

立德樹人 求實創新



Correlation functions in the $TT\bar{b}ar$ deformed CFTs

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Outlook

- **Intro to TTar deformation**
- **Correlation functions in TT deformed theory (Plane, SUSY, Torus)**
- **Chaotic behavior in TT deformed theory**
- **Future Problems**



The $T\bar{T}$ operator

Consider the following **bi-local operator** in a **2d QFT**.

$$T\bar{T}(z, z') = T_{zz}(z)T_{\bar{z}\bar{z}}(z') - T_{z\bar{z}}(z)T_{\bar{z}z}(z') \quad z = x + it$$

[Zamolodchikov]

The **expectation value** of this operator is given by the expectation values of the stress tensor itself and is a **constant**.

$$\langle T\bar{T} \rangle = \langle T_{zz} \rangle \langle T_{\bar{z}\bar{z}} \rangle - \langle T_{z\bar{z}} \rangle^2$$

This is true very generally in a reasonably well behaved 2d QFT which has a local conserved stress tensor.



The operator can also be defined (upto total derivatives)
at coincident points.

$$T\bar{T} \equiv T_{zz}T_{\bar{z}\bar{z}} - T_{z\bar{z}}^2$$

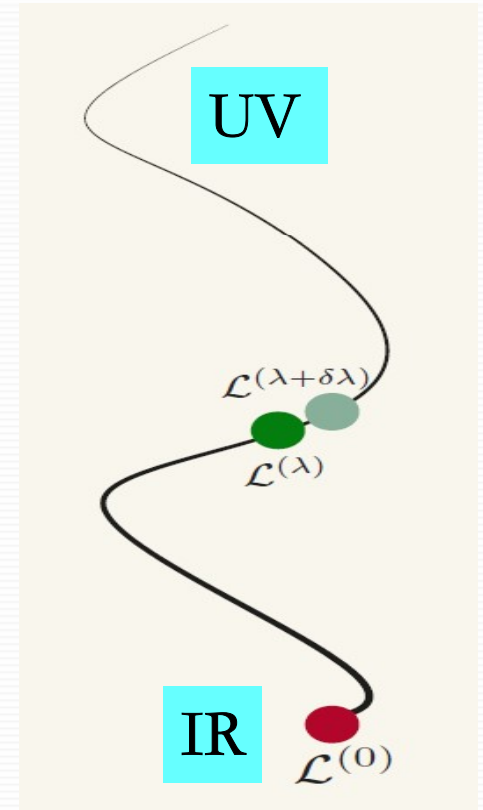
Higher Dimensional Deformation

$$\det T = \frac{1}{2}\epsilon_{ik}\epsilon_{jl}T^{ij}T^{kl}$$

Deformation of $T\bar{T}$

$$\mathcal{L}^{(\lambda+\delta\lambda)} = \mathcal{L}^{(\lambda)} + \delta\lambda T\bar{T}$$

$$\frac{dS(\lambda)}{d\lambda} = \int d^2x T\bar{T}(x).$$



- The **spectrum** of the deformed theory can be solved exactly and non-perturbatively.

[Smirnov-Zamolodchikov; Cavaglia-Negro-Szecsényi-Tateo]

- Deforming an integrable QFT by this operator **preserves integrability**.

[F. A. Smirnov and A. B. Zamolodchikov,16]

- Deforming by $T\bar{T}$ = coupling the theory to **Jackiw-Teitelbohm gravity**

[Dubovsky-Gorbenko-...]

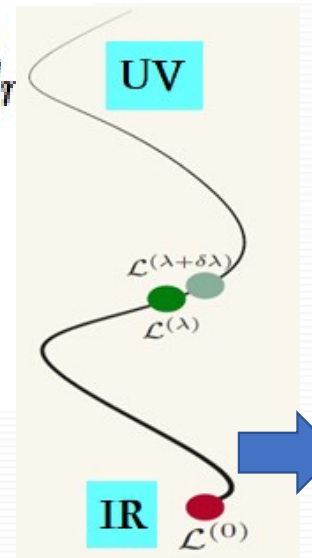
- **Correlation functions in deep UV of the deformed theory given by diffusion equation** [Cardy, 19]

$$\langle \prod_n \Phi_n(x_n) \rangle_\lambda = \int \prod_n G(x_n - y_n; \tilde{\lambda}) \langle \prod_n \Phi_n(y_n) \rangle_0 \prod_n d^2 y_n$$

where

$$G(x - y; \tilde{\lambda}) = (4\pi\tilde{\lambda})^{-1} e^{-(x-y)^2/4\tilde{\lambda}}$$

- **By canonical transformation in phase space, Flow equation of correlation functions** [Jorrit Kruthoff, Onkar Parrikar, 20]





Correlation functions in deformed theory

Zero-pt Correlation function:[Aharony-Shouvik-Giveon-Jiang-Kutasov,18]; [Shouvik-Jiang,18][Cardy,19]...

$$\lambda \int d^2 z \langle T \bar{T}(z) \phi_1(z_1) \dots \phi_n(z_n) \rangle$$

N<5 Point, S. He, Hongfei Shu [1907.12603]



The deformation of correlation function

$$\langle \mathcal{O}_1(z_1, \bar{z}_1) \mathcal{O}_2(z_2, \bar{z}_2) \cdots \mathcal{O}_n(z_n, \bar{z}_n) \rangle_\lambda = \lambda \int d^2z \langle T\bar{T}(z, \bar{z}) \mathcal{O}_1(z_1, \bar{z}_1) \mathcal{O}_2(z_2, \bar{z}_2) \cdots \mathcal{O}_n(z_n, \bar{z}_n) \rangle$$

Energy-Momentum conservation

$$\begin{aligned} \langle \mathcal{O}_1(z_1, \bar{z}_1) \mathcal{O}_2(z_2, \bar{z}_2) \cdots \mathcal{O}_n(z_n, \bar{z}_n) \rangle_\lambda = & \lambda \int d^2z \left(\sum_{i=1}^n \left(\frac{h_i}{(z - z_i)^2} + \frac{\partial_{z_i}}{z - z_i} \right) \right) \left(\sum_{i=1}^n \left(\frac{\bar{h}_i}{(\bar{z} - \bar{z}_i)^2} + \frac{\partial_{\bar{z}_i}}{\bar{z} - \bar{z}_i} \right) \right) \\ & \times \langle \mathcal{O}_1(z_1, \bar{z}_1) \mathcal{O}_2(z_2, \bar{z}_2) \cdots \mathcal{O}_n(z_n, \bar{z}_n) \rangle, \end{aligned}$$

Ward
Identity

The deformed Two-Point Function(Step 1)

$$\lim_{\ell \rightarrow 0} \langle T(z + \ell) \bar{T}(\bar{z} - \ell) \mathcal{O}(z_1, \bar{z}_1) \mathcal{O}(z_2, \bar{z}_2) \rangle$$

$$= \lim_{\ell \rightarrow 0} \left(\sum_{i=1}^2 \left(\frac{\bar{h}}{(\bar{z} - \bar{z}_i - \ell)^2} + \frac{\partial_{\bar{z}_i}}{\bar{z} - \bar{z}_i - \ell} \right) \right) \left(\sum_{i=1}^2 \left(\frac{h}{(z - z_i + \ell)^2} + \frac{\partial_{z_i}}{z - z_i + \ell} \right) \right) \langle \mathcal{O}(z_1, \bar{z}_1) \mathcal{O}(z_2, \bar{z}_2) \rangle$$

Regularization (Step 2)

Log term associated with boundary term of Non local deformation(TTar). Nonlocal divergence given by Peskin 's QFT.

$$\langle T(z) \bar{T}(\bar{z}) \mathcal{O}(z_1, \bar{z}_1) \mathcal{O}(z_2, \bar{z}_2) \rangle_\lambda = \lambda h \bar{h} z_{12}^2 \bar{z}_{12}^2 \mathcal{I}_2(z_1, z_2) \langle \mathcal{O}(z_1, \bar{z}_1) \mathcal{O}(z_2, \bar{z}_2) \rangle$$

$$= \lambda h \bar{h} \frac{8\pi}{|z_{12}|^2} \left(\frac{4}{\epsilon} + 2 \log |z_{12}|^2 + 2 \log \pi + 2\gamma - 5 \right) \langle \mathcal{O}(z_1, \bar{z}_1) \mathcal{O}(z_2, \bar{z}_2) \rangle.$$



Four-Point in CFTs

$$\langle \mathcal{O}^\dagger(z_1, \bar{z}_1) \mathcal{O}(z_2, \bar{z}_2) \mathcal{O}^\dagger(z_3, \bar{z}_3) \mathcal{O}(z_4, \bar{z}_4) \rangle = \frac{G(\eta, \bar{\eta})}{z_{13}^{2h} z_{24}^{2h} \bar{z}_{13}^{2\bar{h}} \bar{z}_{24}^{2\bar{h}}},$$

with the cross ratios

$$\eta = \frac{z_{12} z_{34}}{z_{13} z_{24}}, \quad \bar{\eta} = \frac{\bar{z}_{12} \bar{z}_{34}}{\bar{z}_{13} \bar{z}_{24}}.$$



Four-Point in Deformed-CFTs

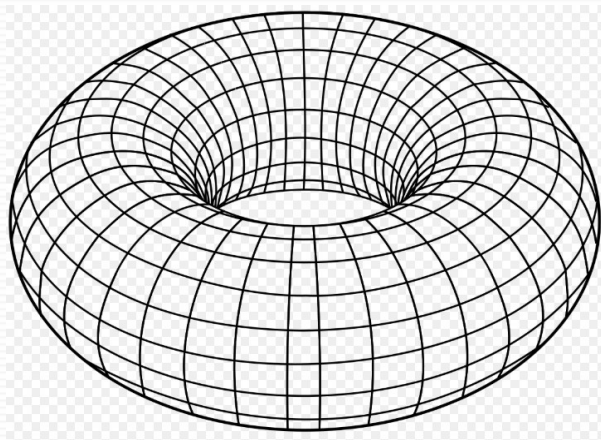
$$\begin{aligned} & \langle \mathcal{O}(z_1, \bar{z}_1) \mathcal{O}(z_2, \bar{z}_2) \mathcal{O}^\dagger(z_3, \bar{z}_3) \mathcal{O}(z_4, \bar{z}_4) \rangle_\lambda \\ &= \lambda \left\{ h \bar{h} z_{13}^2 \bar{z}_{13}^2 \mathcal{I}_{2222}(z_1, z_3, \bar{z}_1, \bar{z}_3) + h \bar{h} z_{24}^2 \bar{z}_{13}^2 \mathcal{I}_{2222}(z_2, z_4, \bar{z}_1, \bar{z}_3) \right. \\ & \quad + h \bar{h} z_{13}^2 \bar{z}_{24}^2 \mathcal{I}_{2222}(z_1, z_3, \bar{z}_2, \bar{z}_4) + h \bar{h} z_{24}^2 \bar{z}_{24}^2 \mathcal{I}_{2222}(z_2, z_4, \bar{z}_2, \bar{z}_4) \\ & \quad + \left(\bar{z}_{13}^2 \mathcal{I}_{111122}(z_1, z_2, z_3, z_4, \bar{z}_1, \bar{z}_3) + \bar{z}_{24}^2 \mathcal{I}_{111122}(z_1, z_2, z_3, z_4, \bar{z}_2, \bar{z}_4) \right) \bar{h} z_{23} z_{14} \frac{\eta \partial_\eta G(\eta, \bar{\eta})}{G(\eta, \bar{\eta})} \\ & \quad + \left(z_{13}^2 \mathcal{I}_{221111}(z_1, z_3, \bar{z}_1, \bar{z}_3, \bar{z}_2, \bar{z}_4) + z_{24}^2 \mathcal{I}_{221111}(z_2, z_4, \bar{z}_2, \bar{z}_4, \bar{z}_1, \bar{z}_3) \right) h \bar{z}_{23} \bar{z}_{14} \frac{\bar{\eta} \partial_{\bar{\eta}} G(\eta, \bar{\eta})}{G(\eta, \bar{\eta})} \\ & \quad \left. + z_{23} z_{14} \bar{z}_{23} \bar{z}_{14} \mathcal{I}_{11111111}(z_1, z_2, z_3, z_4, \bar{z}_1, \bar{z}_3, \bar{z}_2, \bar{z}_4) \eta \bar{\eta} \frac{\partial_\eta \partial_{\bar{\eta}} G(\eta, \bar{\eta})}{G(\eta, \bar{\eta})} \right\} \\ & \langle \mathcal{O}^\dagger(z_1, \bar{z}_1) \mathcal{O}(z_2, \bar{z}_2) \mathcal{O}^\dagger(z_3, \bar{z}_3) \mathcal{O}(z_4, \bar{z}_4) \rangle. \end{aligned}$$

N-point Function, SYSU ...

S. He, Jia-Rui Sun, Yuan Sun [1912.11461]



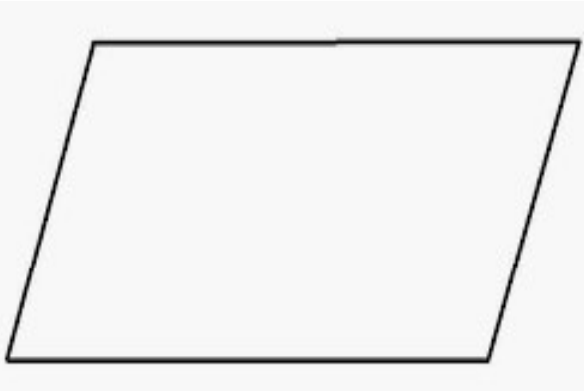
Correlation functions in deformed CFTs on Torus



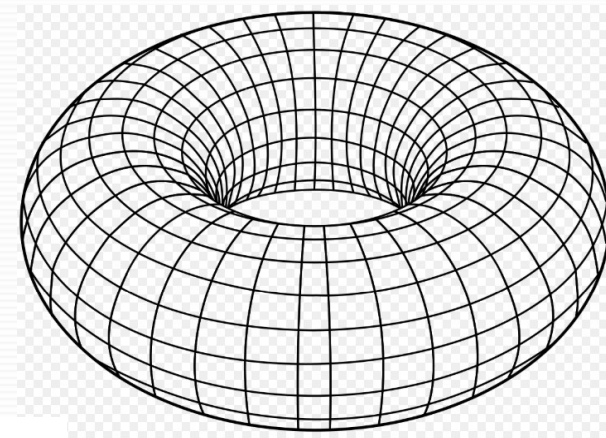
S. He, Yuan Sun [2004.07486]

Correlation function on Torus

S. He, Yuan Sun [2004.07486]



$$z = e^{2\pi i w}$$

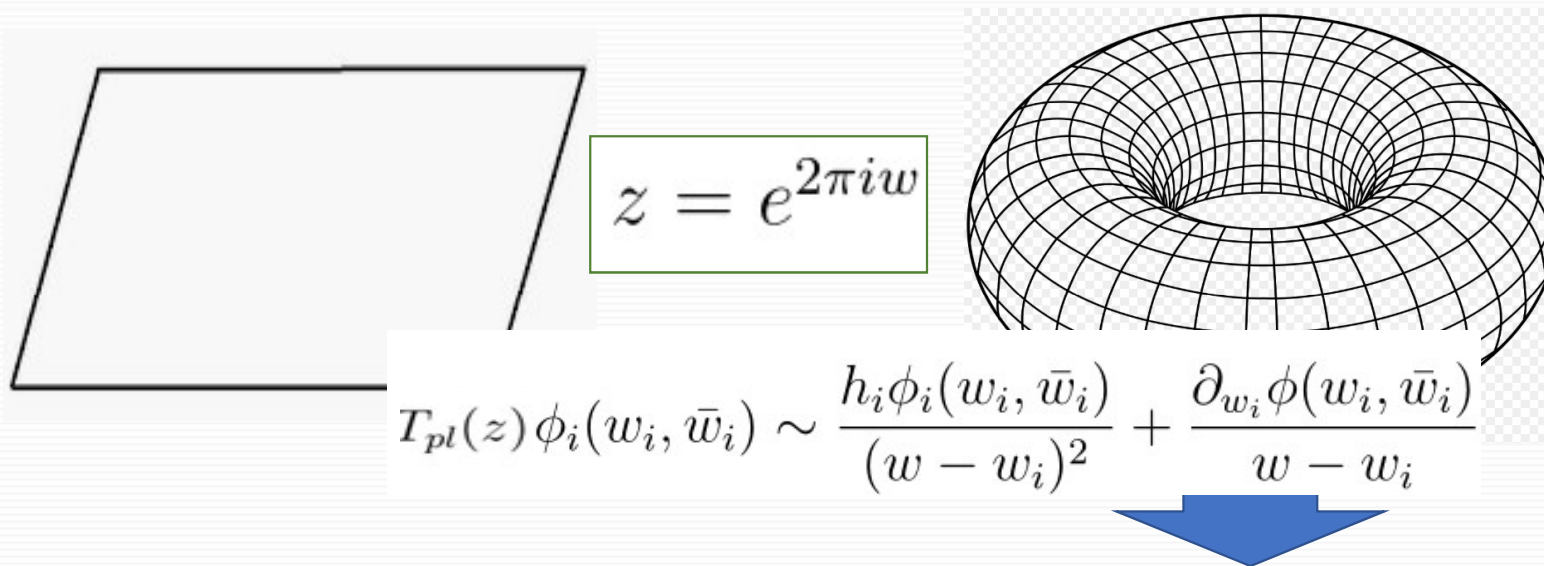


$$z^2 T_{pl}(z) = \frac{1}{(2\pi i)^2} T(w) + \frac{c}{24},$$

$$T_{pl}(z) \phi_i(w_i, \bar{w}_i) \sim \frac{h_i \phi_i(w_i, \bar{w}_i)}{(w - w_i)^2} + \frac{\partial_{w_i} \phi(w_i, \bar{w}_i)}{w - w_i}$$

Correlation function on Torus

S. He, Yuan Sun [2004.07486]



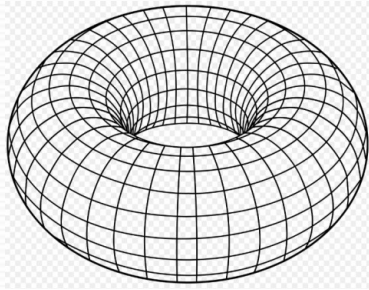
$$\begin{aligned} & \langle T(w)X \rangle - \langle T \rangle \langle X \rangle \\ &= \sum_i \left(h_i (P(w - w_i) + 2\eta_1) + (\zeta(w - w_i) + 2\eta_1 w_i) \partial_{w_i} \right) \langle X \rangle + 2\pi i \partial_\tau \langle X \rangle, \end{aligned}$$

Weierstrass P -function and zeta function

$$P(w) \sim 1/w^2, \zeta(w) \sim 1/w$$

Ward Identity on Torus

S. He, Yuan Sun [2004.07486]



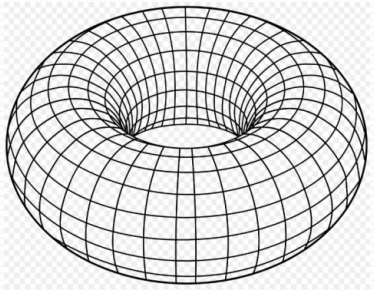
$$\begin{aligned}
 & \langle T(w) \bar{T}(\bar{v}) X \rangle \\
 &= 2\pi i \partial_\tau \langle \bar{T}(\bar{v}) X \rangle + 2\pi i (\partial_\tau \ln Z) \langle \bar{T}(\bar{v}) X \rangle \\
 &+ \sum_i h_i \left(-\zeta'(w_i - w) + 2\eta_1 \right) \langle \bar{T}(\bar{v}) X \rangle \\
 &+ \sum_i \left(-\zeta(w_i - w) + 2\eta_1 w_i - 2\eta_1 w - \pi i \right) \partial_{w_i} \langle \bar{T}(\bar{v}) X \rangle
 \end{aligned}$$

Regularization

$$\lambda \int_{T^2 - \sum_i D(w_i)} d^2 v \langle T(v) \bar{T}(\bar{v}) X \rangle,$$

First order deformation on Torus

S. He, Yuan Sun [2004.07486]



$$\begin{aligned}
 Z' &= \int D\phi e^{-S + \lambda \int d^2 z T \bar{T}(z)} \\
 &= Z \left(1 + \lambda \int d^2 z \langle T \bar{T} \rangle(z) \right) + \frac{1}{2} \lambda^2 \int \int d^2 u_1 d^2 u_2 \langle T \bar{T}(u_1) T \bar{T}(u_2) \rangle + \dots
 \end{aligned}$$



$$\lambda Z \int d^2 z \langle T \bar{T} \rangle(z) = \lambda (2\pi)^2 \tau_2 \partial_\tau \partial_{\bar{\tau}} Z.$$

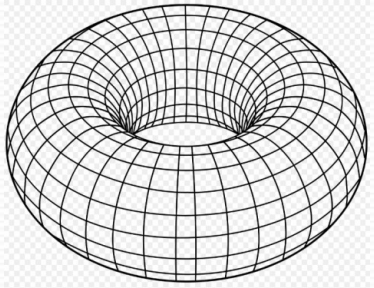
$$Z = \text{tr}(q^{L_0 - c/24} \bar{q}^{\bar{L}_0 - c/24}), \quad q = e^{2\pi i \tau}$$

- Consistent with 1806.07426 [hep-th]
- Correlation function of KdV Operator is Modular invariant.
- Correlation function of Generic Operator may not be Modular invariant.



First order deformation on Torus

S. He, Yuan Sun [2004.07486]



$$\text{tr}(T(w)[T(u_1)\dots T(u_n)\bar{T}(v_1)\dots\bar{T}(v_m)]Xq^{L_0-c/12})$$

- Can be computable but very complicated.
- New recursion relation found!
- Algorithm is offered!

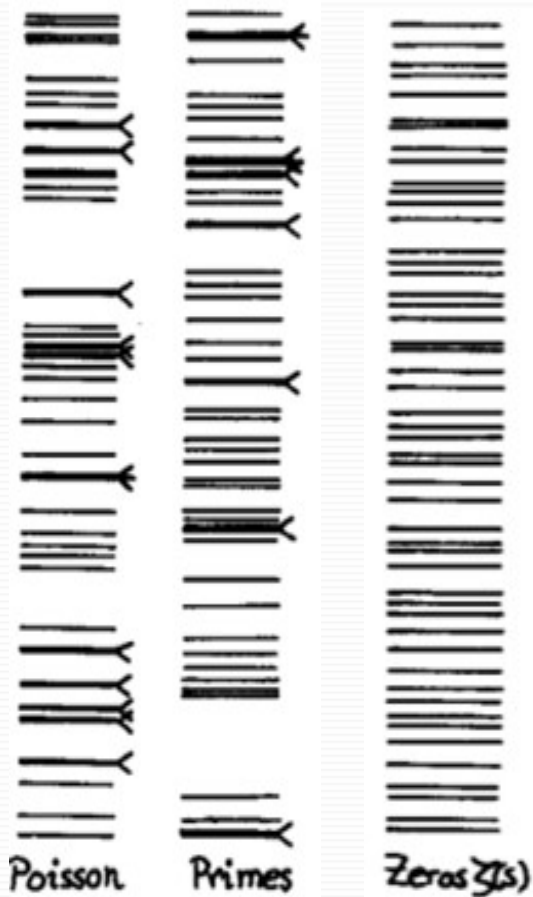
$$\langle T \rangle_\lambda - \langle T \rangle = \lambda \left(-\frac{(2\pi i)^3 \tau_2 \partial_\tau^2 \partial_{\bar{\tau}} Z}{Z} + (2\pi)^3 \frac{\partial_\tau \partial_{\bar{\tau}} Z}{Z} - (2\pi i) \frac{\partial_\tau Z}{Z} \frac{(2\pi)^2 \tau_2 \partial_\tau \partial_{\bar{\tau}} Z}{Z} \right)$$

where $\langle T \rangle = 2\pi i \partial_\tau \ln Z$.



Chaotic Behavior of deformed CFTs

A Chaotic spectrum

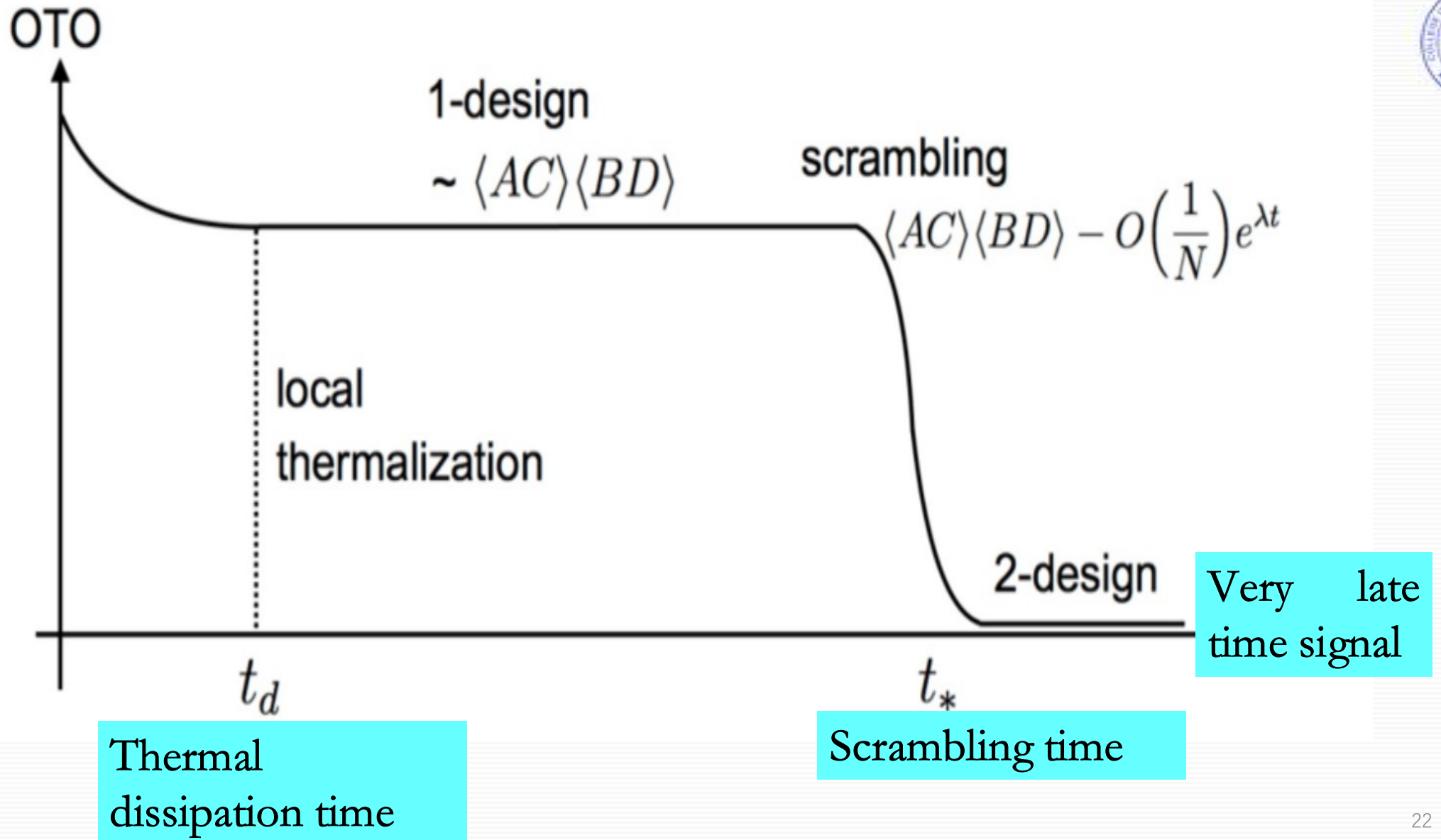


- Spectrum V.S. Quantum Chaos:
- Various spectra, the chaotic ones exhibit level repulsion and spectral rigidity.
- Their unfolded spectral distribution is a Wigner distribution $P(s) \sim s^{\beta} \exp(-s^2)$.
- These properties are shared by various ensembles of random matrices.



OTOC can diagnose the chaotic behavior of Many body system.

- **In Chaotic system, the late time behavior of physical quantities are very sensitive to the early time input.**
- **There are many quantities to capture the behavior:**
Lyapunov parameter, scrambling time scale and Ruelle resonance. [A. Larkin and Y. Ovchinnikov, 1969], [A. Kitaev, 15]
- **For (non) integrable models do not show any chaotic signals, e.g OTOC (2nd REE).** [E. Perlmutter, 16], [Y. Gu and X. L. Qi, 16], [S. He, Feng-Li Lin, J.J. Zhang, JHEP 1708 (2017) 126; JHEP 1712 (2017) 073].
- **One can also look at the spectrum form factor to test the time evolution behavior.**
- **By empirism, Holographic CFTs should have chaotic signals.**
[E. Perlmutter, 16], [J. L. Karczmarek, J. M. Maldacena and A. Strominger, 16], [J. M. Maldacena, D. Stanford, 16].





Late time of OTOC

S. He, Hongfei Shu [1907.12603]

$$\frac{\langle W(w_1, \bar{w}_1) W(w_2, \bar{w}_2) V(w_3, \bar{w}_3) V(w_4, \bar{w}_4) \rangle_\beta}{\langle W(w_1, \bar{w}_1) W(w_2, \bar{w}_2) \rangle_\beta \langle V(w_3, \bar{w}_3) V(w_4, \bar{w}_4) \rangle_\beta} \xrightarrow{T\bar{T}} \frac{\langle W(w_1, \bar{w}_1) W(w_2, \bar{w}_2) V(w_3, \bar{w}_3) V(w_4, \bar{w}_4) \rangle_\beta}{\langle W(w_1, \bar{w}_1) W(w_2, \bar{w}_2) \rangle_\beta \langle V(w_3, \bar{w}_3) V(w_4, \bar{w}_4) \rangle_\beta} \left\{ 1 - \lambda C_1(x) + \lambda C_2(x) e^{-\frac{2\pi}{\beta} t} + \dots \right\},$$

The choices of the sign of λ do not affect the late time behavior $\exp[-2\pi\beta t]$ in above equation.

D. J. Gross, J. Kruthoff, A. Rolph and E. Shaghoulian, 19



Summary

- **Correlation functions in deformed theory on the plane and torus with/without SUSY.**
- **Entanglement, OTOC in deformed theory.**
- **The situation of TJ deformation is parallel to the TT deformed theory.**



Future Problems

- **Higher order correlation functions in deformed theory.** [Yuan, Yu-Xuan]
- **Entanglement entropy, Renyi Entropy, etc. in deformed CFTs**
- **Entanglement of Purification (EOP) of Local excited states. (6-pt)**
- **Complexity, Psuedo Entanglement entropy of deformed theory,**
- **Correlation function in Deep UV [Cardy,19; 2006.03054](Non-local)**
- **Modular structure of higher point correlation functions on torus (Sub class).**
- **HHLL, HHLLLL.....**



Thanks for your attention!