Generalized scalar tensor theories and spinning black holes in three dimensions

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in collaboration with Moisés Bravo-Gaete and Mokhtar Hassaine

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Why three dimensions?

Studying gravity in three dimensions

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- Particularly quantum gravity simplifies drastically in 3D which makes it more interesting to study.
- 3D gravity provides nice laboratory to study the AdS/CFT correspondence.

Scalar-Tensor theories in general

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- In physics it is common to modify theories, using the correspondence principle, to explain new phenomena. Hence it is reasonable to study modified theories of gravity.

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- In physics it is common to modify theories, using the correspondence principle, to explain new phenomena. Hence it is reasonable to study modified theories of gravity.
- Scalar tensor theories constitute one of the simplest extensions/modifications of General Relativity.

Lovelock, Horndeski and DHOST

A brief history

- In 1971 David Lovelock showed that (under certain assumptions like vanishing torsion, etc) the most general action constructed from the metric yielding at most second order field equations in $D \leq 4$ is the Einstein-Hilbert action.

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- His student, Gregory Horndeski, then determined in 1974 the most general such action constructed from the metric tensor and a scalar field.
- The requirement of having at most second order field equations is to avoid so-called Ostrogradsky instabilities (ghosts), which are extra degrees of freedom with negative energy.

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- His student, Gregory Horndeski, then determined in 1974 the most general such action constructed from the metric tensor and a scalar field.
- In 2015 David Langlois and Karim Noui introduced the so-called Degenerate Higher-Order Scalar-Tensor theories, which are of higher order yet still avoid the Ostrogradsky ghosts.

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The model

Action

$$
\begin{aligned}
S &= \int d^3x\sqrt{-g}\mathcal{L} \\
&= \int d^3x\sqrt{-g}\Big[Z(X) + G(X)R \\
&\quad + A_2(X)\left((\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right) + A_3(X)\Box\phi\,\phi^{\mu}\phi_{\mu\nu}\phi^{\nu} \\
&\quad + A_4(X)\phi^{\mu}\phi_{\mu\nu}\phi^{\nu\rho}\phi_{\rho} + A_5(X)\left(\phi^{\mu}\phi_{\mu\nu}\phi^{\nu}\right)^2\Big]
\end{aligned}
$$

$$
\begin{aligned}
\phi_{\mu} &= \nabla_{\mu}\phi \\
\phi_{\mu\nu} &= \nabla_{\mu}\nabla_{\nu}\phi \\
X &= \phi_{\mu}\,\phi^{\mu}
\end{aligned}
$$

Symmetries of the action

Scalar field transformations

- Shift symmetry: $\phi \to \phi + \text{const.} \to$ Noether current
- Discrete symmetry: $\phi \to -\phi$

Symmetries of the action

Disformal transformation

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} + K(X)\phi_\mu\phi_\nu$$

Transforms one DHOST theory into another by mixing the coupling functions in the action, e.g.:

$$Z(X) \to Z(\tilde{X})(1+KX)^{-1/2},\ G(X) \to G(\tilde{X})(1+KX)^{-1/2},$$
$$A_2(X) \to A_2(\tilde{X})(1+KX)^{3/2} + G(\tilde{X})K(1+KX)^{1/2}.$$

Possibly can be used to encounter solutions of one theory by transforming those of another (work in progress).

Symmetries of the action

Kerr-Schild transformation

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = g^{(0)}_{\mu\nu} - a(x) l_\mu l_\nu,$$

with l_μ being a null and geodesic vector field w.r.t. both metrics.

Symmetries of the action

Kerr-Schild transformation

$$f(r) \to f(r) - a(r),\; H(r) \to H(r),\; k(r) \to k(r)$$

Ansatz

$$\begin{aligned} ds^2 &= -f(r)dt^2 + \frac{dr^2}{f(r)} + H^2(r)\left[d\theta - k(r)dt\right]^2 , \\ \phi &= \phi(r). \end{aligned}$$

Symmetries of the action

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$$f(r) \to f(r) - a(r),\ H(r) \to H(r),\ k(r) \to k(r)$$

Invariance of the action

- Action is left invariant under a Kerr-Schild transformation given that the function $a(r)$ satisfies a first order differential equation.
- In particular, if X is constant, the solution to this equation is $a(r) = M$, where M is a constant (mass term of the metric in 3D).

The road towards a solution

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Additional condition

$$\mathcal{Z}_2^2 - 2\mathcal{Z}_1\mathcal{Z}_3 = 0$$

$$
\begin{aligned}
\mathcal{Z}_1 &= G + XA_2, \\
\mathcal{Z}_2 &= 2A_2 + XA_3 + 4G_X, \\
\mathcal{Z}_3 &= A_3 + A_4 + XA_5.
\end{aligned}
$$

The road towards a solution

General strategy

- Insert ansatz into covariant equations of motion

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General strategy

- Insert ansatz into covariant equations of motion
- Equivalently one can vary the one dimensional Lagrangian with respect to f, k and ϕ
- Instead of using the e.o.m for ϕ, use Noether current $J^r = \text{const.}$
- Regularity implies $J_\mu J^\mu = (\text{const.})^2/f(r) \Rightarrow J^r = 0$

The road towards a solution

Equations of motion

$$\left(\mathcal{Z}_1 H^3 k'\right)' = 0$$

$$f' = -\frac{4fH'\mathcal{Z}_1\mathcal{Z}_2 X' + fH\mathcal{Z}_2^2 X'^2 + 4H^3 k'^2 \mathcal{Z}_1^2 - 8HZ\mathcal{Z}_1}{8H'\mathcal{Z}_1^2 + 2H\mathcal{Z}_1\mathcal{Z}_2 X'}$$

$$k\mathcal{Z}_2\left(\mathcal{Z}_1 H^3 k'\right)' + 4H\left[(\mathcal{Z}_1 Z)_X - Z\mathcal{Z}_2\right] = 0$$

Note that we impose $Z \neq 0$ in order to avoid degenerate equation.

The road towards a solution

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$$k\mathcal{Z}_2\left(\mathcal{Z}_1 H^3 k'\right)' + 4H\left[(\mathcal{Z}_1 Z)_X - Z\mathcal{Z}_2\right] = 0$$

$$H'' = 0$$

The road towards a solution

~~Solutions~~

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2\left(d\theta + N^\theta(r)dt\right)^2,$$

$$F = \left(\frac{Z}{2\mathcal{Z}_1}r^2 - M + \frac{J^2}{4r^2}\right), \quad N^\theta = \frac{J}{2r^2}.$$

Note that the solution is completely determined by the previously defined combinations $\mathcal{Z}_1$ and $\mathcal{Z}_2$, hence different functions in the action can lead to the same solution with effective cosmological constant $\Lambda_{\text{eff}} = -Z/2\mathcal{Z}_1$.

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Disformal transformation

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - \frac{K}{1+KX}\phi^\mu\phi^\nu, \quad \tilde{X} = \frac{X}{1+KX},$$

$$\mathcal{Z}_1 \to \mathcal{Z}_1(1+KX)^{\frac{1}{2}}, \quad \mathcal{Z} \to \frac{\mathcal{Z}}{(1+KX)^{\frac{1}{2}}}$$

The road towards a solution

~~Solutions~~

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Other properties

- Equations remain solved by this metric without imposing the condition $\mathcal{Z}_2^2 - 2\mathcal{Z}_1\mathcal{Z}_3 = 0$.
- The ansatz $\phi = qt + \psi(r) + L\theta$ admits the same metric as a solution.

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Euclidean method

First steps

- Obtain Euclidean continuation of the action through: $t = -i\tau$.

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- Obtain Euclidean continuation of the action through: $t = -i\tau$.
- To keep metric real introduce $J_{\text{Eucl}} = -iJ$ (J physical angular momentum).
- Avoid conical singularity by imposing periodic euclidean time with period $\beta = 1/T$, where T is the temperature:

$$T = \frac{F'(r)}{4\pi}\bigg|_{r=r_h} = \frac{1}{4\pi}\left(\frac{2r_h}{L^2} - \frac{J^2}{2r_h^3}\right)$$

$$L^2 = \frac{2\mathcal{Z}_1(X)}{Z(X)}$$

Euclidean method

General procedure

- Compute the euclidean action (up to a boundary term), which is of the form

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General procedure

- Compute the euclidean action (up to a boundary term), which is of the form

$$I_E = (\text{lots of terms}) + B_E.$$

- Fix boundary term by demanding that the action has an extremum, $\delta I_E = 0$.
- Compute boundary term at infinity and at the horizon and read off the thermodynamic quantities from the Gibbs free energy.

Results

Thermodynamic parameters

$$\begin{aligned}
\mathcal{S} &= 8\mathcal{Z}_1\pi^2 r_h, \\
\mathcal{M} &= 2\pi\mathcal{Z}_1 M = 2\pi\mathcal{Z}_1\left(\frac{r_h^2}{L^2} + \frac{J^2}{4r_h^2}\right), \\
\mathcal{J} &= -2\pi\mathcal{Z}_1 J, \qquad \Omega = -\frac{J}{2r_h^2}.
\end{aligned}$$

Gibbs free energy

$$I_E = \beta\mathcal{F} = \beta\mathcal{M} - \mathcal{S} - \beta\Omega\mathcal{J}$$

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First law of thermodynamics holds: $\mathrm{d}\mathcal{M} = T\mathrm{d}\mathcal{S} + \Omega\mathrm{d}\mathcal{J}$!

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Recall: $L^2 = 2\mathcal{Z}_1(X)/Z(X)$, so imposing $\mathcal{Z}_1 > 0$ and $Z > 0$ ensures positive mass and entropy solutions.

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$$

Note that the same results can be obtained through a generalized Cardy formula given that the theory admits a regular scalar soliton. The soliton is identified with the ground state of the theory and its mass is $\mathcal{M}_{\text{sol}} = -2\pi\mathcal{Z}_1$.

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Phase transition

Gibbs free energy (static case)

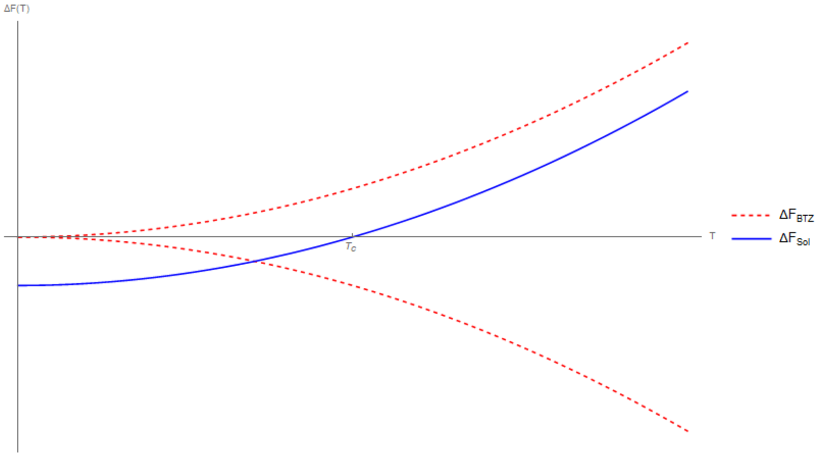
$$\Delta\mathcal{F}_{\text{BTZ}} = \mathcal{F}_{\text{BTZ}} - \mathcal{F} = 16\pi^3 T^2 \left[\frac{\mathcal{Z}_1^2(X)}{\mathcal{Z}(X)} - \frac{\mathcal{Z}_1^2(0)}{\mathcal{Z}(0)}\right]$$

$$\Delta\mathcal{F}_{\text{Sol}} = \mathcal{F}_{\text{Sol}} - \mathcal{F} = 16\pi^3 T^2 \frac{\mathcal{Z}_1^2(X)}{\mathcal{Z}(X)} - 2\pi\mathcal{Z}_1(X)$$

For the soliton there is a Hawking-Page phase transition at

$$T_c = \frac{\sqrt{2}}{4\pi}\sqrt{\frac{\mathcal{Z}(X)}{\mathcal{Z}_1(X)}}$$

Phase transition



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- The thermodynamic properties of the solution correspond exactly to what one would expect from a BTZ-like metric. Further it is consistent with the generalized Cardy formula in CFT.

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- The solution does only depend on a combination of the coupling functions, hence different functions can lead to the same solution.
- The thermodynamic properties of the solution correspond exactly to what one would expect from a BTZ-like metric. Further it is consistent with the generalized Cardy formula in CFT.
- As in the BTZ case there is a phase transition between the black hole and the soliton.

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- Study more thoroughly the disformal transformations.
- What is so special about the condition we imposed on the coupling functions?

That's all folks!

Thank you very much for your attention!