Regular black holes via the Kerr-Schild construction in DHOST theories

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Plan of the talk

1. Kerr-Schild Ansatz : Definitions, Motivations and Examples.
2. Degenerate Higher-Order Scalar Tensor Theories (DHOST).
3. Construction of Regular Black Holes for DHOST Theories.
4. Conclusions and Works in Progress.

Kerr-Schild Ansatz : Definitions, Motivations and Examples

- Kerr-Schild Ansatz

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} - 2H(x)\, l_\mu l_\nu,$$

where $g^{(0)}_{\mu\nu}$ is the seed metric, and l is a null and geodesic vector field with respect to both metrics, i. e.

$$g^{\mu\nu} l_\mu l_\nu = {g^{(0)}}^{\mu\nu} l_\mu l_\nu = 0, \qquad (\nabla_\mu l_\nu) l^\nu = (\nabla^{(0)}_\mu l_\nu) l^\nu = 0.$$

In this representation, the Ricci tensor has the following form

$$R^\mu_\nu = {R^{(0)\mu}}_\nu + 2h^\mu_\sigma {R^{(0)\sigma}}_\nu - \nabla^{(0)}_\nu \nabla^{(0)}_\sigma h^{\sigma\mu} - \nabla^{(0)\mu} \nabla^{(0)}_\sigma h^\sigma_\mu + \Box^{(0)} h^\mu_\nu$$

with $h^{\mu\nu} = H l^\mu l^\nu$. The same effect for linear perturbations about the seed metric $g_{\mu\nu} = g^{(0)}_{\mu\nu} - 2h_{\mu\nu}$

Kerr-Schild Ansatz : Definitions, Motivations and Examples

- Kerr-Schild Ansatz

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} - 2H(x)\, l_\mu l_\nu,$$

1. In the case of BHs, the seed metric corresponds to the asymptotic spacetime (zero mass solution), and for rotating black holes the "angular momentum" is codified in the seed metric.
2. The mass (in case of black holes) is introduced through the function H.

Kerr-Schild Ansatz : Definitions, Motivations and Examples

- Examples of Kerr-Schild metrics

1. The pp wave

$$ds^2 = -F(u,\vec{x})du^2 - 2dudv + d\vec{x}^2 = g_{\mu\nu}^{\text{flat}} - F l_\mu l_\nu$$

where $l^\mu \partial_\mu = \partial_v$ is also a Killing field.

2. The AdS wave

$$\begin{aligned} ds^2 &= \frac{l^2}{y^2}\left[-F(u,y,\vec{x})du^2 - 2dudv + dy^2 + d\vec{x}^2\right] \\ &= g_{\mu\nu}^{\text{AdS}} - \frac{y^2 F}{l^2} l_\mu l_\nu \end{aligned}$$

3. In vacuum, most of the metrics describing black holes are of the Kerr-Schild form (Schwarzschild, Kerr, Kerr-(A)dS in arbitrary D).

4. Five-dimensional black ring solution is not of the Kerr-Schild form. [R. Emparan and H. S. Reall, Phys. Rev. Lett. **88**, 101101 (2002)]

Kerr-(A)dS metric as a Kerr-Schild metric

- The seed flat/(A)dS metric :

$$ds_0^2 = -\frac{(\kappa-\lambda r^2)\Delta(\theta)}{\Xi_a}dt^2 + \frac{\Sigma(r,\theta)dr^2}{(\kappa-\lambda r^2)(r^2+\kappa^2a^2)} + \frac{\Sigma(r,\theta)}{\Delta(\theta)}d\theta^2 + \frac{(r^2+\kappa^2a^2)h_\kappa^2(\theta)}{\Xi_a}d\phi^2.$$

where $\kappa=\pm 1$ or $\kappa=0$, λ is identified with the (A)dS scale radius, and

$$\Delta(\theta) = 1+\kappa\lambda a^2\cos^2(\sqrt{\kappa}\,\theta),\ \Sigma(r,\theta) = r^2+\kappa^2a^2\cos^2(\sqrt{\kappa}\,\theta),$$

$$\Xi_a = 1+\kappa\lambda a^2, \quad h_\kappa^2(\theta) = \frac{\sin^2(\sqrt{\kappa}\theta)}{\kappa}$$

- The Kerr-Schild transformation :

$$ds^2 = ds_0^2 + 2H(r,\theta)\ l\otimes l$$

Kerr-(A)dS metric as a Kerr-Schild metric

- The null and geodesic vector

$$l(\kappa) = \frac{1}{\Xi_a}\left[\Delta(\theta)\sqrt{1+\delta^0_\kappa a^2}\,dt - a\left(\sin^2(\sqrt{\kappa}\,\theta)+\frac{\delta^0_\kappa}{\sqrt{-\lambda}}\right)d\phi\right] + \frac{\Sigma(r,\theta)\,dr}{(\kappa-\lambda r^2)(r^2+\kappa^2 a^2)}$$

- From the circularity theorem (Froebenius integrability condition) and one of the Einstein eq.

$$H(r,\theta) = \frac{r\mathcal{M}}{\Sigma(r,\theta)}.$$

- Boyer-Linquist coordinates

$$t_{BL} = t - \int \frac{(2\mathcal{M}r)\sqrt{1+\delta^0_\kappa a^2}}{(\kappa-\lambda r^2)(r^2+\kappa^2 a^2-2\mathcal{M}r)}\,dr,$$

$$\phi_{BL} = \phi - a\int \frac{(2\mathcal{M}r)\sqrt{1+\delta^0_\kappa a^2}}{(\kappa-\lambda r^2)(r^2+\kappa^2 a^2-2\mathcal{M}r)}\,dr.$$

Kerr-Schild Ansatz in presence of matter source

1. The difficulty with the presence of source is to find an appropriate ansatz for the extra dynamical fields to be fully compatible with the equations of motion.
2. The most appealing example where the Kerr-Schild procedure works is the Kerr-Newman solution in $D = 4$

$$ds^2 = ds_0^2 + \frac{2r}{\Sigma(r,\theta)}\left(\mathcal{M} - \frac{\mathcal{Q}^2}{2r}\right) l \otimes l, \quad A = \frac{r\mathcal{Q}}{\Sigma(r,\theta)} l$$

3. BUT the higher-dimensional Kerr-Newman solution is not yet known ($A \propto l$ is incompatible with the EOM)

Kerr-Schild Ansatz in presence of matter source

• But, instead in five dimensions, consider the Einstein-Maxwell-Chern-Simons action [Z-W. Chong , M. Cvetic, H. Lu and C. N. Pope, Phys. Rev. Lett. **95**, 161301 (2005).]

$$\mathcal{L} = R + 12 - F_{\mu\nu}F^{\mu\nu} - \frac{2}{3\sqrt{3}}\epsilon^{\mu\nu\alpha\beta\sigma}A_\mu F_{\nu\alpha}F_{\beta\sigma},$$

Generalized Kerr-Schild representation of the charged solution

$$\begin{aligned} ds^2 &= d\tilde{s}_0^2 + 2\left(\frac{\mathcal{M}}{\Sigma(r,\theta)} - \frac{\mathcal{Q}^2}{2\Sigma(r,\theta)^2}\right) l \otimes l + \frac{\mathcal{Q}}{\Sigma(r,\theta)}\, l \otimes m \\ A &= \frac{\sqrt{3}}{2\Sigma(r,\theta)} l, \end{aligned}$$

where m is a spacelike vector $m_\mu m^\mu \geq 0$ orthogonal to l and $A \propto l$, [A. N. Aliev and D. K. Ciftci, Phys. Rev. D **79**, 044004 (2009)]

Kerr-Schild Ansatz in presence of matter source

• Investigate the feasibility of the Kerr-Schild procedure in the case of Scalar Tensor Theories (STT), i. e. $\mathcal{L} = \mathcal{L}(g, \phi)$ in order to construct black holes with a scalar field source.

1. From a simple seed configuration (g_0, ϕ_0), generate a nontrivial BH configuration (g, ϕ) by means of the Kerr-Schild procedure.
2. Construct BHs with interesting features (e. g. regular BHs) and with different asymptotics such as flat, (A)dS, Lifshitz or even hyperscaling violation
3. The main idea will be to fix the desired properties of the solution and by "engineering inverse" to determine the STT susceptible to sustain this solution.

Presentation of the Kerr-Schild procedure

1. Consider a STT $\mathcal{L}(g, \partial g, \partial^2 g, \partial\phi, \partial^2\phi)$ free of Ostrogradski ghosts (that will be specify later) and invariant under the shift symmetry of the scalar field $\phi \to \phi+$cst.
2. Implement the KS Ansatz in the static case with a seed configuration $(g_0, \phi^{(0)})$
$$ds_0^2 = -h_0(r)dt^2 + \frac{dr^2}{f_0(r)} + r^2 d\Sigma_2^2,\ \phi^{(0)}(t,r) := qt + \psi^{(0)}(r)$$
3. KS transformation for the metric
$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \mu a(r)\, l_\mu l_\nu,\quad l = dt - \frac{dr}{\sqrt{f_0(r)\, h_0(r)}}$$
where $\mu \propto \mathcal{M}$. The net effect of the KS transf. is
$$h_0(r) \to h_0(r) - \mu a(r),\ \ f_0(r) \to \frac{f_0(r)\,(h_0(r) - \mu a(r))}{h_0(r)}$$

Presentation of the Kerr-Schild procedure

• We have fixed the seed configuration and specify the transformed metric $\Longrightarrow$ Specify the transformed scalar field.

• In the charged cases for which the Kerr-Schild procedure works (the E-M action in $D = 4$ or for the EMCS action in $D = 5$), one observes that

$$(A^{(0)})^2 := g^{(0)\mu\nu} A^{(0)}_\mu A^{(0)}_\nu = A^2 := g^{\mu\nu} A_\mu A_\nu = 0$$

• Working hypothesis : We will demand that the kinetic term of the scalar field remains unchanged (but not necessarily constant) under the Kerr-Schild transformation of the metric

$$X^{(0)} := g^{(0)\mu\nu}\, \partial_\mu \phi^{(0)}\, \partial_\nu \phi^{(0)} = X := g^{\mu\nu}\, \partial_\mu \phi\, \partial_\nu \phi.$$

Presentation of the Kerr-Schild procedure

• Invariance of the kinetic term through the Kerr-Schild transformation

$$X^{(0)} := g^{(0)\mu\nu}\,\partial_\mu\phi^{(0)}\,\partial_\nu\phi^{(0)} = X := g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi.$$

$\longrightarrow$ Sols are such that the kinetic term X is mass independent but not the scalar field ϕ.

$\longrightarrow$ We will say that the action is Kerr-Schild invariant if

$$S(g,\phi)-S(g^{(0)},\phi^{(0)}) = \int dr\,\mathcal{E}\left(r, a(r), a'(r), X(r), X'(r)\right)+b.t.,$$

for a function $a(r) = a(r, X^{(0)}(r))$ such that $\mathcal{E} = 0$.

$\longrightarrow$ This condition seems to be quite restrictive but as shown below, for general DHOST theories that are shift invariant $\phi \to \phi+$constant, the set of EOM is always invariant under this condition for a specific choice of the mass function $a(r)$.

Scalar Tensor Theories (STT)

1. Scalar tensor theories are one of the simplest modified gravity theories which extend GR with one (or more) scalar degrees of freedom.

2. Horndeski theory : The most general (single) scalar-tensor theory with second order equations of motion $\Longrightarrow$ absence of Ostrogradski ghosts [G. W. Horndeski, Int. J. Theor. Phys. **10**, 363 (1974)]. The action is given by $\int d^4x\sqrt{-g}\sum_{i=2}^{5}\mathcal{L}_i$ where

$$\mathcal{L}_2 = K(\phi, X), \qquad \mathcal{L}_3 = -G_3(\phi, X)\Box\phi,$$
$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$
$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}\left[(\Box\phi)^3\right.$$
$$\left.-3\left(\Box\phi\right)\left(\nabla_\mu\nabla_\nu\phi\right)^2 + 2\left(\nabla_\mu\nabla_\nu\phi\right)^3\right]$$

Degenerate Higher Order Scalar Tensor (DHOST)

• $\exists$ higher-order STTs which can propagate healthy degrees of freedom. The most general such Lagrangian depending quadratically on second-order derivatives of a scalar field was dubbed Degenerate Higher Order Scalar Tensor (DHOST) theory [D. Langlois and K. Noui, JCAP **1602** (2016) 034]

• The most general STT which contains up to second-order covariant derivatives of the scalar field

$$\begin{aligned}\mathcal{L} = \quad & K + G\,R + F_1\Box\phi + F_2\,G^{\mu\nu}\phi_{\mu\nu} + A_1\phi_{\mu\nu}\phi^{\mu\nu} + A_2(\Box\phi)^2 \\ & + A_3\Box\phi\,\phi^{\mu}\phi_{\mu\nu}\phi^{\nu} + A_4\phi^{\mu}\phi_{\mu\nu}\phi^{\nu\rho}\phi_{\rho} + A_5\left(\phi^{\mu}\phi_{\mu\nu}\phi^{\nu}\right)^2\end{aligned}$$

where the coupling functions K, G, F_i and A_i are arbitrary functions of ϕ and $X = \phi_\mu\phi^\mu$, and $\phi_{\mu\nu} = \nabla_\mu\nabla_\nu\phi$.

Degenerate Higher Order Scalar Tensor (DHOST)

$\longrightarrow$ The DHOST conditions are given by [D. Langlois and K. Noui, JCAP **1602** (2016) 034 ; M. Crisostomi, K. Koyama and G. Tasinato, JCAP **1604**, 044 (2016)]

$$\begin{aligned}
A_1 &= -A_2 \neq \frac{G}{X}, \\
A_4 &= \frac{1}{8(G-XA_1)^2}\Big\{4G\left[3(-A_1+2G_X)^2-2A_3G\right]-A_3X^2(16A_1G_X+A_3G) \\
&\quad +4X\left[-3A_2A_3G+16A_1^2G_X-16A_1G_X^2-4A_1^3+2A_3GG_X\right]\Big\}, \\
A_5 &= \frac{1}{8(G-XA_1)^2}(2A_1-XA_3-4G_X)\left(A_1(2A_1+3XA_3-4G_X)-4A_3G\right).
\end{aligned}$$

$\longrightarrow$ This theory includes the Horndeski model

$\longrightarrow$ Demanding the gravitational wave to propagate at the speed of light $A_1 = A_2 = 0$.

Degenerate Higher Order Scalar Tensor (DHOST)

- The KS procedure will be applied for $\mathcal{L}$

  1. Invariant under the reflection $\phi \to -\phi$ (i. e. $F_i = 0$),
  2. Shift invariant (i. e. the coupling functions will only depend on X)
  3. And free of Ostrogradski ghosts

Kerr-Schild transformation for DHOST theories

- The Kerr-Schild transf.

$$ds_0^2 = -h_0(r)dt^2 + \frac{dr^2}{f_0(r)} + r^2 d\Sigma_2^2, \quad X^{(0)}(r) \Longrightarrow$$

$$ds^2 = -(h_0 - \mu a)dt^2 + \frac{h_0 dr^2}{f_0(h_0 - \mu a)} + r^2 d\Sigma_2^2, \; X = X^{(0)}$$

- Variation of the DHOST action under the KS transf.

$$\delta S \propto \int dr \sqrt{\frac{f_0(r)}{h_0(r)}} \left[a(r)P(r,X) + a'(r)Q(r,X) \right] + b.t.,$$

- In order for the KS transf. to be a symmetry of the action

$$a(r) = \frac{1}{r} e^{\frac{3}{8}\int dX \frac{B(X)}{H(X)}}, \; B(X) = A_3 X + 4G_X - 2A_1, \; H(X) = A_1 X - G$$

- Standard mass fall off $a \sim \frac{1}{r}$ for $X =$cst or $B = 0$.

Regular black hole solutions

1. The Bardeen regular solution [Bardeen, J., in Proceedings of GR5, Tiflis, U.S.S.R. (1968)] is a counterexample to the possibility that the existence of singularities may be proved in black hole spacetimes without assuming either a global Cauchy hypersurface or the strong energy condition

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Sigma_2^2, \qquad f(r) = 1 - \frac{2mr^2}{(r^2+g^2)^{\frac{3}{2}}}$$

$\longrightarrow$ Regular black hole obeying the weak energy condition, asympto. flat and behaving like the Schwarzschild metric for $r >> \epsilon$, and with a de-Sitter core at the origin

$$f \sim 1 - \frac{2m}{g} r^2, \qquad \text{for} \quad r \sim 0$$

$\longrightarrow$ The Bardeen solution can be interpreted as a magnetic solution to Einstein equations coupled to a nonlinear electrodynamics [E. Ayon-Beato and A. Garcia, Phys. Lett. B **493**, 149-152 (2000)].

Regular black hole solutions

1. The Ayón-Beato-Garcia regular solution [E. Ayon-Beato and A. Garcia, Phys. Rev. Lett. **80**, 5056 (1998)]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Sigma_2^2,$$
$$f(r) = 1 - \frac{2mr^2}{(r^2+q^2)^{\frac{3}{2}}} + \frac{q^2 r^2}{(r^2+q^2)^2}$$

$\longrightarrow$ Regular black hole obeying the weak energy condition, asympto. flat and behaving like the Reissner-Nordstrom metric for $r >> \epsilon$, and with a de-Sitter core at the origin

$$f \sim 1 - \left(\frac{2m}{q} - 1\right)\frac{r^2}{q^2}, \qquad \text{for} \quad r \sim 0$$

Construction of regular black hole solutions for DHOST theories

- Construction of asymptotically flat regular with a flat seed metric $h_0 = f_0 = 1$

1. The final metric obtained through the Kerr-Schild transf. reads
$$h(r) = f(r) = 1 - \frac{\mu}{r} e^{\int dX \, \frac{3B}{8H}}.$$
2. Make the following simple hypothesis (λ is a constant),
$$\frac{3B}{8H} = \frac{\lambda}{X} \Longrightarrow h(r) = f(r) = 1 - \frac{\mu \, X(r)^{\lambda}}{r}.$$
3. The idea will be to choose an appropriate kinetic term $X(r)$ and a parameter λ in order for the metric to be : asymptotically flat and to have an outer event horizon at some finite $r = r_h$ and to satisfy the Sakharov criterion at the origin
$$f(r) \underset{r\sim 0}{\sim} 1 - f_0 r^p, \qquad p \geq 2,$$
$\longrightarrow$ metric function possesses *at least* a de-Sitter core near

Construction of regular black hole solutions for DHOST theories

1. Hence by inverse engineering $\longrightarrow$ specify the corresponding DHOST theory, i. e. K, G, A_1 and A_3 (as functions of X only).
2. For example for $\lambda = 2$ and $X(r) = \frac{r^2}{r^2+\gamma^2}$, one ends with the following regular metric

$$ds^2 = -\left(1 - \frac{\mu r^3}{(r^2+\gamma^2)^2}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{\mu r^3}{(r^2+\gamma^2)^2}\right)} + r^2 d\Sigma_2^2$$

solution of the DHOST theory $K(X)$, $G(X)$, $A_1(X)$ and $A_3(X)$ parameterized by

$$A_3 = -\frac{4G_X}{X} + \frac{2A_1 X}{X} + \frac{16H}{X^2} \qquad A_1 = \frac{H+G}{X}$$

with

$$H(X) = \frac{1}{X^2(4X-1)}, \quad G(X) = -\frac{3\left(8X^2-8X\right)}{X^2(4-1)^2}, \quad K(X) = \frac{8\left(16X^4-54X^3+63X^2-28X+3\right)}{X^3\gamma^2\left(1-4X\right)^2},$$

Construction of regular black hole solutions for DHOST theories

• In a similar way, one can show that the regular Hayward black hole metric [S. A. Hayward, Phys. Rev. Lett. **96**, 031103 (2006)]

$$ds^2 = -\left(1 - \frac{\mu r^2}{r^3 + \gamma^2}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{\mu r^2}{r^3+\gamma^2}\right)} + r^2 d\Sigma_2^2$$

is solution of DHOST theory with $X(r) = r^3/(r^3 + \gamma^2)$ and $\lambda = 1$ and for

$$H(X) = \frac{1}{X^2}, \qquad G(X) = \frac{(5 - 6X)}{X^2},$$

$$K(X) = \frac{(1 - X)^2(3X - 5)}{\gamma^{\frac{4}{3}} X^3} \left(\frac{X}{1 - X}\right)^{\frac{1}{3}},$$

Conclusions and Works in Progress

1. Extension of the Kerr-Schild generating method for shift-invariant DHOST theories by requiring X to remain invariant under the KS transf.
2. The possibility of having BH solutions with different fall off mass term
$$\frac{1}{r}e^{\frac{3}{8}\int dX\frac{B(X)}{H(X)}}$$
For $X =$cst or $B=0$, standard fall off
3. Use the generating method to construct from simple seed configuration, asymptotically flat regular BHs
4. Extend this solution generating method for other sources as for example the generalized Proca Lagrangian
$$\mathcal{L}=G_2(X,F)+G_4(X)R+G_{4,X}\left((\nabla_\mu A^\mu)^2-\nabla_\mu A_\nu\nabla^\nu A^\mu\right)$$
with $X=-A_\mu A^\mu/2$ and F is the standard Maxwell term $F=-F_{\mu\nu}F^{\mu\nu}$

Works in Progress for Spinning DHOST black holes

1. Spinning DHOST solutions that could be obtained via the Kerr-Schild ansatz : find the appropriate shear free and null congruence that will allow a Kerr-Schild representation of the spinning solutions with a kinetic scalar field that remains unchanged along the Kerr-Schild transformation.
2. $\exists$ a stealth solution defined on the Kerr metric for a specific DHOST theory [C. Charmousis, M. Crisostomi, R. Gregory and N. Stergioulas, Phys. Rev. D **100**, no. 8, 084020 (2019)]

$$ds^2_{\text{Kerr}} = ds^2_0 + \mu H(r,\theta) l \otimes l, \qquad X = X_0$$

3. Some results in $D = 3$ have been obtained [O. Baake, M. Bravo-Gaete and M. Hassaine, Phys. Rev. D **102**, no.2, 024088 (2020)], see the talk of Olaf Baake on August 8.