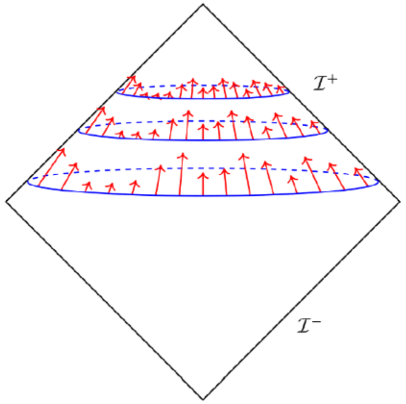
Hawking Flux of Black Hole with nonlinear soft-hairs

Shingo Takeuchi

(Phenikaa University)

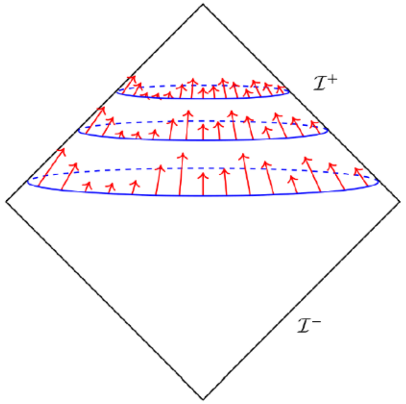
arXiv:2004.07474 (PRD)

With Feng-Li Lin (National Taiwan Normal University)



Asymptotic symmetry

- Supertranslation
  - maps to an asymptotically flat space-times but physically **another solution**.
  - Supertranslated spacetimes are normal (just a Schwartzschild BH is special)



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to 1st order

Soft Hair of Dynamical Black Hole and Hawking Radiation
C.-S. Chu, Y.Koyama (arXiv:1801.03658)

Correction of supertranslation up to 2nd order

$$ds^2 = -\left(1-\frac{2m}{r_s}+\cdots+\mathcal{O}\left(\varepsilon^3\right)\right)dt_s^2+\left(\frac{1}{1-\frac{2m}{r_s}}+\cdots+\mathcal{O}\left(\varepsilon^3\right)\right)dr_s^2+\left(r_s^2+\cdots\right)d\theta_s^2+\left(r_s^2\sin^2\theta+\cdots\right)d\phi_s^2+2\left(\cdots\right)dr_s d\theta$$

$$\boxed{r_h = 2m-\frac{15m\sin^2(2\theta)}{8\pi}\varepsilon^2+O\left(\varepsilon^3\right)}$$

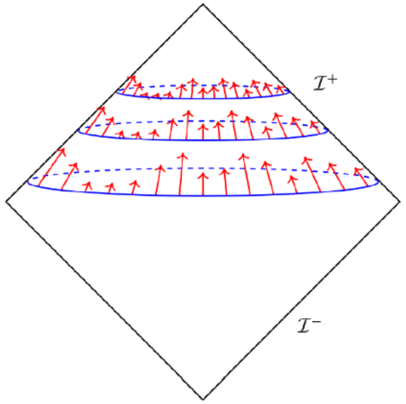
$$g_{tt}=-g_{rr}^{-1}=-\frac{r-r_h}{2m}\left(1-\frac{45\sin^2 2\theta}{8\pi}\varepsilon^2\right)+\mathcal{O}\left(\varepsilon^3\right)$$

$$T_H=\frac{1}{4\pi}\left|\partial_r f(r)\big|_{r=r_h}\right|$$

$$\Rightarrow\quad T_H=\frac{1}{8\pi m}+O\left(\varepsilon^3\right)$$

$$(m_{\mathrm{eff}})_{kn,\,lm}\equiv m+\frac{15m}{8\pi r}\mathcal{I}^C_{kn,\,lm}\varepsilon^2+O\left(\varepsilon^3\right)$$

$$(g_{\mathrm{2D\ eff}})^{tt}_{kn,lm}=-\frac{2m}{r-2m\left(1+\frac{\mathcal{I}^C_{kn,\,lm}}{\Lambda_{kn,\,lm}}\frac{15}{16\pi r}\varepsilon^2\right)}=-\frac{2(m_{\mathrm{eff}})_{kn,\,lm}}{r-2(m_{\mathrm{eff}})_{kn,\,lm}}+O\left(\varepsilon^3\right)$$



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➢ $$ds^2 = -\left(1 - \frac{2m}{r_s} + \cdots + \mathcal{O}\left(\varepsilon^3\right)\right) dt_s^2 + \left(\frac{1}{1 - \frac{2m}{r_s}} + \cdots + \mathcal{O}\left(\varepsilon^3\right)\right) dr_s^2 + \left(r_s^2 + \cdots\right) d\theta_s^2 + \left(r_s^2 \sin^2\theta + \cdots\right) d\phi_s^2 + 2\left(\cdots\right) dr_s d\theta$$

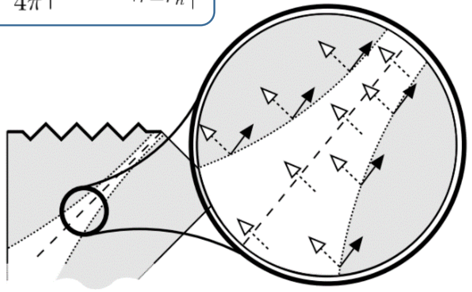
$$\boxed{r_h = 2m - \frac{15 m \sin^2(2\theta)}{8\pi}\varepsilon^2 + O\left(\varepsilon^3\right)}$$

$$g_{tt} = -g_{rr}^{-1} = -\frac{r - r_h}{2m}\left(1 - \frac{45 \sin^2 2\theta}{8\pi}\varepsilon^2\right) + \mathcal{O}\left(\varepsilon^3\right)$$

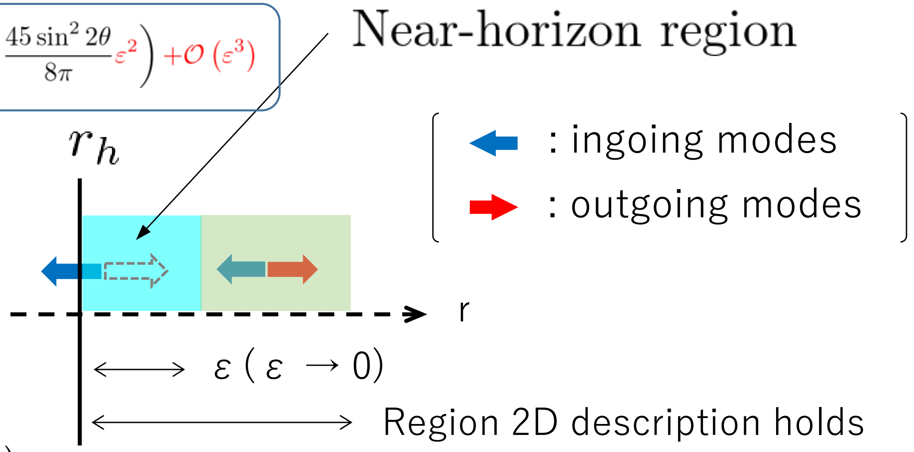
$$T_H = \frac{1}{4\pi}\left|\partial_r f(r)\right|_{r=r_h}$$

$$\Rightarrow T_H = \frac{1}{8\pi m} + O\left(\varepsilon^3\right)$$

$$(m_{\rm eff})_{kn,\,lm} \equiv m + \frac{15m}{8\pi r}\mathcal{I}^C_{kn,\,lm}\varepsilon^2 + O\left(\varepsilon^3\right)$$



Taken from **gr-qc/0502074** (S.P.Robinson, F.Wilczek)



➢ $$S = \int d^4x \sqrt{-g}\, g^{MN} \partial_M \phi \partial_N \phi$$

$$(g_{\rm 2D\ eff})^{tt}_{kn,lm} = -\frac{2m}{r - 2m\left(1 + \frac{\mathcal{I}^C_{kn,\,lm}}{\Lambda_{kn,\,lm}}\frac{15}{16\pi r}\varepsilon^2\right)} = -\frac{2(m_{\rm eff})_{kn,\,lm}}{r - 2(m_{\rm eff})_{kn,\,lm}} + O\left(\varepsilon^3\right)$$

Comment on the general supertranslation

⬇ $r = r_h + \Delta r$ (take near-horizon limit)

$$= -\int d^4x\, \sin(\theta)\,(2m)^2 \left\{1 + \frac{3}{2}\sqrt{\frac{5}{\pi}}\left(1 + 3\cos(2\theta)\right)\varepsilon + \frac{45}{2\pi}\left(\frac{\sin(2\theta)}{4} - \cos^2(\theta) + 3\cos^2(\theta)\cos(2\theta)\right)\varepsilon^2\right\}\phi^*\left(t^{tt}\partial_t\partial_t + \partial_r\left(t^{rr}\partial_r\right)\right)\phi + \cdots$$

Integrate out for angular- direction

$\phi = \phi_{lm} Y^l_m \quad (\phi = \phi_{kn} Y^n_k)$

Highly complicated

I will write this in proceedings in detail

Outline of this talk

1. What’s supertranslation ?

2. The fact that supertranslations exist is ordinary

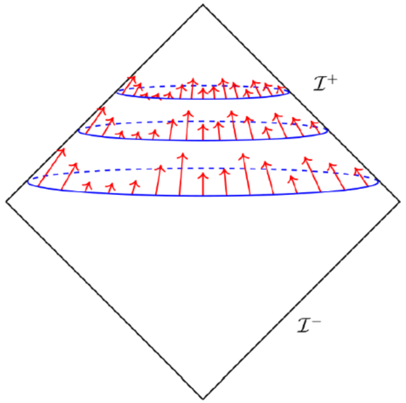
3. How to involve correction of supertranslations

4. Our analysis and result

What’s supertranslation

$$u = t - r, \quad z = e^{i\phi}\cot\frac{\theta}{2}, \quad \gamma_{z\bar z} = \frac{2}{(1+z\bar z)^2}$$

$$ds^2 = \underbrace{-du^2 - 2dudr + 2r^2\gamma_{z\bar z}dzd\bar z}_{\text{Minkowski metric}}$$
$$+ \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + rC_{\bar z\bar z}d\bar z^2 + D^zC_{zz}dudz + D^{\bar z}C_{\bar z\bar z}dud\bar z$$
$$+ \frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}(C_{zz}C^{zz})\right)dudz + \text{c.c.} + \underbrace{\cdots\cdots\cdots}_{\text{sub leadings}}$$



- $\xi = f\partial_u + \frac{1}{r}\left(D^z f\partial_z + D^{\bar z} f\partial_{\bar z}\right) + D^zD_z f\partial_r\ , \quad f = f(z,\bar z)$

arbitrary function

- **preserves** all the gauge and fall-off conditions
- **generalizations** of the four translations (← f = const.)
- transforms into a physically different geometry (**another solution**)

- **A Bondi coordinate**
  - the most usually form near $\mathcal{I}^+$
  - Falloff conditions:

$$g_{uu} = -1 + \mathcal{O}(r^{-1}), \quad g_{ur} = -1 + \mathcal{O}(r^{-2}), \quad g_{uz} = \mathcal{O}(1),$$
$$g_{zz} = \mathcal{O}(r), \quad g_{z\bar z} = r^2\gamma_{z\bar z} + \mathcal{O}(1), \quad g_{rr} = g_{rz} = 0$$

※ there is no priori methods to determine these

What I’ll talk from now are stories in this falling condition

- m_B, N_z and C_{zz} depend on $(u, z, \bar z)$.

$$\int_{S^2} m_B dzd\bar z \qquad \int_{S^2} N_z v^z dzd\bar z$$

$$N_{zz} = \partial_u C_{zz}$$

What's supertranslation

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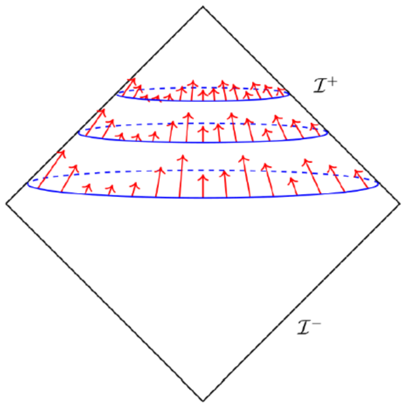

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arbitrary function

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Example

$$ds^2 = -du^2 - dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}\ (= -dt^2 + dx_i^2) \leftarrow m_B = N_z = N_{zz} = C_{zz} = 0$$

$$\mathcal{L}_f N_{zz} = f\partial_u N_{zz},$$
$$\mathcal{L}_f m_B = f\partial_u m_B + \frac{1}{4}\left[N^{zz}D_z^2 f + 2D_zN^{zz}D_zf + c.c.\right],$$
$$\mathcal{L}_f C_{zz} = f\partial_u C_{zz} \ -2D_z^2 f$$

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} + rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2 + \cdots$$

$$\delta_{st.(BMS)} : g_{\mu\nu}(x^\mu) \mapsto \tilde{g}_{\mu\nu}(x^\mu)$$

$$\mathcal{L}_f C = f$$

C is akin to NG boson
(Normally, taken as **spherical harmonics**)

(ex. for SSB of U(1) with $\phi = \phi_0 e^{i\theta}$, $\delta\theta \neq 0$)

- Bondi, van der Burg, Metzner, and Sachs
  - *Proc. Roy. Soc. Lond.* A269 (**1962**)
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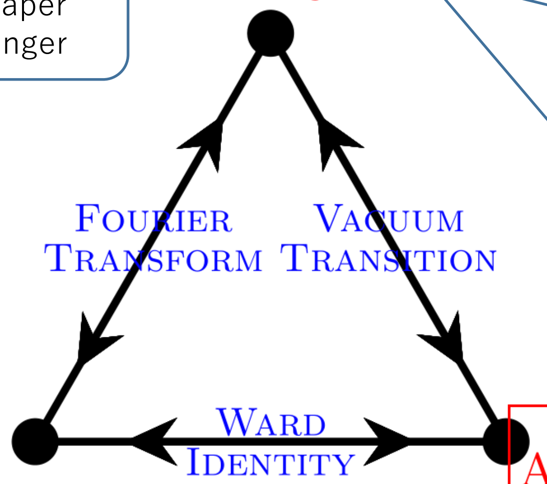
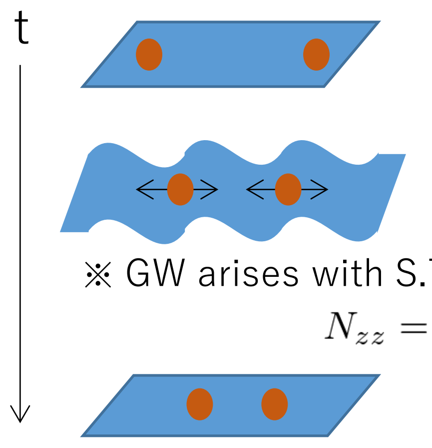
Vacua of the gravitational field
G.Compère, J.Long (**1601.04958**)

arXiv:1703.05448
Review paper
A. Strominger



$$\langle \text{out}| \left(Q_\varepsilon^+ \mathcal{S} - \mathcal{S} Q_\varepsilon^-\right) |\text{in}\rangle = 0$$



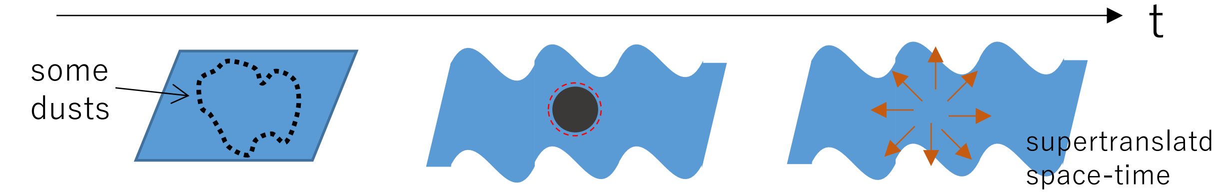
※ GW arises with S.T.

$N_{zz} = \partial_u C_{zz}$

$$ds^2 = \underbrace{-du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}}_{\text{Minkowski metric}} + \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2 + D^z C_{zz}dudz + D^{\bar{z}} C_{\bar{z}\bar{z}}dud\bar{z} + \frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}(C_{zz}C^{zz})\right)dudz + \text{c.c.} + \dots$$

$$\boxed{\Delta x^{\bar{z}} = \frac{\gamma^{z\bar{z}}}{2r}\Delta C_{zz}}$$

Eg. Kamal Hajian, Light speed as a local observable for soft hairs, **arXiv:2001.07540 ; P.Mao**



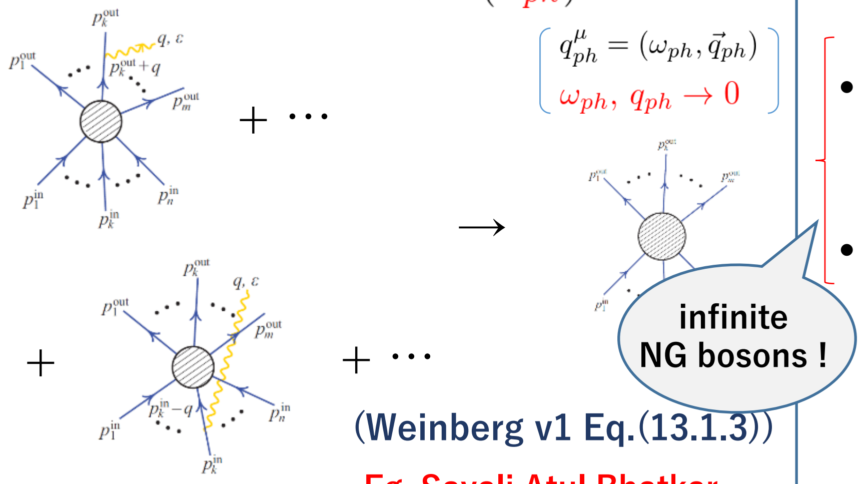
Y.Yokokura's talk; indeed, realistic BH have been just a high dense stars without horizon, but Hawking radiation exists. (arXiv:2002.10331)

(case of QED)

$$\langle \text{out}| a_+^{\text{out}}(\vec{q})\mathcal{S}|\text{in}\rangle = e\frac{Q_k p_k \cdot \varepsilon}{p_k \cdot q_{ph}}\langle \text{out}|\mathcal{S}|\text{in}\rangle + \mathcal{O}(\omega_{ph})$$

$q^\mu_{ph} = (\omega_{ph}, \vec{q}_{ph})$

$\omega_{ph},\ q_{ph} \to 0$



(Weinberg v1 Eq.(13.1.3))

Eg. Sayali Atul Bhatkar

<u>Infinite</u> charges, soft-hair

M.Porrati and A.Strominger

- $$Q_f^{+(-)} = \frac{1}{4\pi G}\int_{\mathcal{I}^+_-(\mathcal{I}^-_+)} d^2z\gamma_{z\bar{z}} f\, m_B \leftarrow$$ Super**translation** charge
- $$Q_Y^{-(+)} = \frac{1}{8\pi G}\int_{\mathcal{I}^-_+(\mathcal{I}^+_-)} dzd\bar{z}\left(Y_{\bar{z}}N_z + Y_z N_{\bar{z}}\right) \leftarrow$$ Super**rotation** charge

Information of initial state may be stored
in how space is deformed by supertranslation.

$$\underbrace{(E,\vec{p}),\ \text{and}\ \vec{J}\ \text{and}\ \vec{K}}_{10\ \text{pieces}} + \underbrace{\text{soft-hair charges}}_{\text{infinite pieces}}$$ ⇒ Information paradox

Fact supertranslated BH space-times are ordinary

Classical static final state of collapse with supertranslation memory, Geoffrey Compère, Jiang Long, 1602.05197 (CQG)

- **Collapse of spherical shell**

$$ds^2 = -\left(1 - \frac{2M\Theta(v)}{r}\right)dv^2 + 2dvdr + r^2 d\Omega^2$$

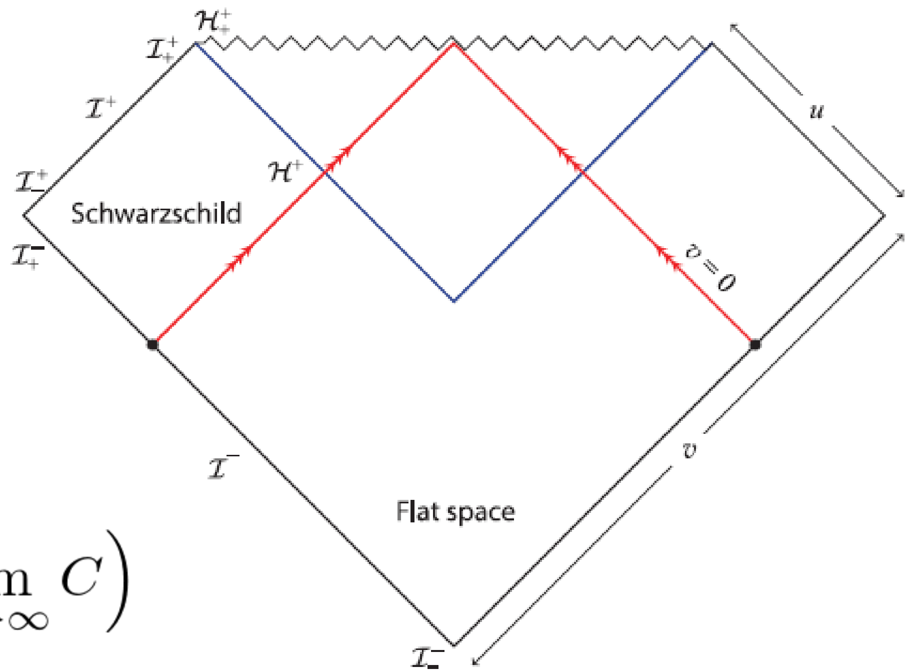
$\delta_{st.(BMS)} : g_{\mu\nu}\,(x$

$\mathcal{L}_f C = f$ C is akir
(Norma

(ex. for SSB of U(1

$$T_{\mu\nu} = \frac{M\delta(v)}{4\pi r^2}\delta^v_\mu \delta^v_\nu \quad \Longrightarrow \quad \text{Schwartzshchild BH with } C_\infty = 0$$

$$\left(C_\infty \equiv \lim_{u\to\infty} C\right)$$



- **Collapse of non-spherical shell**

$$T_{vv} = \left(\frac{MP^{in}(w,\bar{w})}{4\pi r^2} + O(r^{-3})\right)\delta(v), \quad \text{where} \quad \frac{MP^{in}(w,\bar{w})}{4\pi r^2} = \frac{M}{4\pi r^2} + \frac{M}{4\pi r^2}\sum_{l\geq 1,m} P_{l,m} Y_{l,m}$$

$$\mathcal{D}C_\infty = M(P^{in}(w,\bar{w}) - P^{out}(z,\bar{z})) - M \quad \text{where} \quad \int_{-\infty}^{\infty} du T_{uu} = \left(\frac{MP^{out}(z,\bar{z})}{4\pi r^2} + \mathcal{O}(r^{-3})\right)$$

from some combining of Einstein eq.

$$P^{out}(z,\bar{z}) = 0 \quad \text{(case for no outgoing radiation)}$$

For simplicity

$$C_\infty = \sum_{l\le 1,m} C^{(0)}_{l,m} Y_{l,m} + M \sum_{l\ge 2,m} (-1)^l \frac{4}{(l-1)l(l+1)(l+2)} P_{l,m} Y_{l,m} \quad \left[C^{(0)}_{l,m} \text{ are the 4 lowest spherical harmonics} \right]$$

NG field

$$\text{where} \quad T_{vv} = \left(\frac{M P^{in}(w,\bar{w})}{4\pi r^2} + O(r^{-3}) \right) \delta(v), \quad \frac{M P^{in}(w,\bar{w})}{4\pi r^2} = \frac{M}{4\pi r^2} + \frac{M}{4\pi r^2} \sum_{l\ge 1,m} P_{l,m} Y_{l,m}$$

Examples of solution

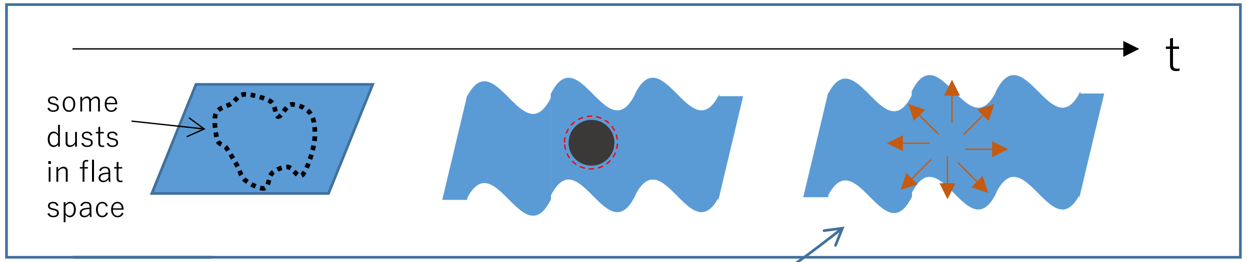
- For $(l,m) = (2,0)$, $\quad C_\infty = \alpha \frac{M}{6}(3\cos^2\theta - 1), \quad -\frac{1}{2} \le \alpha \le 1$
- For $(l,m) = (2,\pm 1)$, $\quad C_\infty = \alpha \frac{M}{6} \sin 2\theta \cos(\phi + \delta), \quad -1 \le \alpha \le 1$

$$\mathcal{L}_f C = f \quad \text{where} \quad C = C(z,\bar{z})$$

$$\xi = f\partial_u + \frac{1}{r}\left(D^z f \partial_z + D^{\bar{z}} f \partial_{\bar{z}}\right) + D^z D_z f \partial_r$$

Akin to NG boson

(ex. for SSB of U(1) with $\phi = \phi_0 e^{i\theta}$, $\delta\theta \neq 0$)



Even in the simple case where the outgoing radiation is **ignored**, non-trivial NG field appears if the initial configuration of dusts is non-spherical

Get the 4D BH space-times with supertranslation corrections to quadratic order

- $$ds^2 = -\frac{\left(1-\frac{m}{2\rho_s}\right)^2}{\left(1+\frac{m}{2\rho_s}\right)^2}dt_s^2 + \left(1+\frac{m}{2\rho_s}\right)^4 \left(d\rho_s^2 + \rho_s^2 d\Omega_s^2\right)$$

- $d\rho_s^2 + \rho_s^2 d\Omega_s^2 = dx_s^2 + dy_s^2 + dz_s^2 \quad \text{and} \quad \rho_s^2 = x_s^2 + y_s^2 + z_s^2$
- $$\begin{aligned} x_s &= (\rho - C)\sin\theta\cos\phi + \frac{\sin\phi}{\sin\theta}\,\partial_\phi C - \cos\theta\cos\phi\,\partial_\theta C, \\ y_s &= (\rho - C)\sin\theta\sin\phi - \frac{\cos\phi}{\sin\theta}\,\partial_\phi C - \cos\theta\sin\phi\,\partial_\theta C, \\ z_s &= (\rho - C)\cos\theta + \cos\theta\cos\phi\,\partial_\theta C \end{aligned}$$
- $C = m\,\varepsilon\,\underline{Y_2^0(\theta,\phi)} = m\,\varepsilon\sqrt{\frac{5}{16\pi}}\left(3\cos^2\theta - 1\right)$

$$ds^2 = -\left(1 - \frac{2m}{r_s} + \cdots\right)dt_s^2 + \left(\frac{1}{1-\frac{2m}{r_s}} + \cdots\right)dr_s^2 + \left(r_s^2 + \cdots\right)d\theta_s^2 + \left(r_s^2\sin^2\theta + \cdots\right)d\phi_s^2 + 2\left(\cdots\right)dr_s d\theta + \mathcal{O}\left(\varepsilon^3\right)$$

Classical static final state of collapse with supertranslation memory
Geoffrey Compère, Jiang Long, 1602.05197 (CQG)

As explained in [45], the shift of ρ by the lowest (constant) spherical harmonic $C_{(0,0)}$ is fixed by requiring the invariance of the radius under a constant time shift $\delta C = \Delta t$. The metric then reads

$$ds^2 = -dt^2 + d\rho^2 + \left(((\rho - E)^2 + U)\gamma_{AB} + (\rho - E)C_{AB}\right) dz^A dz^B. \tag{65}$$

supertranslated

This metric can be compared with the original global Minkowski vacuum written in static coordinates

$$ds^2 = -dt_s^2 + d\rho_s^2 + \rho_s^2 \gamma_{AB} dz_s^A dz_s^B \tag{66}$$

no supertranslated

where $t_s = u_s + \rho_s$. Following the chain of coordinate transformations, we can finally relate these two coordinate systems by the change of coordinates

$$\begin{aligned} t_s &= t + C_{(0,0)}, \\ \rho_s &= \sqrt{(\rho - C + C_{(0,0)})^2 + D_A C D^A C}, \\ z_s &= \frac{(z - \bar{z}^{-1})(\rho - C + C_{(0,0)}) + (z + \bar{z}^{-1})(\rho_s - z\partial_z C - \bar{z}\partial_{\bar{z}} C)}{2(\rho - C + C_{(0,0)}) + (1 + z\bar{z})(\bar{z}\partial_{\bar{z}} C - \bar{z}^{-1}\partial_z C)}. \end{aligned} \tag{67}$$

Transformation rules can be obtained like this

In that sense, (67) is the supertranslation generating coordinate transformation. The equality between (65) and (66) under (67) is identical to the equality (24) using (26), which proves the statement in the main text.

The metric (65) is written in static gauge defined as $g_{\rho A} = 0$, $g_{\rho\rho} = 1$. The generator of supertranslations in that gauge can be written as

$$\xi_T^{(stat)} = T_{(0,0)}\partial_t - (T - T_{(0,0)})\partial_\rho + \frac{C^{AB} D_B T - 2D^A T(\rho - \frac{1}{2}(D^2 + 2)(C - C_{0,0}))}{2((\rho - \frac{1}{2}(D^2 + 2)(C - C_{0,0}))^2 - U)}\partial_A. \tag{68}$$

These generators exactly commute under the adjusted bracket defined in [28]

$$[\xi_1, \xi_2]_{ad} \equiv [\xi_1, \xi_2] - \delta_{\xi_1}\xi_2 + \delta_{\xi_2}\xi_1. \tag{69}$$

Here, the variation δ_{ξ_1} acts on the field C and its p-th derivative, $p = 1, 2, \ldots$ as a derivative operator contracted with the p-th derivative of $\delta_T C(\theta, \phi) = T(\theta, \phi)$. As a consequence of these vanishing commutation relations, the supertranslations act everywhere in the bulk spacetime described by the metric (65) which extends the asymptotic result of [28]. In a group theory language, the metric (65) describes the orbit of Minkowski spacetime under the supertranslation group.

Get the 4D BH space-times with supertranslation corrections to quadratic order

➤ $$ds^2 = -\frac{\left(1-\frac{m}{2\rho_s}\right)^2}{\left(1+\frac{m}{2\rho_s}\right)^2}dt_s^2 + \left(1+\frac{m}{2\rho_s}\right)^4 (d\rho_s^2 + \rho_s^2 d\Omega_s^2)$$

- $ds_{(3)}^2 = dx_s^2 + dy_s^2 + dz_s^2 \quad \text{and} \quad \rho_s^2 = x_s^2 + y_s^2 + z_s^2$

- $$x_s = (\rho - C)\sin\theta\cos\phi + \frac{\sin\phi}{\sin\theta}\partial_\phi C - \cos\theta\cos\phi\,\partial_\theta C,$$
$$y_s = (\rho - C)\sin\theta\sin\phi - \frac{\cos\phi}{\sin\theta}\partial_\phi C - \cos\theta\sin\phi\,\partial_\theta C,$$
$$z_s = (\rho - C)\cos\theta + \cos\theta\cos\phi\,\partial_\theta C$$

- $\boxed{C = m\,\varepsilon\,Y_2^0(\theta,\phi) = m\,\varepsilon\sqrt{\frac{5}{16\pi}}\,(3\cos^2\theta - 1)}$

$$ds^2 = -\left(1-\frac{2m}{r_s}+\cdots\right)dt_s^2 + \left(\frac{1}{1-\frac{2m}{r_s}}+\cdots\right)dr_s^2 + (r_s^2+\cdots)\,d\theta_s^2 + (r_s^2\sin^2$$

This mode is expected to be dominant in the process of forming of a soft-hairy black hole

E. Berti, V. Cardoso and C. M. Will, PRD [gr-qc/0512160]

"On gravitationalwave spectroscopy of massive black holes with the space interferometer LISA,"

(a study on emitted gravitational wave in the merger and ringdown phases).

- T
C

- $r_{h,\,4D} = 2m - \dfrac{15m\sin^2(2\theta)}{8\pi}\varepsilon^2 + O(\varepsilon^3)$
- $(m_{\text{eff}})_{kn,\,lm} \equiv m + \dfrac{15m}{8\pi r}\mathcal{I}^C_{kn,\,lm}\varepsilon^2 + O(\varepsilon^3)$
- $r_{h,\,2D} = 2(m_{\text{eff}})_{kn,\,lm}$

➤ $$S = \int d^4x\sqrt{-g}\,g^{MN}\partial_M\phi\partial_N\phi$$

$r = r_h + \Delta r$ (take near-horizon limit)

$$S = -\int d^4x\,\sin(\theta)\,(2m)^2 \underbrace{\left\{1 + \frac{3}{2}\sqrt{\frac{5}{\pi}}(1+3\cos(2\theta))\,\varepsilon + \frac{45}{2\pi}\left(\frac{\sin(2\theta)}{4} - \cos^2(\theta) + 3\cos^2(\theta)\cos(2\theta)\right)\varepsilon^2\right\}}_{\text{dilaton part}} \phi^*\left(t^{tt}\partial_t\partial_t + \partial_r(t^{rr}\partial_r)\right)\phi + \mathcal{O}(\varepsilon^3)$$

Get 2-dim. action by $\phi = \phi_{lm}Y^l_m$ and $\int d\theta d\phi$

$$= \sum_{l=0}^{l_{max}}\sum_{|m|=0}^{l}\int d^2x\,\Phi_{lm}\left((g_{\text{eff}})^{tt}_{lm}\,\partial_t\varphi^*_{lm}\partial_t\varphi_{lm} + (g_{\text{eff}})^{rr}_{lm}\,\partial_r\varphi^*_{lm}\partial_r\varphi_{lm}\right)$$

$\left(\Phi_{lm} = (2(m_{\text{eff}})_{lm,\,lm})^2\right)$

same with Schwarzschild except for mass

$$
\begin{aligned}
(80) \;=\; & \sum_{l=0}^{l_{max}-4} \sum_{m=-l}^{l} \int d^2x\, \Omega_{lm} \left(\partial_t \phi_{lm} + \Gamma^{(4)}_{lm}\right)^* \left(\partial_t \phi_{lm} + \Gamma^{(4)}_{lm}\right) \\
& + \sum_{l=l_{max}-3}^{l_{max}-2} \sum_{m=-l}^{l} \int d^2x\, \Omega_{lm} \left(\partial_t \phi_{lm} + \Gamma^{(2)}_{lm}\right)^* \left(\partial_t \phi_{lm} + \Gamma^{(2)}_{lm}\right) \\
& + \sum_{l=l_{max}-1}^{l_{max}} \sum_{m=-l}^{l} \int d^2x\, \Omega_{lm} \partial_t \phi^*_{lm} \partial_t \phi_{lm}. \qquad (84)
\end{aligned}
$$

Since it can be written as follows:

$$
\partial_t \phi_{lm} + \Gamma^{(4)}_{lm} = \partial_t \left(\phi_{lm} + \overline{\Lambda}^{(2)}_{lm} \phi_{l+2\,m} + \overline{\Lambda}^{(4)}_{lm} \phi_{l+4\,m}\right), \qquad (85a)
$$

$$
\partial_t \phi_{lm} + \Gamma^{(2)}_{lm} = \partial_t \left(\phi_{lm} + \overline{\Lambda}^{(2)}_{lm} \phi_{l+2\,m}\right), \qquad (85b)
$$

let us perform the redefinition of the fields as

- $\varphi_{lm} \equiv \phi_{lm} + \overline{\Lambda}^{(2)}_{lm} \phi_{l+2\,m} + \overline{\Lambda}^{(4)}_{lm} \phi_{l+4\,m}$ for $l = 0, 1, \cdots, l_{max} - 4$, (86a)
- $\varphi_{lm} \equiv \phi_{lm} + \overline{\Lambda}^{(2)}_{lm} \phi_{l+2\,m}$ for $l = l_{max} - 3, l_{max} - 2$, (86b)
- $\varphi_{lm} \equiv \phi_{lm}$ for $l = l_{max} - 1, l_{max}$, (86c)

where m above are $0, \pm 1, \cdots, \pm(l-2)$ for each l. The leadings of $\overline{\Lambda}^{(K)}_{lm}$ is $\varepsilon^{K/2}$.

$$
(84) = \sum_{l=0}^{l_{max}} \sum_{|m|=0}^{l} \int d^2x\, \Phi_{lm} \left((g_{\rm eff})^{tt}_{lm}\, \partial_t \varphi^*_{lm} \partial_t \varphi_{lm} + (g_{\rm eff})^{rr}_{lm}\, \partial_r \varphi^*_{lm} \partial_r \varphi_{lm} \right), \qquad (87)
$$

$$
\phi_{lm} \to \frac{\phi_{lm}}{\left(\Theta^{(0)}_{lm}\right)^{1/2}} \quad \text{for all } l,\, m,
$$

To appear in the conference proceedings of this

$$
\frac{(g_{\rm eff})^{rr}_{l+K\,m}}{(g_{\rm eff})^{rr}_{lm}} \sim 1 + \varepsilon^2, \quad \Lambda^{(K)}_{lm} \sim \varepsilon^{K/2}, \quad \Theta^{(0)}_{lm} \sim 1 + \varepsilon + \left(1 + \frac{1}{r}\right)\varepsilon^2, \quad \frac{\Lambda^{(K)}_{lm}}{\Theta^{(0)}_{lm}} \sim \varepsilon^{K/2}, \quad (70)
$$

$$
\begin{aligned}
(68) \;=\; & \sum_{l=0}^{l_{max}-4} \sum_{|m|=0}^{l} \int d^2x\, \Omega_{lm} \left(\left(\partial_t \phi_{lm} + \Gamma^{(4)}_{lm}\right)^* \left(\partial_t \phi_{lm} + \Gamma^{(4)}_{lm}\right) - \left(\overline{\Lambda}^{(2)}_{lm}\right)^2 \partial_t \phi^*_{l+2m} \partial_t \phi_{l+2m} \right) \\
& + \sum_{l=l_{max}-3}^{l_{max}-2} \sum_{|m|=0}^{l} \int d^2x\, \Omega_{lm} \left(\left(\partial_t \phi_{lm} + \Gamma^{(2)}_{lm}\right)^* \left(\partial_t \phi_{lm} + \Gamma^{(2)}_{lm}\right) - \left(\overline{\Lambda}^{(2)}_{lm}\right)^2 \partial_t \phi^*_{l+2m} \partial_t \phi_{l+2m} \right) \\
& + \sum_{l=l_{max}-1}^{l_{max}} \sum_{|m|=0}^{l} \int d^2x\, \Omega_{lm} \partial_t \phi^*_{lm} \partial_t \phi_{lm} + O\left(\varepsilon^3\right). \qquad (75)
\end{aligned}
$$

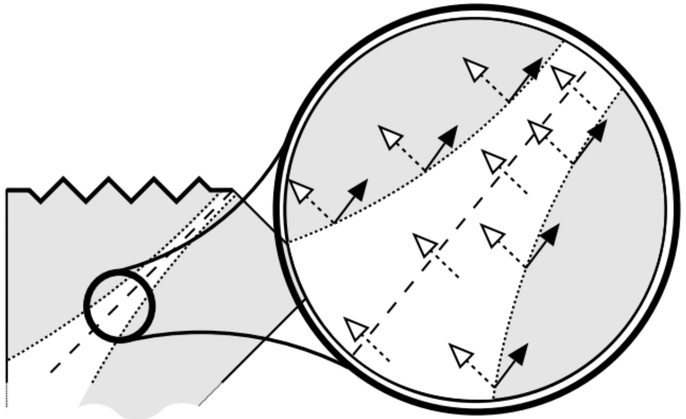
- $\Omega_{lm} \left(\overline{\Lambda}^{(2)}_{lm}\right)^2 \partial_t \phi^*_{l+2\,m} \partial_t \phi_{l+2\,m} \to \Omega_{l+2\,m} \Xi_{l+2\,m} \partial_t \phi^*_{l+2\,m} \partial_t \phi_{l+2\,m}$ for all l, m. (79)
- Therefore, uniformly sliding each "$\Omega_{lm} \left(\overline{\Lambda}^{(2)}_{lm}\right)^2 \partial_t \phi^*_{l+2m} \partial_t \phi_{l+2m}$" by 2 regarding l in (75),

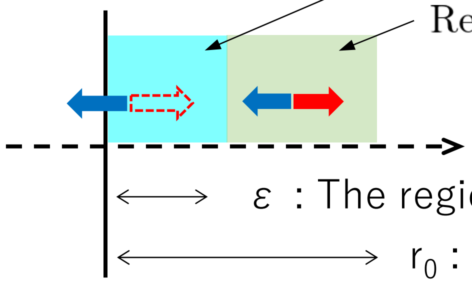
$$
\begin{aligned}
(75) \;=\; & \sum_{l=0}^{1} \sum_{|m|=0}^{l} \int d^2x\, \Omega_{lm} \left(\partial_t \phi_{lm} + \Gamma^{(4)}_{lm}\right)^* \left(\partial_t \phi_{lm} + \Gamma^{(4)}_{lm}\right) \\
& + \sum_{l=2}^{l_{max}-4} \sum_{|m|=0}^{l-2} \int d^2x\, \Omega_{lm} \left(\left(\partial_t \phi_{lm} + \Gamma^{(4)}_{lm}\right)^* \left(\partial_t \phi_{lm} + \Gamma^{(4)}_{lm}\right) - \Xi_{lm}\, \partial_t \phi^*_{lm} \partial_t \phi_{lm} \right) \\
& + \sum_{l=2}^{l_{max}-4} \sum_{|m|=l-1}^{l} \int d^2x\, \Omega_{lm} \left(\partial_t \phi_{lm} + \Gamma^{(4)}_{lm}\right)^* \left(\partial_t \phi_{lm} + \Gamma^{(4)}_{lm}\right) \\
& + \sum_{l=l_{max}-3}^{l_{max}-2} \sum_{|m|=0}^{l-2} \int d^2x\, \Omega_{lm} \left(\left(\partial_t \phi_{lm} + \Gamma^{(2)}_{lm}\right)^* \left(\partial_t \phi_{lm} + \Gamma^{(2)}_{lm}\right) - \Xi_{lm}\, \partial_t \phi^*_{lm} \partial_t \phi_{lm} \right) \\
& + \sum_{l=l_{max}-3}^{l_{max}-2} \sum_{|m|=l-1}^{l} \int d^2x\, \Omega_{lm} \left(\partial_t \phi_{lm} + \Gamma^{(2)}_{lm}\right)^* \left(\partial_t \phi_{lm} + \Gamma^{(2)}_{lm}\right) \\
& + \sum_{l=l_{max}-1}^{l_{max}} \sum_{|m|=0}^{l-2} \int d^2x\, \Omega_{lm} \left(\partial_t \phi^*_{lm} \partial_t \phi_{lm} - \Xi_{lm}\, \partial_t \phi^*_{lm} \partial_t \phi_{lm}\right) \\
& + \sum_{l=l_{max}-1}^{l_{max}} \sum_{|m|=l-1}^{l} \int d^2x\, \Omega_{lm} \partial_t \phi^*_{lm} \partial_t \phi_{lm} + O\left(\varepsilon^3\right). \qquad (80)
\end{aligned}
$$

$$
\phi_{l\,m} \to \frac{\phi_{l\,m}}{\left(1 - \Xi_{lm}\right)^{1/2}} \quad \text{for } l = 2, 3, \cdots, l_{max} \ (l = 0, 1 \text{ are not included}) \text{ and } |m| = 0, 1, \cdots, l-2 \text{ for each } l. \qquad (81)
$$

Hawking flux by anomaly cancellation method

Anomalies, Hawking Radiations and Regularity in Rotating Black Holes, S.Iso, H.Umetsu, F.Wilczek, hep-th/0606018 (PRL)





: Ingoing modes

: Outgoing modes

Total flux = + = 0

$$Z\left[(g_{\rm eff})^{\mu\nu}_{lm}, \Phi_{lm}\right] = \int \mathcal{D}\varphi_{lm} \exp iS_{\rm 2D}\left((g_{\rm eff})^{\mu\nu}_{lm}, \Phi_{lm}, \varphi_{lm}\right)$$

$$x^\mu \mapsto x'^\mu = x$$

$$(g_{\rm eff})_{tt,lm} = -\frac{r - 2m - \frac{\mathcal{I}^C_{lm,\,lm}}{\Lambda_{lm,\,lm}}\frac{15m\varepsilon^2}{8\pi r}}{2m} + O\left(\varepsilon^3\right)$$

$$(\delta Z)_{lm} = \left(\delta_L (g_{\rm eff})^{\mu\nu}_{lm}\frac{\delta}{\delta_L (g_{\rm eff})^{\mu\nu}_{lm}} + \delta_L \Phi_{lm}\frac{}{\delta_L \Phi_{lm}}\right)$$

$$(\delta S_{\rm 2D})_{lm} = -\int d^2x\sqrt{-(g_{\rm eff})_{lm}}\,\eta^\nu \nabla_{\mu,\,lm} T^\mu{}_{\nu,\,lm}$$

$$(c_o)^r{}_{t,\,lm} = (c_H)^r{}_{t,\,lm} - N^r{}_{t,\,lm}\Big|_{r=(r_{h\,(eff)})_{lm}}$$
$$= \frac{\pi}{12}T_H^2$$

$$\nabla_\mu T^\mu{}_{\nu,\,lm} = -\frac{\partial_\nu \Phi_{lm}}{\sqrt{-(g_{\rm eff})_{lm}}}\frac{\delta S_{\rm 2D}}{\delta_L \Phi_{lm}} + \text{both/either } \mathcal{A}^{\pm}_{\nu,\,lm} = \pm\partial_r N^r{}_{t,lm}$$

$$\nabla_\mu \widetilde{T}^\mu{}_{\nu,\,lm} = -\frac{\partial_\nu \Phi_{lm}}{\sqrt{-(g_{\rm eff})_{lm}}}\frac{\delta S_{\rm 2D}}{\delta_L \Phi_{lm}} + \text{both/either } \widetilde{\mathcal{A}}^{\mp}_{\nu,\,lm} = \pm\partial_r \widetilde{N}^r{}_{t,lm}$$

$$N^r{}_{t,\,lm} = \frac{1}{192\pi}\left(f'^2 + ff''\right)$$

$$\widetilde{N}^r{}_{t,\,lm} = \frac{1}{96\pi}\left(ff'' - (f')^2/2\right)$$

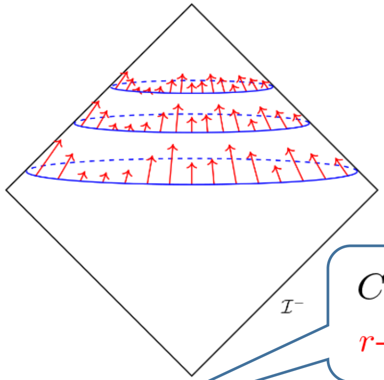
$$f = -(g_{\rm eff})_{tt,\,lm} \qquad \lim_{r\to r_h} f = 0$$

Conclusion

S... ...otic symmetry ➡ Information paradox

$$Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\phi}$$

...dinarly



- ... : $g_{\mu\nu}(x^\mu) \mapsto \tilde{g}_{\mu\nu}(x^\mu)$, g and $\tilde{g}$ are physically

$C = C(z,\bar{z})$
r-independence

If ϕ mixes, it becomes the form where formulas are unavilable. At this time, even if we can say this, result is unclear as anomaly cancellation is unavailable.

- $(m_{\text{eff}})_{kn,\,lm} \equiv m + \frac{15m}{8\pi r}\mathcal{I}^C_{kn,\,lm}\varepsilon^2 + O(\varepsilon^3)$

some coefficient

$r_{h,\,2D} = 2(m_{\text{eff}})_{kn,\,lm}$

- $(c_o)^r{}_{t,\,lm} = \frac{\pi}{12}T_H^2$

➢ $C = m\,\varepsilon\, Y_2^0(\theta,\phi) = m\,\varepsilon\sqrt{\frac{5}{16\pi}}\left(3\cos^2\theta - 1\right)$

$$\begin{cases} \mathcal{L}_f C = f & \text{where } C = C(z,\bar{z}) \\ \xi = f\partial_u + \frac{1}{r}\left(D^z f \partial_z + D^{\bar{z}} f \partial_{\bar{z}}\right) \end{cases}$$

$$ds^2 = -\left(1 - \frac{2m}{r_s} + \cdots + \mathcal{O}(\varepsilon^3)\right)dt_s^2 + \left(\frac{1}{1-\frac{2m}{r_s}} + \cdots + \mathcal{O}(\varepsilon^3)\right)dr_s^2 + \left(r_s^2 + \cdots\right)d\ldots$$

Conclusion
We can always get this Hawking flux for arbitaraly C if it is quaratic order

and if it is independet of ϕ.

➢ $S = \int d^4x\sqrt{-g}\,g^{MN}\partial_M\phi\partial_N\phi$

$$(g_{\text{eff}})_{tt,lm} = -\frac{r - 2m - \frac{\mathcal{I}^C_{lm,\,lm}}{\Lambda_{lm,\,lm}}\frac{15m}{8\pi r}\varepsilon^2}{2m} + O(\varepsilon^3)$$

$r = r_h + \Delta r$ (near-horizon limit)

$$= -\int d^4x\,\sin(\theta)\,(2m)^2\left\{1 + \frac{3}{2}\sqrt{\frac{5}{\pi}}\left(1 + 3\cos(2\theta)\right)\varepsilon + \frac{45}{2\pi}\left(\frac{\sin(2\theta)}{4} - \cos^2(\theta) + 3\cos^2(\theta)\cos(2\theta)\right)\varepsilon^2\right\}\phi^*\left(t^{tt}\partial_t\partial_t + \partial_r(t^{rr}\partial_r)\right)\phi + \cdots$$

Get 2-dim. action by $\phi = \phi_{lm}Y_m^l$ and $\int d\theta d\phi$

$$= \sum_{l=0}^{l_{max}}\sum_{|m|=0}^{l}\int d^2x\,\Phi_{lm}\left((g_{\text{eff}})^{tt}_{lm}\,\partial_t\varphi^*_{lm}\partial_t\varphi_{lm} + (g_{\text{eff}})^{rr}_{lm}\,\partial_r\varphi^*_{lm}\partial_r\varphi_{lm}\right)$$

If the order of ε raises, it is unclear if we can rewrite to this form or not.

- $\cos(2\theta) = \frac{8}{3}\sqrt{\frac{\pi}{5}}Y_2^0 - \frac{2\sqrt{\pi}}{3}Y_0^0$
- $\int d\Omega\,(Y_{l_1}^{m_1})^*(Y_{l_2}^{m_2})^*Y_L^M = \sqrt{\frac{(2l_1+1)(2l_2+1)}{4\pi(2L+1)}}\langle l_1 0\, l_2 0|L0\rangle\langle l_1 m_1\, l_2 m_2|LM\rangle$
- $Y_{l_1}^{m_1}Y_{l_2}^{m_2} = \sum_{L,M}\sqrt{\frac{(2l_1+1)(2l_2+1)}{4\pi(2L+1)}}\langle l_1 0\, l_2 0|L0\rangle\langle l_1 m_1\, l_2 m_2|LM\rangle Y_L^M.$

wiki/Clebsch-Gordan coefficients