Bulk Reconstruction and Gauge Invariance

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and a paper to appear

Introduction

One way to study quantum gravity is AdS/CFT duality

Maldacena

Quantum gravity on AdS

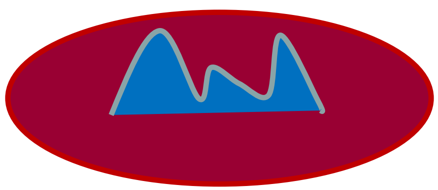
= conformal field theory (CFT)

In operator formalism, AdS/CFT is

equivalence or duality between
Hilbert spaces and Hamiltonians
of gravity on AdS and CFT

Low energy states of bulk theory and CFT

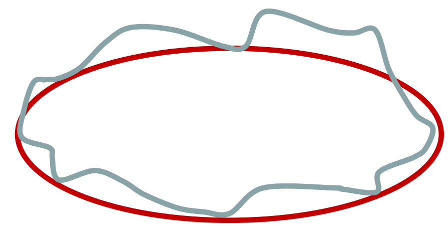
Bulk theory



states on disk

1 to 1

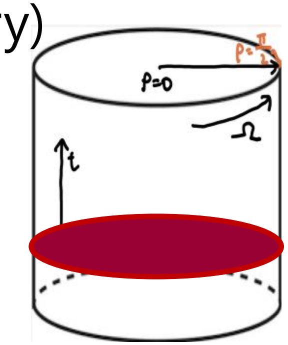
CFT



states on S^1

(CFT is NOT living on boundary,
CFT corresponds to whole bulk theory)

They are on a fixed time slice of AdS or cylinder

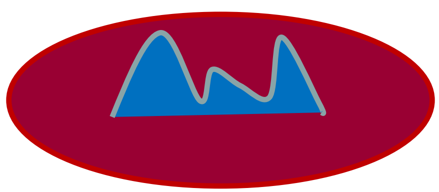


Bulk reconstruction is a map from CFT operator to bulk operator

$\phi(x) \Longleftarrow O(X)$

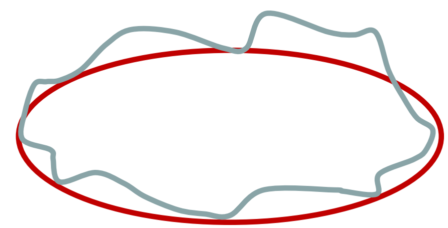
Bulk theory

CFT



$\Longleftrightarrow$

1 to 1



states on disk

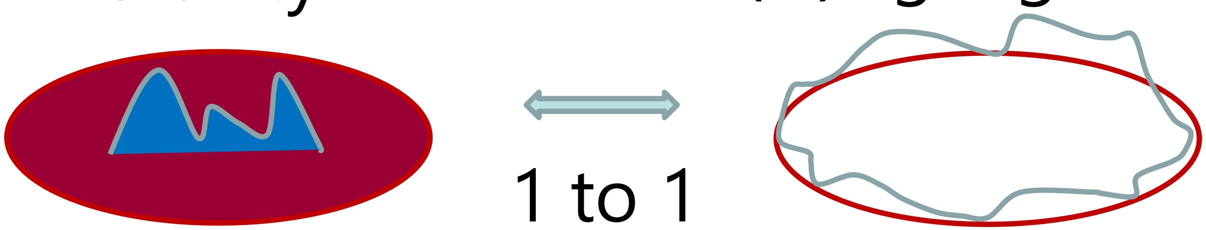
states on S^1

To understand bulk reconstruction is to understand AdS/CFT itself

Newton constant

$$\frac{1}{(G_N)^{d-1}} \Longleftrightarrow N$$

Gravity ”SU(N)" gauge theory

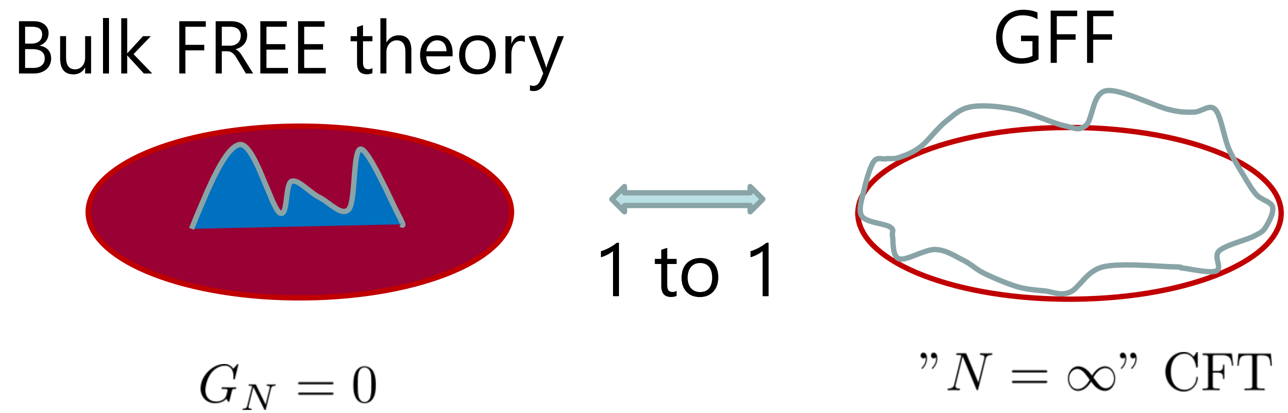


1 to 1

We set $l_{AdS} = 1$

free bulk gravity is $N = \infty$

For N=∞,



Generalized free field (GFF) is
“dimensional reduction” of bulk free theory.

But, GFF violates fundamental properties of QFT!
just a approximation of large, but finite N

Another important subject in AdS/CFT: Subregions in bulk and boundary

Subregions are important for quantum information theoretical aspects of AdS/CFT

Black hole is related to subregion because outside observers can see a subregion of spacetime

Subregion is related to horizon and black hole

half subregion of a time slice of Minkowski space

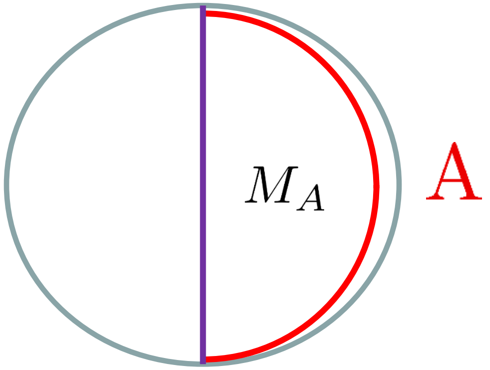
⇕

Rindler space

Near horizon of (outside) Black hole = Rindler space

In particular,
Rindler patch of AdS = "Black hole" of hyperbolic space

Entanglement wedge reconstruction (EWR) is a bulk reconstruction for subregion



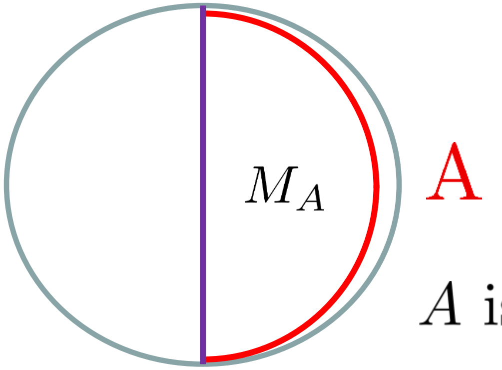
$t = 0$ slice of AdS_3

Essentially, EWR claim that
bulk operators on M_A can be reconstructed
from CFT oeprators on A.

Subregion duality

Simplest example:

$t = 0$ slice of AdS_3



A is a half-circle

M_A is a half-disk (Rindler patch of AdS) surrounded by Ryu-Takayanagi surface

Correspondence: $A \to M_A$

In this talk,

by considering the gauge symmetry (diffeomorphism) in gravitational theory,

we demonstrate that the widely believed properties of bulk reconstruction are either incorrect or significantly modified,

These includes holographic error correction code and entanglement wedge reconstruction

There is a crucial gap
between $N=\infty$ and finite N
because of the gauge invariance in gravity

Gravitational dressing of “local” operator
can not be negelected!

Plan

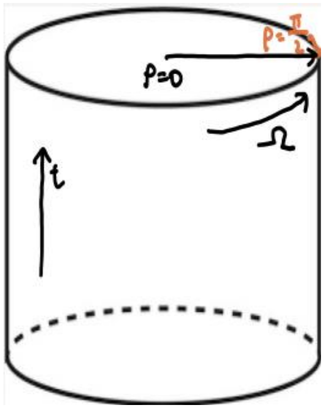
1. **Introduction**
2. **(HKLL) bulk reconstruction**
3. **Bulk reconstruction for subregion**
4. **Gauge invariant operator and gravitational dressing**
5. **Conclusion**

(HKLL) Bulk reconstruction

(Global) AdS_{d+1}

The metric of global AdS_{d+1} ($l_{AdS} = 1$) is

$$ds^2_{AdS} = \frac{1}{\cos^2(\rho)} \left(-dt^2 + d\rho^2 + \sin^2(\rho) d\Omega^2_{d-1}\right)$$



where $0 \leq \rho < \pi/2$

For N=∞,

large N limit of holographic CFT_d

⇕ **equivalent!**

Free bulk theory on AdS_{d+1}

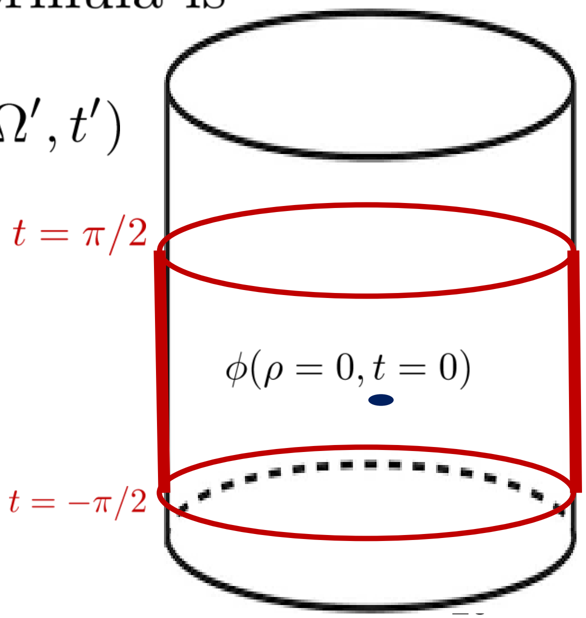
HKLL reconstruction formula

Then, the HKLL bulk reconstruction formula is

$$\phi(\rho=0,t=0)=\int_{-\frac{\pi}{2}\le t\le\frac{\pi}{2}} dt'\,d\Omega'\,K(\Omega',t')\mathcal{O}(\Omega',t')$$

where $K(\Omega,t)\sim\dfrac{1}{(\cos t)^{d-\Delta}}$

(K is given for any ρ.)



(Global) HKLL bulk reconstruction

Now $\mathcal{O}(\theta, t)$ denotes the CFT operator and we define

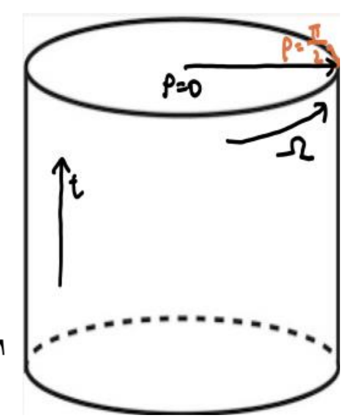
$$\phi^G(t,\rho,\theta) \equiv \int dt'd\theta' K(\theta',t';t,\rho,\theta)\mathcal{O}(\theta',t')$$

Then, we can show

$$\langle 0|\phi(t,\rho,\theta)\phi(t',\rho',\theta')|0\rangle = \langle 0|\phi^G(t,\rho,\theta)\phi^G(t',\rho',\theta')|0\rangle$$

i.e. bulk 2-point function
is reproduced by CFT operator
(of any CFT)

because 2pt func. is universal for CFT



Comment 1:

Large N factorization of holographic CFT with $N = \infty$:

vanishing of connected n-pont func. for $n > 2$

i.e. $\langle \mathcal{O}_1 \mathcal{O}_2 \cdots \mathcal{O}_n \rangle = \sum \langle \mathcal{O}_1 \mathcal{O}_2 \rangle \langle \mathcal{O}_3 \mathcal{O}_4 \rangle \cdots \langle \mathcal{O}_{l-1} \mathcal{O}_l \rangle$

Those of GFF is also factorized because it is free theory

⇓

Bulk n-point function is reproduced by the CFT operator for holographic CFT

Comment 2:

The smearing function $K(\theta, t)$ in

$$\phi^G(t,\rho,\theta) \equiv \int dt' d\theta' K(\theta', t')\mathcal{O}(\theta', t')$$

has ambiguity.

$$K(\theta', t') \to K(\theta', t') + \delta K(\theta', t')$$

is OK as a bulk operator if

$$\int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dt\, e^{i\omega t + im\theta}\, \delta K(\theta, t) = 0$$

for $\omega = 2n + |m| + \Delta$ where n is non-negative integer

Comment 3:

HKLL bulk recosntruction can be extended to to include $1/N$ corrections (=interaction) as

$$\phi^G(t,\rho,\theta) = \int dt' d\theta' K(\theta',t')\mathcal{O}(\theta',t') + \frac{1}{N}\int dt' dt'' d\theta' d\theta''\, K^1(\theta',t',\theta'',t'')\mathcal{O}(\theta',t')\mathcal{O}(\theta'',t'') + \cdots$$

by requiring e.o.m. or a kind of micro causality.

Bulk reconstruction for subregion

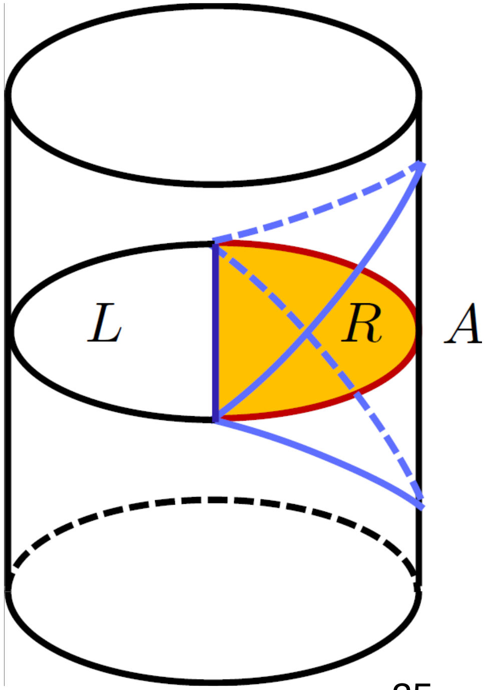
Rindler patch of AdS_3

The metric of Rindler patch of AdS_3 $(l_{AdS} = 1)$ is

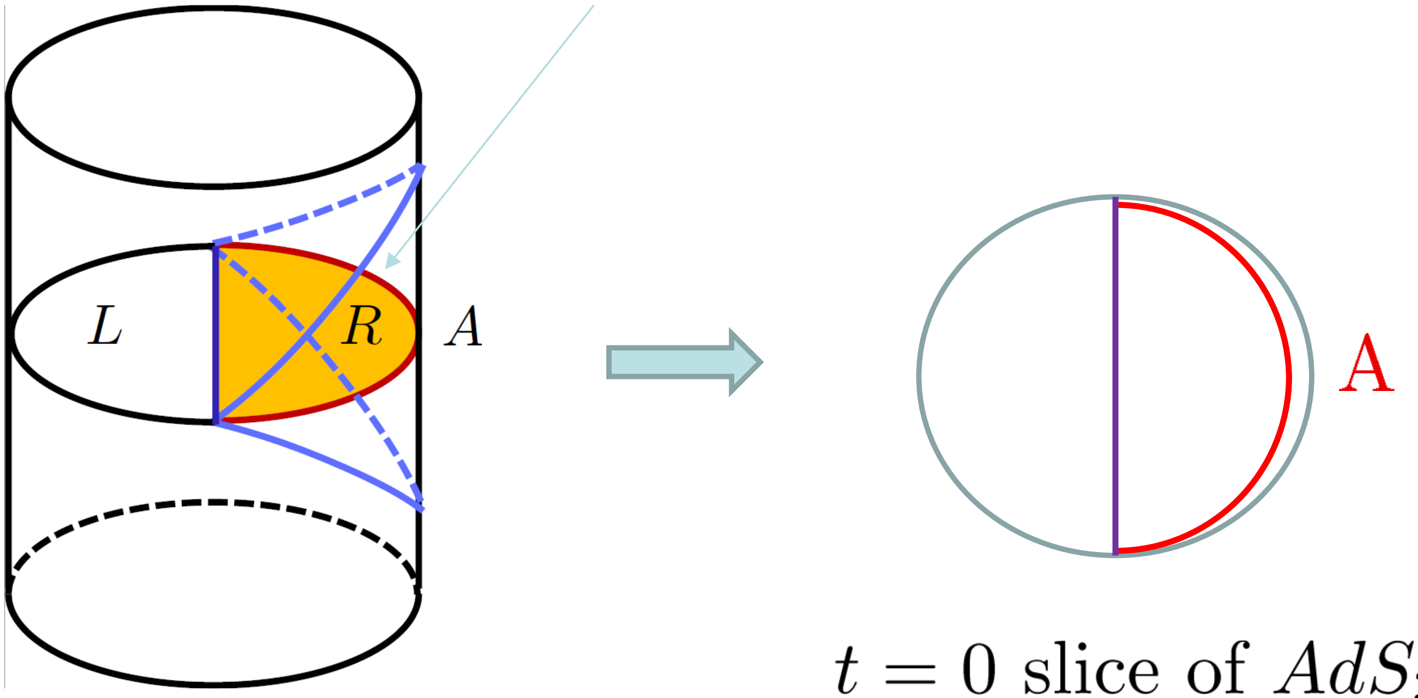
$$ds^2 = -\xi^2 dt_R^2 + \frac{d\xi^2}{1+\xi^2} + (1+\xi^2)d\chi^2$$

where $-\infty < t_R < \infty$, $-\infty \leq \chi < \infty$

$$0 \leq \xi < \infty$$



Entanglement wedge of A (denoted by M_A) =AdS-Rindler patch



$t = 0$ slice of AdS_3

AdS-Rindler HKLL bulk reconstruction

Finally, we regard $O(t_R, \chi)$ as CFT operator and define

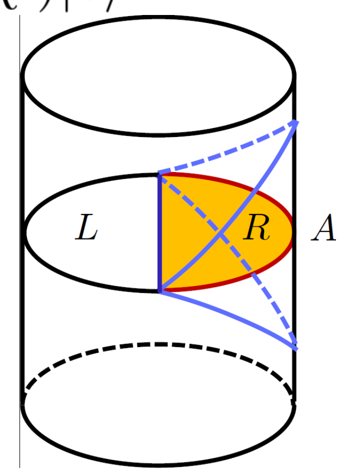
$$\phi^R(t_R, \xi, \chi) \equiv \int dt'_R d\chi' K^R(\chi', t'_R)\mathcal{O}(\chi', t'_R)$$

Then, if $\{t_R, \xi, \chi\}$ are in AdS-Rindler patch,

$$\langle 0|\phi(t_R, \xi, \chi)\phi(t'_R, \xi', \chi')|0\rangle = \langle 0|\phi^R(t_R, \xi, \chi)\phi^R(t'_R, \xi', \chi')|0\rangle$$

i.e. bulk correlation function in M_A
is reproduced by CFT operator in A

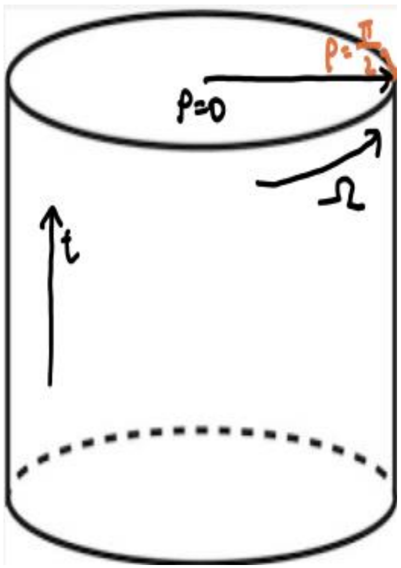
(It may possible that $K^R = K + \delta K$.)



This suggests

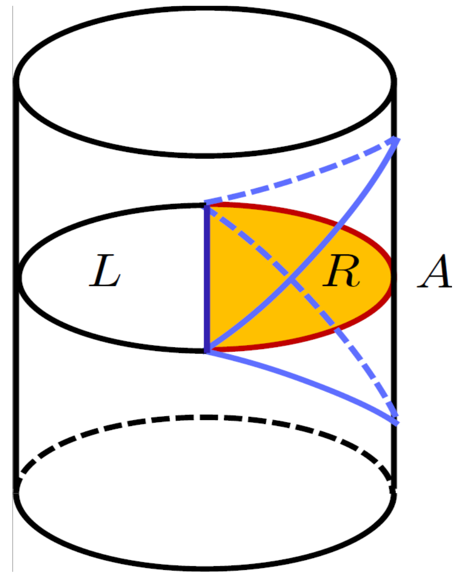
both quantum gravity on Global and Rindler paches

are consistent



Global

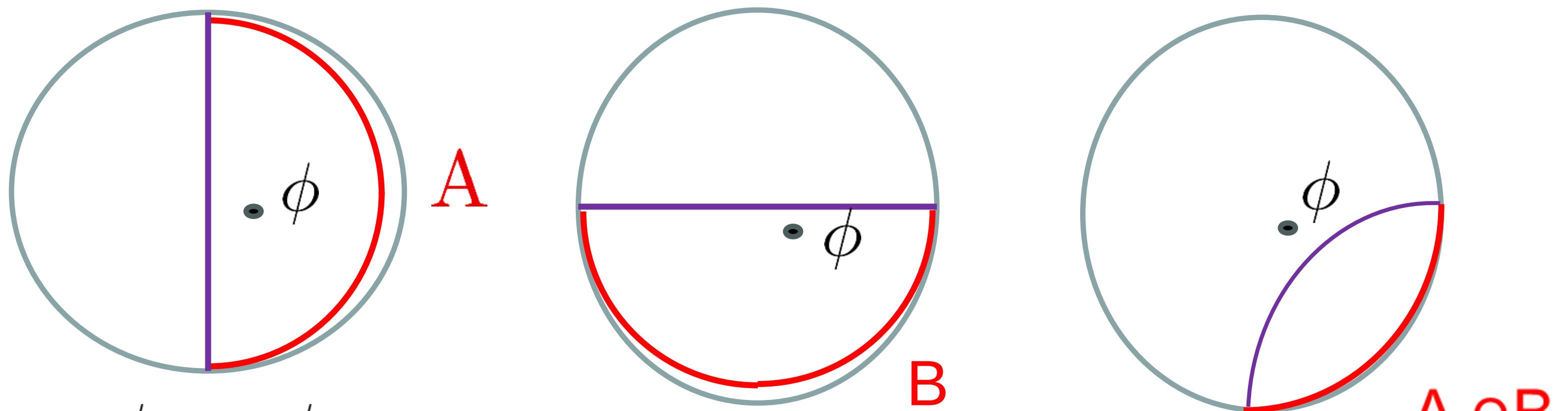
(”Kruskal coord.”)



Rindler

(”Schwarzsschild coord.”)

"Paradox" for CFT with $N < \infty$



$\phi \to \phi_A$ $\phi \to \phi_B$ $\phi \to ?$

Bulk local operator ϕ can be reconsructed
by CFT operator on A, and on B, but not on $A \cap B$.

⇓

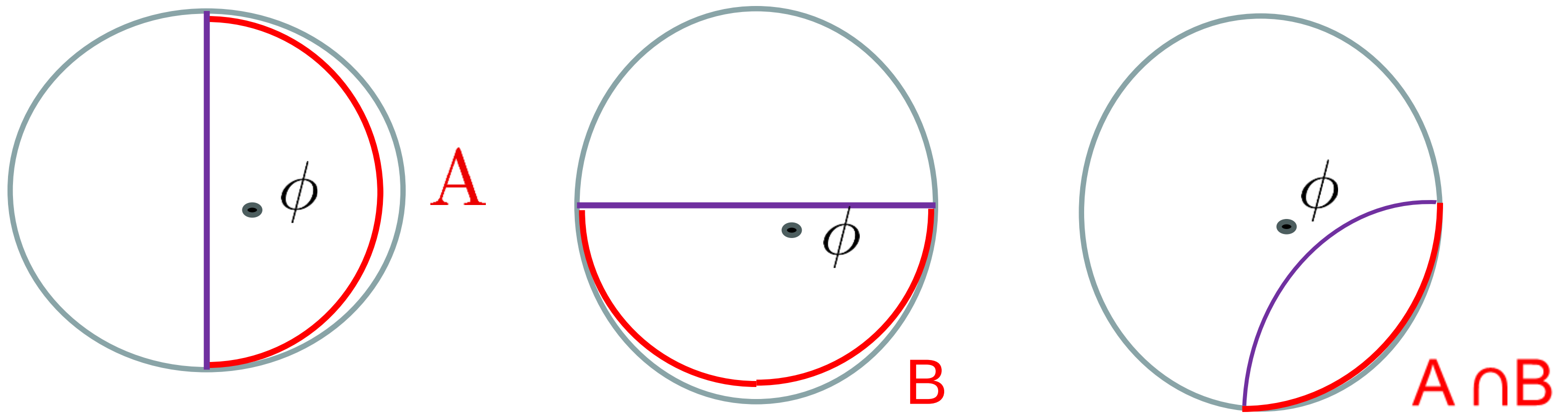
Holographic quantum error correction code proposal

Almheiri-Dong-Harlow, HaPPY model

$\phi_A \neq \phi_B$, but $\phi_A = \phi_B$ in the code (low energy) subspace

(without concrete evidences)

Our claim:



$\phi \to \phi_A$ $\phi \to \phi_B$ $\phi \to ?$

$\phi_A \neq \phi_B$ even in the code (low energy) subspace

(bulk local operator ϕ depends on the subregion:
"subregion comlementariy")

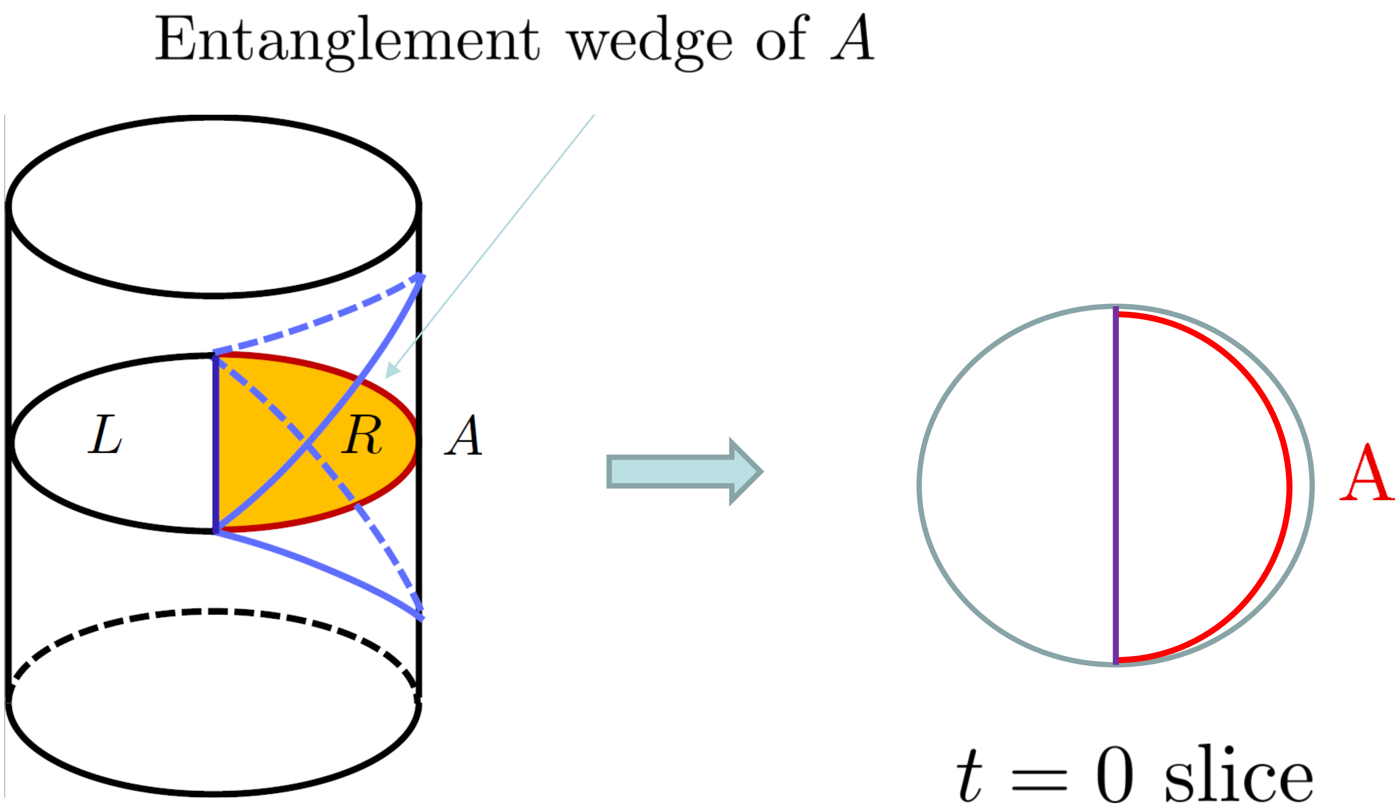
No paradox, No holographic quantum error correction code,
No entanglement wedge reconstruction (in Dong-Harlow-Wall)

As an example, we will show that

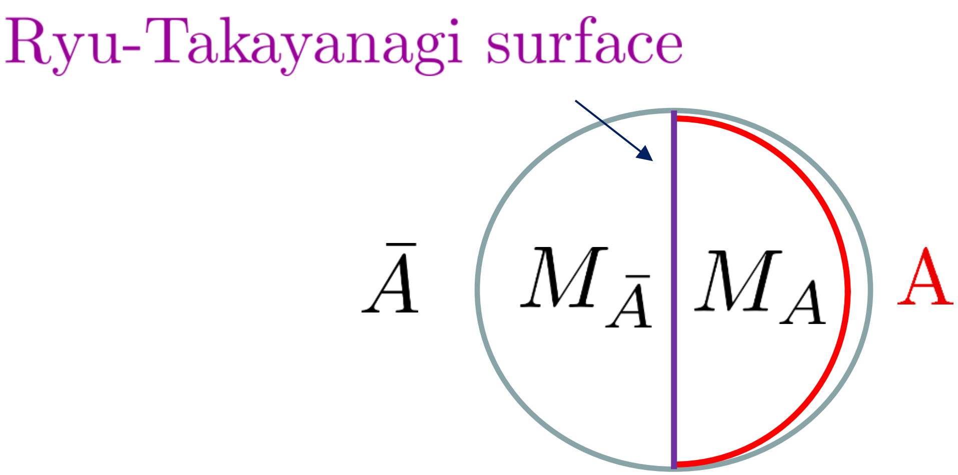
$\phi^G \neq \phi^R$ even in the low energy subspace

even if we include $1/N$ corrections

What is entanglement wedge reconstrcution?



Decompositions for Bulk and CFT



$$\text{CFT space } (= S^{d-1}) = A + \bar{A}$$

$$\text{Bulk space} = M_A + M_{\bar{A}}$$

M_A =Entanglement wedge of A

Subregion duality:

For density matrices ρ, σ,

$$\rho_A = \sigma_A \Leftrightarrow \rho_{M_A} = \sigma_{M_A}$$

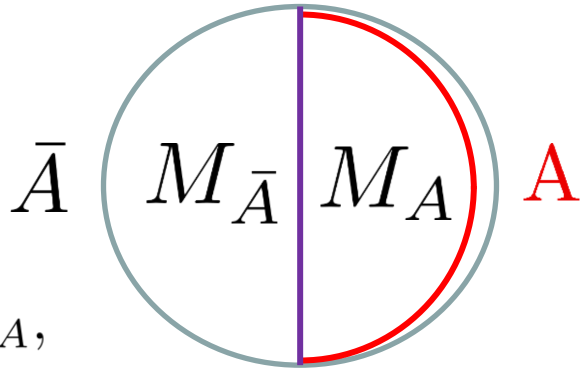
where $\rho_A = \mathrm{tr}_{\bar{A}}(\rho), \rho_{M_A} = \mathrm{tr}_{M_{\bar{A}}}(\rho)$

Entanglement wedge reconstruction:

Dong-Harlow-Wall

For low enegy state $|\phi\rangle$,

$$^{\forall}\mathcal{O}_{M_A}|\phi\rangle = {}^{\exists}\mathcal{O}_A|\phi\rangle,$$



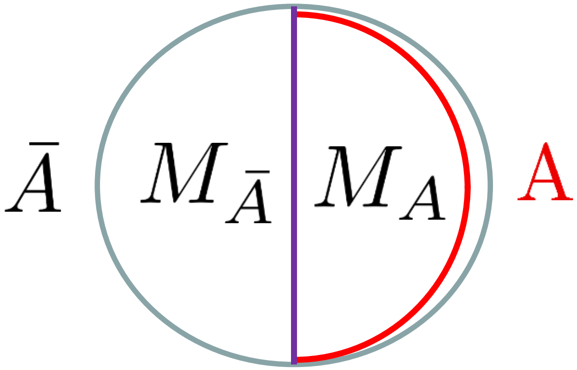
$\mathcal{O}_{M_A}$ is bulk operator supported in M_A,

$\mathcal{O}_A$ is CFT operator supported in A

For GFF case (=free bulk theory $= N = \infty$ CFT),

with M_A = AdS-Rindler case,

entanglement wedge reconstruction
and subregion duality are
(seemingly) satisfied



because bulk operators on M_A
and CFT operators on A are identcal.

(More presicely, bulk operators on M_A
and CFT operators on $D(A)$ are identcal.)

"Derivation"

Jafferis-Lewkowycz-Maldacena-Suh (JLMS) showed

CFT relative entropy = bulk relative entropy

$\Downarrow$

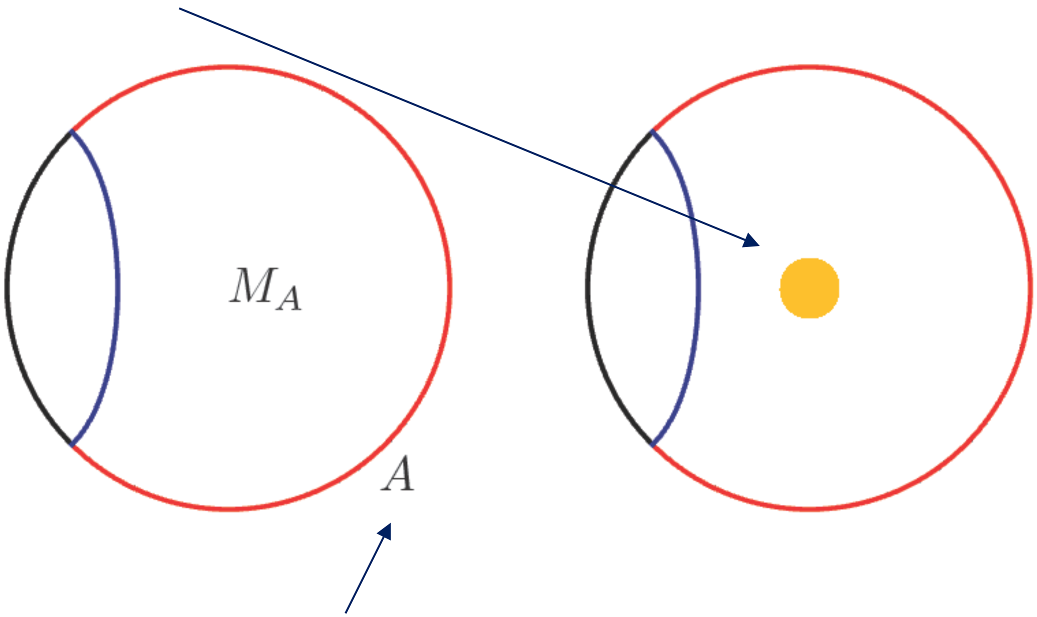
Subregion duality

$\Downarrow$

Entanglement wedge reconstruction

Couterexample of entanglement wedge reconstruction

bulk operator ϕ supported on small region around the center such that ϕ is spherically symmetric and Hermite



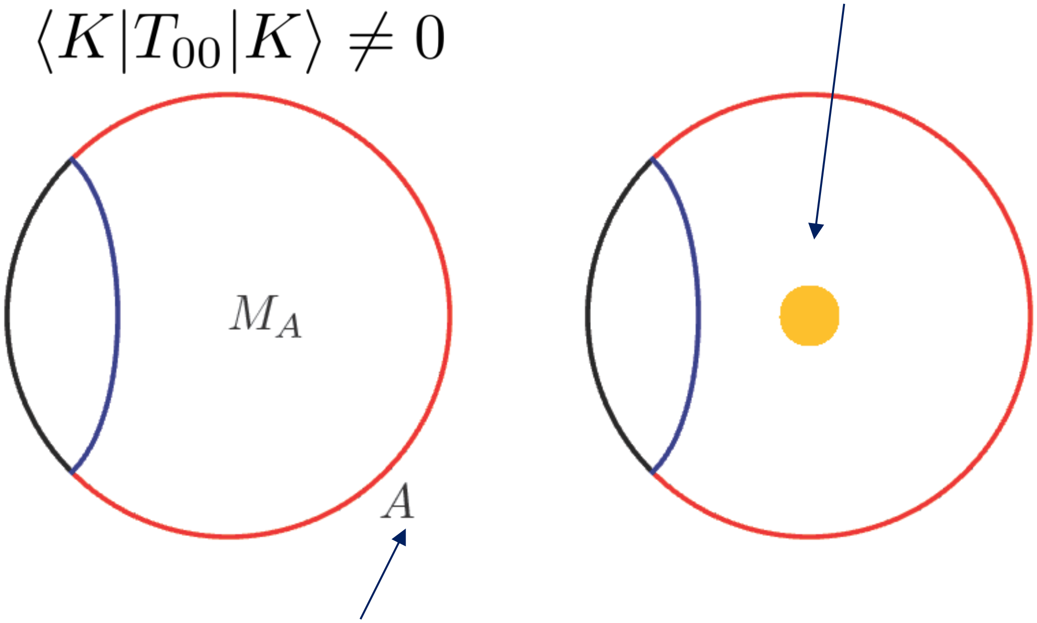
a CFT subregion A such that its causal wedge M_A includes the bulk region around the center

Consider the state $|K\rangle = e^{i\phi}|0\rangle$ (or $(1 + i\phi - \frac{1}{2}\phi^2)|0\rangle$)

Violation of entanglement wedge reconstruction

Clearly, expectation value of the CFT stress tensor for $|K\rangle$ is spherical symmetric and nonzero everywhere.

i.e. $\langle K|T_{00}|K\rangle \neq 0$

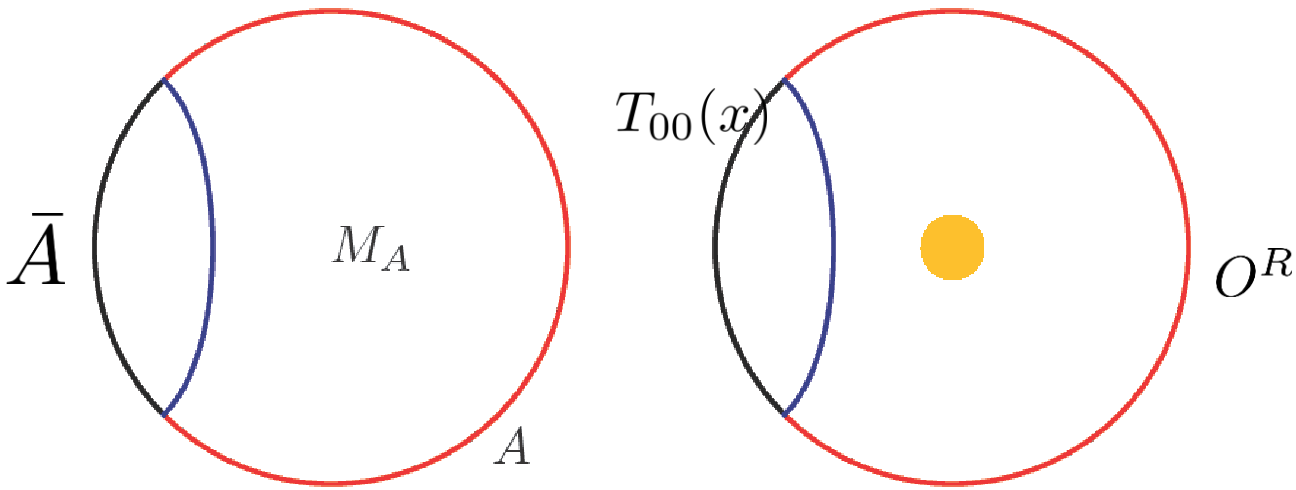


Can we find CFT operator ϕ^R, which is supported only on A, such that $e^{i\phi^R}|0\rangle = |K\rangle$?

The answer is NO!

Violation of entanglement wedge reconstruction

The answer is no because
$\langle 0|e^{-iO^R}T_{00}(x)e^{iO^R}|0\rangle = 0$ where $x \in \bar{A}$
by the causality $[T_{00}(x), O^R] = 0$.



There is no O^R such that $e^{iO^R}|0\rangle = e^{i\phi}|0\rangle$.
Thus, the entanglement wedge reconstruction is not valid.

Remarks:

(1) By smearing of the operators in spacetime, everything here is considered low energy.

(2) Reeh–Schlieder theorem gives any state, but not operator. Thus, it is not relevant here.

(3) Energy density here is $\mathcal{O}(N^0)$.

The difference in energy density means difference in 3-point functions $\langle 0|[\phi, [T_{00}(x), \phi]]|0\rangle$.

The leading order of 3-point function is $\mathcal{O}(1/N)$, i.e. $\langle OOO \rangle = \mathcal{O}(1/N)$, in $1/N$ expansion.

Here, $\langle O\, T_{\mu\nu} O \rangle = \mathcal{O}(1)$ because $T_{\mu\nu} \sim N h_{\mu\nu}$ where $h_{\mu\nu}$ satisfies $\langle\, h_{\mu\nu} h_{\mu'\nu'} \rangle = \mathcal{O}(1)$.

Thus, the diffrence betwen ϕ^G and ϕ^R is 3-point function, which is exactly zero for GFF.

The interaction (=3-point function) fundamentally change the bulk reconstruction!

Gauge invariant operator and Gravitational dressing

Bulk local operator is NOT gauge invariant

Under infinitesimal diffeo $x \to x + \xi$,

$$\phi(x) \to \phi(x) + \frac{\partial \xi \mu}{\partial x^{\nu}} \partial_{\mu} \phi(x)$$

No local gauge invariant operator in gravity theory except free "gravity" theory.

Gauge invariant operator can be constructed by gravitational dressing.

First, let us consider gauge field case, instead of gravity.

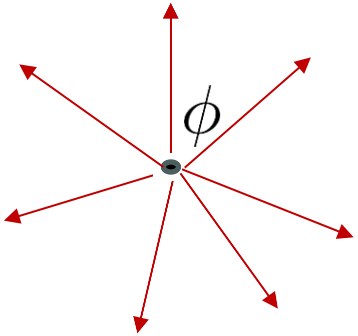
Charged operator with Wilson line is gauge invariant,

$$\phi(x)e^{ie\int_C A}$$

where C is a curve from x to boundary

We can average the direction of Wilson line.

This gives Coulomb potential, related to charge conservation



Gravity case

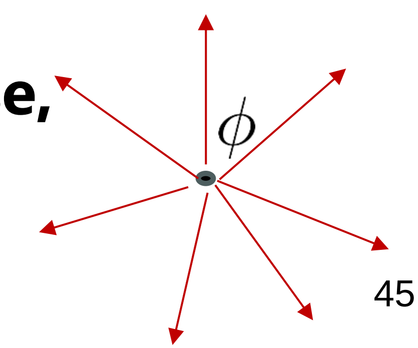
Operator with gravitaional Wilson line is gauge invariant,

$$\phi(x)e^{i\int_C f(h)}$$

where C is a curve from x to boundary

We can average the direction of Wilson line.

This gives Schwarzschild metric, in a sense, related to energy conservation



Gauge invariant operator in gravity need boundary

(which implies that "Island" is inconsistent

except for massive gravity)

Geng-Karch-Perez-Pardavila-Raju-Randall-Riojas-Shashi

To consider gauge invariant operator,
subregion including boundary is needed

Why $\phi^G \neq \phi^R$ in bulk theory?

Their gravitational dressings are different!

correponding to the difference in 3-point functions $\langle 0|\phi\,\phi\,T_{00}(x)|0\rangle$.

Gauge invariance is crucial and essential, thus
Naive picture or models from $N = \infty$ is not good!

Entanglement entropy for gravity theory

Gauge invariant operators are non-local and Hilbert space is not factorized.

It is natural to consider subalgebra on entanglement wedge because entanglement entropy for subalgebra is known.

Entanglement wedge reconstruction based on subalgebra

If we assume that relative entropy in Jafferis-Lewkowycz-Maldacena-Suh may be also for this subalgebra ver. although they used replica trick, then, the entanglement wedge reconstruction of the algebraic version will be derived.

Harlow

But, this version is very different from the usual one.

Moreover, this version is consistent with the weak entanglement wedge reconstruction which follows from the "simple bulk reconstruction"

Conclusion

- **By considering the gauge symmetry (diffeomorphism) in gravitational theory, we demonstrate that the widely believed properties of bulk reconstruction are either incorrect or significantly modified.**

- **Many interesting future directions**

Fin

backups

AdS/CFT for subregion

Subregion complementarity

Both of ϕ^G, ϕ^R, reconstructed by CFT operator,
give bulk 2-point function for $X, X' \in M_A$:

$$\langle 0|\phi(X)\phi(X')|0\rangle = \langle 0|\phi^G(X)\phi^G(X')|0\rangle = \langle 0|\phi^R(X)\phi^R(X')|0\rangle$$

Thus, both describe bulk physics in M_A.

Nevertheless, these are different: $\phi^R(X)|0\rangle \neq \phi^G(X)|0\rangle$

Same bulk operator in different coordinate patches
are realized in CFT differently!

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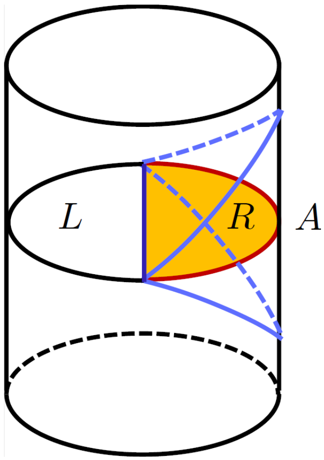
Subregion complementarity
(similar(?) to Black hole complementarity)



What is black hole complementarity

Consider a black hole, and two observers:

Infalling observer:	Outside observer:
According to an infalling observer, nothing special happens at the event horizon itself	infalling information heats up the stretched horizon, which then reradiates it as Hawking radiation

Black hole complementarity:

These two observers see different physics, but no problems because they can not communicate.

Subregion complementarity may be similar to this.