A 3D flat-space holography inspired by AdS3/CFT2 duality

Nabamita Banerjee
IISER, Bhopal

11th August 2020
Holography, Hanoi

Motivation : Why 3D ?

- Gravity in three space time dimensions is special, there is no dynamical graviton !
- But a large variety of gravitational solutions exists whenever global topological structures, such as the holonomy of the manifold are considered.
- 3D gravity is renormalizable, thus a perfect toy model to study some quantum aspects of gravity.
- Holography is well understood in the context of AdS3 gravity.
- 3D gravity gravity can also be alternately viewed as a pure grage theory, known as Chern-Simons theory.

We shall use the Chern-Simons formulation to look for a holographic dual of 3D Flat (super)gravity theories.

Motivation : Why 3D ?

- Gravity in three space time dimensions is special, there is no dynamical graviton !
- But a large variety of gravitational solutions exists whenever global topological structures, such as the holonomy of the manifold are considered.
- 3D gravity is renormalizable, thus a perfect toy model to study some quantum aspects of gravity.
- Holography is well understood in the context of AdS3 gravity.
- 3D gravity gravity can also be alternately viewed as a pure grage theory, known as Chern-Simons theory.

We shall use the Chern-Simons formulation to look for a holographic dual of 3D Flat (super)gravity theories.

Motivation : Why 3D ?

- Gravity in three space time dimensions is special, there is no dynamical graviton !
- But a large variety of gravitational solutions exists whenever global topological structures, such as the holonomy of the manifold are considered.
- 3D gravity is renormalizable, thus a perfect toy model to study some quantum aspects of gravity.
- Holography is well understood in the context of AdS3 gravity.
- 3D gravity gravity can also be alternately viewed as a pure grage theory, known as Chern-Simons theory.

We shall use the Chern-Simons formulation to look for a holographic dual of 3D Flat (super)gravity theories.

Motivation : Why 3D ?

- Gravity in three space time dimensions is special, there is no dynamical graviton !
- But a large variety of gravitational solutions exists whenever global topological structures, such as the holonomy of the manifold are considered.
- 3D gravity is renormalizable, thus a perfect toy model to study some quantum aspects of gravity.
- Holography is well understood in the context of AdS3 gravity.
- 3D gravity gravity can also be alternately viewed as a pure grage theory, known as Chern-Simons theory.

We shall use the Chern-Simons formulation to look for a holographic dual of 3D Flat (super)gravity theories.

Lessons from AdS3/CFT2 correspondence :

3D gravity in asymptotically AdS space is dual to a CFT2, one of the most well understood holographic duality.

Witten, Maldacena-Ooguri, Brown-Henneaux, Strominger, Giveon-Kutasov-Seiberg.

- Asymptotic symmetry of AdS3 gravity is centraly extended Virasoro symmetry $\equiv$ symmetry of dual CFT2.
- There exist a BTZ Black Hole solution which is globally different form AdS3. Its entropy is related to the central charge of the Virasoro algebra.
- The dual CFT can be constructed as a Liouville theory.
- Supersymmetric extensions gave further clarifications.
- The nontrivial global dofs of 3D gravity can be identified in string theory on AdS3 as holomorphic (or anti-holomorphic) vertex operators integrated over contours on the worldsheet.

Lessons from AdS3/CFT2 correspondence :

3D gravity in asymptotically AdS space is dual to a CFT2, one of the most well understood holographic duality.

Witten,Maldacena-Ooguri,Brown-Henneaux,Strominger,Giveon-Kutasov-Seiberg.

- Asymptotic symmetry of AdS3 gravity is centraly extended Virasoro symmetry $\equiv$ symmetry of dual CFT2.
- There exist a BTZ Black Hole solution which is globally different form AdS3. Its entropy is related to the central charge of the Virasoro algebra.
- The dual CFT can be constructed as a Liouville theory.
- Supersymmetric extensions gave further clarifications.
- The nontrivial global dofs of 3D gravity can be identified in string theory on AdS3 as holomorphic (or anti-holomorphic) vertex operators integrated over contours on the worldsheet.

3D gravity in asymptotically AdS space is dual to a CFT2, one of the most well understood holographic duality.

Witten,Maldacena-Ooguri,Brown-Henneaux,Strominger,Giveon-Kutasov-Seiberg.

- Asymptotic symmetry of AdS3 gravity is centraly extended Virasoro symmetry $\equiv$ symmetry of dual CFT2.
- There exist a BTZ Black Hole solution which is globally different form AdS3. Its entropy is related to the central charge of the Virasoro algebra.
- The dual CFT can be constructed as a Liouville theory.
- Supersymmetric extensions gave further clarifications.
- The nontrivial global dofs of 3D gravity can be identified in string theory on AdS3 as holomorphic (or anti-holomorphic) vertex operators integrated over contours on the worldsheet.

Lessons from AdS3/CFT2 correspondence :

3D gravity in asymptotically AdS space is dual to a CFT2, one of the most well understood holographic duality.

Witten, Maldacena-Ooguri, Brown-Henneaux, Strominger, Giveon-Kutasov-Seiberg.

- Asymptotic symmetry of AdS3 gravity is centraly extended Virasoro symmetry $\equiv$ symmetry of dual CFT2.
- There exist a BTZ Black Hole solution which is globally different form AdS3. Its entropy is related to the central charge of the Virasoro algebra.
- The dual CFT can be constructed as a Liouville theory.
- Supersymmetric extensions gave further clarifications.
- The nontrivial global dofs of 3D gravity can be identified in string theory on AdS3 as holomorphic (or anti-holomorphic) vertex operators integrated over contours on the worldsheet.

Lessons from AdS3/CFT2 correspondence :

3D gravity in asymptotically AdS space is dual to a CFT2, one of the most well understood holographic duality.

Witten,Maldacena-Ooguri,Brown-Henneaux,Strominger,Giveon-Kutasov-Seiberg.

- Asymptotic symmetry of AdS3 gravity is centraly extended Virasoro symmetry $\equiv$ symmetry of dual CFT2.
- There exist a BTZ Black Hole solution which is globally different form AdS3. Its entropy is related to the central charge of the Virasoro algebra.
- The dual CFT can be constructed as a Liouville theory.
- Supersymmetric extensions gave further clarifications.
- The nontrivial global dofs of 3D gravity can be identified in string theory on AdS3 as holomorphic (or anti-holomorphic) vertex operators integrated over contours on the worldsheet.

3D gravity in asymptotically AdS space is dual to a CFT2, one of the most well understood holographic duality.

Witten,Maldacena-Ooguri,Brown-Henneaux,Strominger,Giveon-Kutasov-Seiberg.

- Asymptotic symmetry of AdS3 gravity is centraly extended Virasoro symmetry $\equiv$ symmetry of dual CFT2.
- There exist a BTZ Black Hole solution which is globally different form AdS3. Its entropy is related to the central charge of the Virasoro algebra.
- The dual CFT can be constructed as a Liouville theory.
- Supersymmetric extensions gave further clarifications.
- The nontrivial global dofs of 3D gravity can be identified in string theory on AdS3 as holomorphic (or anti-holomorphic) vertex operators integrated over contours on the worldsheet.

GOAL

We are in search of a similar duality for asymptotically flat 3D gravity !

Plan of The talk

- Why 3D ?
- Lessons From AdS3/CFT2
- BMS from Chern-Simons Formulation of 3D gravity
- Supersymmetric extensions of BMS_3 :
  - Asymptotic symmetry analysis of 3dimensional supergravity theories
  - Bosonic solutions with non trivial topologies
  - Outlook so far
- 2D field theories dual to 3D Flat gravity
  - WZW like theories
  - Flat limit of Liouville Theories
- Outlook and Open questions

What is BMS ?

- The symmetry algebra at null infinity ($\mathcal{I}_+$ or $\mathcal{I}_-$) for gravity theories with flat-space asymptotics is known as Bondi-van der Burg-Metzner-Sachs (BMS) algebra.

- In three and four space time demensions, BMS algebra is infinite dimensional and transformation generators are known as supertranslation and superrotation generators.

- In higher (> 4) even space time dimensions, with a relaxed boundary condition on metric, one gets supertranslation and Lorentz transformations as the asymptotic symmetry generators .

In this talk we shall be interested in asymptotically flat gravity in three space time dimentions.

What is BMS ?

- The symmetry algebra at null infinity ($\mathcal{I}_+$ or $\mathcal{I}_-$) for gravity theories with flat-space asymptotics is known as Bondi-van der Burg-Metzner-Sachs (BMS) algebra.

- In three and four space time demensions, BMS algebra is infinite dimensional and transformation generators are known as supertranslation and superrotation generators.

- In higher (> 4) even space time dimensions, with a relaxed boundary condition on metric, one gets supertranslation and Lorentz transformations as the asymptotic symmetry generators .

In this talk we shall be interested in asymptotically flat gravity in three space time dimentions.

What is BMS ?

- The symmetry algebra at null infinity ($\mathcal{I}_+$ or $\mathcal{I}_-$) for gravity theories with flat-space asymptotics is known as Bondi-van der Burg-Metzner-Sachs (BMS) algebra.
- In three and four space time demensions, BMS algebra is infinite dimensional and transformation generators are known as supertranslation and superrotation generators.
- In higher (> 4) even space time dimensions, with a relaxed boundary condition on metric, one gets supertranslation and Lorentz transformations as the asymptotic symmetry generators .

In this talk we shall be interested in asymptotically flat gravity in three space time dimentions.

What is BMS ?

- The symmetry algebra at null infinity ($\mathcal{I}_+$ or $\mathcal{I}_-$) for gravity theories with flat-space asymptotics is known as Bondi-van der Burg-Metzner-Sachs (BMS) algebra.

- In three and four space time demensions, BMS algebra is infinite dimensional and transformation generators are known as supertranslation and superrotation generators.

- In higher (> 4) even space time dimensions, with a relaxed boundary condition on metric, one gets supertranslation and Lorentz transformations as the asymptotic symmetry generators .

In this talk we shall be interested in asymptotically flat gravity in three space time dimentions.

For flat space, it is most elegantly described using Penrose compactipication of null infinity . Assumption of an asymptotic series expansion in $1/r$ becomes a smoothness condition at $\mathcal{I}_+$.

- 3D gravity : $S = \int d^3x\sqrt{-g}R$
- Consider a metric $g_{\mu\nu}(x)$ with proper fall off that describes a geometry of an asymptotically flat manifold $g_{\mu\nu}(x) \xrightarrow{r\sim\infty} \eta_{\mu\nu}$
- Next we look for the isometries ξ that they leave the asymptotic form of the metric invariant, $\mathcal{L}_\xi g_{\mu\nu}(x) \xrightarrow{r\sim\infty} g_{\mu\nu}(x)$.
- These killing fields ξ^a are the generators of the asymptotic symmetry algebra.

Implementation of the above criteria is much easier and illuminating in the Chern-Simons formulation of 3D gravity.

How do we get asymptotic symmetry algebra ? 1609.01731

For flat space, it is most elegantly described using Penrose compactipication of null infinity . Assumption of an asymptotic series expansion in $1/r$ becomes a smoothness condition at $\mathcal{I}_+$.

- 3D gravity : $S = \int d^3x \sqrt{-g}R$
- Consider a metric $g_{\mu\nu}(x)$ with proper fall off that describes a geometry of an asymptotically flat manifold $g_{\mu\nu}(x) \xRightarrow{r\sim\infty} \eta_{\mu\nu}$
- Next we look for the isometries ξ that they leave the asymptotic form of the metric invariant, $\mathcal{L}_\xi g_{\mu\nu}(x) \xRightarrow{r\sim\infty} g_{\mu\nu}(x)$.
- These killing fields ξ^a are the generators of the asymptotic symmetry algebra.

Implementation of the above criteria is much easier and illuminating in the Chern-Simons formulation of 3D gravity.

How do we get asymptotic symmetry algebra ? 1609.01731

For flat space, it is most elegantly described using Penrose compactipication of null infinity . Assumption of an asymptotic series expansion in $1/r$ becomes a smoothness condition at $\mathcal{I}_+$.

- 3D gravity : $S = \int d^3x\sqrt{-g}R$
- Consider a metric $g_{\mu\nu}(x)$ with proper fall off that describes a geometry of an asymptotically flat manifold $g_{\mu\nu}(x) \xRightarrow{r\sim\infty} \eta_{\mu\nu}$
- Next we look for the isometries ξ that they leave the asymptotic form of the metric invariant, $\mathcal{L}_\xi g_{\mu\nu}(x) \xRightarrow{r\sim\infty} g_{\mu\nu}(x)$.
- These killing fields ξ^a are the generators of the asymptotic symmetry algebra.

Implementation of the above criteria is much easier and illuminating in the Chern-Simons formulation of 3D gravity.

Pure Chern-Simons theory is given as,

$$S_{CS} = \frac{k}{4\pi} \int_M \langle A, dA + \frac{2}{3} A^2 \rangle$$

- k is level of the theory, M is the spacetime manifold.
- A is a Lie-algebra-valued one form : $A = A^a_\mu T_a dx^\mu$.
- $\langle , \rangle$ represents a non-degenerate invariant bilinear form taking values on the Lie algebra space.
- The Equation of motion is: $F \equiv dA + [A, A] = 0$.
- The solutions are pure gauge $A = G^{-1} dG$ where G is a Lie group element.

Pure Chern-Simons theory is given as,

$$S_{CS} = \frac{k}{4\pi} \int_M \langle A, dA + \frac{2}{3} A^2 \rangle$$

- k is level of the theory, M is the spacetime manifold.
- A is a Lie-algebra-valued one form : $A = A^a_\mu T_a dx^\mu$.
- $\langle , \rangle$ represents a non-degenerate invariant bilinear form taking values on the Lie algebra space.
- The Equation of motion is: $F \equiv dA + [A, A] = 0$.
- The solutions are pure gauge $A = G^{-1} dG$ where G is a Lie group element.

Background : Chern-Simons Theory in 3D

Pure Chern-Simons theory is given as,

$$S_{CS} = \frac{k}{4\pi} \int_M \langle A, dA + \frac{2}{3} A^2 \rangle$$

- k is level of the theory, M is the spacetime manifold.
- A is a Lie-algebra-valued one form : $A = A_\mu^a T_a dx^\mu$.
- $\langle , \rangle$ represents a non-degenerate invariant bilinear form taking values on the Lie algebra space.
- The Equation of motion is: $F \equiv dA + [A, A] = 0$.
- The solutions are pure gauge $A = G^{-1} dG$ where G is a Lie group element.

Chern-Simons Theory in 3D :

- Onshell CS theory does not have any bulk dynamical dof and hence it is a topological field theory.
- For gauge-fixing condition $\partial_\phi A_r = 0$, the solution looks like

$$A = b^{-1}(a + d)b, \quad A_r(r) = b(r)^{-1}\partial_r b(r),$$

- Here $a(\phi, u)$ represents the residual part of the gauge field that can not be fixed by eoms.
- Conserved charges that correspond to the residual global part of the gauge symmetry will produce a centrally extended symmetry algebra at the boundary. Brown Heanneaux.

Chern-Simons Theory in 3D :

- Onshell CS theory does not have any bulk dynamical dof and hence it is a topological field theory.
- For gauge-fixing condition $\partial_\phi A_r = 0$, the solution looks like

$$A = b^{-1}(a + d)b, \quad A_r(r) = b(r)^{-1}\partial_r b(r),$$

- Here $a(\phi, u)$ represents the residual part of the gauge field that can not be fixed by eoms.
- Conserved charges that correspond to the residual global part of the gauge symmetry will produce a centrally extended symmetry algebra at the boundary. Brown Heanneaux.

Pure 3D gravity as a CS Theory :

- 3D pure gravity is also a topological theory. The symmetry group here is non-compact group $ISO(2,1)$.
- The non-zero commutators of the Lie-algebra are:

$$[J_a, J_b] = \epsilon_{abc} J^c, \; [J_a, P_b] = \epsilon_{abc} P^c.$$

- Expanding the CS gauge field as $A_\mu = e^a_\mu P_a + \omega^a_\mu J_a$, the CS action boils down to

$$S = \frac{1}{16\pi G} \int 2e^a R_a \,, \quad R^a = \mathrm{d}\omega^a + \frac{1}{2} \varepsilon^a{}_{bc}\, \omega^b \omega^c \,,$$

- $k = \frac{1}{4G}$, and $M = \Sigma \times R$.
- Classical equivalence !

Classical symmetries are also identical.

- A generic gauge transformation in CS Theory by parameter $\Lambda = \zeta^a P_a + \Upsilon^a J_a$ would be

$$\delta A_\mu = \partial_\mu \Lambda + [A_\mu, \Lambda].$$

- This gauge transformation of Chern Simons theory is identical to local Lorentz and diffeomorphism transformation of 3D Gravity On-shell.
- Thus the residual boundray symmetry of CS theory must identically imply the same for the corresponding gravity theory.
- The algebra of the corresponding global charges at null infinity is identified with BMS_3.

Pure 3D gravity as a CS Theory : Further similarities

Classical symmetries are also identical.

- A generic gauge transformation in CS Theory by parameter $\Lambda = \zeta^a P_a + \Upsilon^a J_a$ would be

$$\delta A_\mu = \partial_\mu \Lambda + [A_\mu, \Lambda].$$

- This gauge transformation of Chern Simons theory is identical to local Lorentz and diffeomorphism transformation of 3D Gravity On-shell.
- Thus the residual boundray symmetry of CS theory must identically imply the same for the corresponding gravity theory.
- The algebra of the corresponding global charges at null infinity is identified with BMS_3.

Pure 3D gravity as a CS Theory : Further similarities

Classical symmetries are also identical.

- A generic gauge transformation in CS Theory by parameter $\Lambda = \zeta^a P_a + \Upsilon^a J_a$ would be

$$\delta A_\mu = \partial_\mu \Lambda + [A_\mu, \Lambda].$$

- This gauge transformation of Chern Simons theory is identical to local Lorentz and diffeomorphism transformation of 3D Gravity On-shell.
- Thus the residual boundray symmetry of CS theory must identically imply the same for the corresponding gravity theory.
- The algebra of the corresponding global charges at null infinity is identified with BMS_3.

Asymptotic symmetry of Gravity theory Using CS formulation :

We denote the three manifold at null infinity by (u, r, ϕ).

- The fallof of the residual gauge field at null infinity is :

$$a = [\sqrt{2}J_1 + \frac{\pi}{k}\mathcal{P}J_0 + \frac{\pi}{k}\mathcal{J}P_0]d\phi + [\sqrt{2}P_1 + \frac{\pi}{k}\mathcal{P}P_0]du,$$

- This gives Bondi metric :

$$ds^2 = \mathcal{P}du^2 - 2dudr - \mathcal{J}dud\phi + r^2 d\phi^2$$

- The gauge symmetry transformation parameter has the form ,

$$\Lambda = \zeta^a P_a + \Upsilon^a J_a,$$

- $\mathcal{P}, \mathcal{J}, \zeta, \Upsilon$ are functions of (u, ϕ) and are constrained by,

$$da + \frac{1}{2}[a, a] = 0, \delta a = d\Lambda + [a, \Lambda].$$

- The global charges in this gauge theory are defined as ,

$$\delta Q(\Lambda) = -\frac{\kappa}{4\pi}\int < \Lambda, \delta A_\phi > d\phi.$$

Asymptotic symmetry of Gravity theory Using CS formulation :

We denote the three manifold at null infinity by (u, r, ϕ).

- The fallof of the residual gauge field at null infinity is :

$$a = [\sqrt{2}J_1 + \frac{\pi}{k}\mathcal{P}J_0 + \frac{\pi}{k}\mathcal{J}P_0]d\phi + [\sqrt{2}P_1 + \frac{\pi}{k}\mathcal{P}P_0]du,$$

- This gives Bondi metric :

$$ds^2 = \mathcal{P}du^2 - 2dudr - \mathcal{J}dud\phi + r^2 d\phi^2$$

- The gauge symmetry transformation parameter has the form ,

$$\Lambda = \zeta^a P_a + \Upsilon^a J_a,$$

- $\mathcal{P}, \mathcal{J}, \zeta, \Upsilon$ are functions of (u, ϕ) and are constrained by,

$$da + \frac{1}{2}[a, a] = 0, \delta a = d\Lambda + [a, \Lambda].$$

.

- The global charges in this gauge theory are defined as ,

$$\delta Q(\Lambda) = -\frac{\kappa}{4\pi}\int < \Lambda, \delta A_\phi > d\phi.$$

Asymptotic symmetry of Gravity theory Using CS formulation :

We denote the three manifold at null infinity by (u, r, ϕ).

- The fallof of the residual gauge field at null infinity is :

$$a = [\sqrt{2}J_1 + \frac{\pi}{k}\mathcal{P}J_0 + \frac{\pi}{k}\mathcal{J}P_0]d\phi + [\sqrt{2}P_1 + \frac{\pi}{k}\mathcal{P}P_0]du,$$

- This gives Bondi metric :

$$ds^2 = \mathcal{P}du^2 - 2dudr - \mathcal{J}dud\phi + r^2 d\phi^2$$

- The gauge symmetry transformation parameter has the form ,

$$\Lambda = \zeta^a P_a + \Upsilon^a J_a,$$

- $\mathcal{P}, \mathcal{J}, \zeta, \Upsilon$ are functions of (u, ϕ) and are constrained by,

$$da + \frac{1}{2}[a, a] = 0, \delta a = d\Lambda + [a, \Lambda].$$

.

- The global charges in this gauge theory are defined as ,

$$\delta Q(\Lambda) = -\frac{\kappa}{4\pi}\int < \Lambda, \delta A_\phi > d\phi.$$

Asymptotic symmetry of Gravity theory Using CS formulation :

- Integrating the above relation one gets the conserved charges that generate the asymptotic symmetry algebra as :

$$\{\mathcal{Q}[\lambda_1], \mathcal{Q}[\lambda_2]\}_{PB} = \delta_{\lambda_1} \mathcal{Q}[\lambda_2] \; ,$$

- Defining modes for various independent components of the residual gauge fields, one finally gets the infinite dimensional symmetry algebra of the conserved charges.

$$[J_m, J_n] = (m-n)\, J_{m+n}, \quad [P_m, P_n] = 0$$
$$[J_m, P_n] = (m-n)\, P_{m+n} + \frac{c_2}{12}\; m^3\, \delta_{m+n,0}.$$

- This is BMS_3 algebra. Here J_m are supertranslation generators and P_m are superotation generators.

3D Flat Gravity theory From CS formulation :

- Thus CS formulation of 3D gravity has been a very useful tool to understand various properties of gravity.
- This was first established by Witten and later rigorously used by Henneaux mostly in the context of asymptotically AdS gravity theories.
- In the past decade, Barnich et.al. started using the same tool for understanding asymptotic properties of pure Gravity as centrally extended BMS_3 symmetry.
- The technique is highly successful by now and all possible supersymmetric and higher-spin extension of BMS_3 are known. Barnich et.al, Troncoso et.al, Grumiller et.al, Banerjee et.al

Alternate way to get BMS_3 algebra :

- Alternately, BMS_3 algebra can be obtained as a flat limit of AdS_3 asymptotic algebra, namely the Virasoro algebra

$$[L^{\pm}_m, L^{\pm}_n] = (m-n)\, L^{\pm}_{m+n} + \frac{c^{\pm}}{12}\, m^3\, \delta_{m+n,0}$$

- Redefine generators and central charges as

$$P_m = \lim_{\epsilon \to 0} \epsilon\,(L^+_m + L^-_{-m})\,, \quad J_m = \lim_{\epsilon \to 0} (L^+_m - L^-_{-m})$$

$$c_1 = \lim_{\epsilon \to 0}(c^+ - c^-)\,, \quad c_2 = \lim_{\epsilon \to 0} \epsilon\,(c^+ + c^-)$$

- The contracted algebra looks like,
$[J_m, J_n] = (m-n)\, J_{m+n} + \frac{c_1}{12}\, m^3\, \delta_{m+n,0}$,
$[J_m, P_n] = (m-n)\, P_{m+n} + \frac{c_2}{12}\, m^3\, \delta_{m+n,0}$, $[P_m, P_n] = 0$.

Alternate way to get BMS_3 algebra :

- Alternately, BMS_3 algebra can be obtained as a flat limit of AdS_3 asymptotic algebra, namely the Virasoro algebra

$$[L^{\pm}_m, L^{\pm}_n] = (m-n)\, L^{\pm}_{m+n} + \frac{c^{\pm}}{12}\, m^3\, \delta_{m+n,0}$$

- Redefine generators and central charges as

$$P_m = \lim_{\epsilon\to 0} \epsilon\, (L^{+}_m + L^{-}_{-m})\,, \quad J_m = \lim_{\epsilon\to 0} (L^{+}_m - L^{-}_{-m})$$

$$c_1 = \lim_{\epsilon\to 0}(c^{+} - c^{-})\,, \quad c_2 = \lim_{\epsilon\to 0} \epsilon\, (c^{+} + c^{-})$$

- The contracted algebra looks like,
$[J_m, J_n] = (m-n)\, J_{m+n} + \frac{c_1}{12}\, m^3\, \delta_{m+n,0}$,
$[J_m, P_n] = (m-n)\, P_{m+n} + \frac{c_2}{12}\, m^3\, \delta_{m+n,0}$, $[P_m, P_n] = 0$.

Alternate way to get BMS_3 algebra :

- Alternately, BMS_3 algebra can be obtained as a flat limit of AdS_3 asymptotic algebra, namely the Virasoro algebra

$$[L^{\pm}_m, L^{\pm}_n] = (m-n)\, L^{\pm}_{m+n} + \frac{c^{\pm}}{12}\, m^3\, \delta_{m+n,0}$$

- Redefine generators and central charges as

$$P_m = \lim_{\epsilon\to 0} \epsilon\,(L^+_m + L^-_{-m})\;, \quad J_m = \lim_{\epsilon\to 0} (L^+_m - L^-_{-m})$$

$$c_1 = \lim_{\epsilon\to 0}(c^+ - c^-)\;, \quad c_2 = \lim_{\epsilon\to 0}\, \epsilon\,(c^+ + c^-)$$

- The contracted algebra looks like,
$[J_m, J_n] = (m-n)\, J_{m+n} + \frac{c_1}{12}\, m^3\, \delta_{m+n,0}$,
$[J_m, P_n] = (m-n)\, P_{m+n} + \frac{c_2}{12}\, m^3\, \delta_{m+n,0}$, $[P_m, P_n] = 0$.

Supersymmetric extensions of BMS_3 algebra

- In JHEP 1606 (2016) 024 and JHEP 1611 (2016) 059 we presented all possible supersymmetric extensions of BMS_3 algebra along with their free field realisations , using the contraction technique.
- There are distinct ways of taking the limit and all corresponds to valid two dimensional algebras.
- Imposing unitarity constraints puts certain restrictions to its validity, but still one gets multiple infinite algebras corresponding to a fixed number of supercharges.
- Thus a direct asymptotic symmetry analysis of the corresponding supergravity theory (using CS formulation) is required to uniquely identify the symmetry algebra.
- In JHEP 1911 (2019) 122, Phys.Rev. D96 (2017) no.6, 066029, JHEP 1901 (2019) 115 we have presented the proper analysis to identify the right superBMS_3 algebras.

Supersymmetric extensions of BMS_3 algebra

- In JHEP 1606 (2016) 024 and JHEP 1611 (2016) 059 we presented all possible supersymmetric extensions of BMS_3 algebra along with their free field realisations , using the contraction technique.
- There are distinct ways of taking the limit and all corresponds to valid two dimensional algebras.
- Imposing unitarity constraints puts certain restrictions to its validity, but still one gets multiple infinite algebras corresponding to a fixed number of supercharges.
- Thus a direct asymptotic symmetry analysis of the corresponding supergravity theory (using CS formulation) is required to uniquely identify the symmetry algebra.
- In JHEP 1911 (2019) 122, Phys.Rev. D96 (2017) no.6, 066029, JHEP 1901 (2019) 115 we have presented the proper analysis to identify the right superBMS_3 algebras.

Supersymmetric extensions of BMS_3 algebra

- In JHEP 1606 (2016) 024 and JHEP 1611 (2016) 059 we presented all possible supersymmetric extensions of BMS_3 algebra along with their free field realisations , using the contraction technique.
- There are distinct ways of taking the limit and all corresponds to valid two dimensional algebras.
- Imposing unitarity constraints puts certain restrictions to its validity, but still one gets multiple infinite algebras corresponding to a fixed number of supercharges.
- Thus a direct asymptotic symmetry analysis of the corresponding supergravity theory (using CS formulation) is required to uniquely identify the symmetry algebra.
- In JHEP 1911 (2019) 122, Phys.Rev. D96 (2017) no.6, 066029, JHEP 1901 (2019) 115 we have presented the proper analysis to identify the right superBMS_3 algebras.

Supersymmetric extensions of BMS_3 algebra

- In JHEP 1606 (2016) 024 and JHEP 1611 (2016) 059 we presented all possible supersymmetric extensions of BMS_3 algebra along with their free field realisations , using the contraction technique.
- There are distinct ways of taking the limit and all corresponds to valid two dimensional algebras.
- Imposing unitarity constraints puts certain restrictions to its validity, but still one gets multiple infinite algebras corresponding to a fixed number of supercharges.
- Thus a direct asymptotic symmetry analysis of the corresponding supergravity theory (using CS formulation) is required to uniquely identify the symmetry algebra.
- In JHEP 1911 (2019) 122, Phys.Rev. D96 (2017) no.6, 066029, JHEP 1901 (2019) 115 we have presented the proper analysis to identify the right superBMS_3 algebras.

- In JHEP 1606 (2016) 024 and JHEP 1611 (2016) 059 we presented all possible supersymmetric extensions of BMS_3 algebra along with their free field realisations , using the contraction technique.
- There are distinct ways of taking the limit and all corresponds to valid two dimensional algebras.
- Imposing unitarity constraints puts certain restrictions to its validity, but still one gets multiple infinite algebras corresponding to a fixed number of supercharges.
- Thus a direct asymptotic symmetry analysis of the corresponding supergravity theory (using CS formulation) is required to uniquely identify the symmetry algebra.
- In JHEP 1911 (2019) 122, Phys.Rev. D96 (2017) no.6, 066029, JHEP 1901 (2019) 115 we have presented the proper analysis to identify the right superBMS_3 algebras.

Asymptotic symmetry of $\mathcal{N} = (2,0)$ Sugra :

Let us consider $\mathcal{N} = (2,0)$ Super-Poincaré algebra as the gauge algebra for CS theory:

$$
\begin{aligned}
[J_a, J_b] &= \epsilon_{abc} J^c \qquad [J_a, P_b] = \epsilon_{abc} P^c \\
[J_a, Q^i_\alpha] &= \tfrac{1}{2}(\Gamma^a)^\beta_\alpha Q^i_\beta \qquad [Q^i_\alpha, T] = \epsilon^{ij} Q^j_\alpha \\
\{Q^i_\alpha, Q^j_\beta\} &= -\tfrac{1}{2}\delta^{ij}(C\Gamma^a)_{\alpha\beta} P_a + C_{\alpha\beta}\epsilon^{ij} Z.
\end{aligned}
$$

- The non-degenerate invariant bilinears are :

$$
< J_a, P_b >= \eta_{ab}, < J_a, J_b >= \mu\eta_{ab}, < Q^I_\alpha, Q^J_\beta >= \delta^{IJ} C_{\alpha\beta},
$$

$$
< T, Z >= -1, < T, T >= \bar{\mu}.
$$

- The lie algebra valued gauge field :

$$
A = e^a P_a + \hat{\omega}^a J_a + \psi^\alpha_i Q^i_\alpha + BT + CZ
$$

- With this identification, the CS action reduces to

$$
I[A] = \frac{k}{4\pi}\int [2e^a \hat{R}_a + \mu L(\hat{\omega}_a) - \bar{\psi}^i_\beta \nabla \psi^\beta_i - 2BdC + \bar{\mu} BdB]
$$

Asymptotic symmetry of $\mathcal{N} = (2,0)$ Sugra:JHEP 1911

- The three manifold at null infinity has coordinates (u, r, ϕ).
- The asymptotic fallof of the residual gauge field :

$$
\begin{aligned}
a =& \sqrt{2}[J_1 + \frac{\pi}{k}(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2)J_0 + \frac{\pi}{k}(\mathcal{J} + \frac{2\pi}{k}\tau\mathcal{Z})P_0 - \frac{\pi}{k}\psi_i Q^i_+ \\
& - \frac{2\pi}{k}\mathcal{Z}T - \frac{2\pi}{k}\tau Z]d\phi + [\sqrt{2}P_1 + \frac{8\pi}{k}\mathcal{Z}Z + \frac{\pi}{k}(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2)P_0]du,
\end{aligned}
$$

- The gauge symmetry transformation parameter has the form ,

$$
\Lambda = \zeta^n P_n + \Upsilon^n J_n + \lambda T + \tilde{\lambda} Z + \zeta^i_+ Q^i_+ + \zeta^i_- Q^i_- \ ,
$$

- $\mathcal{P}, \mathcal{J}, \mathcal{Z}, \tau, \psi_i, \zeta, \Upsilon, \lambda, \tilde{\lambda}, \zeta^i_\pm$ are functions of (u, ϕ).

For $\mathcal{N}=(2,0)$ asymptotically flat Supergravity theory, the asymptotic symmetry algebra looks as :

$$
\begin{aligned}
&[P_n, J_m] = (n-m)P_{n+m} + n^3 k\ \delta_{n+m,0}, \quad [J_n, J_m] = (n-m)J_{n+m} + n^3 \mu\ k\ \delta_{n+m,0} \\
&[P_n, R_m] = -4mS_{n+m}, \quad [J_n, R_m] = -mR_{n+m}, \quad [J_n, S_m] = -mS_{n+m} \\
&[R_n, S_m] = n\ k\ \delta_{n+m,0}, \quad [R_n, R_m] = n\ \bar{\mu}\ k\ \delta_{n+m,0} \\
&[J_n, \mathcal{G}^i_m] = \left(\frac{n}{2} - m\right) \mathcal{G}^i_{n+m}, \quad (i = 1, 2) \\
&[R_n, \mathcal{G}^1_m] = \mathcal{G}^1_{n+m}, \quad [R_n, \mathcal{G}^2_m] = -\mathcal{G}^2_{n+m} \\
&\{\mathcal{G}^1_n, \mathcal{G}^2_m\} = P_{n+m} + 2kn^2\delta_{n+m,0} + (n-m)S_{n+m}
\end{aligned}
$$

This is the most generic $\mathcal{N}=(2,0) BMS_3$ algebra.

For $\mathcal{N}=(2,0)$ asymptotically flat Supergravity theory, the asymptotic symmetry algebra looks as :

$$
\begin{aligned}
&[P_n, J_m] = (n-m)P_{n+m} + n^3 k\, \delta_{n+m,0}, \quad [J_n, J_m] = (n-m)J_{n+m} + n^3 \mu\, k\, \delta_{n+m,0} \\
&[P_n, R_m] = -4mS_{n+m}, \quad [J_n, R_m] = -mR_{n+m}, \quad [J_n, S_m] = -mS_{n+m} \\
&[R_n, S_m] = n\, k\, \delta_{n+m,0}, \quad [R_n, R_m] = n\, \bar{\mu}\, k\, \delta_{n+m,0} \\
&[J_n, \mathcal{G}^i_m] = \left(\frac{n}{2} - m\right) \mathcal{G}^i_{n+m}, \quad (i = 1,2) \\
&[R_n, \mathcal{G}^1_m] = \mathcal{G}^1_{n+m}, \quad [R_n, \mathcal{G}^2_m] = -\mathcal{G}^2_{n+m} \\
&\{\mathcal{G}^1_n, \mathcal{G}^2_m\} = P_{n+m} + 2kn^2 \delta_{n+m,0} + (n-m)S_{n+m}
\end{aligned}
$$

This is the most generic $\mathcal{N}=(2,0)BMS_3$ algebra.

Nontrivial Bosonic solutions and their Thermodynamic properties : JHEP 1901

Can there be generic asymptotically flat bosonic solutions ? YES !

- Since asymptotic symmetry is fixed, we keep a_ϕ unchanged.
- Time evolution of a_ϕ is similar to gauge transformation, thus a_u must have similar form as gauge transformation parameter.
- Incorporate the chemical potentials into the system to give vacuum expectation value to the time component of the gauge field a_u.
- Finally one fixes the chemical potentials by demanding regularity of the solution.

The stationary circular symmetric metric takes the form

$$ds^2 = (\mathcal{P}' + r^2\mu_J^2)du^2 - 2\mu_P dudr - (\mathcal{J}' + 2r^2\mu_J^2)dud\phi + r^2 d\phi^2.$$

Csmological solutions, with a cosmological horizon and entropy.

Nontrivial Bosonic solutions and their Thermodynamic properties : JHEP 1901

Can there be generic asymptotically flat bosonic solutions ? YES !

- Since asymptotic symmetry is fixed, we keep a_ϕ unchanged.
- Time evolution of a_ϕ is similar to gauge transformation, thus a_u must have similar form as gauge transformation parameter.
- Incorporate the chemical potentials into the system to give vacuum expectation value to the time component of the gauge field a_u.
- Finally one fixes the chemical potentials by demanding regularity of the solution.

The stationary circular symmetric metric takes the form

$$ds^2 = (\mathcal{P}' + r^2\mu_J^2)du^2 - 2\mu_P du dr - (\mathcal{J}' + 2r^2\mu_J^2)du d\phi + r^2 d\phi^2.$$

Csmological solutions, with a cosmological horizon and entropy.

Nontrivial Bosonic solutions and their Thermodynamic properties : JHEP 1901

Can there be generic asymptotically flat bosonic solutions ? YES !

- Since asymptotic symmetry is fixed, we keep a_ϕ unchanged.
- Time evolution of a_ϕ is similar to gauge transformation, thus a_u must have similar form as gauge transformation parameter.
- Incorporate the chemical potentials into the system to give vacuum expectation value to the time component of the gauge field a_u.
- Finally one fixes the chemical potentials by demanding regularity of the solution.

The stationary circular symmetric metric takes the form

$$ds^2 = (\mathcal{P}' + r^2\mu_J^2)du^2 - 2\mu_P du dr - (\mathcal{J}' + 2r^2\mu_J^2)du d\phi + r^2 d\phi^2.$$

Csmological solutions, with a cosmological horizon and entropy.

Outlook so far and Open Questions

So far we have understood :

- CS formulation of 3D gravity can be used to understand the asymptotic symmetry structure of space time at null infinity.
- New bosonic solutions can be obtained that has a different topology than the minkowski space-time.
- The dynamics of these solutions lies in the boundary of the 3D manifold M.

Can we find the boundary theory ? Will it be dual to 3D gravity ?

- We shall exploit the equivalence of CS theory and the Wess-Zumino-Witten models.

Outlook so far and Open Questions

So far we have understood :

- CS formulation of 3D gravity can be used to understand the asymptotic symmetry structure of space time at null infinity.
- New bosonic solutions can be obtained that has a different topology than the minkowski space-time.
- The dynamics of these solutions lies in the boundary of the 3D manifold M.

Can we find the boundary theory ? Will it be dual to 3D gravity ?

- We shall exploit the equivalence of CS theory and the Wess-Zumino-Witten models.

Outlook so far and Open Questions

So far we have understood :

- CS formulation of 3D gravity can be used to understand the asymptotic symmetry structure of space time at null infinity.
- New bosonic solutions can be obtained that has a different topology than the minkowski space-time.
- The dynamics of these solutions lies in the boundary of the 3D manifold M.

Can we find the boundary theory ? Will it be dual to 3D gravity ?

- We shall exploit the equivalence of CS theory and the Wess-Zumino-Witten models.

CS $\iff$ WZW equivalence

A WZW model is a 2D non-linear sigma model with a 3D term:

$$S_{WZW} = \frac{1}{4a^2}\int_{\partial\Sigma} d^2x Tr[\partial^\mu g \partial_\mu g^{-1}] + \kappa \int_\Sigma Tr[(G^{-1}dG)^3]$$

- a, κ are constants. g is dynamical matrix valued field on $\partial\Sigma$.
- G is the extension of g to the volume Σ.
- It posses Conserved Currents $J(z) = -\kappa\partial_z g g^{-1} = \sum_a \mathcal{J}^a T_a$ which gives rise to Current Algebra.
- Bilinears of these currents gives energy-momentum tensor through Sugawara Construction.

$$T(z) = \sum_{a,b} K^{-1}_{ab}(\mathcal{J}^a\mathcal{J}^b)(z)$$

A WZW model is a 2D non-linear sigma model with a 3D term:

$$S_{WZW} = \frac{1}{4a^2} \int_{\partial\Sigma} d^2x Tr[\partial^\mu g \partial_\mu g^{-1}] + \kappa \int_\Sigma Tr[(G^{-1} dG)^3]$$

- a, κ are constants. g is dynamical matrix valued field on $\partial\Sigma$.
- G is the extension of g to the volume Σ.
- It posses Conserved Currents $J(z) = -\kappa \partial_z g g^{-1} = \sum_a \mathcal{J}^a T_a$ which gives rise to Current Algebra.
- Bilinears of these currents gives energy-momentum tensor through Sugawara Construction.

$$T(z) = \sum_{a,b} K^{-1}_{ab} (\mathcal{J}^a \mathcal{J}^b)(z)$$

CS $\Longleftrightarrow$ WZW equivalence

- CS theory on a manifold with boundary generically imposes boundary conditions on the gauge field.
- Boundary conditions of $A \longmapsto$ Constraints in Phase space
- With Identification $A = G^{-1}dG$ along with the constraints, it can be shown that onshell:

$$S_{CS} + S_{bdy} = S_{WZW}$$

- The asymptotic gauge field of the corresponding CS theory is highly constrained at the boundary:

$$e^a_u = \omega^a_\phi, \qquad \omega^a_u = 0, \qquad \psi^\pm_{Iu} = 0, \qquad B_u = 0, \qquad -4B_\phi = C_u.$$

- These requires the action to be improved by a boundary term:

$$S_{imp} = S_{CS} - \frac{k}{4\pi}\int_{\partial M} dud\phi[\omega^a_\phi \omega_{a\phi} + 4B^2_\phi]$$

- Next we evaluate the above action on a general solutions of EOM for the component fields.

- A generic gauge fixed solution looks as,

$$
\begin{aligned}
\Lambda &= \lambda(u,\phi)\zeta(u,r) \\
\tilde{B} &= a(u,\phi) + \tilde{a}(u,r), \quad \tilde{C} = c(u,\phi) + \tilde{c}(u,r) + \bar{d}_2\lambda\tilde{d}_1 - \bar{d}_1\lambda\tilde{d}_2 \\
\eta_1 &= e^{ia}(\lambda\tilde{d}_1(u,r) + d_1(u,\phi)), \quad \eta_2 = e^{-ia}(\lambda\tilde{d}_2(u,r) + d_2(u,\phi)) \\
b &= \lambda E(u,r)\lambda^{-1} - \frac{1}{2}(d_1\bar{\tilde{d}}_2\lambda^{-1} - d_1\bar{\tilde{d}}_2\lambda^{-1}\mathbf{I}) - \frac{1}{2}(d_2\bar{\tilde{d}}_1\lambda^{-1} - d_2\bar{\tilde{d}}_1\lambda^{-1}\mathbf{I}) + F(u,\phi),
\end{aligned}
$$

- Putting this in the improved action, we get

$$
\begin{aligned}
I =& \frac{k}{4\pi}\{\int du d\phi Tr[-4\dot{\lambda}\lambda^{-1}\beta' - 2(\lambda^{-1}\lambda')^2 + 2\mu\lambda^{-1}\lambda'\lambda^{-1}\dot{\lambda} \\
& - 2(\bar{d}_1{}'\dot{d}_2 + \bar{d}_2{}'\dot{d}_1) - 2ia'(\bar{d}_{1\alpha}\dot{d}_2 - \bar{d}_{2\alpha}\dot{d}_1 + \dot{\lambda}\lambda^{-1}(d_2\bar{d}_1 - d_1\bar{d}_2)) - 4(a')^2 \\
& - 4\dot{\lambda}\lambda^{-1}(\frac{1}{2}(d_1\bar{d}_2' - d_1\bar{d}_2{}'\mathbf{I}) + \frac{1}{2}(d_2\bar{d}_1' - d_2\bar{d}_1{}'\mathbf{I})) \\
& + 2i\dot{a}\bar{C}' + \bar{\mu}a'\dot{a}] + \frac{2\mu}{3}\int Tr[(d\Lambda\Lambda^{-1})^3]\}
\end{aligned}
$$

- The above action describes a chiral WZW model with gauge group supersymmetric $ISO(2,1)$.

Some properties of the dual Chiral WZW Model :

- The gauge and global symmetries of the above Chiral WZW model can be obtained.
- In particular the global symmetry corresponds to a current algebra.
- With proper modified Sugawara construction, the corresponding Stress tensor can be obtained.
- It can further be shown that the properly constructed current bilinears actually forms the $\mathcal{N} = (2,0) BMS_3$ algebra that we obtianed earlier.
- To construct these bilinears, one has to further use the intricate details of the boundary gauge fields.

Equivalent form of the Dual Theory : Phys.Rev. D100 (2019)

- The dual WZW model needs to be gauged to get the correct $\mathcal{N} = (2,0) BMS_3$ isometry.
- Thus the phase space and the dynamics get reduced .
- With proper phase reduction, the gauged WZW model reduces to a flat limit of a (super)Liouville like theory.

Outlook and Open questions :

Outlook: Thus, we have constructed a two dimensional theory, sitting at the null infinity of 3D space time that has

- exactly the same isometries as the asymptotic symmetry of the bulk.
- describes the same classical solutions as that of the residual dofs of 3D gravity.
- can describe the dynamics of these configurations.

Hence, we conclude that the Liouville like theories as constructed above are the 3D gravity duals.

Outlook and Open questions :

Outlook: Thus, we have constructed a two dimensional theory, sitting at the null infinity of 3D space time that has

- exactly the same isometries as the asymptotic symmetry of the bulk.
- describes the same classical solutions as that of the residual dofs of 3D gravity.
- can describe the dynamics of these configurations.

Hence, we conclude that the Liouville like theories as constructed above are the 3D gravity duals.

Outlook and Open questions :

Open Questions:

- What can we say at the semi-classical level ?
- can String theory help ? can Vortex operators in string theory in flat space time be identified with the global dofs ?
- equivalence of string S-matrix with correlations functions on Liouville like field theory ?

Thank You

Open Questions:

- What can we say at the semi-classical level ?
- can String theory help ? can Vortex operators in string theory in flat space time be identified with the global dofs ?
- equivalence of string S-matrix with correlations functions on Liouville like field theory ?

Thank You