2024.08.22
7th International Conference on Holography and
String Theory in Da Nang@ Da Nang, Vietnam

Nonlinear conducting in holographic Weyl semi-metal from probe branes

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arXiv: 2405.06484 [hep-th]

Outline

- **Introduction**
  - About Weyl semi-metals
  - Simplest field theory for Weyl semi-metals
- **Method**
  - Holographic models for Weyl semi-metals
  - Holographic Weyl semi-metals from probe branes
- **Results**
  - Nonlinear conductivity ($T = 0$)
  - Nonlinear conductivity ($T \neq 0$) and nonequilibrium phase transitions
  - Critical phenomena
- **Summary and Outlook**

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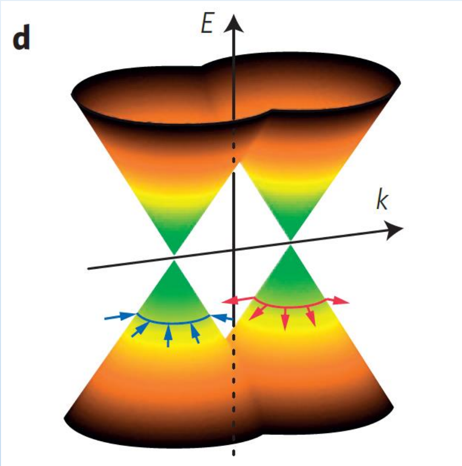
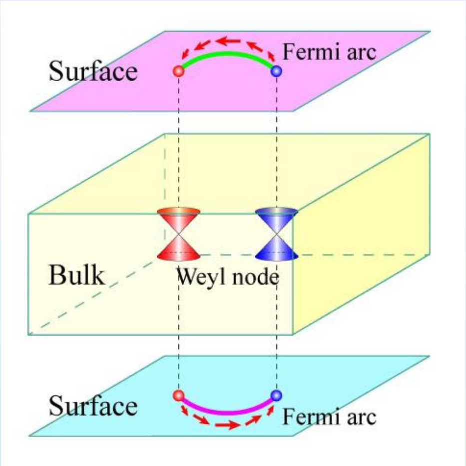
Introduction: Weyl semi-metals

- Weyl nodes in momentum space

  - Two electric bands touch at isolated points in momentum space at the Fermi surface.

  - Time reversal or parity symmetry breaking split the Dirac fermion into left- and right-handed Weyl fermions.



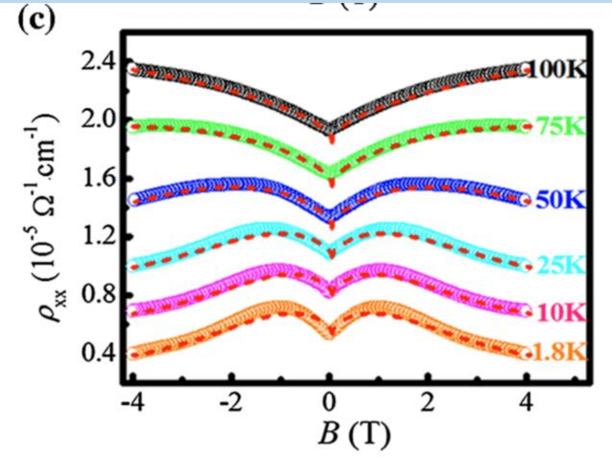


Figures from Lv, etal., (2015).

- Transport response and chiral anomaly

  - The chiral anomaly is an unique property of a Weyl semi-metal.

  - This leads to interesting transport properties, such as negative magneto-resistivity, chiral magnetic effect, and anomalous Hall effect.



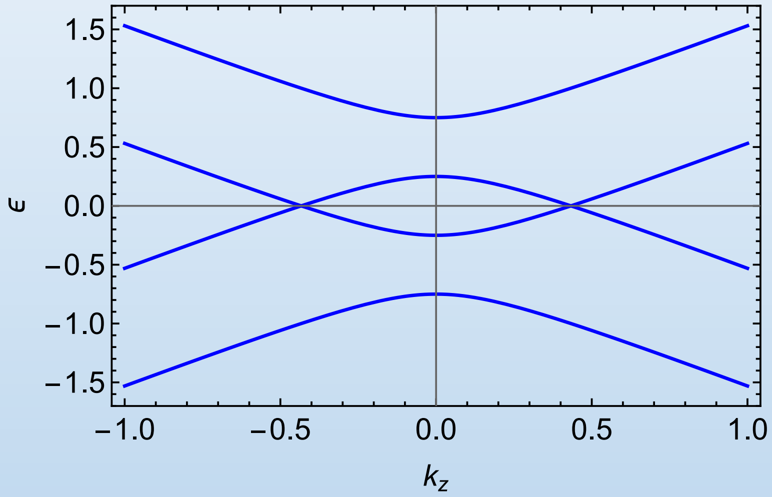
Negative magneto-resistivity in TaAs

Huang, etal., (2015).

Introduction: Simplest field theory for Weyl semi-metals

- ❑ Free Dirac fermion with non-dynamical axial vector field

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - m + A^5_j \gamma^j \gamma^5\right)\psi$$



Choosing $A^5_z = b/2$, the energy of the Dirac fermion is

$$\epsilon = \pm\sqrt{k_x^2 + k_y^2 + \left(\frac{b^2}{4} \pm \sqrt{k_z^2 + m^2}\right)^2}$$

- $|m/b| < 2$: two levels are crossed (Weyl nodes) ➡ **Weyl semi-metal**
- $|m/b| > 2$: energy gap appears ➡ **insulator**

- ➢ Quantum phase transition is second order
- ➢ Anomalous Hall effect

$$J^y = \sigma_{yx} E_x, \qquad \sigma_{yx} = -\sigma_{xy} = \frac{1}{4\pi^2}\sqrt{b^2 - 4m^2}\,\Theta(|b| - 2|m|)$$

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Method: Holographic models for Weyl semi-metals

- Bottom-up approach
  - Einstein-Hilbert gravity in (4+1)-dim AdS, complex scalar, two U(1) gauge fields with Chern-Simons term

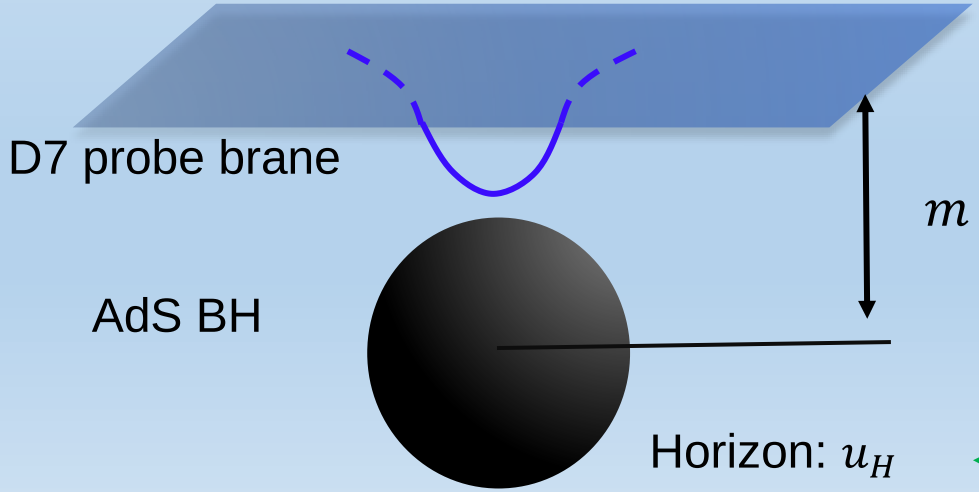
Landsteiner, Liu (2016). Landsteiner, Liu, Sun (2016).

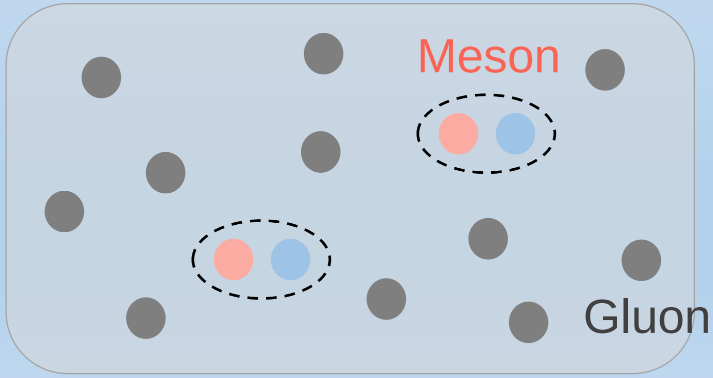
- Top-down approach
  - Probe brane model

e.g.) D3/D7 model

AdS black hole + D7 probe brane ⟷ (3+1) dim. N=4 SYM + N=2 hypermultiplets

Karch, Katz (2003).





Quark mass: m

Heat bath temperature: $T \sim u_H^{-1}$

Method: Holographic Weyl semi-metals from probe branes

❑ Holographic Weyl semi-metals in D3/D7 model

$$f(u) = 1 - \frac{u^4}{u_{\mathrm{H}}^4}, \quad h(u) = 1 + \frac{u^4}{u_{\mathrm{H}}^4}$$

- Background metric: $\mathrm{d}s^2 = \frac{L^2}{u^2}\left(-\frac{f(u)^2}{h(u)}\mathrm{d}t^2 + h(u)\mathrm{d}\vec{x}^2\right) + \frac{L^2}{u^2}\mathrm{d}u^2 + L^2\mathrm{d}\Omega_5^2$

$u = u_H$: Black hole horizon
$u = 0$: AdS boundary

- Probe D7-brane action:

$$S_{D7} = \underbrace{-T_{D7}\int d^8\xi\sqrt{-\det\left(g_{ab} + (2\pi\alpha')F_{ab}\right)}}_{\text{Dirac-Infeld-Born (DBI) action}} + \underbrace{\frac{(2\pi\alpha')^2}{2}T_{D7}\int P[C^{(4)}]\wedge F\wedge F}_{\text{Wess-Zumino term}}$$

Dirac-Infeld-Born (DBI) action

Wess-Zumino term ➡ axial anomaly

4-form: $C^{(4)} = \frac{L^4}{u^4}\mathrm{d}t\wedge\mathrm{d}x\wedge\mathrm{d}y\wedge\mathrm{d}z - L^4\cos^4\theta\mathrm{d}\psi\wedge\omega(S^3)$

- D3/D7-branes intersection:

	t	x	y	z	u	S^3	θ	ψ
D3	×	×	×	×				
D7	×	×	×	×	×	×		

- Two scalars determine the configuration of D7-brane in the AdS geometry.
- ψ introduces the finite contribution of the Wess-Zumino term and axial anomaly.

Method: Holographic Weyl semi-metals from probe branes

- Ansatz of scalars for WSM: $\theta = \theta(u), \quad \psi = bz$

Fadafan, O'Bannon, Rodgers, Russel (2021).

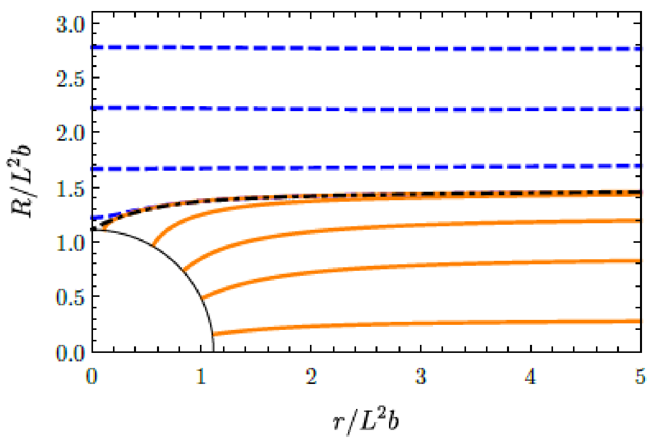
near $u = 0$ → $\sin\theta(u) = mu + cu^3 + \cdots$

Dual field theory

- m: quark mass, c: quark condensate
- The potential term for the complex hypermultiplet: $V \supset \bar{\psi}_f \left(m - \frac{b}{2}\gamma^z\gamma^5 \right) \psi_f$

External axial gauge potential: $A^5_z = b/2$

- Solutions of $\theta(u)$ and phase transitions



Fixed T/b

Insulator phase, $\sigma_{xy} = 0$, for larger m/b.

WSM phase, $\sigma_{xy} \propto b$, for larger m/b.

1st order phase transition

Method: Holographic Weyl semi-metals from probe branes

Our interest: How response with respect to an external electric field?

- Ansatz of gauge fields: $A_x = -Et + a_x(u), \quad A_y = a_y(u)$

Electric field

near $u = 0$

$$a_x(u) = a_x^{(0)} + \frac{j_x}{2} u^2 + \cdots ,$$
$$a_y(u) = a_y^{(0)} + \frac{j_y}{2} u^2 + \cdots ,$$

Dual field theory

- j_x: longitudinal current density

$$j_x = \frac{f}{u_*^2} \sqrt{b^2 \sin^2\theta + \frac{h}{u_*^2}} \cos^3\theta$$

where

$$u_* = \frac{\sqrt{2}}{\pi T} \sqrt{\frac{E}{(\pi T)^2} + \sqrt{1 + \frac{E^2}{(\pi T)^4}}}$$

Effective horizon on the probe brane
➡ the system is the steady state

- j_y: transverse current density

$$j_y = -bE\cos^4\theta$$

The current density shows a highly nonlinear response with respect to E.
We focus on the only longitudinal current density.

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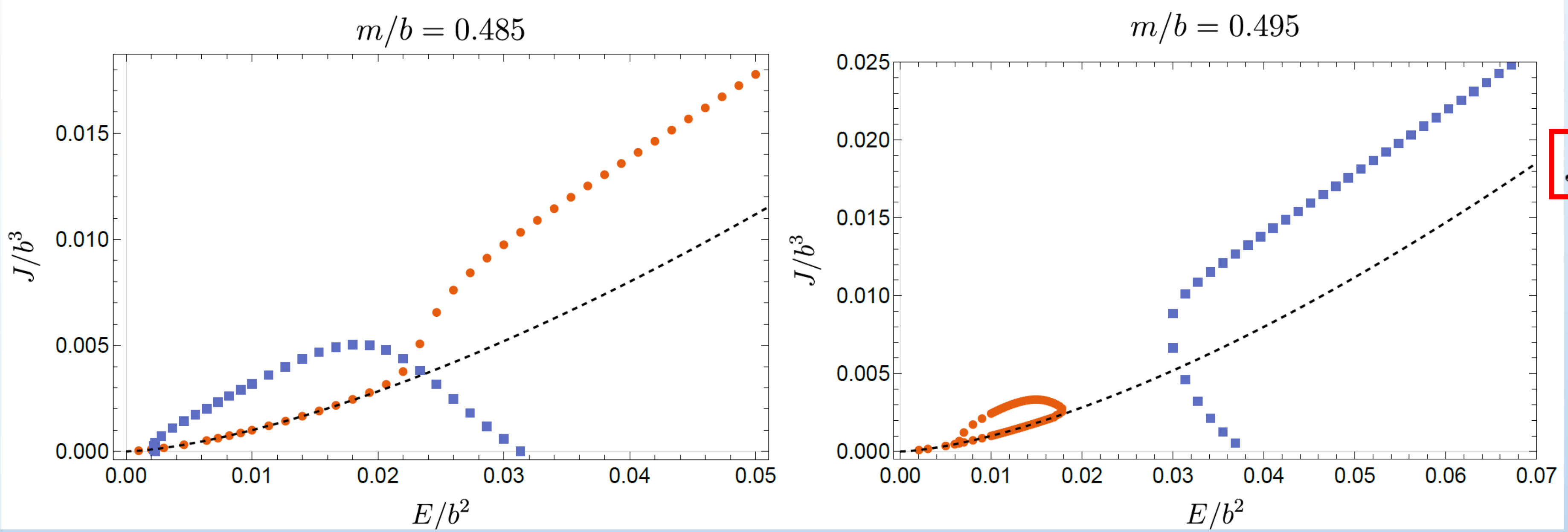
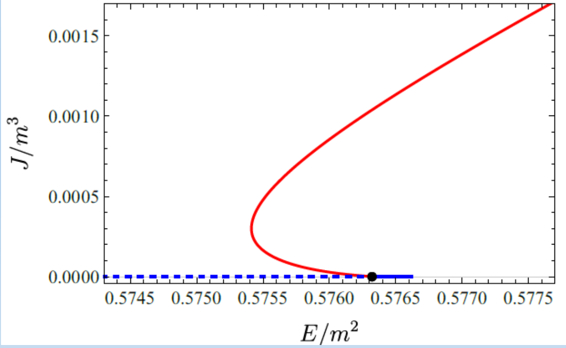
Results: Nonlinear conductivity ($T = 0$)

- J-E characteristics at zero temperature



$J = E^{3/2}$

- For small E, the current is well described by the analytic form, $J = E^{3/2}$.
- Changing m/b, there is a “reconnection” transition between two branches.
- It is due to the competition between the WSM-like behavior and ordinary conducting behavior.

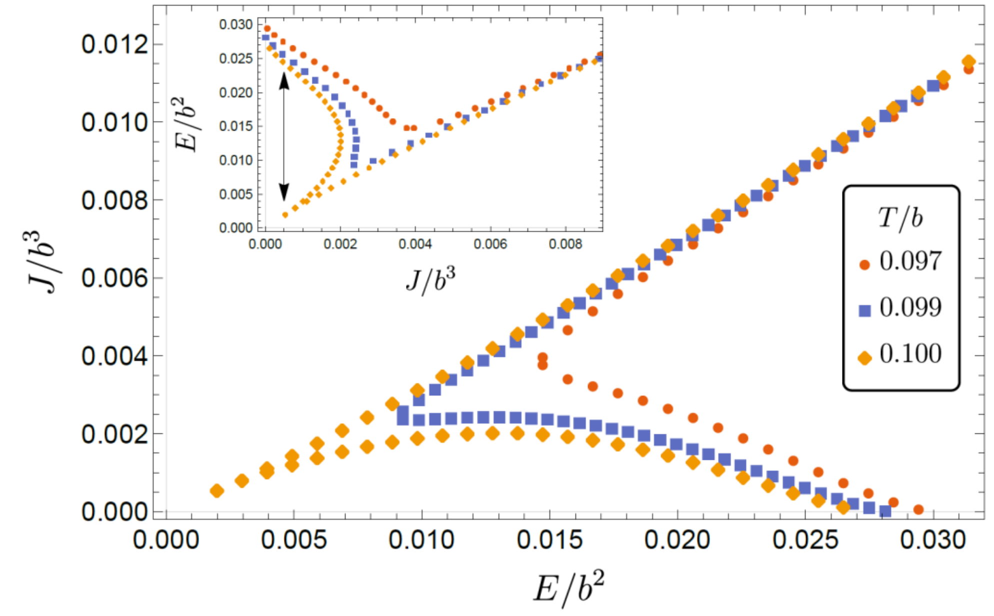


cf.) J-E relation without b

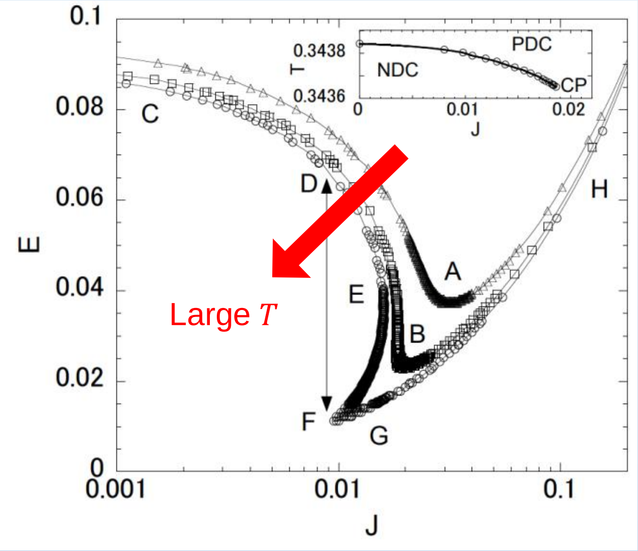
Results: Nonlinear conductivity ($T \neq 0$) and nonequilibrium phase transitions

- J-E characteristics at finite temperature

$m/b = 0.5$



$b = 0$, normalized by m



Nakamura (2012).

- For large m/b and increasing T/b, one branch cannot be found.
- The nonlinear behavior of J-E characteristics is similar to the system without b.
- Given J fixed, E is multivalued. ➡ <u>Current-driven non-equilibrium phase transitions</u>

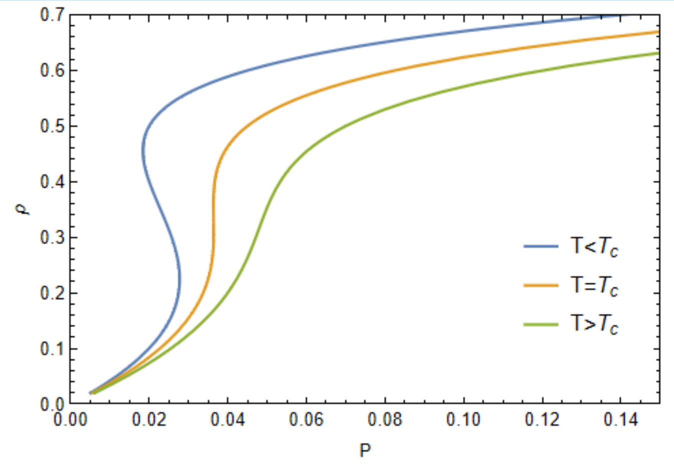
Results: Nonlinear conductivity ($T \neq 0$) and nonequilibrium phase transitions

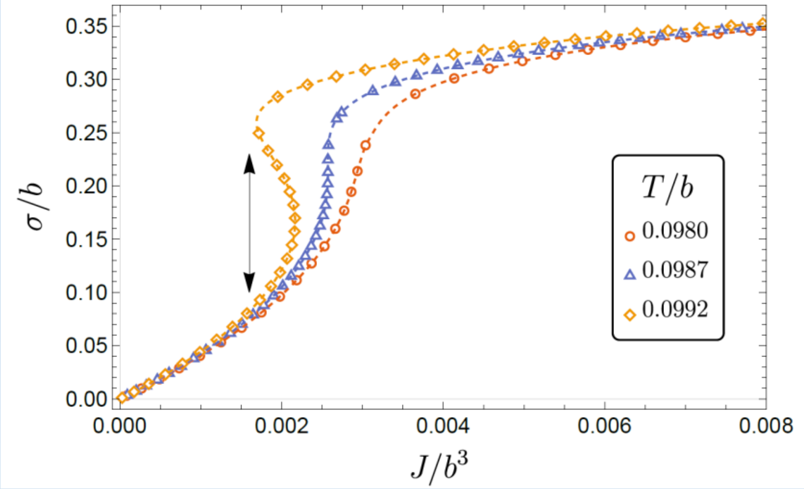
In analogy with equilibrium phase transitions and following the previous analysis, define order parameter.

Nakamura (2012). MM, Nakamura (2018)

	Ferromagnet (equilibrium)	**Liquid-Gas (equilibrium)**	**Our case (Non-eq. steady state)**
Order parameter	Magnetization (M)	Density (ρ)	Conductivity (σ)
Control parameter	Temperature (T)	Temperature (T)	Temperature (T)
External source	Magnetic field (H)	Pressure (P)	Current density (J)

Van der Waals theory





Results: Nonlinear conductivity ($T \neq 0$) and nonequilibrium phase transitions

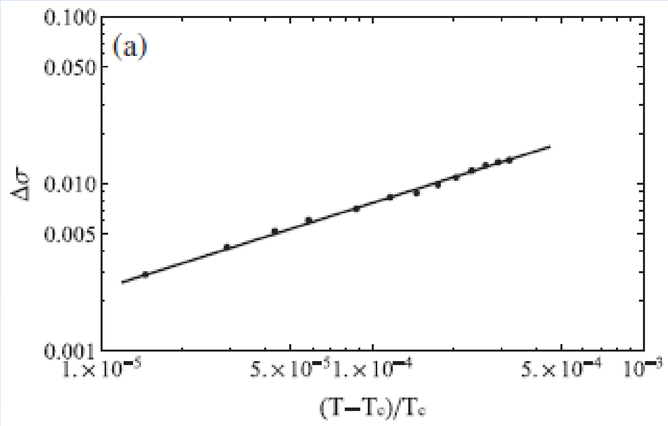
To study critical phenomena, define the critical exponents:

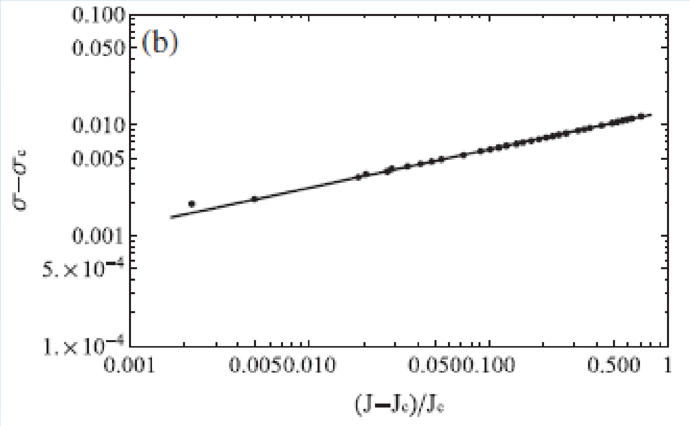
$$\Delta\sigma \propto |T - T_c|^{\beta}, \quad (T > T_c)$$
$$|\sigma - \sigma_c| \propto |J - J_c|^{1/\delta}, \quad (T = T_c)$$

Ferromagnetic phase transition

$$\Delta M \propto |T - T_c|^{\beta}, \quad (T < T_c)$$
$$|M - M_c| \propto |H - H_c|^{1/\delta}, \quad (T = T_c)$$
$$\vdots$$

- In the case of non-WSM system, the values of critical exponents agree with the mean-field values.





$$\beta = \frac{1}{2}, \quad \delta = 3$$

Nakamura (2012). MM, Nakamura (2018)

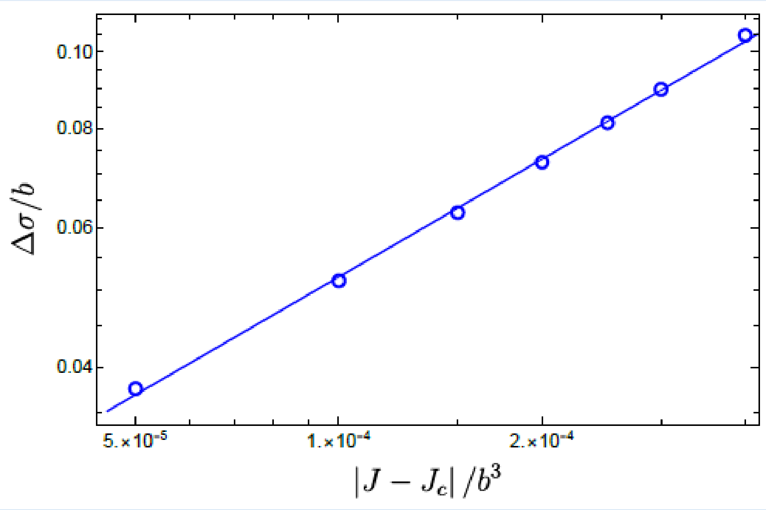
Results: Nonlinear conductivity ($T \neq 0$) and nonequilibrium phase transitions

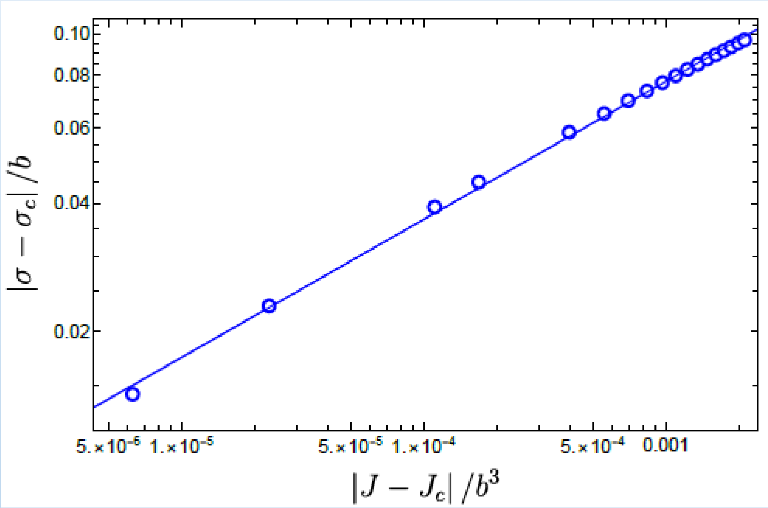
Compute the critical exponents

$$\beta \approx 0.4941, \quad \delta \approx 3.087$$

which also agree with the mean field values

$$\beta = \frac{1}{2}, \quad \delta = 3$$





- ➢ Even in the WSM system, the critical exponents of current-driven phase transitions agree with the mean-field values.
- ➢ The axial gauge potential does not change the critical behavior of this phase transition.

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Summary

- Study the nonlinear J-E characteristics in the holographic Weyl semi-metals from the probe brane model.
- The "reconnection" transition is found due to the competition between the WSM-like behavior and ordinary conducting behavior.
- At finite temperature, we study the critical phenomena of the current-driven phase transition emerged by the nonlinear behavior.

Outlook

- Dynamical stability of the two branches?
- Magnetic response?
- Experimental realization?
- …

Thank you!