Phases and Stability of Non-Uniform Black Strings

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August 5, 2020

Based on:

Roberto Emparan, Keisuke Izumi, Raimon Luna, Ryotaku Suzuki, and Kentaro Tanabe, JHEP, vol. 06, p. 117, 2016. [arXiv:1602.05752 [hep-th]]

Roberto Emparan, Raimon Luna, Marina Martínez, Ryotaku Suzuki, and Kentaro Tanabe, JHEP, vol. 05, p. 104, 2018. [arXiv:1802.08191 [hep-th]]

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European Research Council

$$R_{\mu\nu} = 0$$

$$\mu, \nu = 0, \ldots, D-1$$

$$R_{\mu\nu} = 0^*$$

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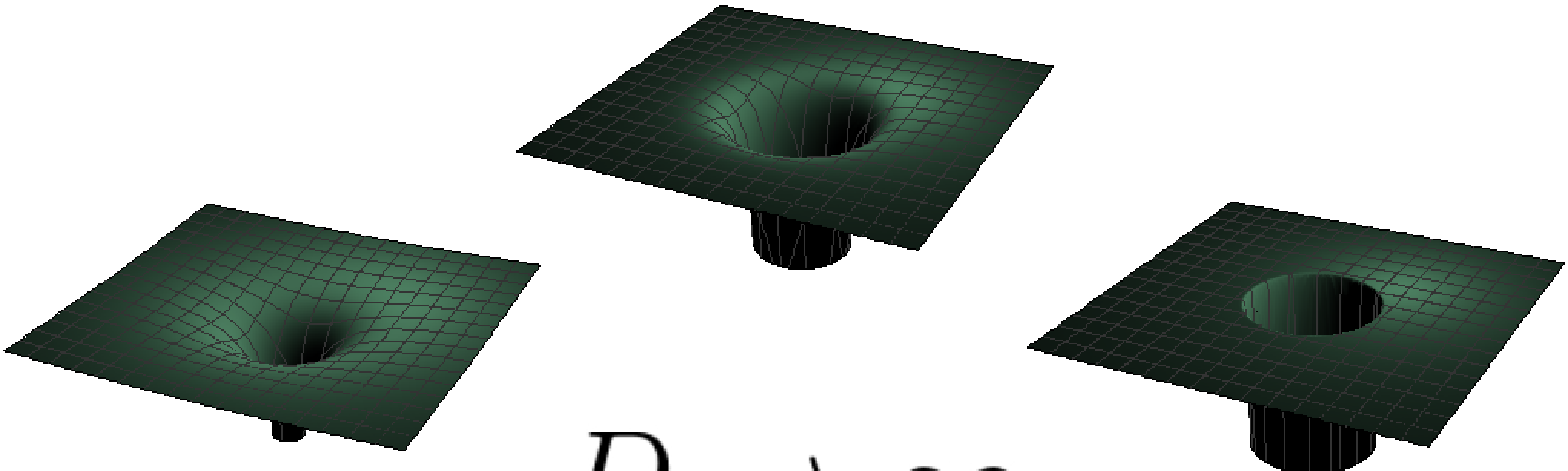
$$\mu, \nu = 0, \ldots, D-1$$

$$f = f_0 + \frac{f_1}{D} + \frac{f_2}{D^2} + \ldots$$



$$D \to \infty$$

Many directions to interact: Mean Field?
Infinite number of graviton polarizations

Schwarzschild-Tangherlini black brane (AF):

$$ds^2 = 2dtdr - \left(1 - \frac{r_o^n}{r^n}\right) dt^2 + \delta_{ij} d\sigma^i d\sigma^j + r^2 d\Omega_{n+1}$$

$$i, j = 1, \ldots, p$$

$$D = n + p + 3$$

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Natural Expansion Parameter

Large *D* ansatz (AF):

$$ds^2 = -2\left(u_t dt + \frac{u_i}{n} d\sigma^i\right) dr - A dt^2 - \frac{2}{n} C_i d\sigma^i dt + \frac{1}{n} G_{ij} d\sigma^i d\sigma^j + r^2 d\Omega_{n+1}$$

New radial coordinate:

$$\mathsf{R} = r^n$$

Solve order by order in *1/D*

Leading order solution:

$$ds^2 = -2\left(u_t dt + \frac{u_i}{n} d\sigma^i\right) dr - A dt^2 - \frac{2}{n} C_i d\sigma^i dt + \frac{1}{n} G_{ij} d\sigma^i d\sigma^j + r^2 d\Omega_{n+1}$$

$$A = 1 - \frac{m(t,\sigma)}{\mathsf{R}} \qquad C_i = \frac{p_i(t,\sigma)}{\mathsf{R}}$$

$$G_{ij} = \delta_{ij} + \frac{1}{n} \frac{p_i(t,\sigma) p_j(t,\sigma)}{m(t,\sigma)\mathsf{R}}$$

Leading order effective equations:

$$\partial_t m - \partial_i \partial^i m = -\partial_i p^i$$

$$\partial_t p_i - \partial_j \partial^j p_i = \partial_i m - \partial^j \left(\frac{p_i p_j}{m} \right)$$

p + 1 effective equations

Radial dependence solved analytically

No constraints to be solved

Invariant under Galilean boosts

Hydrodynamic form: $p_i = mv_i + \partial_i m$

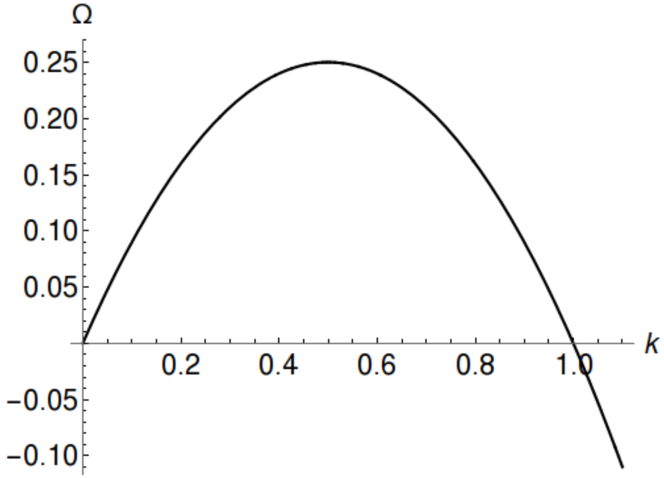
$$\boxed{\begin{gathered}\partial_t m + \partial_i \left(mv^i\right) = 0 \\ \partial_t(mv^i) + \partial_j \left(mv^i v^j + \tau^{ij}\right) = 0 \\ \hline \tau_{ij} = -m\delta_{ij} - 2m\partial_{(i}v_{j)} - m\partial_j\partial_i \ln m\end{gathered}}$$

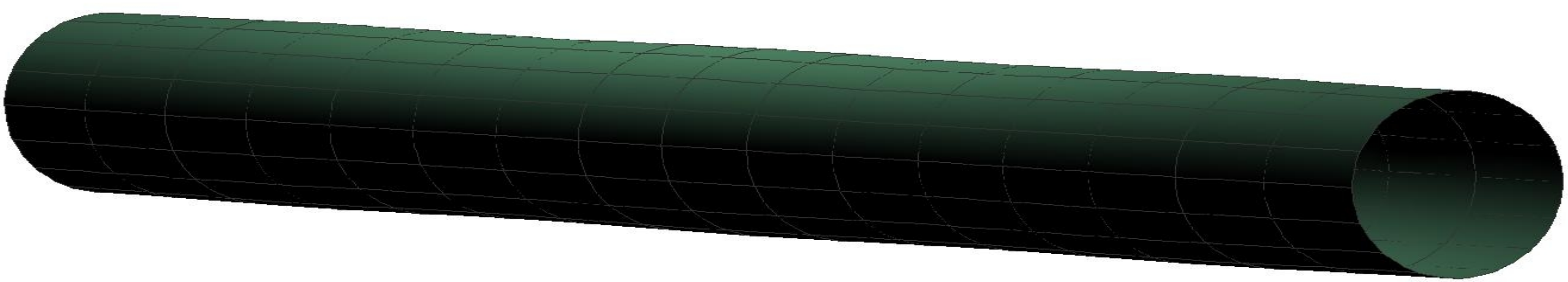
$$P = -m, \qquad s = 4\pi m, \qquad T = \frac{1}{4\pi}$$

$$\eta = \frac{4\pi}{s}, \qquad \zeta = \frac{2}{p}\eta$$

The Gregory-Laflamme instability

$$\Omega = k(1-k)$$





R. Gregory and R. Laflamme (1993)

The Gregory-Laflamme instability

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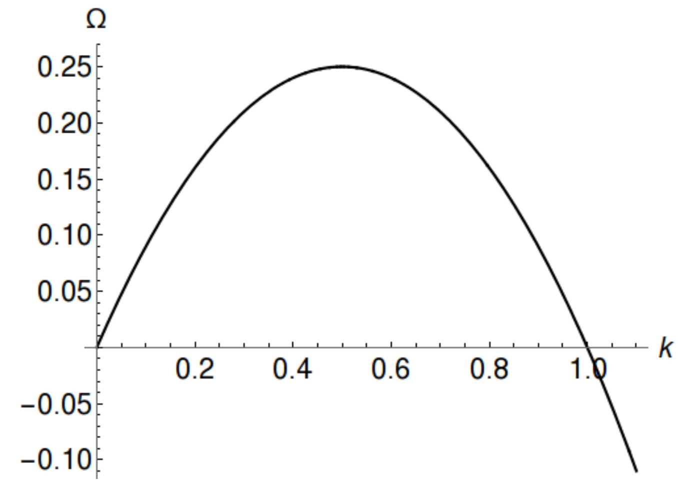
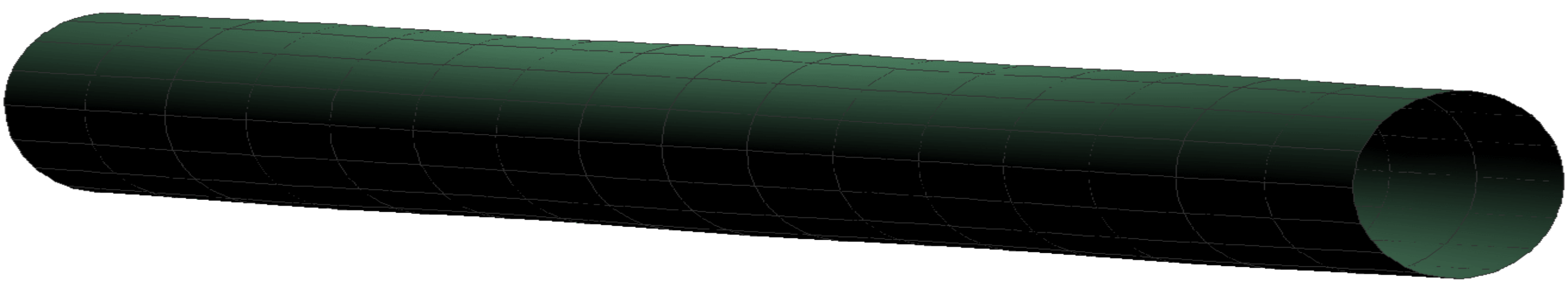
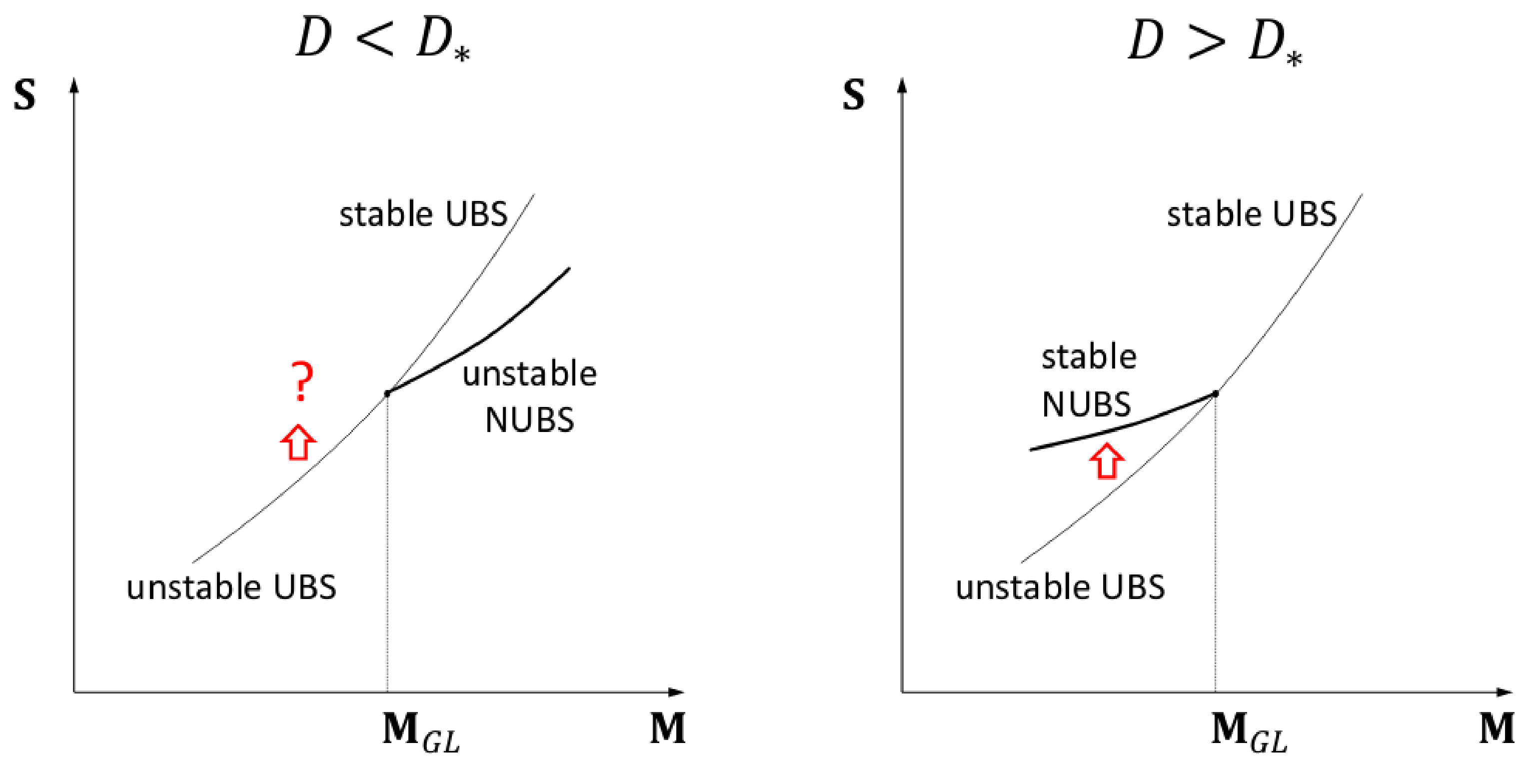




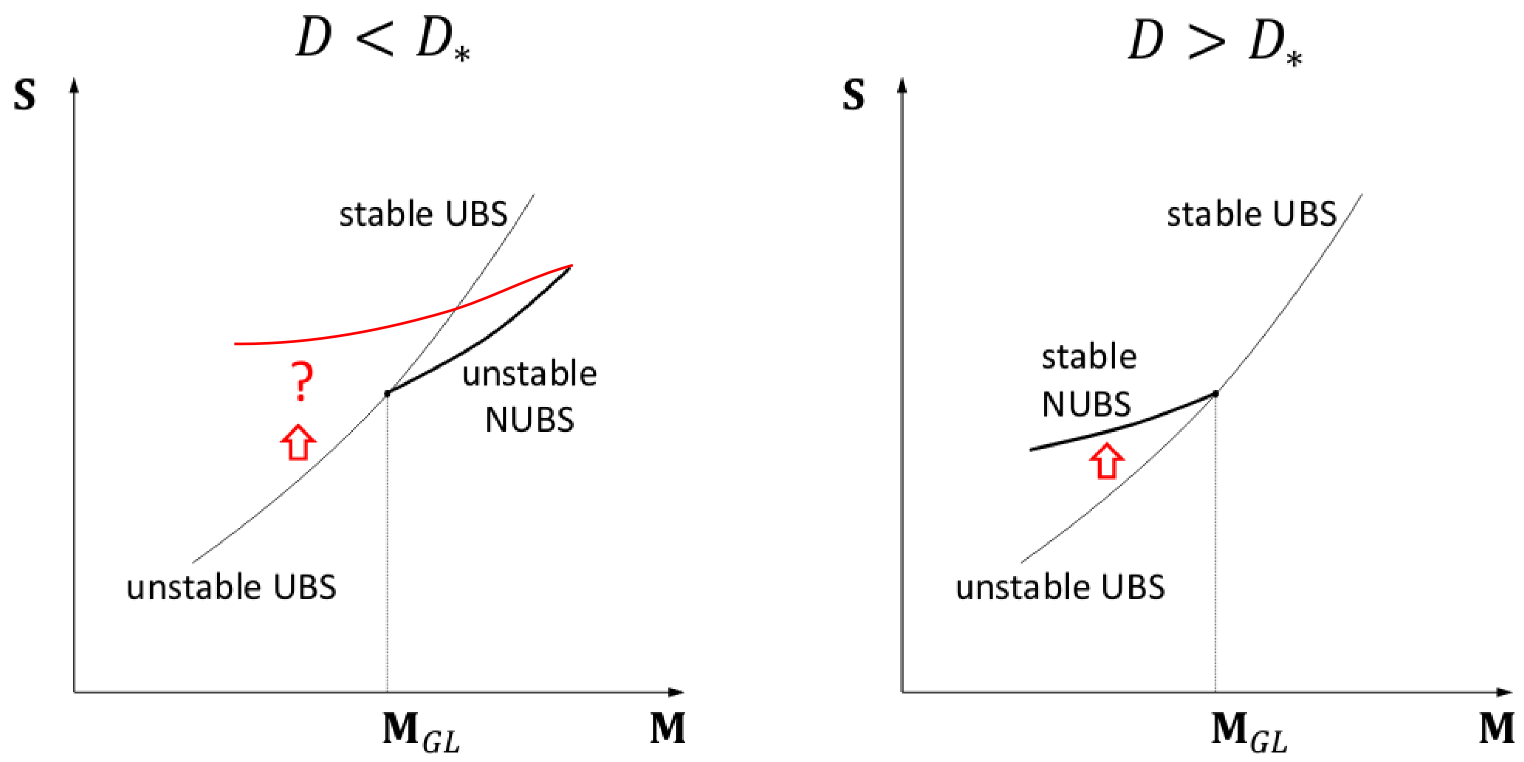
L. Lehner and F. Pretorius (2010)

Sorkin's critical dimension $D_* = 13.59$



E. Sorkin (2004)

Stable NUBS below the critical dimension



P. Figueras, K. Murata and H. S. Reall (2012)

Onset of the Gregory-Laflamme instability: Beginning of the NUBS branch

$$k_{\mathrm{GL}} = 1 - \frac{1}{2n} + \frac{7}{8n^2} + \frac{-\frac{25}{16} + 2\zeta(3)}{n^3} + \frac{\frac{363}{128} - 5\zeta(3)}{n^4}$$

Parametrization of the NUBS branch

$$k_{\mathrm{GL}} = 1 - \frac{1}{2n} + \frac{7}{8n^2} + \frac{-\frac{25}{16} + 2\zeta(3)}{n^3} + \frac{\frac{363}{128} - 5\zeta(3)}{n^4}$$

$$k(\epsilon) = k_0(\epsilon)\left(1 + \frac{\alpha_1(\epsilon)}{n} + \frac{\alpha_2(\epsilon)}{n^2} + \frac{\alpha_3(\epsilon)}{n^3} + \frac{\alpha_4(\epsilon)}{n^4} + \ldots\right)$$

Parameter along NUBS branch

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Parameter along NUBS branch

$$k_0(0) = 1\,, \quad \alpha_1(0) = -\frac{1}{2}\,, \quad \alpha_2(0) = \frac{7}{8}\,,$$

$$\alpha_3(0) = -\frac{25}{16} + 2\zeta(3)\,, \quad \alpha_4(0) = \frac{363}{128} - 5\zeta(3)\,, \ \dots$$

The nested character of the effective equations

$$m(\epsilon, z) = \sum_{N=0}^{\infty} \frac{m_N(\epsilon, z)}{n^N}, \quad z \sim z + \frac{2\pi}{k_0(\epsilon)}$$

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$$\left(1 - \frac{2m_0''(z)}{m_0(z)}\right) m_0'(z) + \frac{m_0'(z)^3}{m_0(z)^2} + m_0{}^{(3)}(z) = 0$$

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$$\mathcal{F}_1[m_1, m_0; \alpha_1] = 0 \qquad \longrightarrow \qquad m_1(\epsilon, z),\ \alpha_1(\epsilon)$$

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$$\mathcal{F}_2[m_2, m_1, m_0; \alpha_1, \alpha_2] = 0 \qquad \longrightarrow \qquad m_2(\epsilon, z),\ \alpha_2(\epsilon)$$

$$\cdots$$

The nested character of the effective equations

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$$m_N(\epsilon, z) = \delta_{0N} + \sum_{q=1}^{\infty} \epsilon^q \mu_N^q(\epsilon) \cos(q k_0(\epsilon) z)$$

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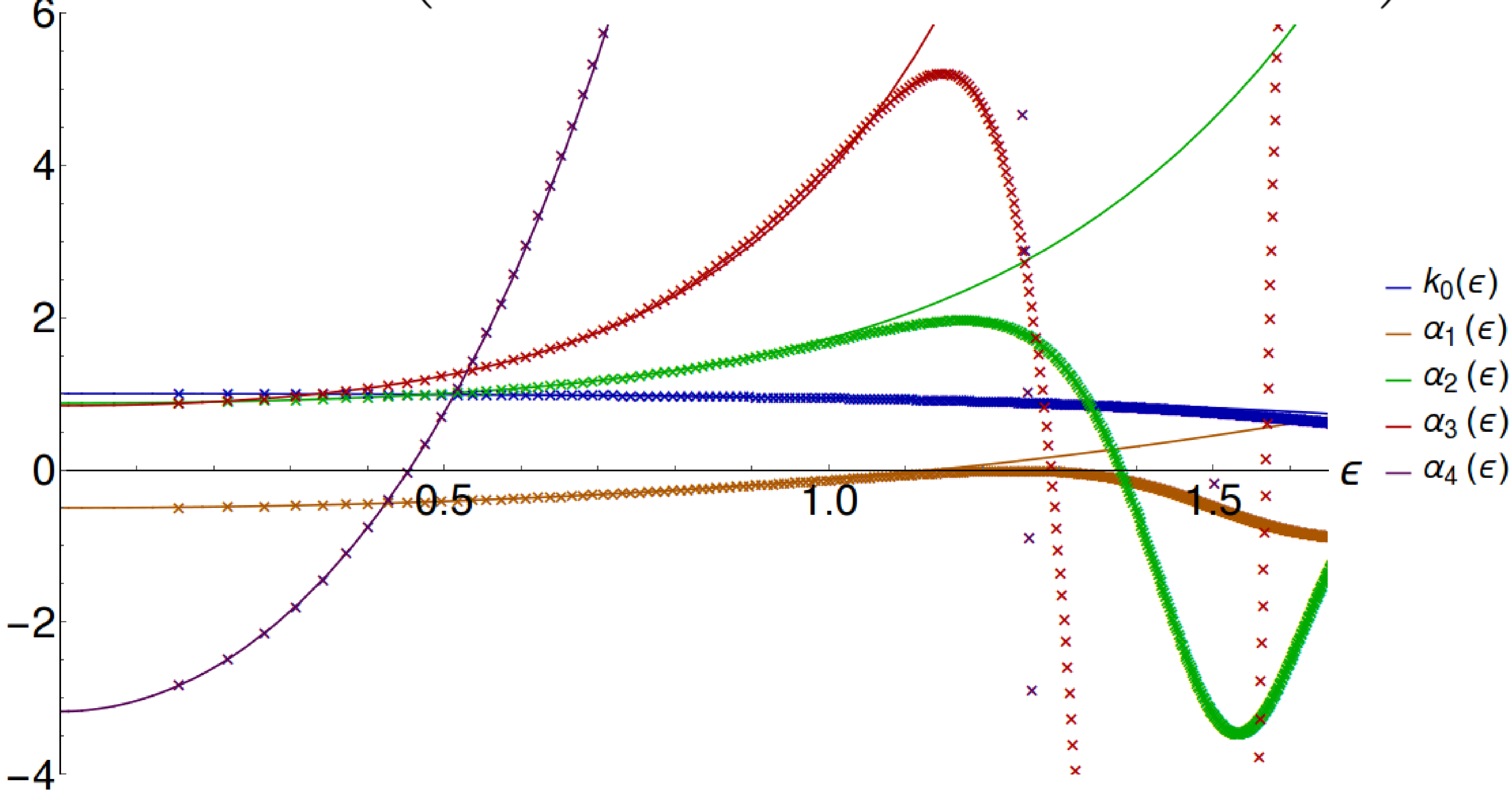
$\mu_N^q(\epsilon)\,, k_0(\epsilon)\,, \alpha_N(\epsilon)$ → Perturbatively in ϵ

$\mu_N^q(\epsilon)\,, k_0(\epsilon)\,, \alpha_N(\epsilon)$ → Numerically

$$k(\epsilon) = k_0(\epsilon)\left(1 + \frac{\alpha_1(\epsilon)}{n} + \frac{\alpha_2(\epsilon)}{n^2} + \frac{\alpha_3(\epsilon)}{n^3} + \frac{\alpha_4(\epsilon)}{n^4} + \dots\right)$$



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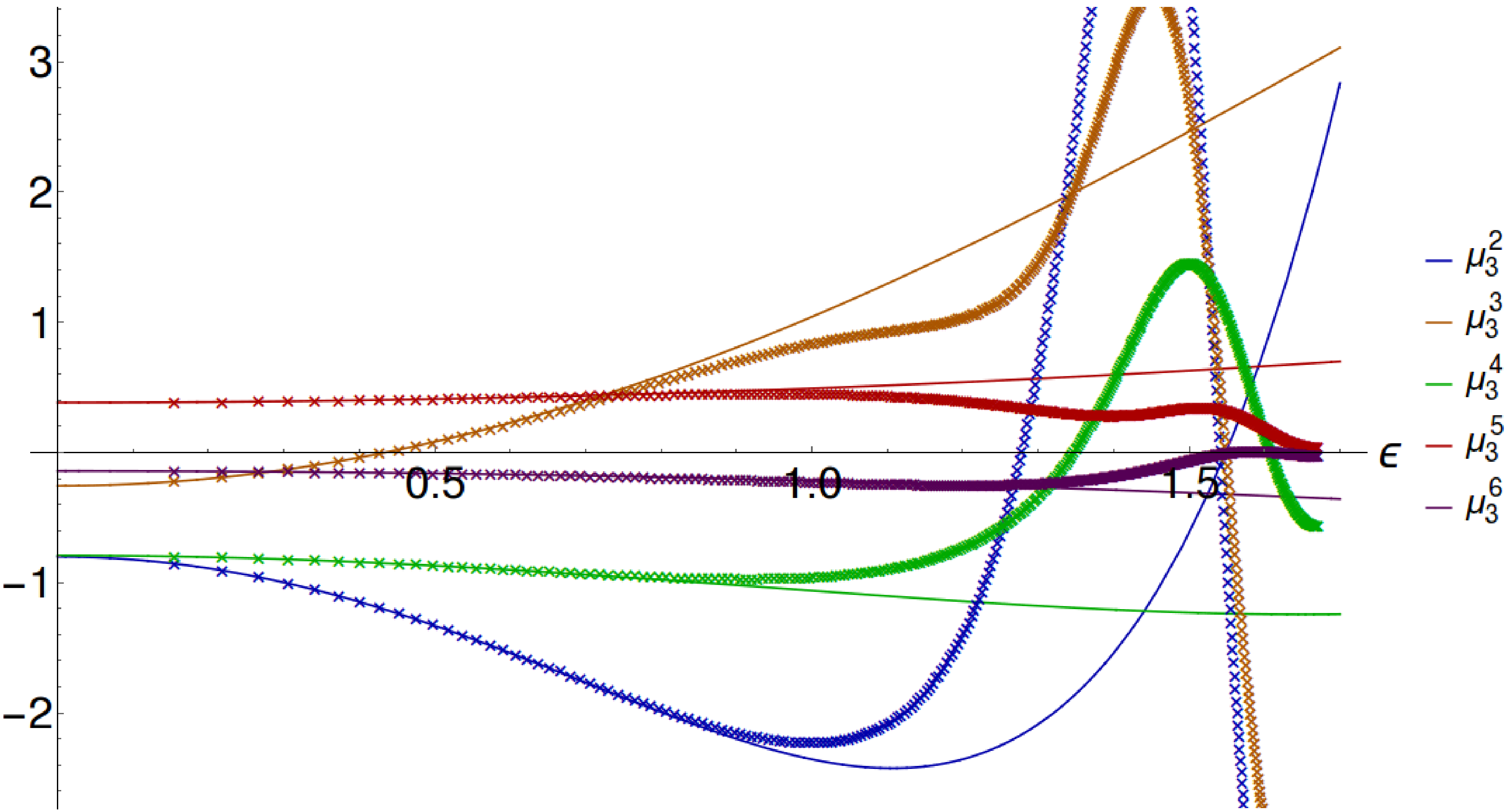
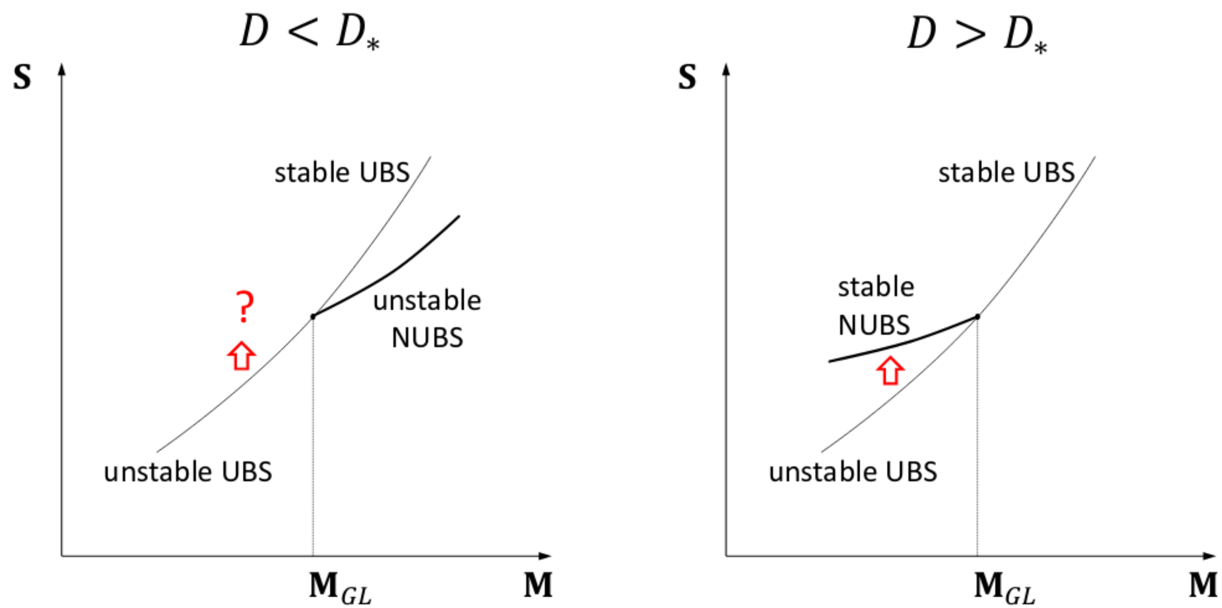


Sorkin's critical dimension (static)



$$\frac{\Delta \mathbf{S}}{\mathbf{S}} = \frac{\mathbf{S}}{\mathbf{S}_{\mathrm{UBS}}} - 1 = \left(1 - \frac{7}{n} - \frac{22}{n^2} - \frac{8(1+2\zeta(3))}{n^3}\right)\frac{\epsilon^4}{96n} + \mathcal{O}\left(\epsilon^5\right)$$

$$D_* = 13.59$$

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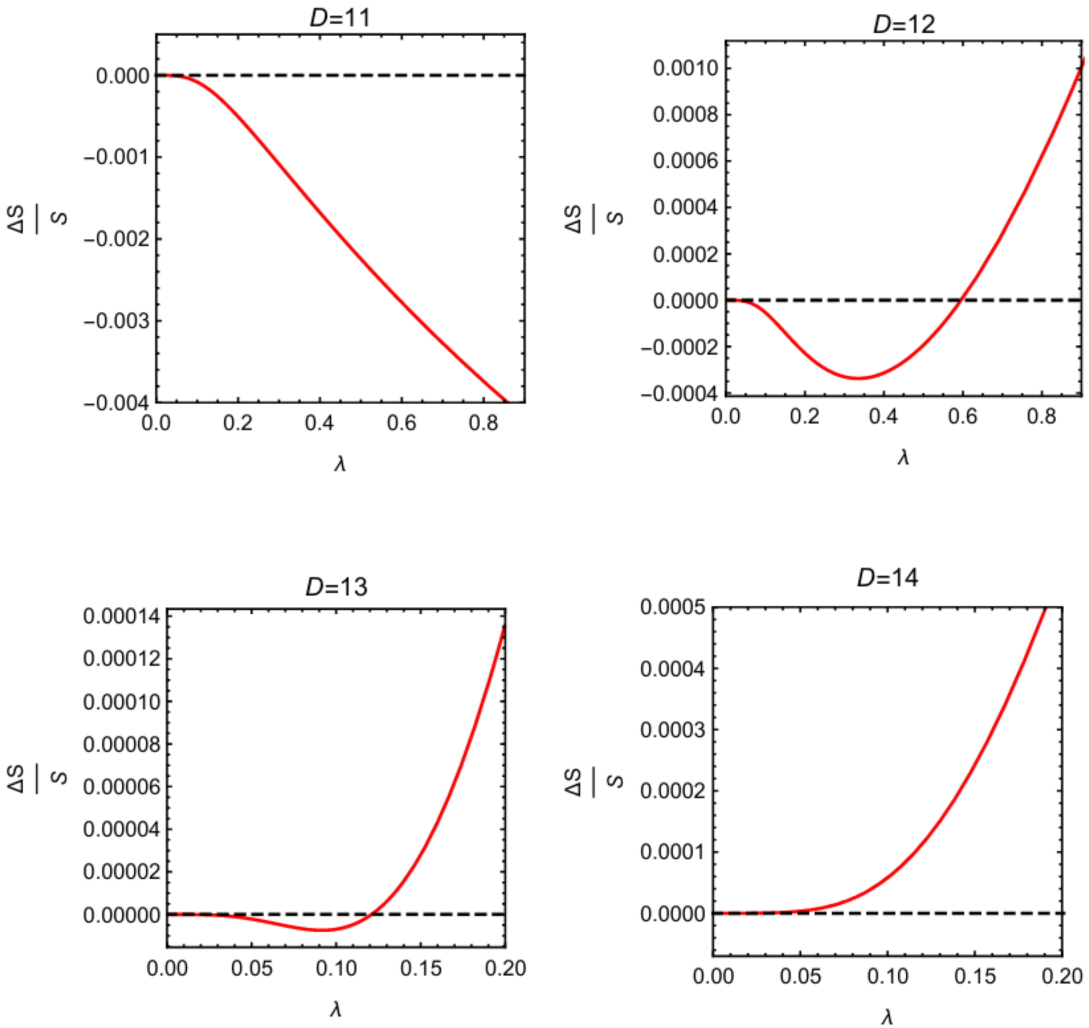
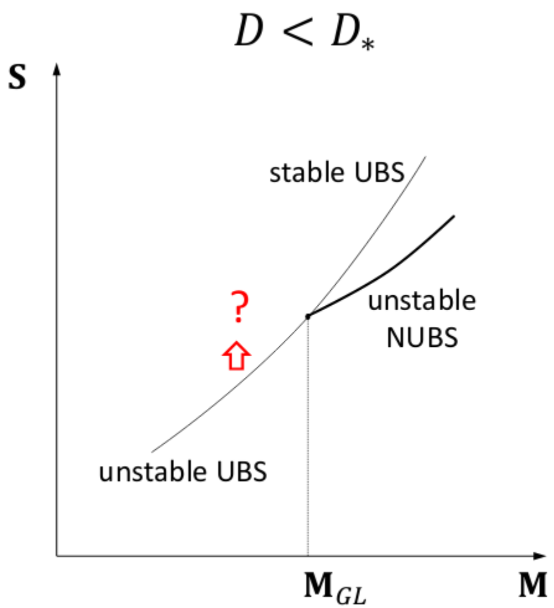
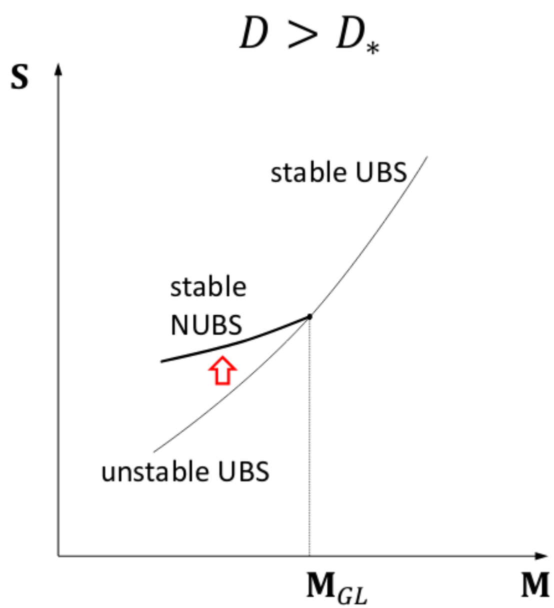


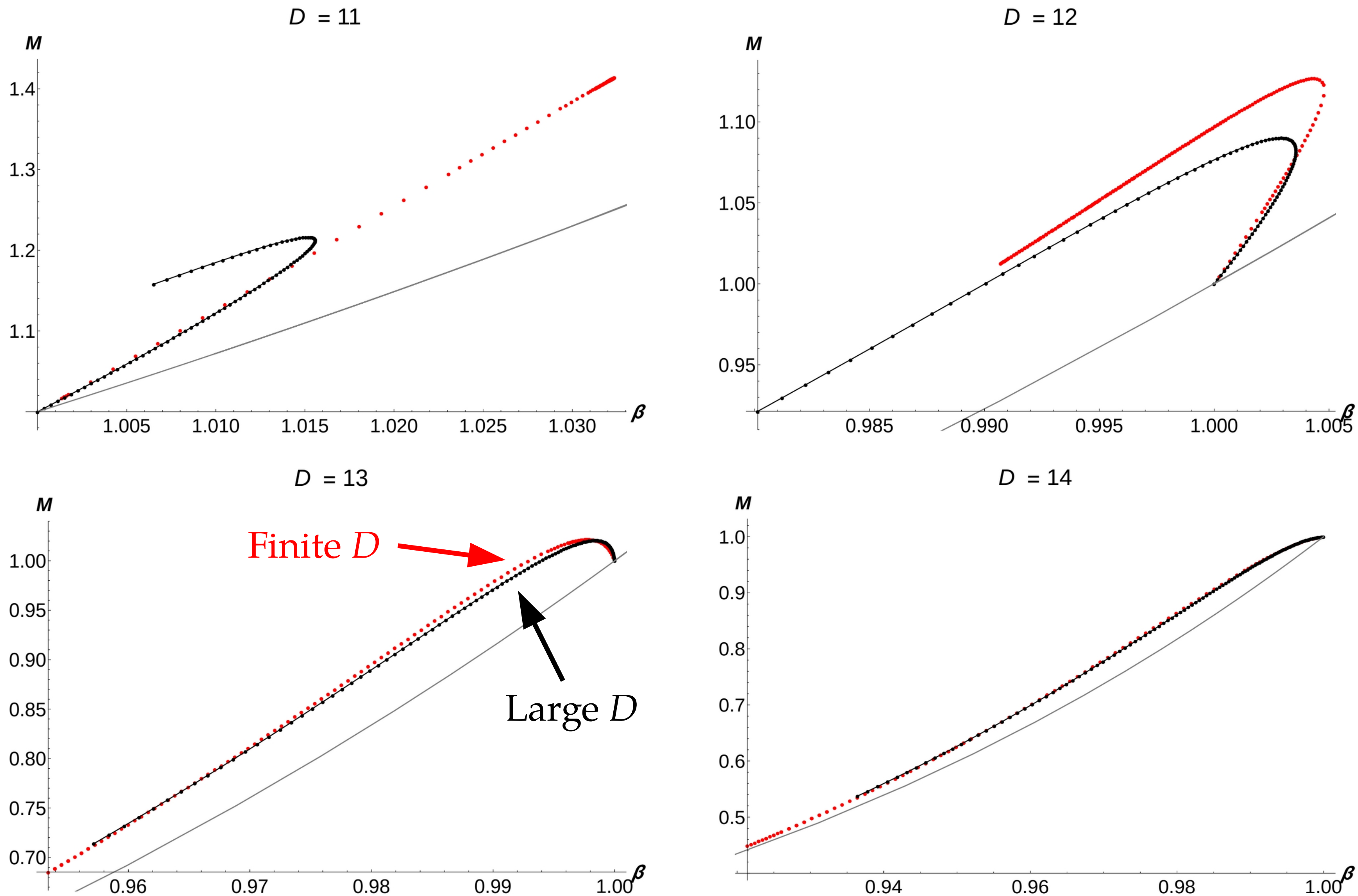
Sorkin's critical dimension (dynamic)



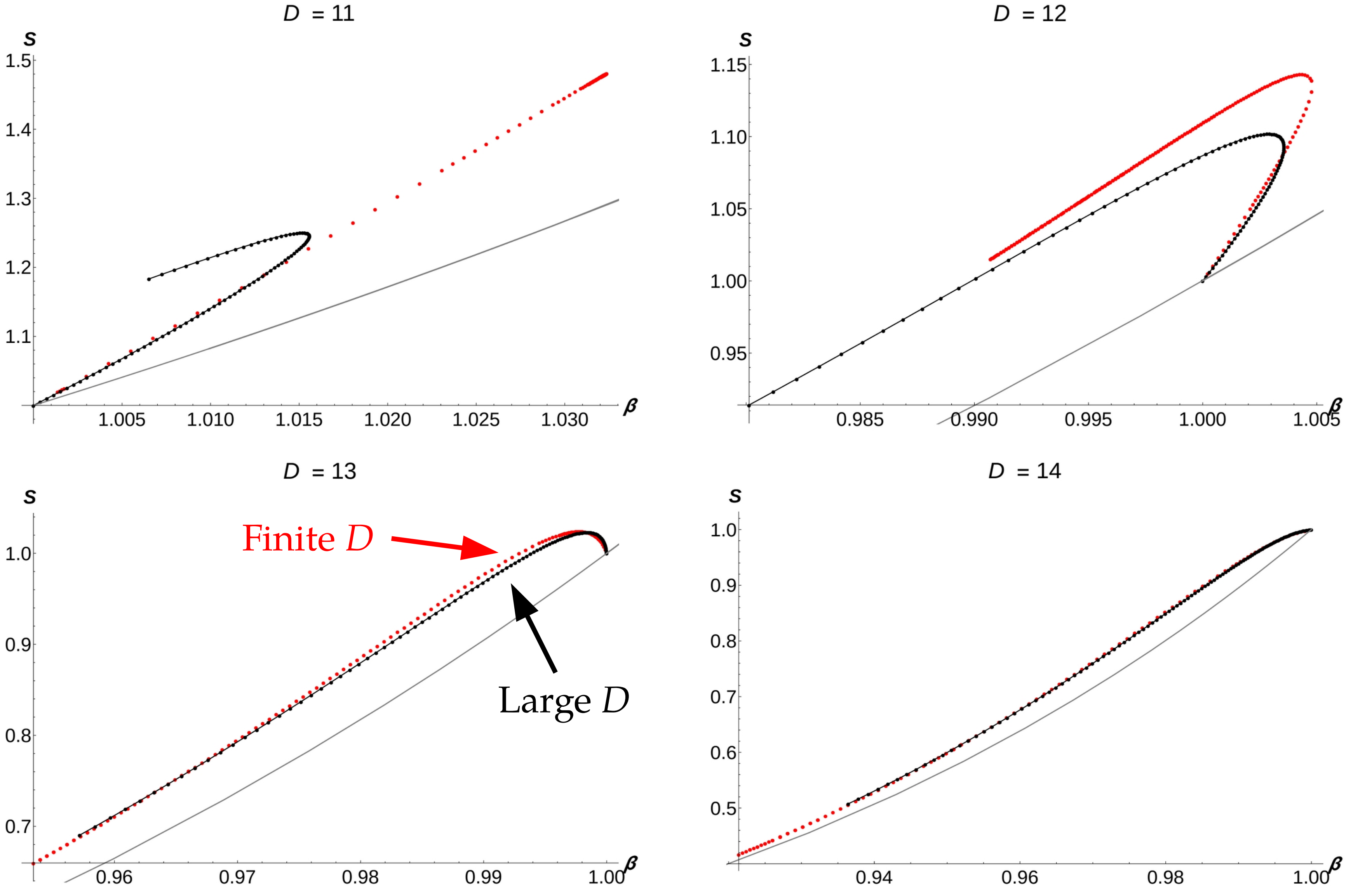


$$\boldsymbol{\Omega} = -\frac{\epsilon^2}{12}\left(1 - \frac{10}{n} + \frac{6-2\zeta(2)}{n^2} - \frac{6-4\pi^2+20\zeta(3)}{n^3}\right) + \mathcal{O}\left(\epsilon^4\right)$$

$$D_* = 13.62$$



Red dots: Data by P. Figueras, K. Murata and H. S. Reall (2012)



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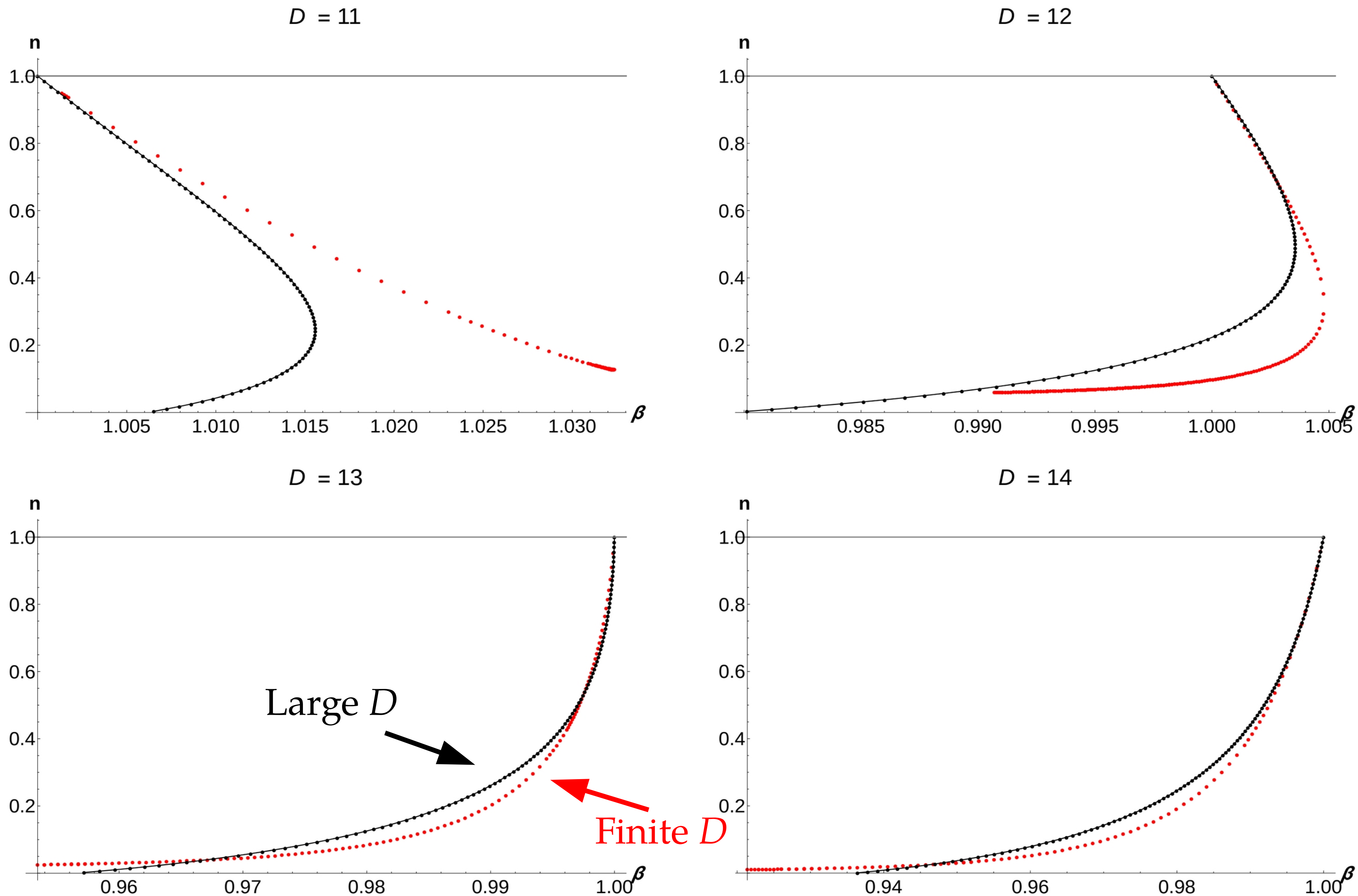

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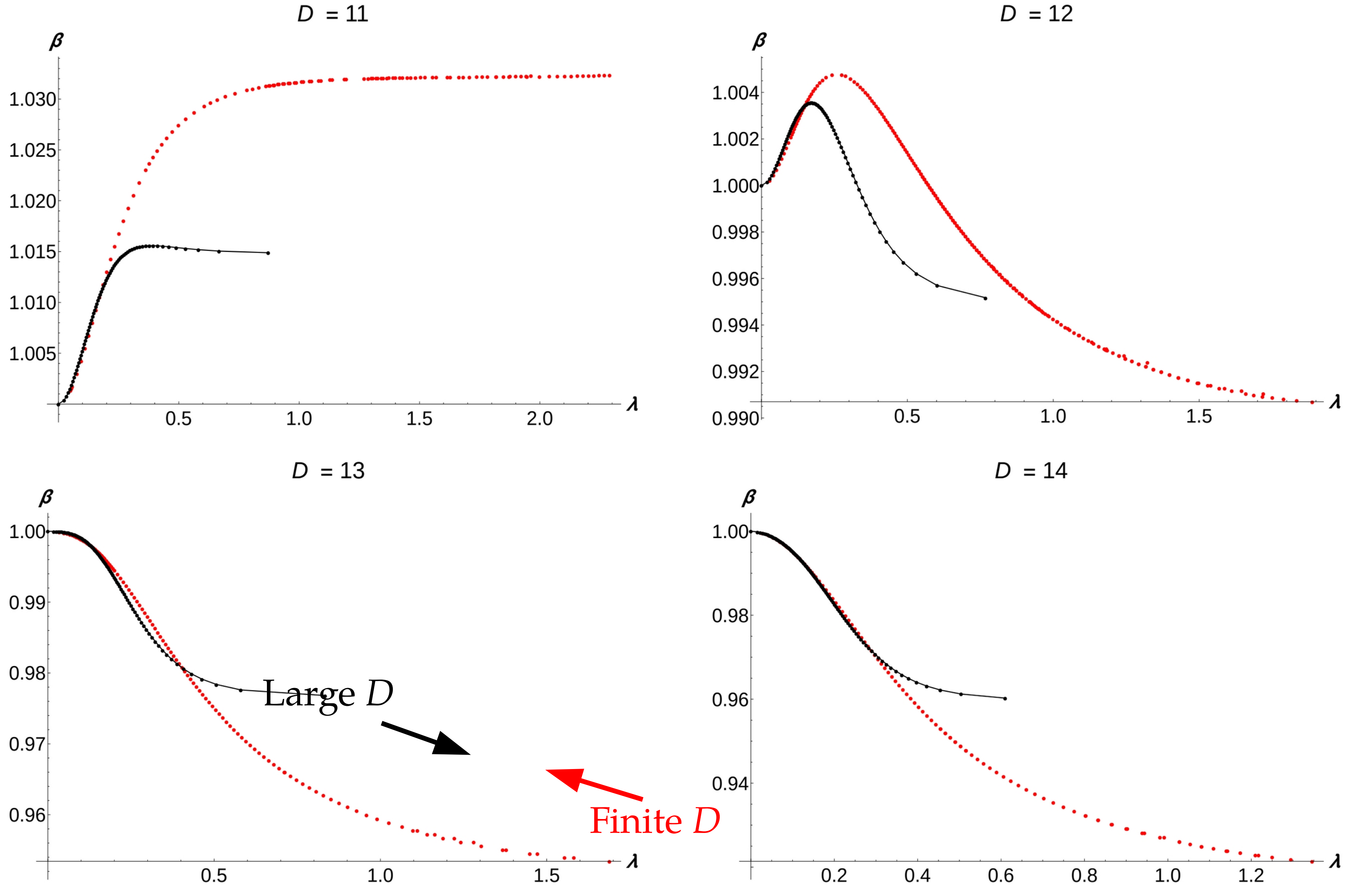

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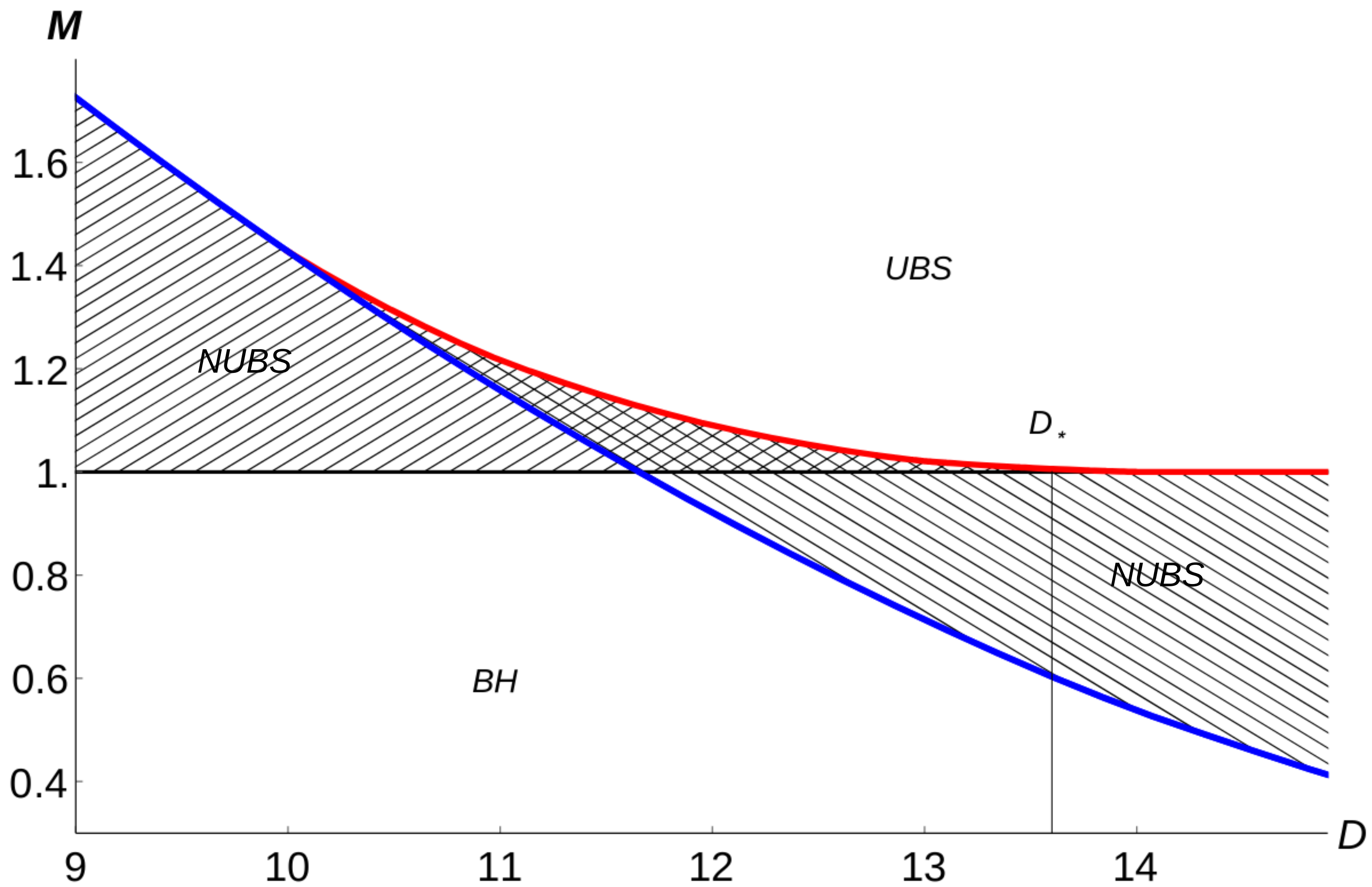
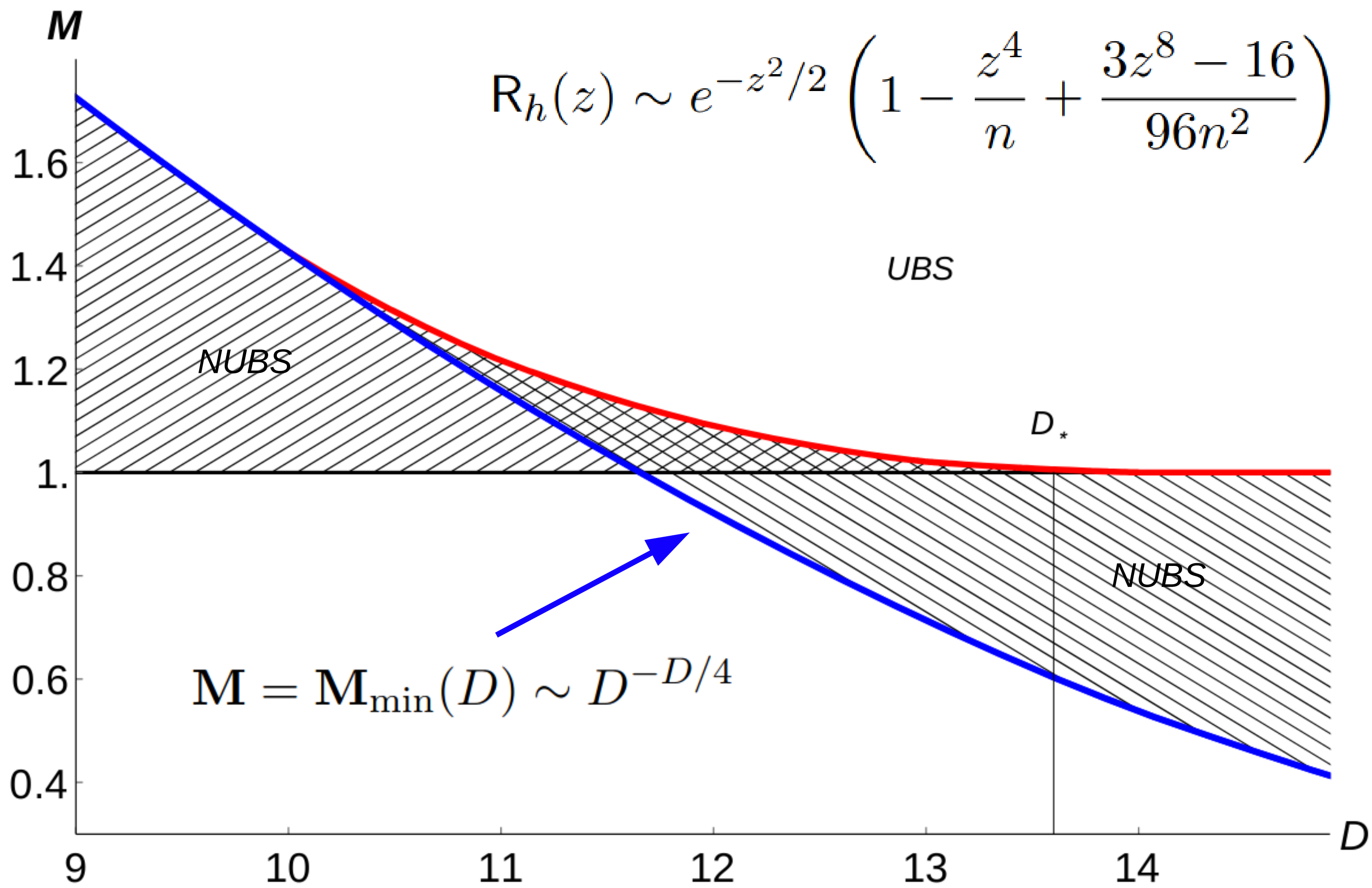
Static solutions up to *1/D⁴*



Breakup estimate: Zero tension solution



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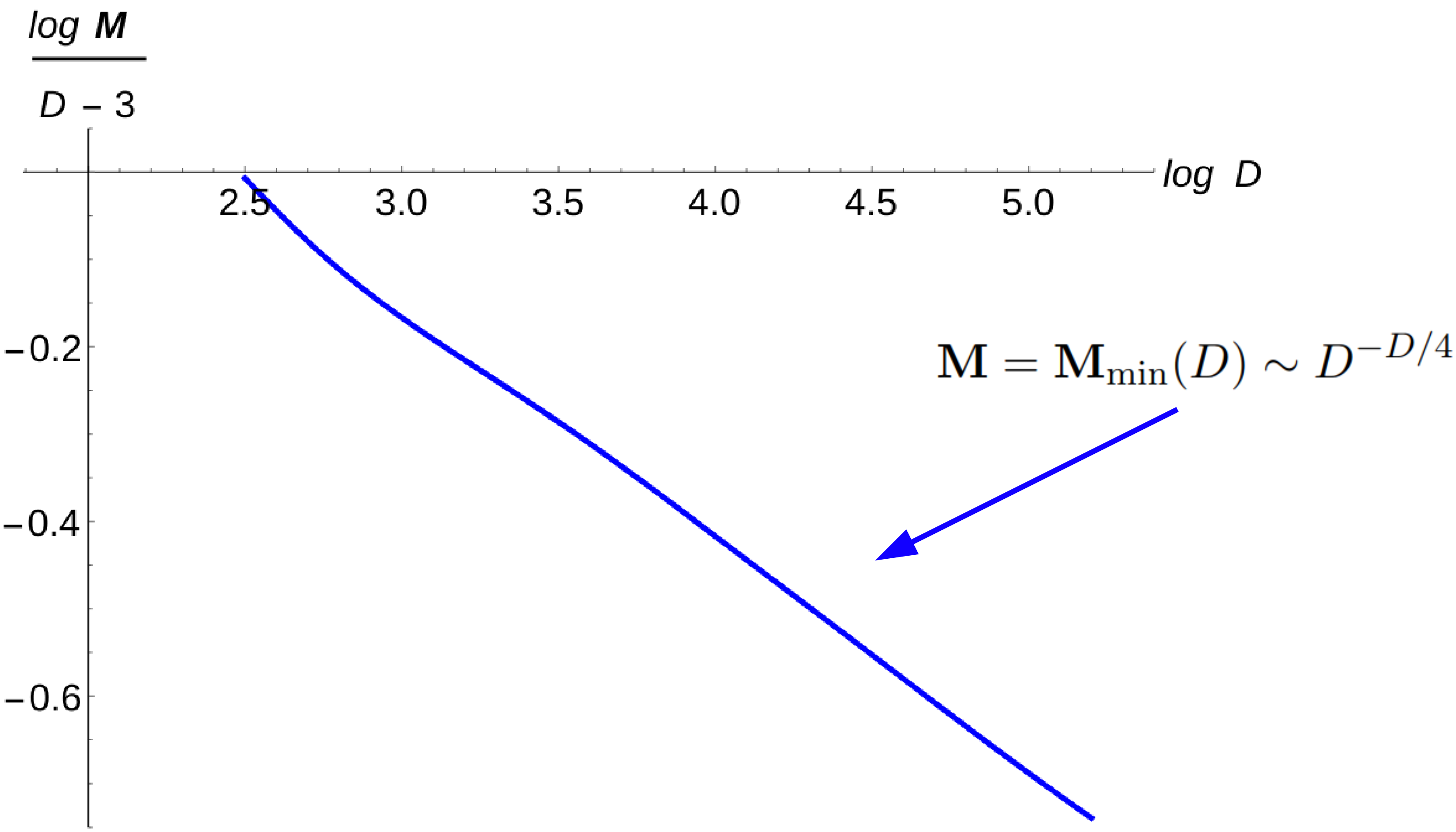

Conclusions

Black branes at large-D can be described by effective, hydrodynamic equations during their time evolution. Once they settle down to a stationary configuration, they can be seen as elastic membranes.

The effective equations are much simpler than the full Einstein's Equations and can be treated with modest numerical methods, or even analytically.

The large-D expansion is able to correctly capture black string physics at and even below the critical dimension.

We have been able to compute the critical dimension analytically, with all the 3NLO results converging on the value D = 13.6