Fermi seas from Bose condensates and a bosonic exclusion principle

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Based on

- 2008.00024 (SM, Amiya Mishra, Naveen Prabhakar)
- See also 1511.04772 M.Geracie, M. Goykhman and D. Son
- Also I. Halder, L. Janagal, SM, N. Prabhakar, D. Radicevic and T. Sharma, to appear

Intro: Gauge Theories at large N

- Well known that ‘vector’ like large N limits are easy to solve, but ‘matrix’ like large N limits are typically intractable. Large N $SU(N)$ gauge theories always have gauge bosons which are matrix like fields. Typically hard to solve.
- Exception. Pure Chern Simons theory in $d = 3$. Solvable at finite N. So also at large N. Price you pay for this: the theory is topological.
- Now consider CS theories coupled to matter in the fundamental rep. Now genuine QFT. Realized in 2011 theory still solvable at large N
- Large N solution much studied in last 9 years in the limit $N \to \infty$, $k \to \infty$ N/k held fixed. Results nontrivial function of this ratio. Study of these exactly solved field theories continues to lead to new qualitative insights. (Hopefully) new such lesson in this talk.

Intro: 'Quasi Fermionic' theories

- Simplest and best studied large N matter CS theories. So called Quasi Fermionic theories. This talk: single matter flavour. (Generalization to finite number of flavour N_f also solvable and studied. Ignore in this talk. Generalization to N_f order N not solvable using our large N techniques).
- At least order by order in $1/N$, theories defined by path integral using actions

$$SU(N_F)_{\left(k_F - \frac{\mathrm{sgn}(k_F)}{2}\right)} + \int \bar{\psi} D_\mu \gamma^\mu \psi + m_F^{\mathrm{reg}} \bar{\psi}\psi$$

$$U(N_B)_{(k_B, k_B)} + \int D_\mu \bar{\phi} D^\mu \phi + \sigma_B \left(\bar{\phi}\phi + \frac{N_B}{4\pi} m_B^{\mathrm{cri}} \right)$$

- Refer to these as the 'regular fermion' and critical boson theories respectively. Can also interchange fermionic SU and bosonic U theories- or study U U theories with shifted $U(1)$ levels. All very similar at large N.

Intro: Duality map and notation

- These two classes of theories are conjecturally dual.
- Notation

$$\kappa_B = \mathrm{sgn}(k_B)(N_B + |k_B|), \quad \lambda_B = \frac{N_B}{\kappa_B}, \quad (\text{and} \quad B \leftrightarrow F)$$

- Conjectured duality map

$$k_B = -\mathrm{sgn}(k_F)N_F, \quad N_B = |k_F|, \quad m_B^{\text{cri}} = \left(\frac{|k_F| + N_F}{k_F}\right) m_F^{\text{reg}}$$

- Equivalently

$$\kappa_B = -\kappa_F, \quad \lambda_F = \lambda_B - \mathrm{sgn}(\lambda_B), \quad -\lambda_B m_B^{\text{cri}} = m_F^{\text{reg}}$$

- Level and rank duality map believed to be exact. Mass map known only at leading order in large N. Note large N results nontrivial function of effective 't Hooft coupling λ.

Intro: ‘Quasi Fermionic’: Phases 1

- Simple gauge invariant operators J_μ^s are bilinears and so always bosonic. Obscures Bose Fermi nature of duality.
- Motivates the study of massive phases where gauge charged particle like excitations appear meaningful.
- Mass term only relevant operator. Phase diagram with two distinct massive phases (sign of deformation) separated by a second order phase transition.
- Bosonic side. Positive mass deformation $m_B^{\rm cri} > 0$. $SU(N_B)$ ‘spins’ in the paramagnetic phase, gauged. Elementary excitations the $SU(N_B)_{k_B}$ ‘spins’ created by the boson ϕ^a.
- Negative mass deformation $m_B^{\rm cri} < 0$. CS gauged $SU(N_B)_{k_B}$ ‘spins’ in the ferromagnetic phase. Higgs phenomenon. Use unitary gauge to put ϕ in $N_B^{\rm th}$ direction. ϕ degrees of freedom eaten up. $SU(N_B-1)_{k_B}$ CS gauge fields coupled to a massive fundamental W_μ boson plus massive neutral Z_μ boson. ‘Vector Excitations’.

Intro: 'Quasi Fermionic': Phases 2

- $m_B^{\rm cri} > 0$ maps to $m_F^{\rm reg} k_F > 0$. Two phases, massive fermions with masses of opposite signs. Low energy theories topological.

$$U(N_F)_{(k_F,k_F)} \leftrightarrow SU(N_B)_{k_B}$$

$$U(N_F)_{(\tilde{k}_F,\tilde{k}_F)} \leftrightarrow SU(N_B-1)_{k_B}$$

$$\tilde{k}_F = \mathrm{sgn}(k_F)(|k_F|-1), \quad \leftrightarrow \quad \text{means level rank dual to}$$

- Excitations on both signs are the elementary fermions.
- How do the elementary charged excitations map across duality?
- Claim. First phase the fermions map to the elementary bosonic spins. Second phase the fermions map to the W_μ bosons. Return to this point below.

Spins: The puzzle

- Puzzle. ϕ is a scalar field. Its excitations classically have spin zero.
- W_μ is a vector field. Classically, the excitation created by it can easily be shown to carry spin of modulus unity. More precisely the spin of this excitation classically equals $\mathrm{sgn}(k_B)$.
- On the other hand ψ excitations classically have spin of modulus $\frac{1}{2}$. More precisely the classical spin of these excitations is $\frac{\mathrm{sgn}(m_F)}{2}$
- Given all these facts, how can ϕ and W_μ excitations possibly be dual to ψ excitations as we have claimed above?

Spins: The resolution

- Answer. Intrinsic (or classical) spins are additively renormalized by a statistical Chern Simons (analogy $E \times B$) contribution. $s_{stat} = \frac{c_2(R)}{2\kappa}$. Physical requirement

$$s^B_{intrinsic} + s^B_{stat} = s^F_{intrinsic} + s^F_{stat} \tag{1}$$

- It turns out (group theory)

$$s^F_{stat} - s^B_{stat} = \frac{\mathrm{sgn}(k_B)}{2} = -\frac{\mathrm{sgn}(k_F)}{2}$$

- Follows that (1) works provided

$$s^B_{intrinsic} = \frac{1}{2}\left(\mathrm{sgn}(m_F) - \mathrm{sgn}(k_F)\right).$$

But easy to see its true in both phases. Indeed the exact solution for ϕ, W_μ and ψ propagators demonstrates that excitation masses also match across duality.

Exchange statistics: the puzzle

- Have seen that proposed dual excitations have equal values of all spacetime quantum numbers (spins and masses) across duality. Fantastic.
- However this is not enough to resolve all paradoxes. Recall that two (and in general multi) particle boson/fermion states are necessary antisymmetric under exchange. How, then, can the spectrum of multiparticle states match across duality?
- Might at first suspect that the resolution to this puzzle lies in anyonic phases. Perhaps the anyonic phases equalize statistics. At large N (in fact at any $N \neq 1$) this is not the case as we now explain.(Sense in which it is true at $N = 1, k = 1$.)

Statistics: Irrelevance of anyonic phases at large N

- To see clearly consider the non relativistic limit.
- Problem equivalent to scattering of a non relativistic particle of a flux tube of magnitude

$$\nu = \frac{c_2(R_1) + c_2(R_2) - c_2(R)}{\kappa}.$$

- $2\pi\nu =$ effective anyonic phase seen by the S matrix. Turns out $\nu = \mathcal{O}(1/N)$ in both $FF \to FF$ channels. Scattering of two identical fundamentals effectively non anyonic.
- As an aside we note that at large N the only effectively anyonic scattering channel is $AF \to AF$ in the singlet sector. As an aside we note that there is an interesting related issue - the usual rules of S matrix crossing symmetry are violated in this channel. Unrelated to question of this talk - will not elaborate - move on.

Statistics: Resolution

- Resolution to this puzzle obtained from the results of computation of the the exact (large N) S matrix of two fundamental fermions or bosons.
- Two inequivalent channels of scattering:

$$FF \to FF\ (sym), \qquad FF \to FF\ (as)$$

- Direct (all orders) computation.

$$S^{Boson} = a\mathcal{P}_{sym} + b\mathcal{P}_{as}, \quad S^{Fermion} = b\mathcal{P}_{sym} + a\mathcal{P}_{as}.$$

- Lesson: the difference between Bose and Fermi statistics is compensated by nontrivial duality action on 'hidden' gauge indices. A state symmetric in gauge indices. Allows for matching of statistics for 'non hidden' indices. Physical 'explanation' of the well known map between representations of Wilson loops under level rank duality.

Statistics: Loose End

- There is an obvious issue with the discussion of the last 5 slides. How can (for instance) the ϕ particles map to ψ particles in the unHiggsed phase when there are N_B ϕ particles but N_F ψ particles?
- CS theories on R^2 need to be carefully defined (what are the boundary conditions on gauge fields at infinity? Are there WZW type edge modes on $\mathcal{I}^+$ and $\mathcal{I}^-$). A safe (if bit boring) way to define the theory is by regarding R^2 as S^2 in the limit of an infinitely large radius.
- With this definition single particle states don't exist. Gauss law. Contradiction disappears at level of states with few particles. One might, however, suspect that the contradiction will return when the number of particles becomes much larger than N_B or N_F. Turn to study of thermodynamics on S^2 - and to the question of quasi particle occupation numbers. Will lead to next puzzle.

Review: Thermal partition function I

- The thermal partition function of (for instance) the regular boson theory, $\mathcal{Z}$ on $S^2 \times S^1$ is given as follows.
- $$\mathcal{Z}_{S^2\times S^1} = \int [dU]_{\rm CS}\, e^{-\mathcal{V}_2 T^2 v[\rho]}\ , \qquad (2)$$
- Here the unitary matrix U is the zero mode of the gauge field holonomy around the thermal circle S^1. $\mathcal{V}_2$ is the volume of S^2 and T is the temperature. $[dU]_{\rm CS}$ is a Chern-Simons modified Haar measure. ρ is the holonomy eigenvalue distribution function. $v[\rho]$ is defined by
- $$e^{-\mathcal{V}_2 T^2 v[\rho]} = \int_{\mathbb{R}^2\times S^1} [d\phi]\, e^{-S[\phi,\rho]}\ , \qquad (3)$$

  $[d\phi]$ is the integral over all field theory modes other than the holonomy zero mode.

Review: Thermal partition function I

- The thermal partition function of (for instance) the critical boson theory, $\mathcal{Z}$ on $S^2 \times S^1$ is given as follows.
- $$\mathcal{Z}_{S^2\times S^1} = \int [dU]_{\text{CS}}\, e^{-\mathcal{V}_2 T^2 v[\rho]} \,, \tag{4}$$
- Here the unitary matrix U is the zero mode of the gauge field holonomy around the thermal circle S^1. $\mathcal{V}_2$ is the volume of S^2 and T is the temperature. $[dU]_{\text{CS}}$ is a Chern-Simons modified Haar measure. ρ is the holonomy eigenvalue distribution function. $v[\rho]$ is defined by
- $$e^{-\mathcal{V}_2 T^2 v[\rho]} = \int_{\mathbb{R}^2\times S^1} [d\phi]\, e^{-S[\phi,\rho]} \,, \tag{5}$$

  $[d\phi]$ is the integral over all field theory modes other than the holonomy zero mode.

Review: Thermal Partition Function II

- The great thing is that $v[\rho]$ is effectively computable in the large N limit. The computation proceeds by setting up and exactly solving a Schwinger Dyson equation for the exact thermal propagator and then 'bootstrapping' the result for this two point function into a result for the thermal free energy.
- Final result for $v[\rho]$ for the Critical Boson theory is the extremum over the auxiliary variables $\tilde{\mathcal{S}}$ and $\hat{c}_B$ of the so called 'off shell free energy'.
-

$$
\begin{aligned}
&F_{\rm CB}(c_B, \tilde{\mathcal{S}}) \\
&= \frac{N_B}{6\pi\beta^3}\Bigg[\frac{3}{2}\hat{c}_B^2\hat{m}_B^{\rm cri} - 4\lambda_B^2\left(\tilde{\mathcal{S}} - \tfrac{1}{2}\hat{m}_B^{\rm cri}\right)^3 + 6|\lambda_B|\hat{c}_B\left(\tilde{\mathcal{S}} - \tfrac{1}{2}\hat{m}_B^{\rm cri}\right)^2 \\
&- \hat{c}_B^3 + 3\int_{\hat{c}_B}^{\infty} d\hat{\epsilon}\,\hat{\epsilon}\int_{-\pi}^{\pi} d\alpha\,\rho_B(\alpha)\left(\log(1-e^{-\hat{\epsilon}+\hat{\mu}+i\alpha}) + \log(1-e^{-\hat{\epsilon}-\hat{\mu}-i\alpha})\right)\Bigg]\,. \qquad (6)
\end{aligned}
$$

Review: Thermal Partition Function III

- Though I will not display the detailed expressions, it turns out that the exact solution for the thermal two point function from which this expression follows is very simple. In particular the only non analyticities of this propagator are poles (there are no cuts or any other complicated analytic structures).
- It follows that the thermal ensemble described by our exact all orders result is very simple. Its given by a collection of quasiparticles that are infinitely long lived in the large N limit. The masses of these quasiparticles are dynamically determined by extremizing the off shell functional (6).

Review: Thermal Partition Function IV

- Actually the previously computed expression (6) had been computed only for a range of chemical potentials. (6) is correct only when $|\mu| < c_B$. In today's talk we will be very interested in the complementary range of parameters, i.e. chemical potentials that are larger than the thermal mass.
- So the first technical job we had to undertake in the current paper was to generalize (6) to this complementary range of chemical potentials. I will spare you the details of how we obtained this generalization, and simply present our final results in a special limit (the limit in which the volume of the sphere is taken to infinity, i.e. the limit of primary physical interest).

Corrected results for Thermal Partition Function

- $$
\begin{aligned}
&F_{\rm CB}(c_B, \tilde{\mathcal{S}}) \\
&= \frac{N_B}{6\pi\beta^3}\Bigg[\frac{3}{2}\hat{c}_B^2\hat{m}_B^{\rm cri} - 4\lambda_B^2\left(\tilde{\mathcal{S}} - \tfrac{1}{2}\hat{m}_B^{\rm cri}\right)^3 + 6|\lambda_B|\hat{c}_B\left(\tilde{\mathcal{S}} - \tfrac{1}{2}\hat{m}_B^{\rm cri}\right)^2 \\
&\qquad - \hat{c}_B^3 + \frac{3}{2\pi|\lambda_B|}\int_{\hat{c}_B}^{\infty} d\hat{\epsilon}\,\hat{\epsilon}\int_{-\pi|\lambda_B|}^{\pi|\lambda_B|} d\alpha\,\left(\log(1 - e^{-\hat{\epsilon}+\hat{\mu}+i\alpha}) + \log(1 - e^{-\hat{\epsilon}-\hat{\mu}-i\alpha})\right) \\
&\qquad - \Theta(|\mu| - c_B)\frac{(|\hat{\mu}| - \hat{c}_B)^2(|\hat{\mu}| + 2\hat{c}_B)}{2|\lambda_B|}\Bigg]\,. \qquad (7)
\end{aligned}
$$
- The last line of (7) - proportional to $\Theta(|\mu| - c_B)$ - is new. As you will see it will play a crucial role in the story that follows.
- The regular boson theory has a similar correction term. The fermionic theories, on the other hand, receive no theta function type corrections to their off-shell free energy formulae in the $V \to \infty$ limit They do receive such corrections - which we have determined - at finite V, but I will not describe them further in this talk.
- The function $v[\rho]$ obtained on extremizing the simple expression (7) w.r.t auxiliary variables is a complicated function of temperature, chemical potential and couplings.

Occupation Numbers: The puzzle

- With the technicalities under control we can now return to reconciling the difference between fermions and bosons.
- We have seen that the thermal ensemble of the CB theory (and though we have not given details, also of the dual RF theory) is effectively an ensemble of free quasi particles. As reviewed above, the quantum numbers of all quasi particles match perfectly across the duality.
- How is all this consistent with the fact that the occupation number of an ensemble of effectively free Fermions and bosons to have effectively different single particle occupation numbers for single particle states of energy ϵ and charge q (in our theories $q = \pm 1$ for fundamental / antifundamental particles)

$$\bar{n}_F(\epsilon, \mu) = \frac{1}{e^{\beta(\epsilon - q\mu)} + 1}, \quad \bar{n}_B(\epsilon, \mu) = \frac{1}{e^{\beta(\epsilon - q\mu)} - 1} \qquad (8)$$

Occupation Numbers: Resolution I

- In order to resolve this paradox we simply use the explicit results of the off-shell free energy to compute the charge of our ensembles.
- Recall that the free energy $\mathcal{F}$ is simply the extremal value of the off-shell free energy F w.r.t. auxiliary variables and holonomies. Schematically

$$\mathcal{F}(\mu) \equiv F(\varphi_i^*(\mu), \mu) \ . \tag{9}$$

- It follows from the chain rule that

$$-Q = \left(\frac{\partial \mathcal{F}(\mu)}{\partial \mu} \right)_\beta = \frac{\partial F}{\partial \mu}\bigg|_{\varphi_i = \varphi_i^*} + \sum_i \frac{\partial F}{\partial \varphi_i}\bigg|_{\varphi_i = \varphi_i^*} \frac{\partial \varphi_i^*}{\partial \mu} \ . \tag{10}$$

- However $\mathcal{F}$ is obtained from F precisely by extremizing F w.r.t. ϕ. It follows that $\frac{\partial F}{\partial \varphi_i}$ vanish on-shell.

- It follows that on-shell

$$Q = -\frac{\partial F}{\partial \mu}\Big|_{\varphi_i=\varphi_i^*} . \quad (11)$$

So we the charge can be written in terms of the simple off-shell free energy rather than in terms of the horrendously complicated on-shell free energy.

- A simple computation demonstrates that the net charge of our system is indeed given by the sum over charges associated with each quasiparticle state, provided the quasiparticle occupation numbers $\bar{n}_F(\epsilon, \mu)$ and $\bar{n}_B(\epsilon, \mu)$ are give by:

Corrected Fermion Occupation Numbers

- $$\bar{n}_F(\epsilon,\mu) = \frac{1}{2\pi|\lambda_F|}\int_{-\pi|\lambda_F|}^{\pi|\lambda_F|} d\alpha\, \frac{1}{e^{\beta(\epsilon-q\mu)-iq\alpha}+1}\,, \qquad (12)$$
$$= \frac{1}{2} - \frac{1}{\pi|\lambda_F|}\tan^{-1}\left(\frac{e^{\beta(\epsilon-q\mu)}-1}{e^{\beta(\epsilon-q\mu)}+1}\tan\frac{\pi|\lambda_F|}{2}\right).$$
- The first line of (12) was obtained by Geracie, Goykhman and Son. The formula on the second was obtained in our paper.
- (12) is a one parameter generalization of the famous occupation number formula of Fermi statistics cited above. It reduces to the standard formula in the limit $|\lambda_F| \to 0$.

Corrected Boson Occupation Numbers

- $$\bar{n}_B(\epsilon,\mu) = \frac{1}{2\pi|\lambda_B|}\int_{-\pi|\lambda_B|}^{\pi|\lambda_B|} d\alpha \frac{1}{e^{\beta(\epsilon-q\mu)-iq\alpha}-1} + \frac{1}{|\lambda_B|}\Theta(q\mu-\epsilon)\,, \tag{13}$$
$$= \frac{1-|\lambda_B|}{2|\lambda_B|} - \frac{1}{\pi|\lambda_B|}\tan^{-1}\left(\frac{e^{\beta(\epsilon-q\mu)}-1}{e^{\beta(\epsilon-q\mu)}+1}\cot\frac{\pi|\lambda_B|}{2}\right),$$
- This new result for the bosonic occupation number is a one parameter generalization of the famous Bose Einstein occupation number formula (see above). For states with $\epsilon > q|\mu|$ it reduces to this well known formula in the limit $\lambda_B \to 0$. See below for more discussion of states with $\epsilon < |\mu|$.

Map of Occupation Numbers under Duality

- The occupation numbers listed above do *not* map to each other under duality. Instead they obey the following slightly more involved relation
- $$N_F \bar{n}_F(\epsilon, \mu) = N_B \bar{n}_B(\epsilon, \mu) \ . \qquad (14)$$
- But this is precisely what should happen in order that the net charge agree across duality.
- Once again we see that the invisible 'gauge' quantum numbers are the key to reconciling duality and statistics.

Zero Temperature Limit: Fermions

- It is instructive to examine what happens to the modified occupation number formulae in the limit that the temperature is taken to zero.
- In this limit the fermionic occupation number simplifies to

$$\bar{n}_F(\epsilon, \mu) = \Theta(q\mu - \epsilon) \ , \tag{15}$$

- At zero temperature we see every quasi fermionic state is occupied exactly once. It follows, in other words, that the zero temperature phase of our fermionic quasi particles is simply a garden variety Fermi Sea.

Zero Temperature Limit: Bosons

- For bosons, on the other hand, we find that at zero temperature

$$\bar{n}_B(\epsilon, \mu) = \bar{n}\, \Theta(q\mu - \epsilon)\ , \quad \text{with} \quad \bar{n} = \frac{1 - |\lambda_B|}{|\lambda_B|}\ . \qquad (16)$$

- To understand the interpretation of this formula, recall that in a truly free bosonic theory characterized by the ensemble $\mathrm{Tr}\, e^{-\beta(\epsilon - q\mu)}$, every quasiparticle state with $q\mu > \epsilon$ is infinitely populated (and all other states are unpopulated).
- In the limit $|\lambda_B| \to 0$ this is exactly how (16) behaves. It follows that the zero temperature bosonic phase is that of a Bose condensate. The run away condensation of a free Bose theory is stabilized by the Chern Simons interactions.

Bose Exclusion Principle

- The Fermi exclusion principle asserts that no one particle fermion state can be occupied by more than one fermion. The results of the previous slide suggest that Chern Simons coupled bosons obey an analogous Bose exclusion principle. No single particle boson state can have occupation number greater than

$$\bar{n} = \frac{1 - |\lambda_B|}{|\lambda_B|} = \frac{|k_B|}{N_B} \; . \tag{17}$$

- As the RHS of (17) is, in general, fractional, the meaning of this principle seems quite mysterious. Note however that $N_B \bar{n} = |k_B|$ is an integer.
- This suggests that we should word the Bose exclusion principle in the following way: the net occupation number of any single particle state, once we sum over all colours, cannot exceed $|k_B|$.

- Worded in this manner, the Bose exclusion principle is strongly reminiscent of the fact that one is not allowed to completely symmetrize more than $|k|$ fundamental indices in $SU(N)_k$ WZW theory. Though we do not understand the details yet, it seems likely to us that this observation will provide an 'explanation' of the Bose exclusion principle.
- Note that the Bose exclusion principle stated is this manner fits perfectly with the ideas of level rank duality - which - recall, roughly speaking interchanges rows and columns of a Tableaux.

Which Bosons Condense?

- Recall that zero temperature zero chemical potential phase diagram of the CB theory is given by

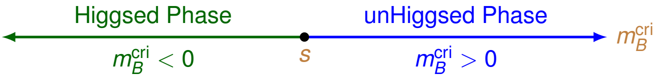

Figure: Phase diagram of the critical boson theory as a function of $m_B^{\rm cri}$ at $|\mu| = T = 0$. Here, s marks the origin of the $m_B^{\rm cri}$ axis at which point the theory undergoes a second order phase transition.

- The charged excitations of the Higgsed phase are W_μ bosons. The charged excitations of the un Higgsed phase are simple bosons. Might thus seem there are two inequivalent Bose condensed phases.

Critical Boson Phase diagram

- Not difficult to use the exact above to work out the free energy in all phases. We find the following phase diagram.

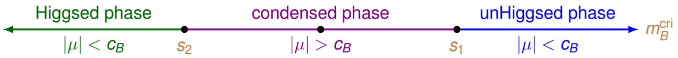

Figure: Phase diagram of critical boson theory as a function of m_B^{cri} at fixed μ. At the points $s_1 = |\mu|$, $s_2 = -|\mu|\left(\frac{2-|\lambda_B|}{|\lambda_B|}\right)$ the theory undergoes a second order phase transition. The point inside the condensed phase corresponds to $m_B^{\text{cri}} = -|\mu|\left(\frac{1-|\lambda_B|}{|\lambda_B|}\right)$ and denotes a change in description from that a phase of condensed scalars to one of phase of condensed W-bosons though the condensed phase is itself unique.

- We see the two condensed phases are simply analytic continuations of each other - they are the same phase!

Phase diagram of the Regular Boson theory

- In addition to the CB and RF duality, it has also been conjectured that so called Regular Bosons and Critical Fermions are dual to each other. We have also worked out the more complicated finite chemical potential phase diagram of these more complicated theories. To end this talk I simply flash results.

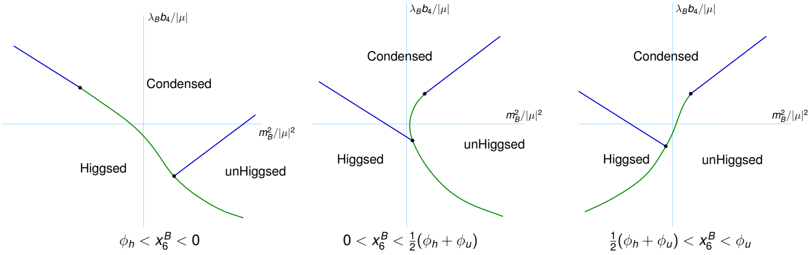

Figure: Phase diagram for the regular boson theory at zero temperature for various subranges of the stable range $\phi_h < x_6^B < \phi_u$. The blue lines are second order transitions while the green lines are first order transitions.

Discussion

- It would be very interesting to re-derive the effective occupation number formulae presented in this talk from a Hamiltonian (Schrodinger Equation) viewpoint. Would help us better understand their origin.
- Would be useful to understand the Bose exclusion principle from several points of view, including the connection to WZW theory.
- The Bose condensate encountered in our analysis is an extremely simple stabilization of the run away instability of free theory. The sharp cut off at k_F plus the Bose exclusion principle gives this phase all the properties of a Fermi Sea.
- It would be interesting to investigate the dynamical implications of the Bose condensation principle. Cut off lasers?
- ...