A domain wall solution in the D3-D7 model with magnetic catalysis

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Collaboration with Masataka Matsumoto (SJTU) and
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Introduction: Inhomogeneous condensates

- Inhomogeneous superconducting states (FFLO state) Fulde, Ferrell (1964), Larkin, Ovchinikov (1964)

- Inhomogeneous chiral condensates

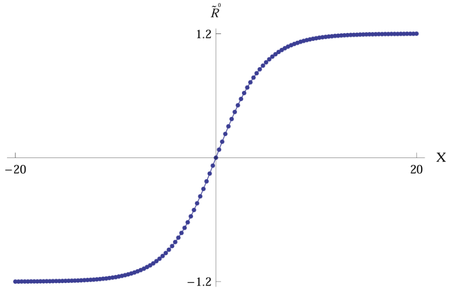
Review: Buballa, Carignano (2014)

In holography…

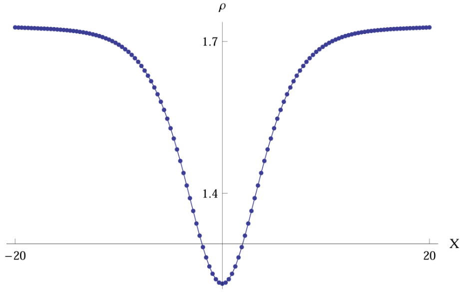
Ex) single kink (dark soliton)

Keranen, Keski-Vakkuri, Nowling, Yogendran (2010)

Scalar condensate



Charge density

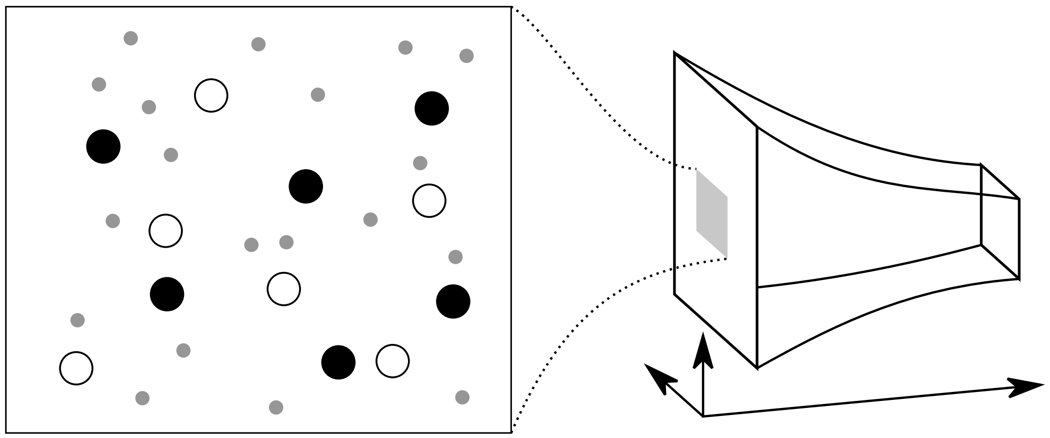


Vortex, (complex) multi-kinks, … and many applications!

This work: single kink in a **top-down** holographic model (D3/D7 model)

AdS/CFT correspondence

d-dim.
gauge theory



$(d+1)$-dim.
gravity theory

Partition function of d-dim. gauge theory

$$Z_{\mathrm{gauge}_d} = Z_{\mathrm{AdS}_{d+1}}$$

Partition function of (d+1)-dim. supergravity theory

Large-N limit

Saddle point approximation

GKP/W relation

$$\left\langle \exp\left(\int \mathrm{d}^d x \mathcal{O}\phi_{(0)}\right)\right\rangle = \exp\left(-\bar{S}_{\mathrm{grav}}\right)\Big|_{\tilde{\phi}(z=0,x)=\phi_{(0)}(x)} .$$

Probe brane model: D3/D7 model

SAdS$_5$×S^5 background
$$ds_{10}^2 = \frac{1}{u^2}\left(-f(u)\,dt^2 + d\vec{x}^2 + \frac{du^2}{f(u)}\right) + d\Omega_5{}^2\,, \quad f(u) = 1 - \frac{u^4}{u_H^4}$$

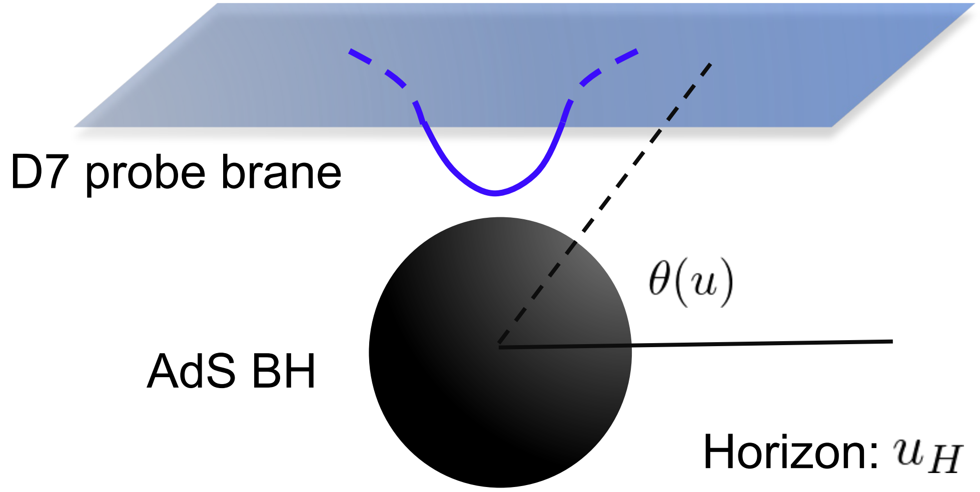
D7 probe-brane's action
$$S_{D7} = T_{D7}\int d^8\xi\,\sqrt{-\det(g_{ab} + 2\pi\alpha' F_{ab})} + S_{WZ}$$

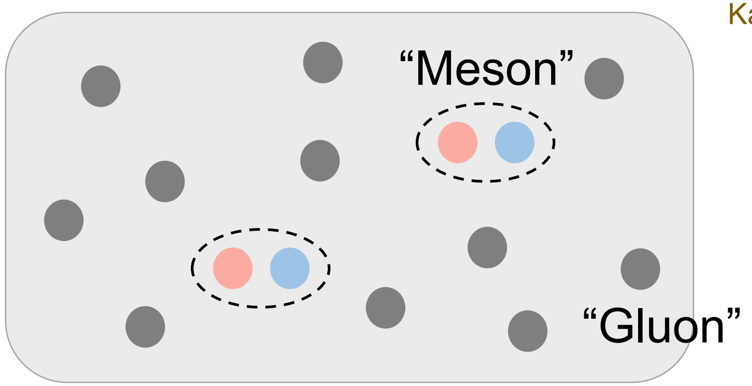
X^M	t	$\vec{x}$	u	Ω_3	θ	φ
D7	$\checkmark$	$(\checkmark)^3$	$\checkmark$	$(\checkmark)^3$		

co-dimension 2

AdS black hole + D7 probe brane ⟺ (3+1) dim. N=4 SYM + N=2 hypermultiplet

Karch, Katz (2003).





Heat-bath temperature: $T = \frac{1}{\pi u_H}$

Probe brane model: D3/D7 model

General equations of motion (w/o WZ term):

$$0 = \frac{1}{\sqrt{-\gamma}}(\sqrt{-\gamma} s^{ab} g_{MN} X^N{}_{,b})_{,a} - \frac{1}{2} s^{ab} g_{PQ,M} X^P{}_{,a} X^Q{}_{,b}$$

$$0 = \frac{1}{\sqrt{-\gamma}}(\sqrt{-\gamma} g^{ac} F_{cd} s^{db})_{,a}$$

$$(2\pi\alpha' = 1)$$

$$\gamma_{ab} = g_{ab} + F_{ab}$$
$$s_{ab} = g_{ab} - F_{ac} g^{cd} F_{gb}$$
$$g_{ab} = g_{MN} X^M{}_{,a} X^N{}_{,b}$$

The system is highly non-linear.

	t	$\vec{x}$	u	Ω_3	θ	φ
$M, N, \cdots$	✓	$(\checkmark)^3$	✓	$(\checkmark)^3$	✓	✓
$a, b, \cdots$	✓	$(\checkmark)^3$	✓	$(\checkmark)^3$		
$m, n, \cdots$					✓	✓
$\mu, \nu, \cdots$	✓	$(\checkmark)^3$				

Dual operators:

$$X^m = \begin{pmatrix} \theta \\ \varphi \end{pmatrix} \leftrightarrows \begin{pmatrix} \langle \bar{q} q \rangle \\ i \langle \bar{q} \gamma^5 q \rangle \end{pmatrix}$$ quark condensate

$$A_\mu \leftrightarrows \langle \bar{q} \gamma^\mu q \rangle$$ current density

Chiral symmetry breaking in D3/D7

Embedding function: $\theta = \theta(u), \ \varphi = 0$

Gauge field on brane:

$$\begin{aligned} A_t &= a_t(u), \\ A_y &= Bx, \\ A_x &= A_z = 0 \end{aligned}$$

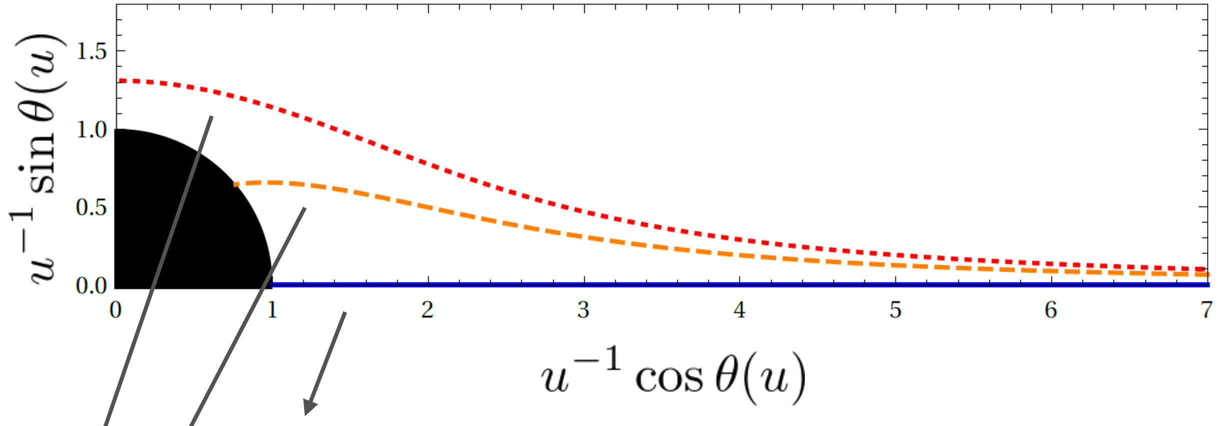
asymptotic forms at boundary

$$\begin{aligned} \frac{\sin\theta(u)}{u} &= m + cu^2 + \cdots \\ a_t(u) &= \mu - \frac{\rho}{2}u^2 + \cdots \end{aligned}$$

Quark mass (m), Chiral condensate (c), Chemical potential (μ), Charge density (ρ)

The external magnetic field *B* becomes a scale of the system.

Set $m = 0$ because we are interested in a spontaneous chiral symmetry breaking.
In bulk, this corresponds to imposing the boundary condition for $\theta(u)$ at the boundary ($u = 0$).

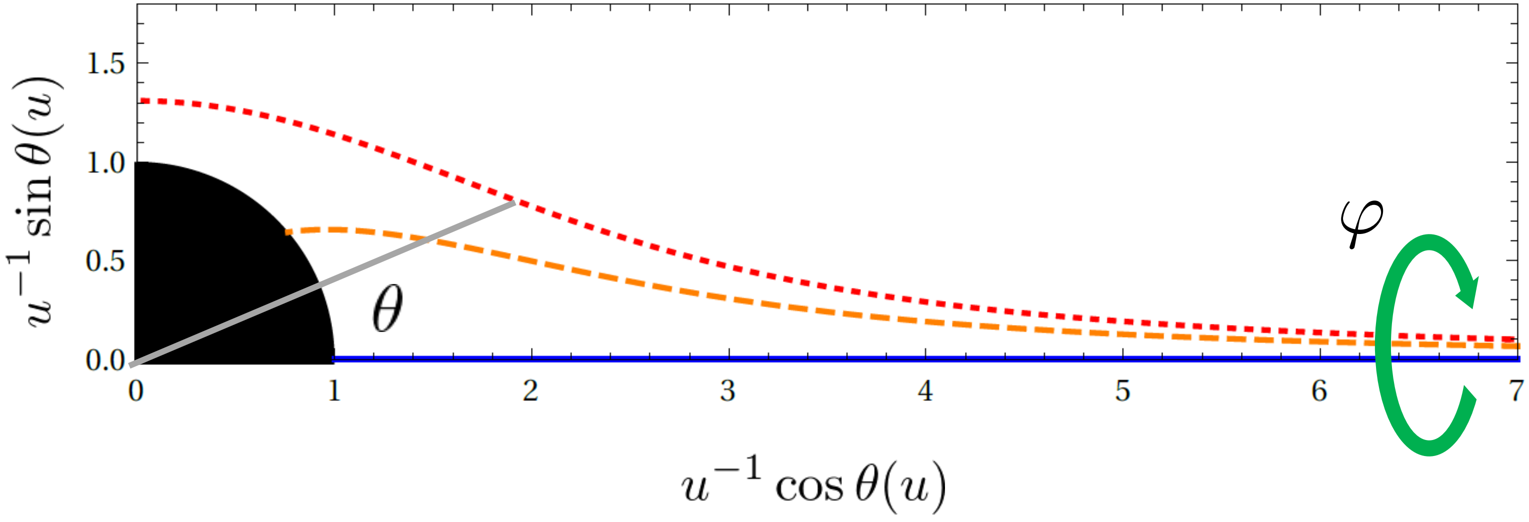


black hole embedding, chiral symmetry restored

black hole embedding, chiral symmetry broken

Minkowski embedding, chiral symmetry broken

Chiral symmetry breaking in D3/D7



Rotation in the co-dimentional plane

$SO(2) \sim U(1)_{chiral}$

$$\frac{\sin\theta(u)}{u} = \cancel{m} + cu^2 + \cdots$$

Chiral condensate (order parameter)

chiral trans. $q \to e^{i\alpha\gamma^5} q = \begin{pmatrix} e^{+i\alpha} q_+ \\ e^{-i\alpha} q_- \end{pmatrix}$

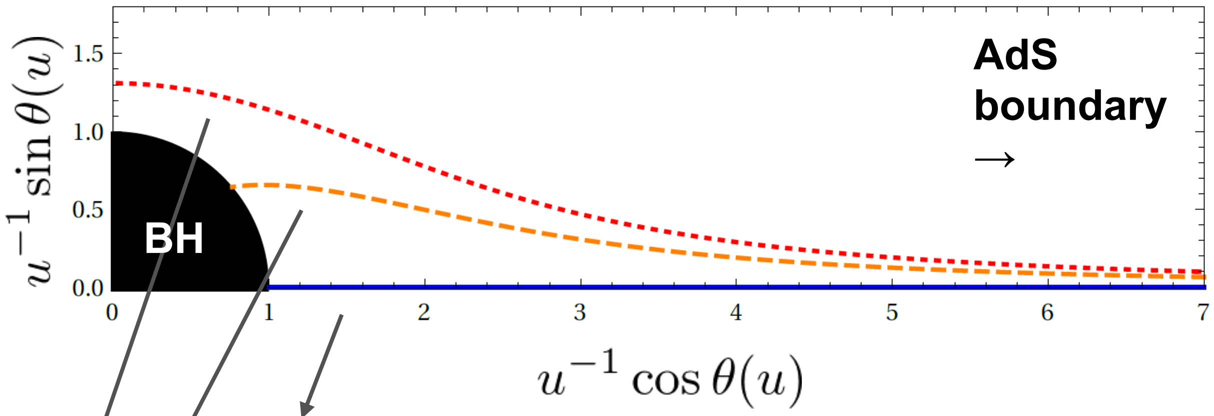
$\mathcal{L} = i\bar{q}\partial\!\!\!/ q - \cancel{m\bar{q}q}$ chiral limit

- Trivial (flat) solution : $\theta(u) = 0 \to \langle\bar{q}q\rangle \sim c = 0$ chiral symmetry restored
- Non-trivial solution : $\theta(u) \neq 0 \to \langle\bar{q}q\rangle \sim c \neq 0$ chiral symmetry broken

Chiral symmetry breaking in D3/D7

Bulk picture:

D7-brane embedding with a magnetic field B



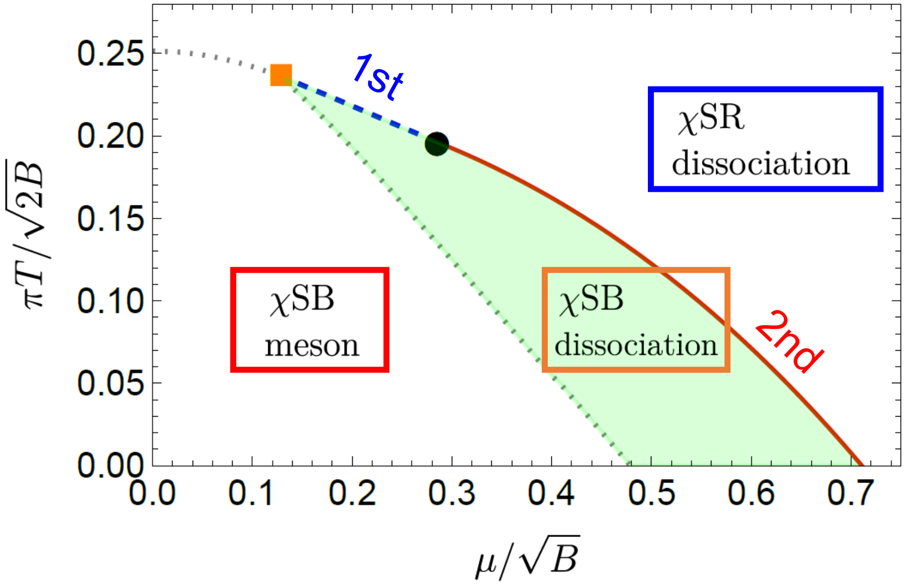
chiral symmetry restored (χSR)

chiral symmetry broken (χSB)

no dissipation / chiral symmetry broken (χSB)

Dual picture:

Order parameter:
chiral condensate $\langle \bar{q} q \rangle$



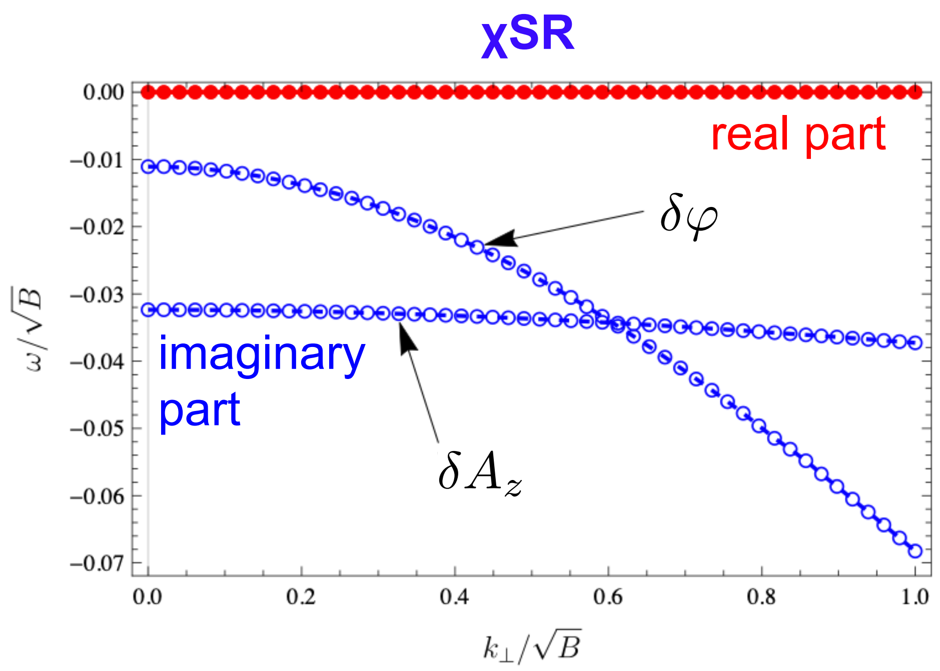
Phase diagram for homogeneous solutions

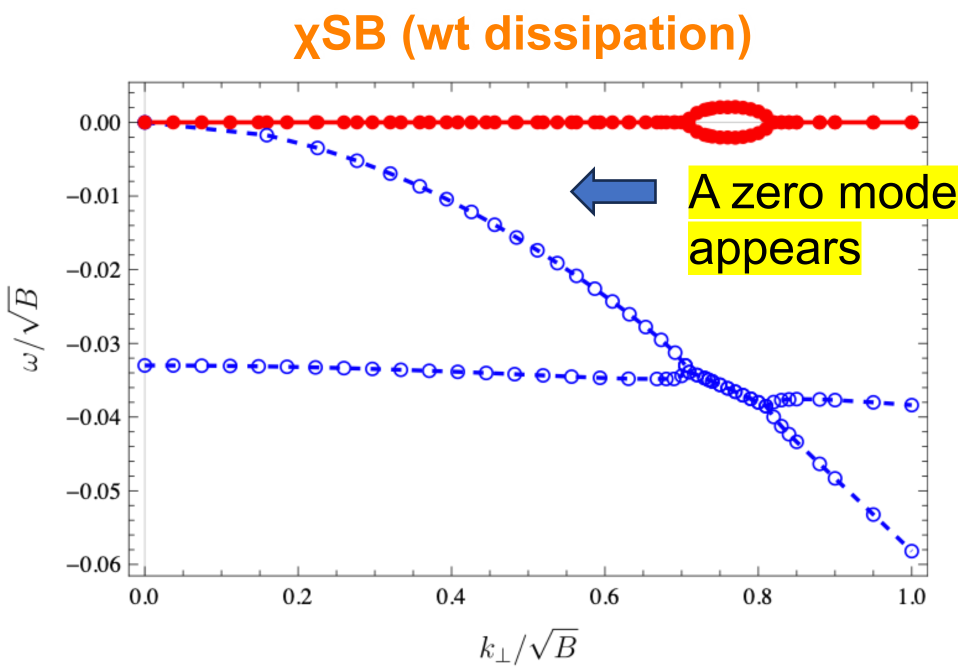
Evans, Gebauer, Kim, Magou (2010)

Related topic: Nambu-Goldstone modes

A fluctuation of φ corresponds to the NG modes in the SSB.

Dispersion relation





SI, M. Matsumoto (2021) arXiv:2012.01177 **[hep-th]**

Main differences from holographic superfluid

1. Dual description is clear: $\mathcal{N} = 4$ SYM $+\mathcal{N} = 2$ hypermultiplets
   ➡ broken symmetry is chiral U(1) (, not gauge symmetry)

2. External magnetic field is necessary for χSB.
   ➡ called “magnetic catalysis”

3. Both of 1st and 2nd order phase transitions appear.

Inhomogeneous solutions

Inhomogeneous solution with χSB

- Consider spatially dependent solution on x.
- Spatial modulation perpendicular to magnetic field B
- Ansatz:

$$\theta(u,x),\ \ A_t(u,x),\ \ A_y(u,x)$$

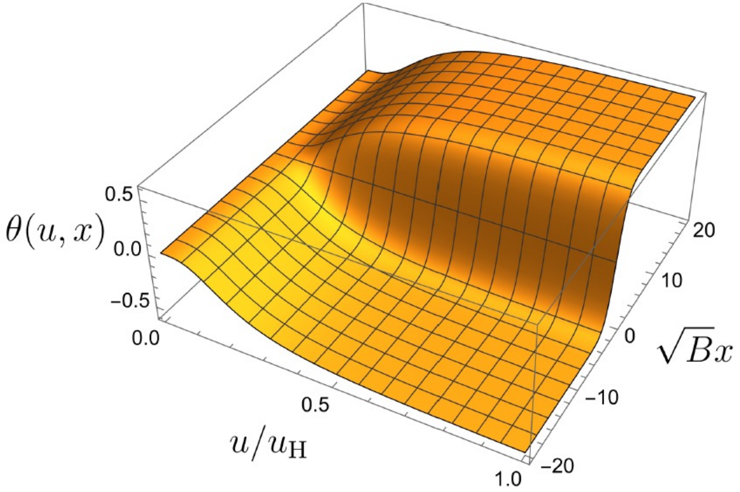
⇓

implying non-trivial current density is induced

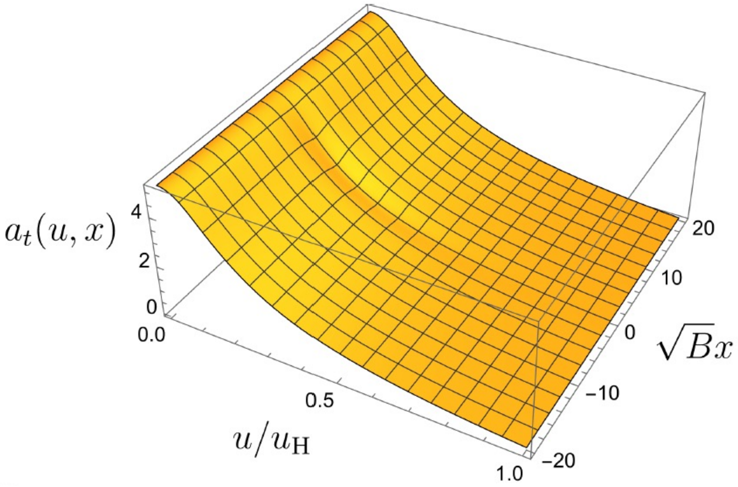
Typical solutions

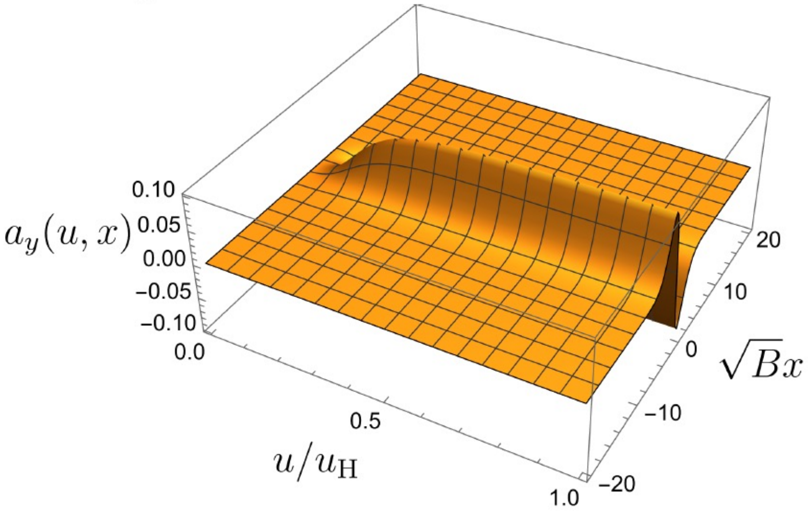
arXiv:2402.04657 [hep-th]

D7-brane configuration

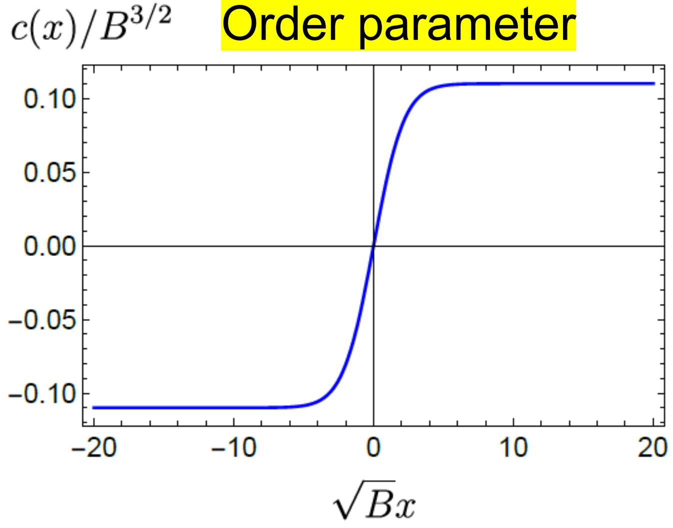


Gauge fields configuration

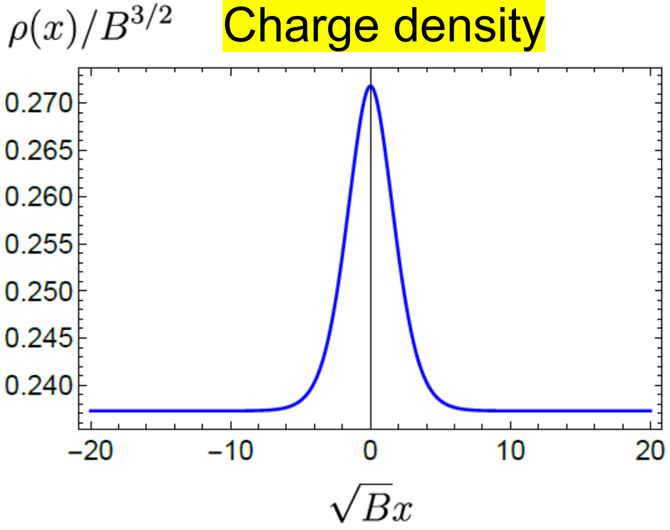




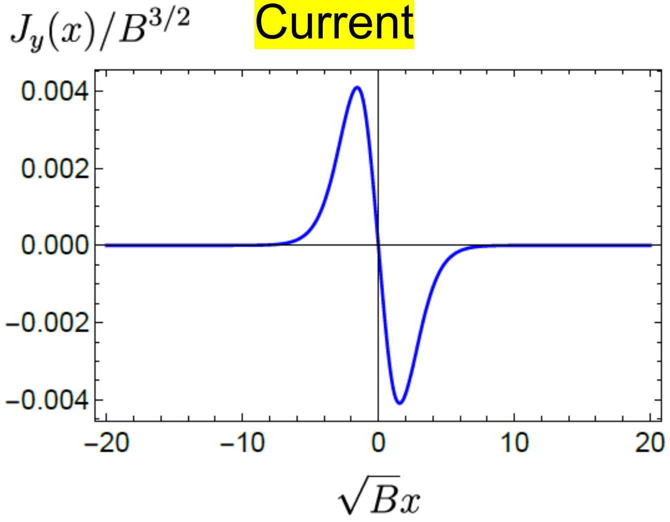
Order parameter



Charge density



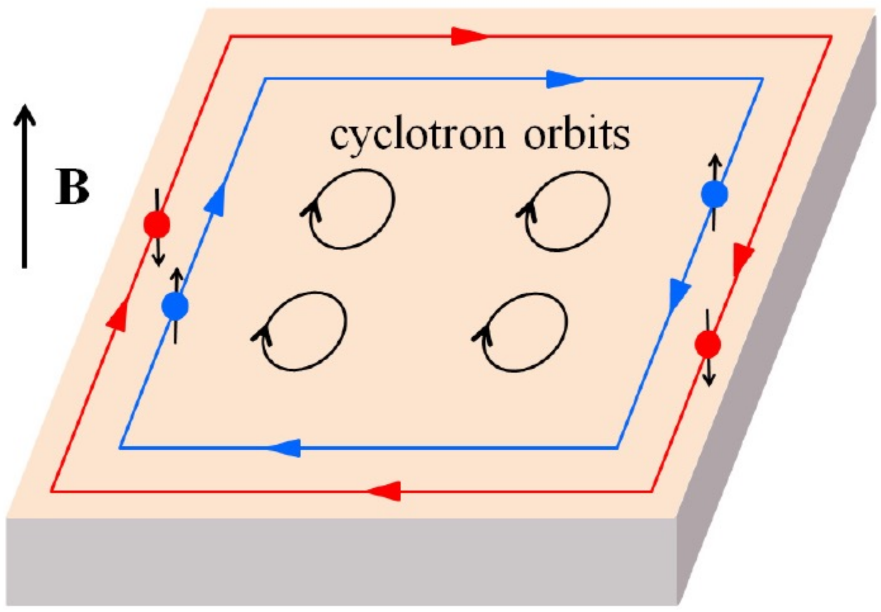
Current



Chiral edge current

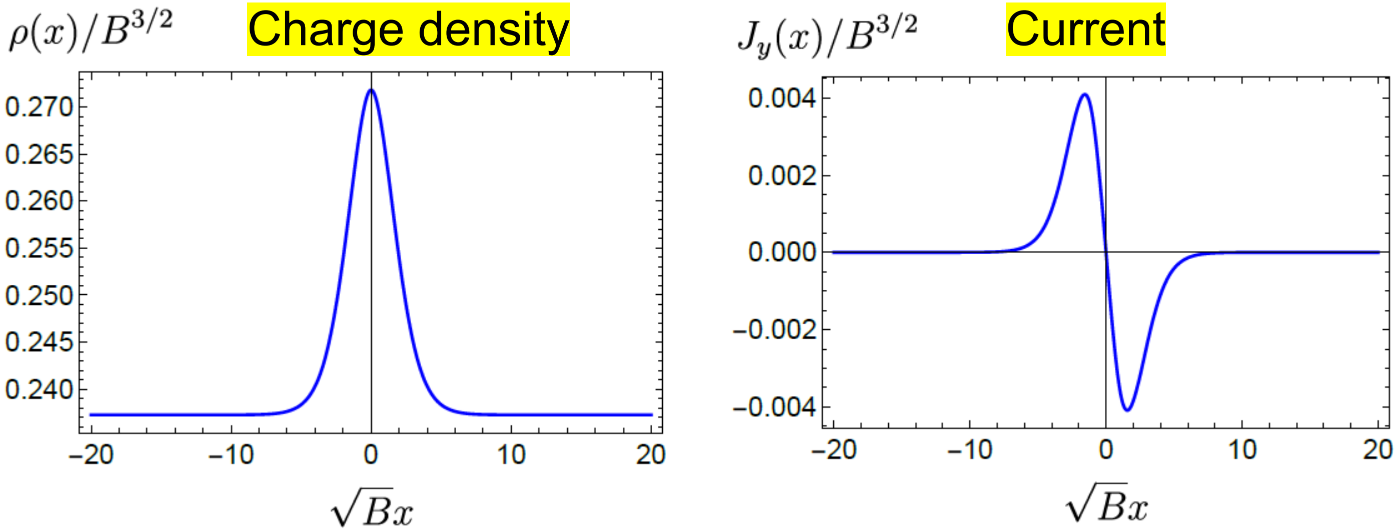
arXiv:2402.04657 [hep-th]

Magnetic field induces a chiral edge current



(a)

from review by B. Liu and W. Zhang (2023)



Displacement of charge density ~ edge

$$J_y(x) \propto \partial_x \rho(x)$$

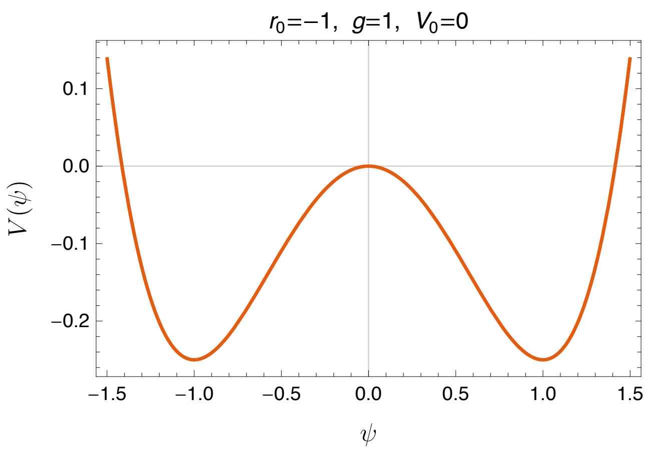
Fitting with Ginzburg-Landau theory

A domain wall solution in GL theory

Ginzburg-Landau theory describes the second order phase transition.

Effective thermodynamic potential from the 4th order GL theory

$$\Omega[\psi] = \int \mathrm{d}^3x \left\{ \frac{1}{2}|\vec{\nabla}\psi|^2 + \frac{r_0}{2}|\psi|^2 + \frac{g}{4}|\psi|^4 + V_0 \right\}$$



Stationary equation:
$$\frac{\delta\Omega}{\delta\psi^*} = -\frac{1}{2}\vec{\nabla}^2\psi + \frac{1}{2}r_0\psi + \frac{1}{2}g\psi|\psi|^2 = 0$$

Inhomogeneous solution:
$$\psi(x) = \Delta \tanh\left(\frac{x}{\xi}\right)$$

In holographic superfluid:

[V. Keranen, E. Keski-Vakkuri, S. Nowling, K. P. Yogendran (2009)]

Let’s compare this solution with our results!

Near 2nd order PT point ($\frac{\pi T}{\sqrt{2B}} \approx 0.071$)

GL theory's solution: $\psi(x) = \Delta \tanh\left(\frac{x}{\xi}\right)$

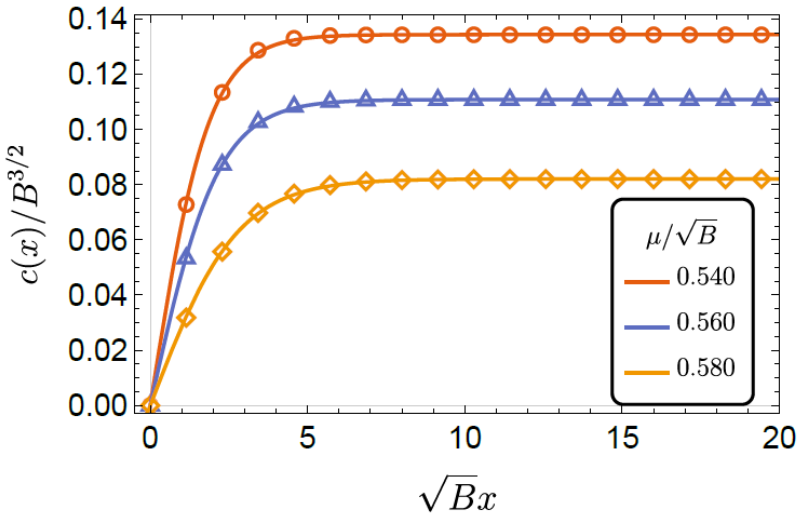
assumption: $\rho(x) \sim c(x)^2$

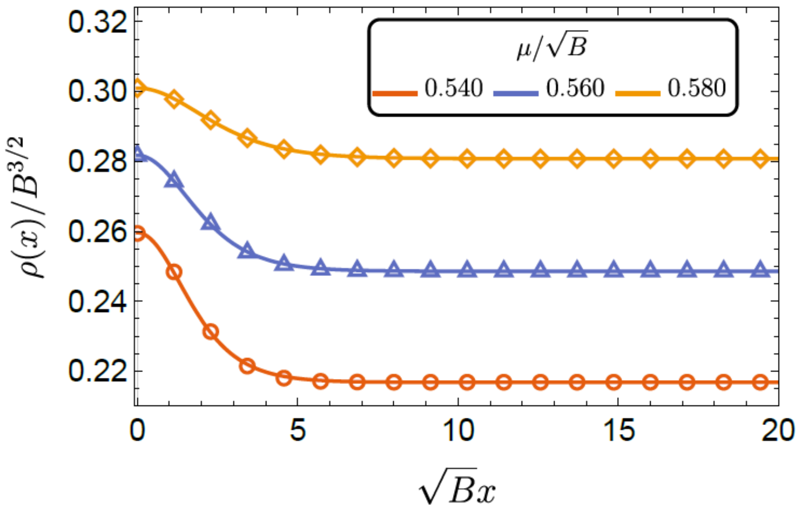
fitting parameters: $\xi_c,\ \xi_\rho,\ \rho_0$

fitting functions:
(solid curves)

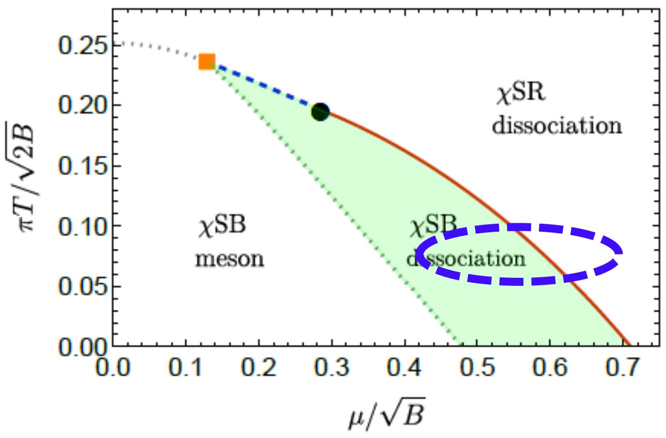
$$c(x) = c_{\text{homo}} \tanh\left(\frac{x}{\xi_c}\right), \quad \rho(x) = \rho_{\text{homo}} - (\rho_{\text{homo}} - \rho_0)\,\text{sech}^2\left(\frac{x}{\xi_\rho}\right)$$

Profiles of order parameter and charge density





T-*μ* phase diagram



The results are well-fitted!

A domain wall solution in 6th order GL theory

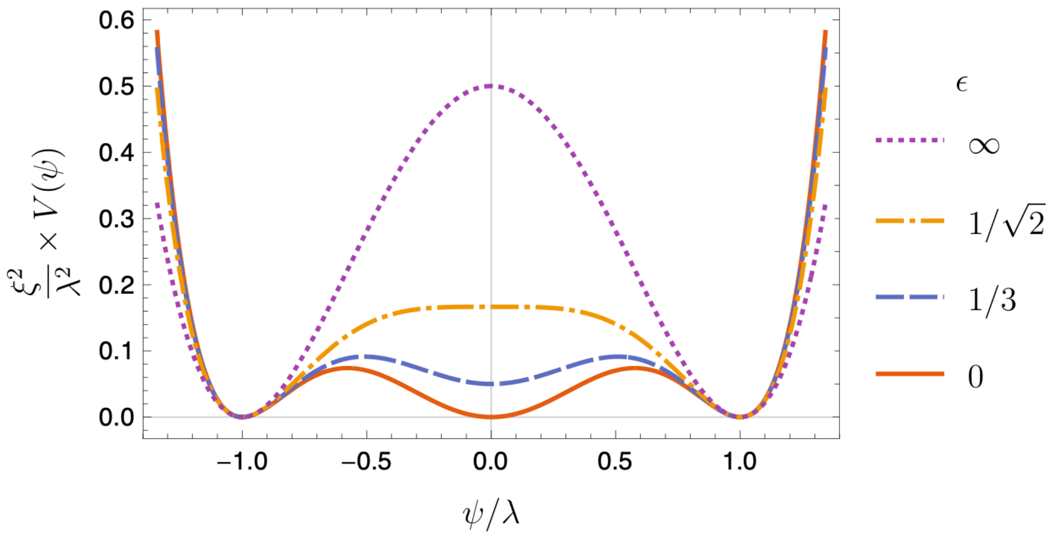
Near the 1st order PT, the fitting gets fails.
➡ We need 6th order GL theory.

Effective thermodynamic potential

$$\Omega[\psi] = \int \mathrm{d}^3x \left\{ \frac{1}{2}|\vec{\nabla}\psi|^2 + V_0 + \right.$$

$$\left. + \frac{1}{2\xi^2\lambda^4(1+\epsilon^2)} \left(|\psi|^2 + \epsilon^2\lambda^2\right) \left(|\psi|^2 - \lambda^2\right)^2 \right\}$$

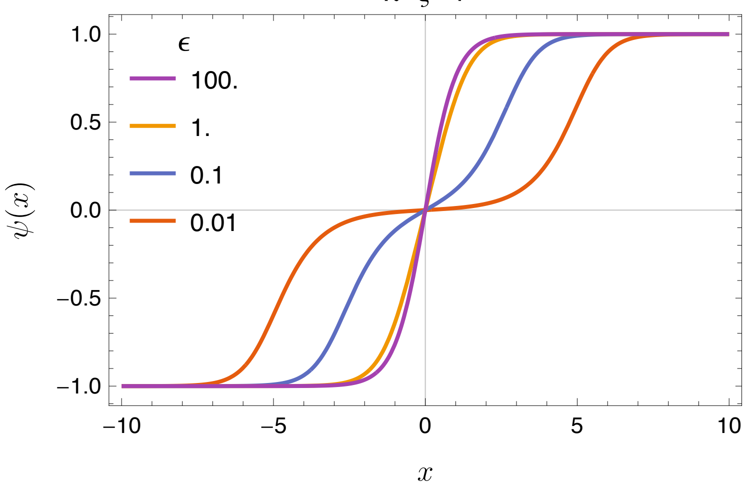
potential's shape



Inhomogeneous solution: *Bound pair solution*

$$\psi(x) = \lambda \frac{\sinh(x/\xi)}{\sqrt{1 + \epsilon^{-2} + \sinh^2(x/\xi)}}$$

λ=ξ=1



Review:
[Saxena,Christov, Khare arXiv:1806.06693 [hep-th].]

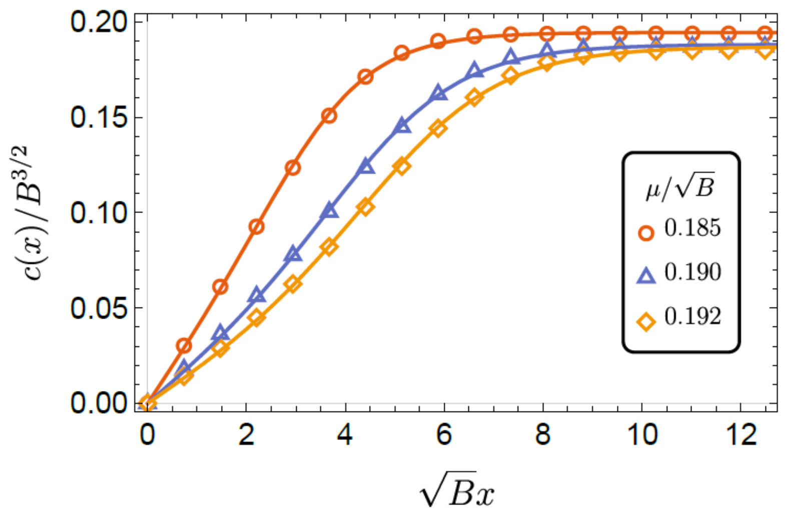
Near 1st order PT point ($\frac{\pi T}{\sqrt{2B}} = 0.22$)

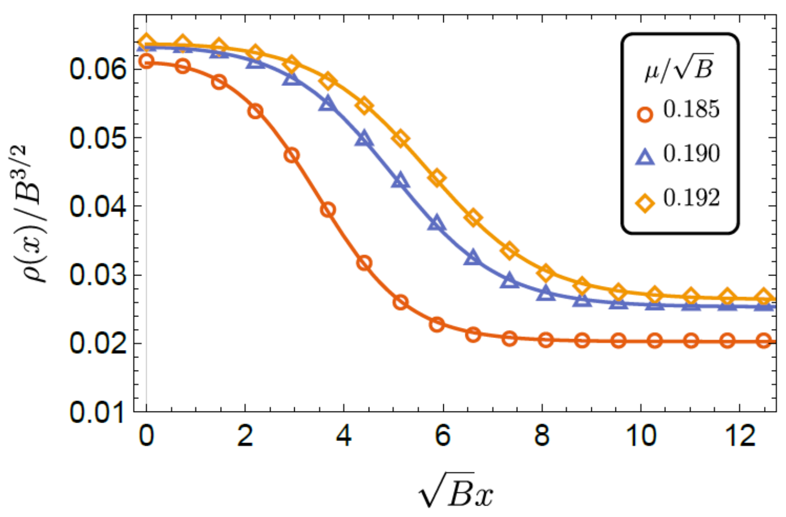
GL theory's solution: $$\psi(x) = \lambda \frac{\sinh(x/\xi)}{\sqrt{1+\epsilon^{-2}+\sinh^2(x/\xi)}}$$

$$c(x) = c_{\rm homo}\frac{\sinh(x/\xi_c)}{\sqrt{1+\epsilon_c^{-2}+\sinh^2(x/\xi_c)}}, \quad \rho(x) = \rho_0-(\rho_0-\rho_{\rm homo})\frac{\sinh^2(x/\xi_\rho)}{1+\epsilon_\rho^{-2}+\sinh^2(x/\xi_\rho)}$$ (solid curves)

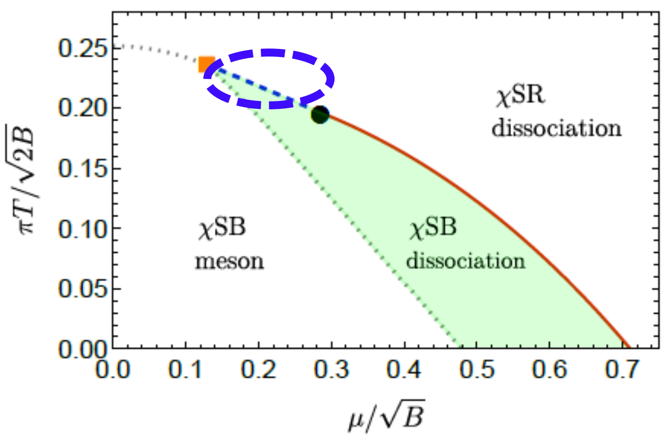
fitting parameters: $\xi_{c,\rho},\ \epsilon_{c,\rho},\ \rho_0$

Profiles of order parameter and charge density



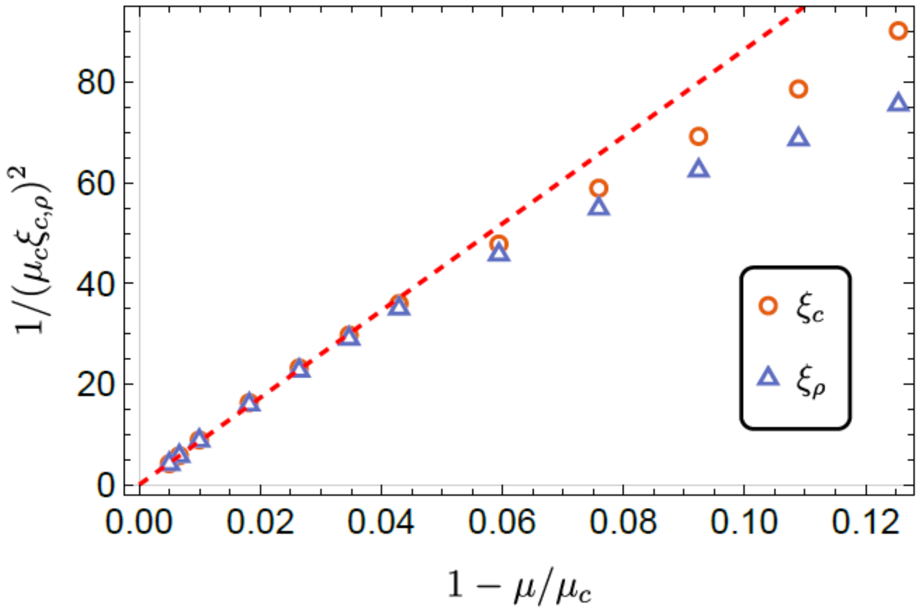


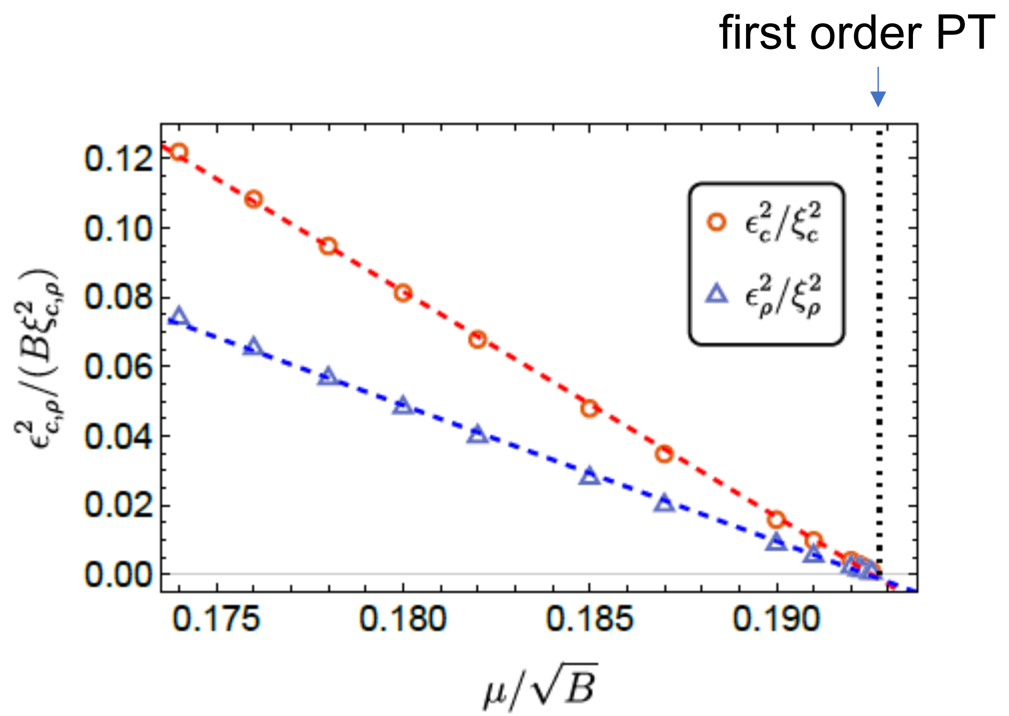
T-μ phase diagram

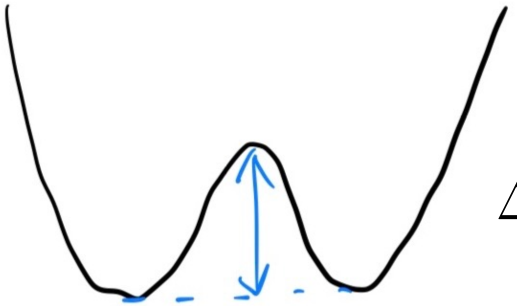


The results are well-fitted!

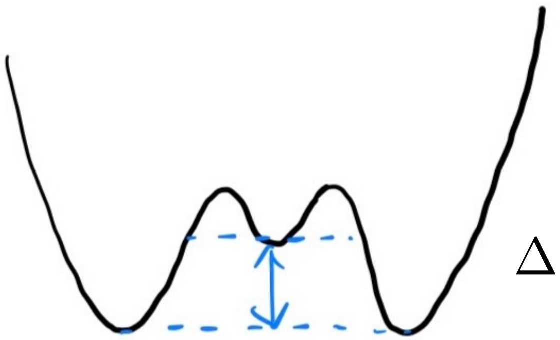
Behaviors of the fitting parameters







$$\Delta V \propto \frac{1}{(\mu\xi)^2} \propto 1 - \frac{\mu}{\mu_c}$$

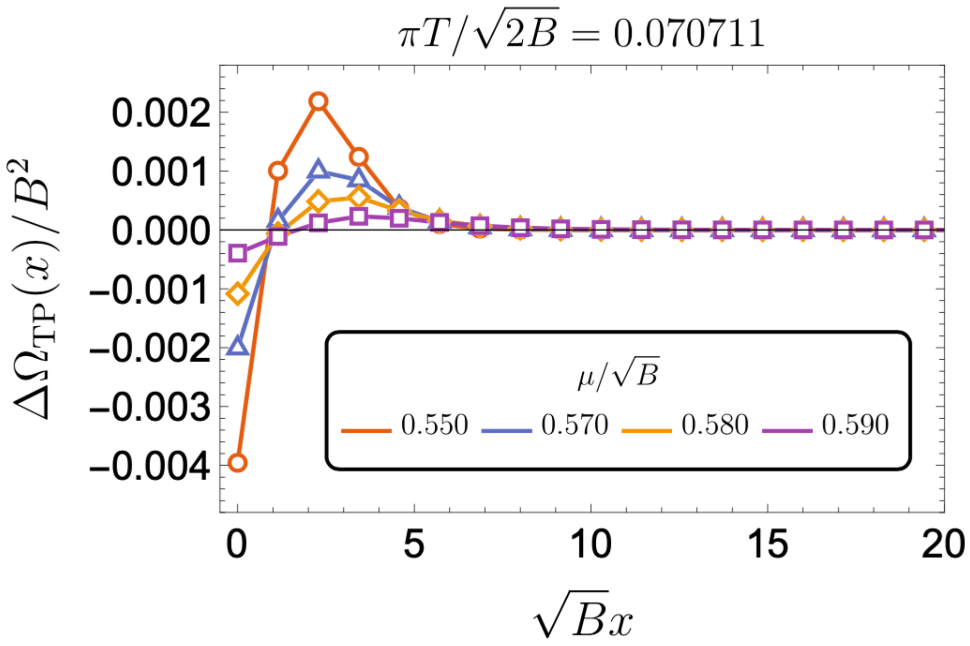


$$\Delta V \propto \frac{\lambda^2\epsilon^2}{\xi^2} \propto 1 - \frac{\mu}{\mu_{1\mathrm{st}}}$$

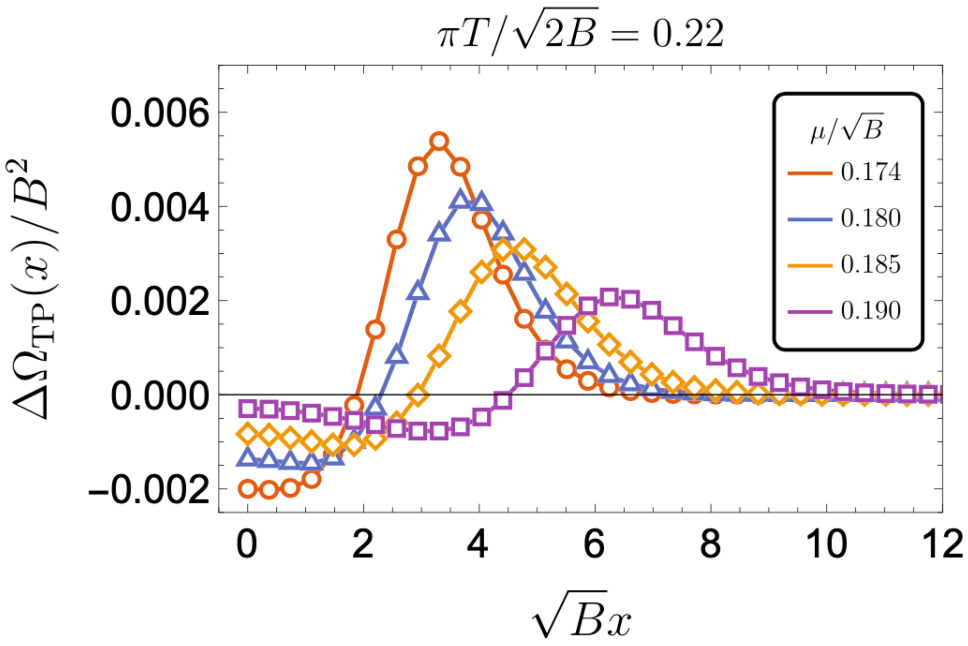
Thermodynamic stability

$$\Omega_{\mathrm{TP}}/T = -S^E|_{\mathrm{onshell}} \qquad \Delta\Omega_{\mathrm{TP}} = \Omega_{\mathrm{TP}}^{\mathrm{kink}} - \Omega_{\mathrm{TP}}^{\chi\mathrm{SB}}$$

Near the second order PT



Near the first order PT



The total energy always higher than the homogenous states.
However, the kink states may be realized during a thermalization process.

Summary

arXiv:2402.04657 [hep-th]

- ❑ Single kink solutions accompanied with chiral edge currents in a top-down D3/D7 model

- ❑ Well-described by Ginzburg-Landau effective theory
  - 2nd order PT ↔ 4th order GL
  - 1st order PT ↔ 6th order GL

Outlook

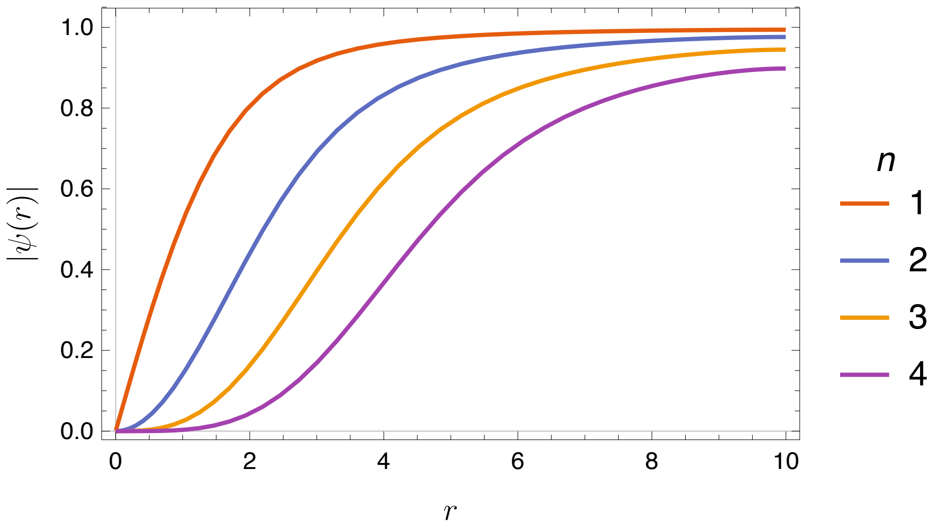
- ❑ Other solutions: Vortex, complex kink, …
- ❑ Nambu-Goldstone modes associated with translation SB
- ❑ Drive system to steady state

Ex. Vortex?

To avoid a singular behavior, a radially symmetric order parameter must behaves as

$$\psi \sim \lambda(r)e^{in\varphi} \qquad \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} r \\ \varphi \end{pmatrix}$$

The GL theory admits such a solution.



The D3-D7 model probably has a similar solution, but we have not obtained.