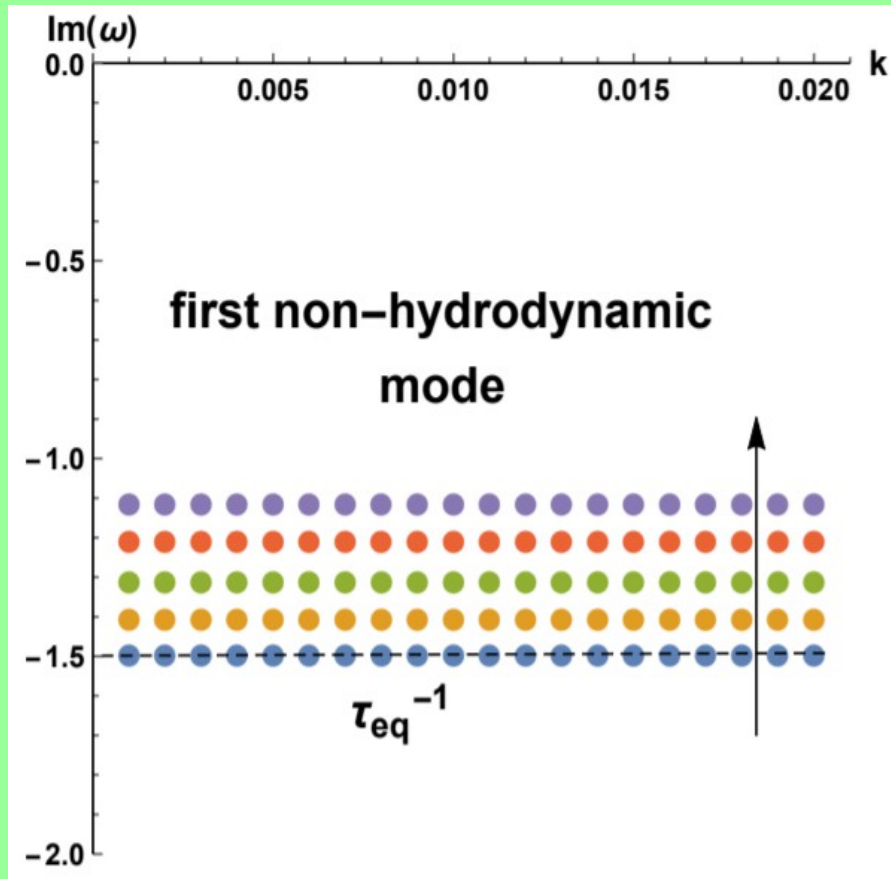
$$\omega^2 + i \omega \tau_\pi^{-1} = v^2 k^2, \quad v^2 = \frac{\eta}{(\epsilon + p) \tau_\pi}$$



$$v < c \rightarrow \sqrt{\frac{D}{\tau_\pi}} < c.$$

$$D \leq c^2 \tau_\pi.$$

HOLOGRAPHIC RESULTS

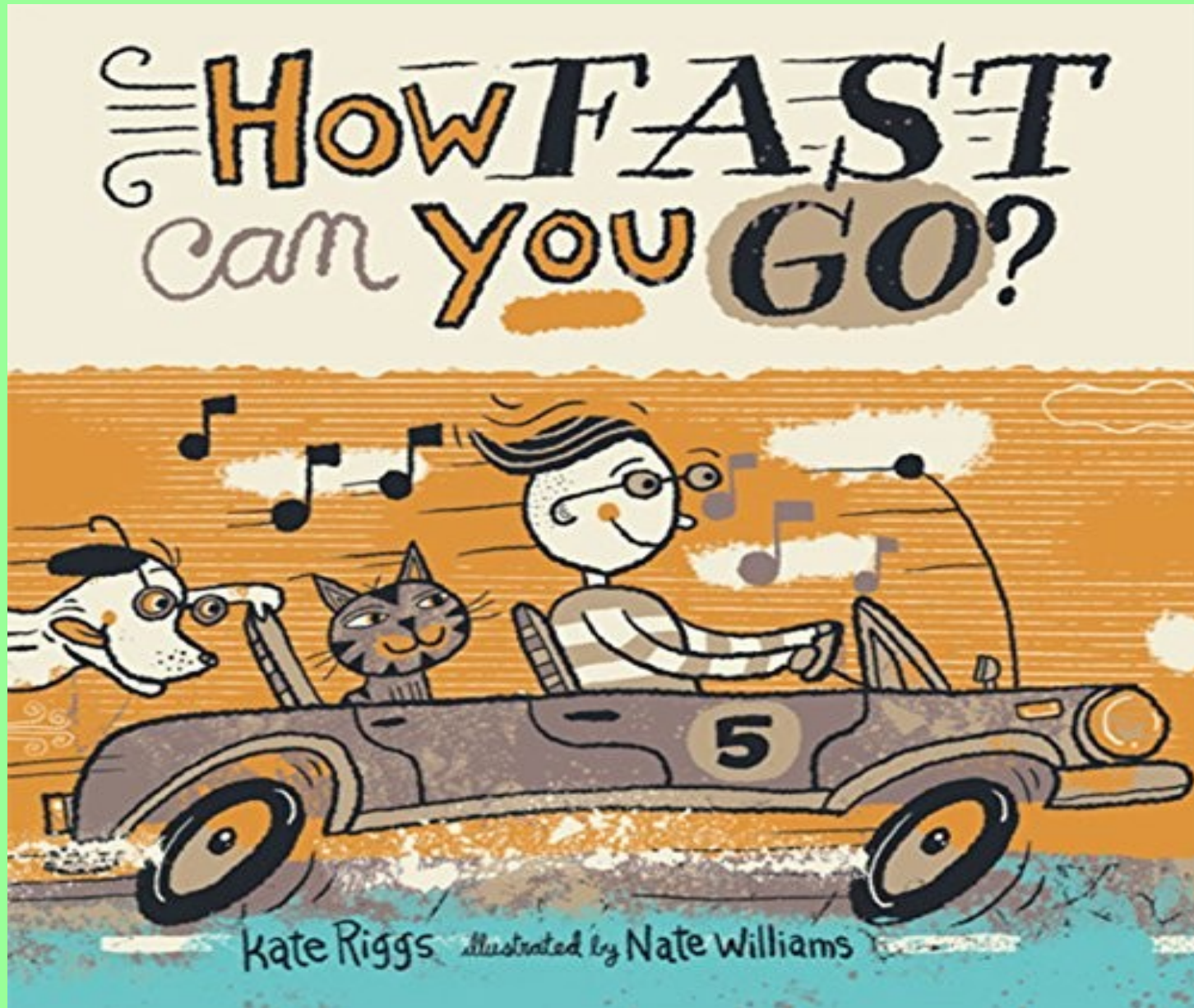


$$\tau_{eq}^{-1} \equiv \text{Im} \omega_{\text{QNM}},$$

$$D_{1,2} \leq \mathfrak{N} v_L^2 \tau_{eq}$$



WHAT ABOUT SOUND ?

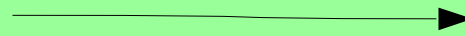


OLD IDEAS

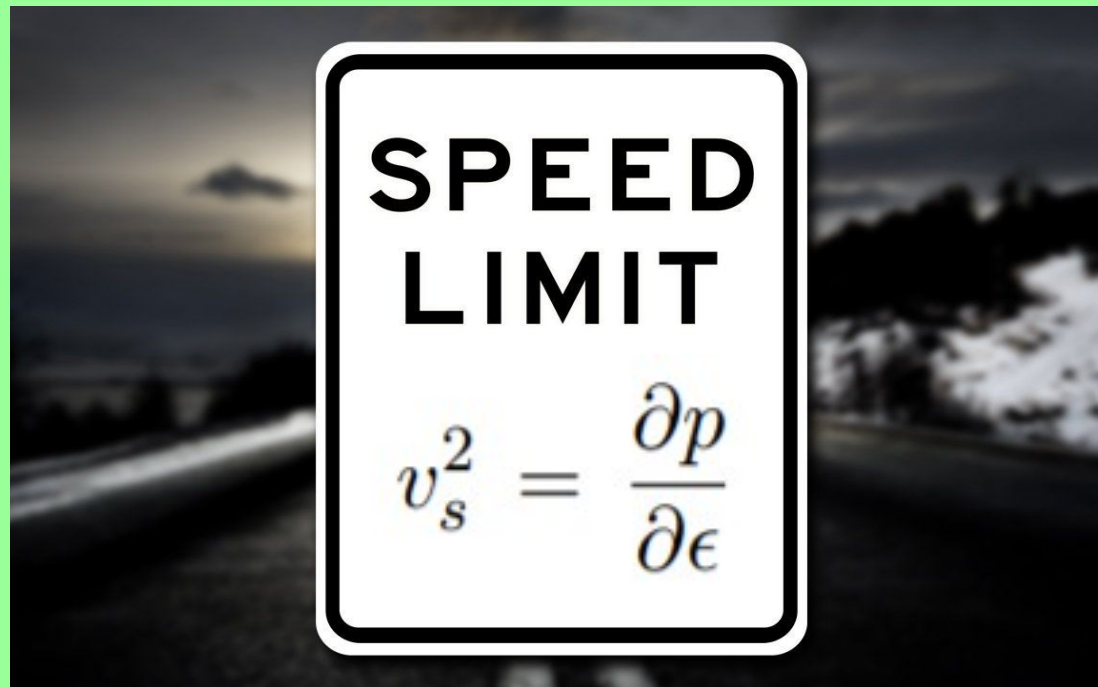
[Hohler, Stephanov 2009 PRD]

[Cherman, Cohen, Nellore 2009 PRD]

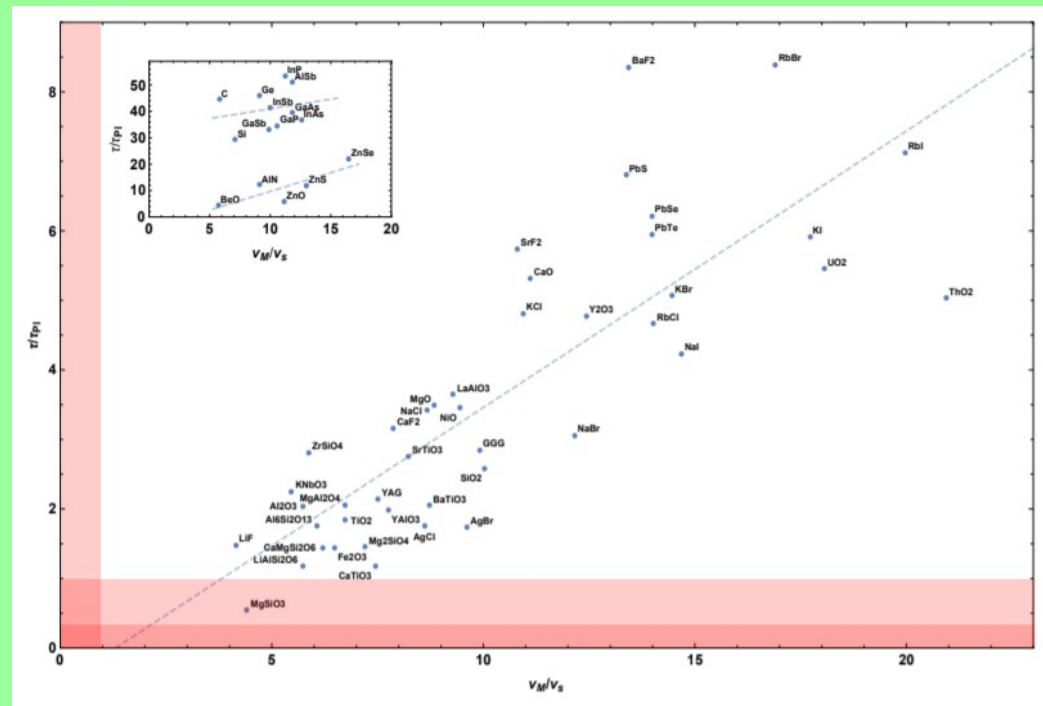
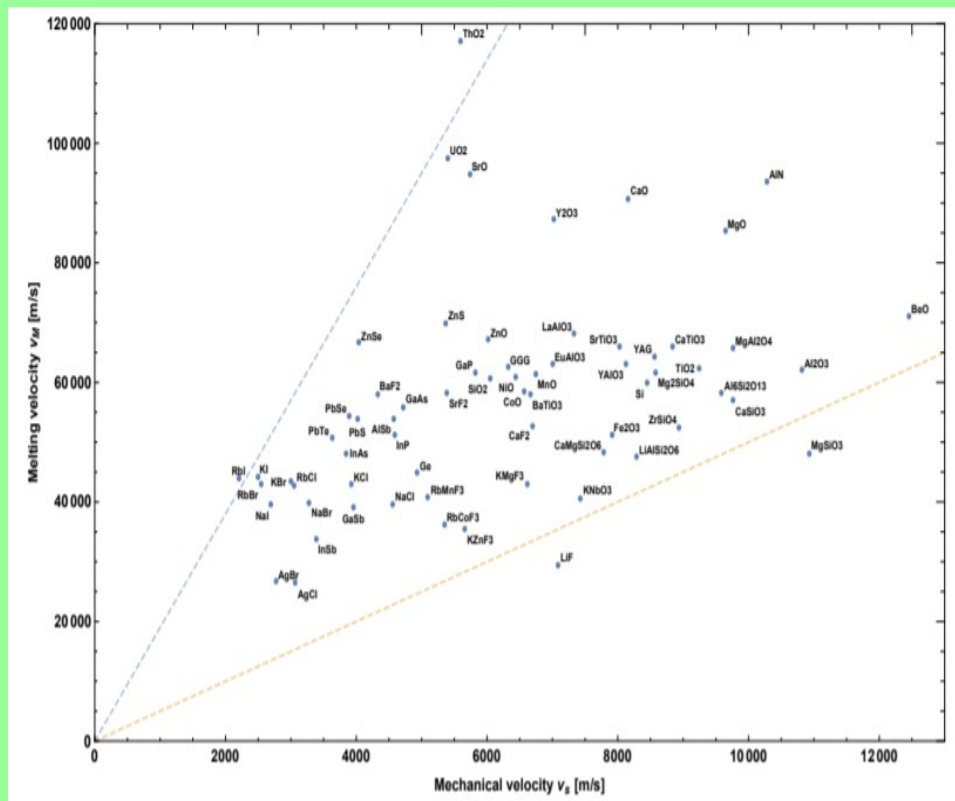
CFT



$$v_c^2 \equiv \frac{1}{d-1} c^2$$



[Mousatov, Hartnoll 2019]



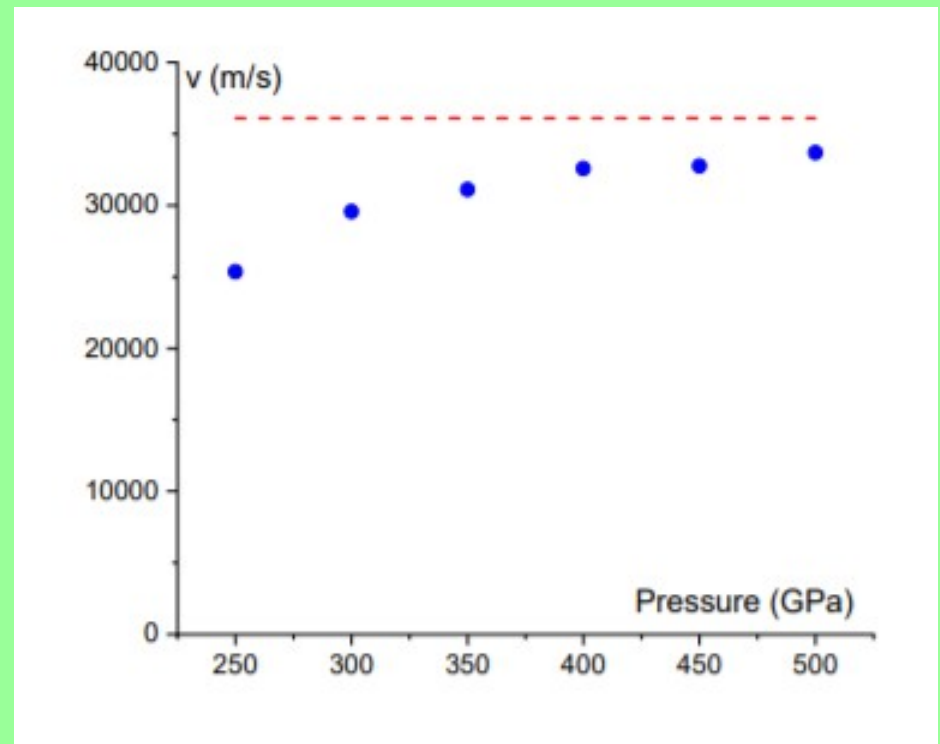
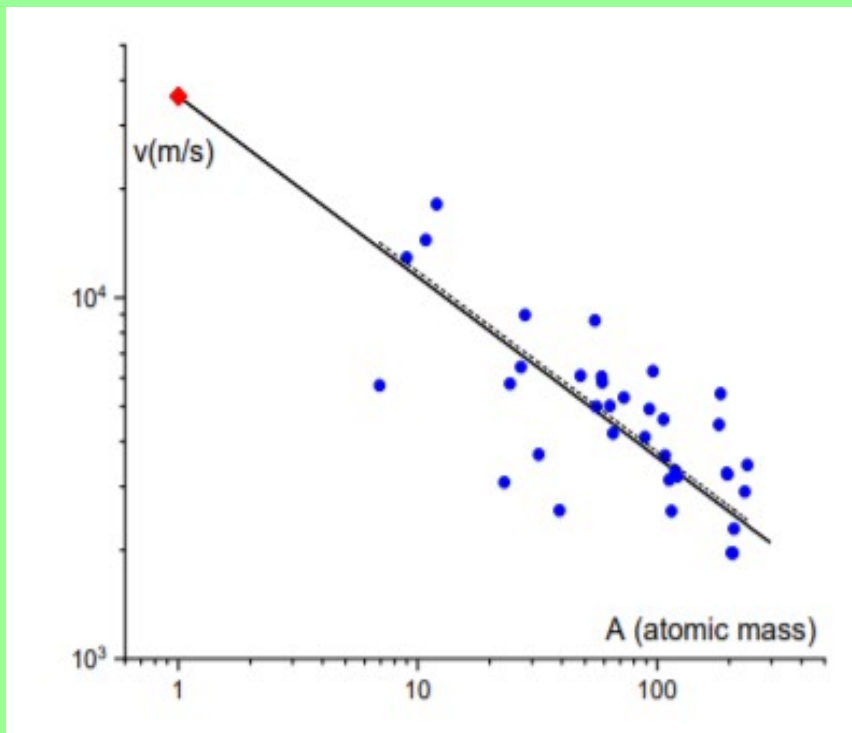
$$v_s \leq v_m \equiv \frac{k_B T_m a}{\hbar},$$

$$\frac{\tau}{\tau_{\text{Pl}}} \sim \frac{v_M}{v_s} \lesssim 1$$

NEW IDEAS

[Trachenko, Monserrat, Pickard, Brazhkin 2020]

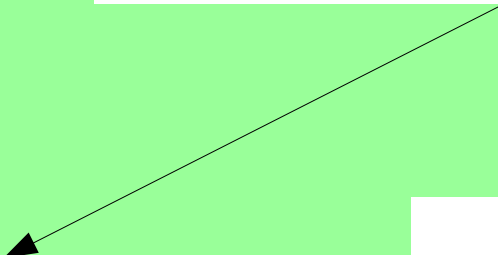
$$v_s \leq \alpha \left(\frac{m_e}{2 m_p} \right)^{1/2} c$$



A CONFORMAL SOLID

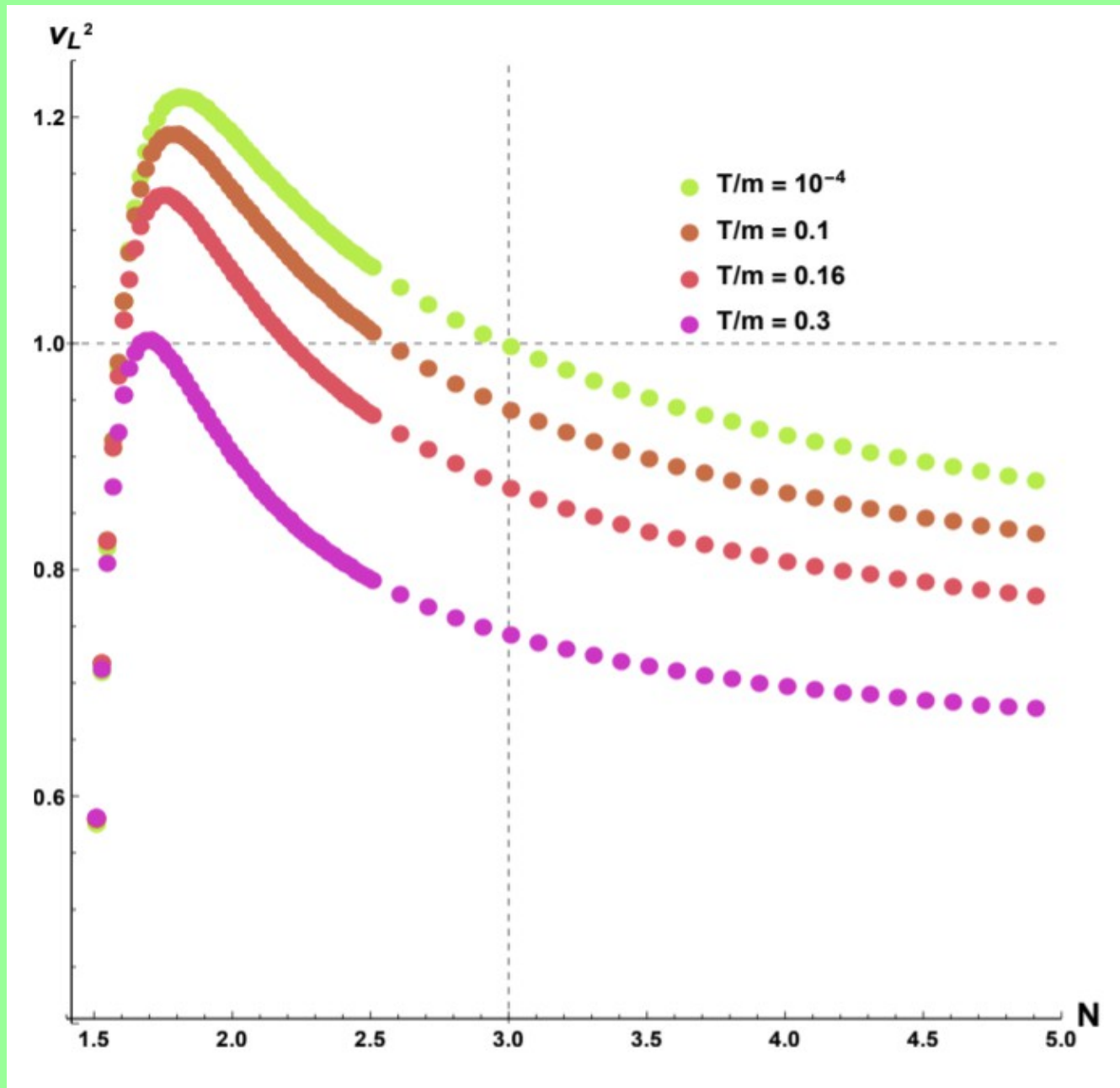
[Esposito, Garcia-Saenz, Nicolis, Penco, JHEP 2017]

$$v_L^2 = \frac{1}{2} + \frac{G}{\chi_{\pi\pi}}$$


$$v_L^2 \geq \frac{1}{2}$$



HOLOGRAPHIC RESULTS



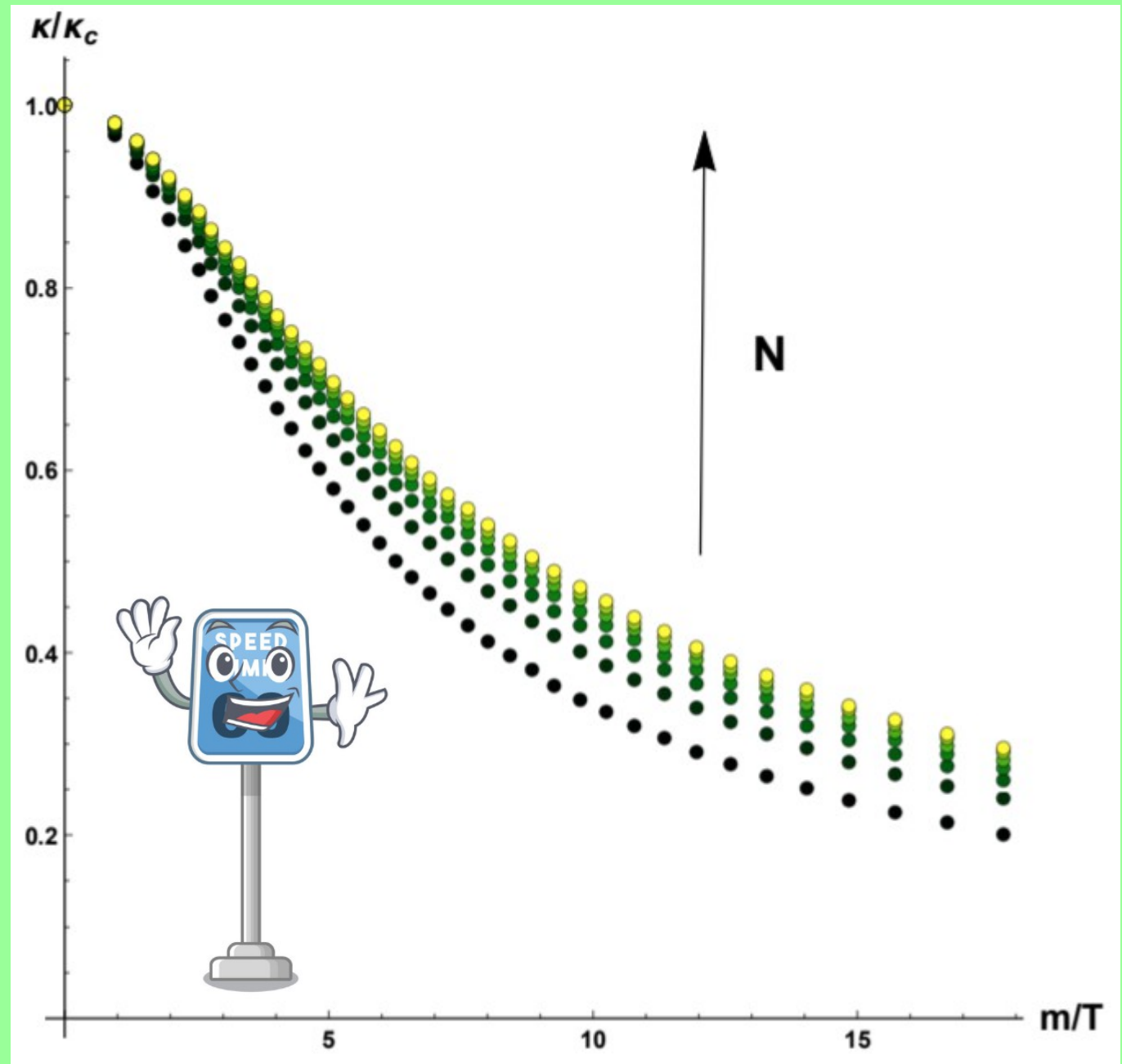
BUT

STIFFNESS

$$\kappa \equiv \frac{\partial p}{\partial \epsilon}$$

$$\kappa_C \equiv \frac{1}{d-1}$$

$$\kappa \leq \kappa_c$$



CONCLUSIONS



Forget about

$$\frac{\eta}{s}$$



Universality in the
Diffusion constants



Evidences from holography and
Experimental data (QGP, insulators)

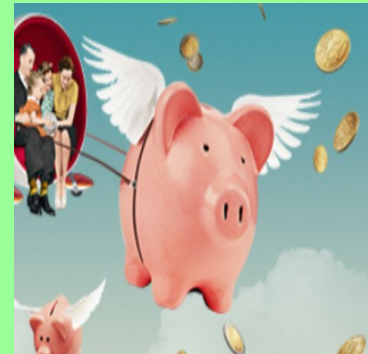
CONCLUSIONS



**Upper bound on diffusion from causality
Respected in holographic models**



**In systems with no UV cutoff there seems to
Be no bound for the speed of sound**



**What is bounded from above
Is the stiffness !**



FUTURE

Electronic thermal conductivity

Solids VS Liquids VS glasses

Standard VS exotic materials

Anisotropy and magnetic field

[See Dimitrios next talk]



**4th INTERNATIONAL CONFERENCE
on HOLOGRAPHY, STRING THEORY and DISCRETE APPROACH
in HANOI, VIETNAM**

**THANKS A LOT
(waiting for next year to visit Vietnam again)**

