Superradiant instability and black resonators in AdS

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Superradiant instability of Myers-Perry AdS BH

Outline

Rotating black holes in asymptotically AdS space show **superradiant instability.**

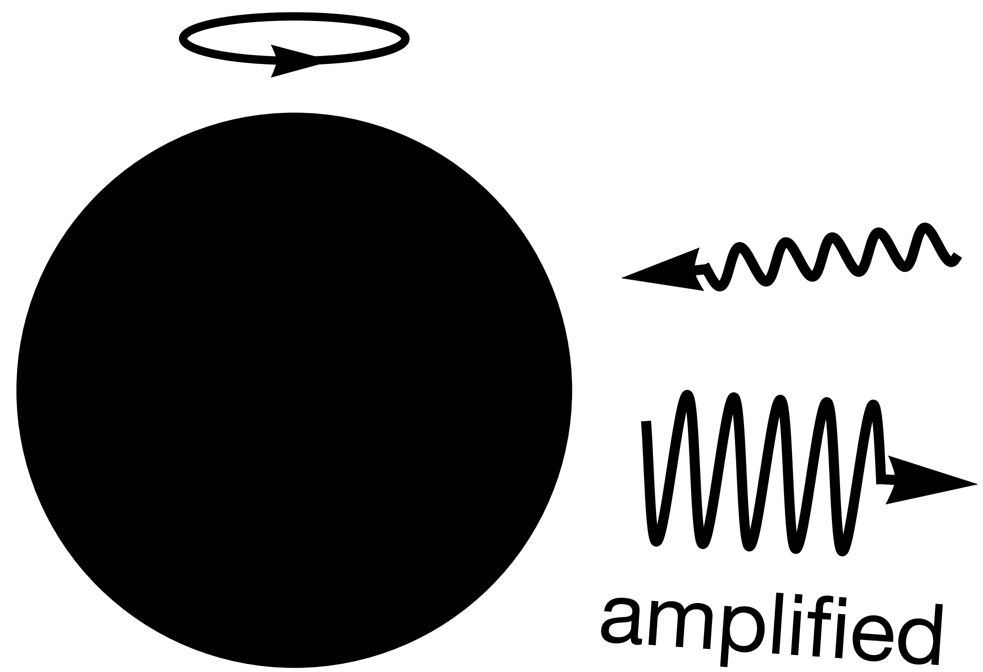
Myers-Perry AdS BH with equal angular momenta provides a simple setup to study this phenomenon.

The perturbation for the superradiant instability **breaks the U(1) isometry** of MPAdS BH.

Superradiance

Rotational superradiance: waves can be amplified near a rotating black hole.

[Review: 1501.06570]

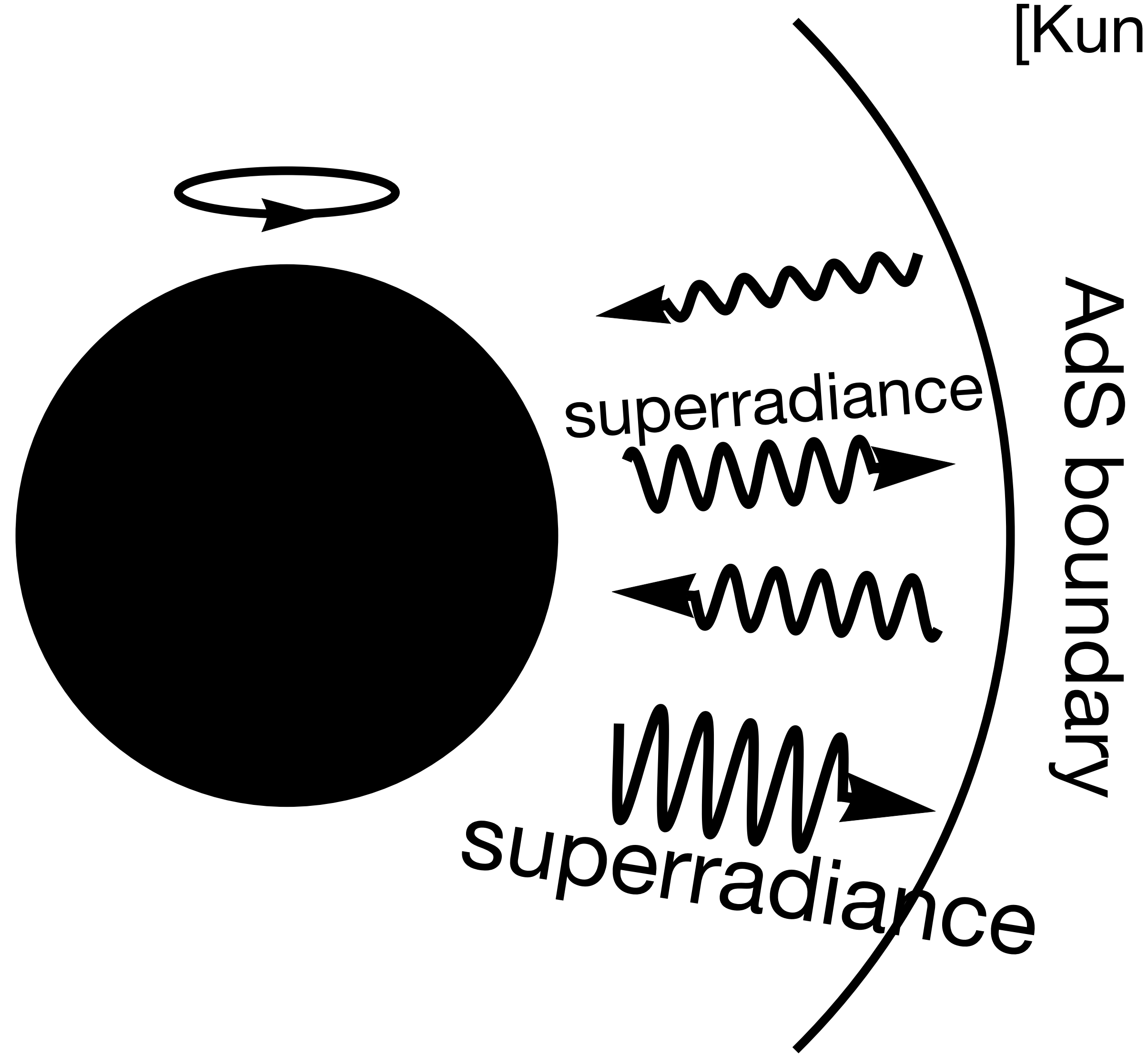


c.f.) charged superradiance for charged BH

Superradiant instability

In AdS space, amplified waves are confined in the geometry, and **superradiant instability** is induced.

[Kunduri-Lucietti-Reall]



5D MPAdS BH

5D Myers-Perry AdS black hole with equal angular momenta is given by a simple metric.

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g}\,[R - 2\Lambda]$$

$$ds^2 = -(1+r^2)f(r)d\tau^2 + \frac{dr^2}{(1+r^2)g(r)} + \frac{r^2}{4}\left[\sigma_1^2 + \sigma_2^2 + \beta(r)(\sigma_3 + 2h(r)d\tau)^2\right]$$

Metric components are functions of 1 variable.

$$f(r) = \frac{g(r)}{\beta(r)}, \quad g(r) = 1 - \frac{2\mu(1-a^2)}{r^2(1+r^2)} + \frac{2a^2\mu}{r^4(1+r^2)}$$

$$\beta(r) = 1 + \frac{2a^2\mu}{r^4}, \quad h(r) = \Omega - \frac{2\mu a}{r^4 + 2a^2\mu}$$

Isometry of the equal rotation solution

When the two rotations are equal, the $R_\tau \times U(1) \times U(1)$ isometries of general MP BH are enhanced to $R_\tau \times U(2)$.

$$ds^2 = -(1+r^2)f(r)d\tau^2 + \frac{dr^2}{(1+r^2)g(r)} + \frac{r^2}{4}\left[\underbrace{\sigma_1^2 + \sigma_2^2}_{S^2} + \beta(r)\underbrace{(\sigma_3 + 2h(r)d\tau)^2}_{S^1 \text{ fiber}}\right]$$

SU(2) invariant 1-forms (θ,ϕ,χ: angles of S^3)

$$\sigma_1 = -\sin\chi d\theta + \cos\chi \sin\theta d\phi$$
$$\sigma_2 = \cos\chi d\theta + \sin\chi \sin\theta d\phi$$
$$\sigma_3 = d\chi + \cos\theta d\phi$$

U(1) shift: $\chi \to \chi + \text{const.}$

Superradiant instability of MPAdS

We focus specifically on the following perturbation.

[Murata]

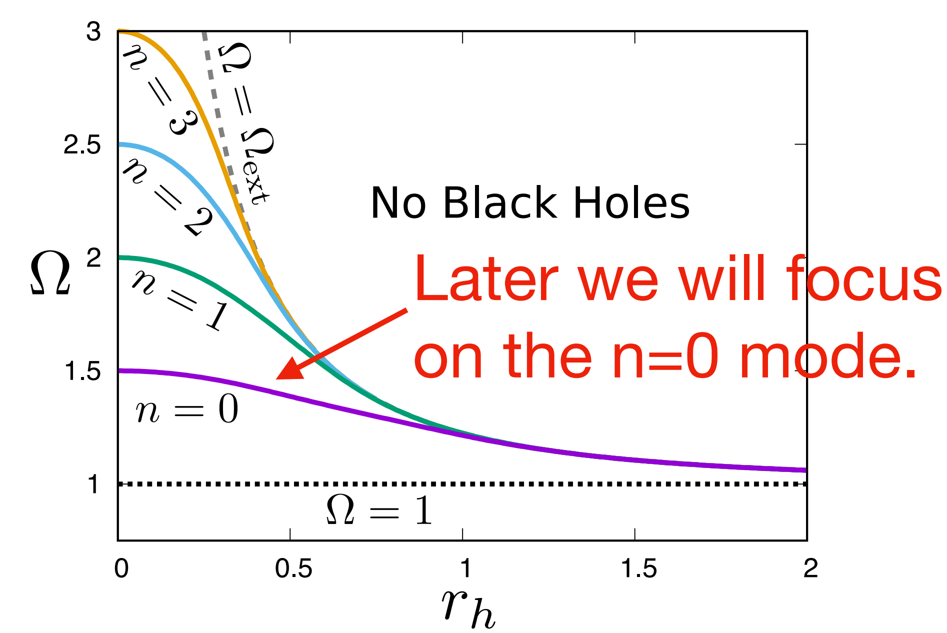
$$\delta g_{\mu\nu}dx^\mu dx^\nu = \frac{r^2}{4}\delta\alpha(r)(\sigma_1^2 - \sigma_2^2)$$

This gives a decoupled perturbation equation:

$$\delta\alpha'' + \left(\frac{g'}{g} + \frac{3+5r^2}{r(1+r^2)}\right)\delta\alpha' + \frac{8}{(1+r^2)g}\left(\frac{\beta-2}{r^2\beta} + \frac{2\beta h^2}{(1+r^2)g}\right)\delta\alpha = 0$$

Onset of superradiant instability:

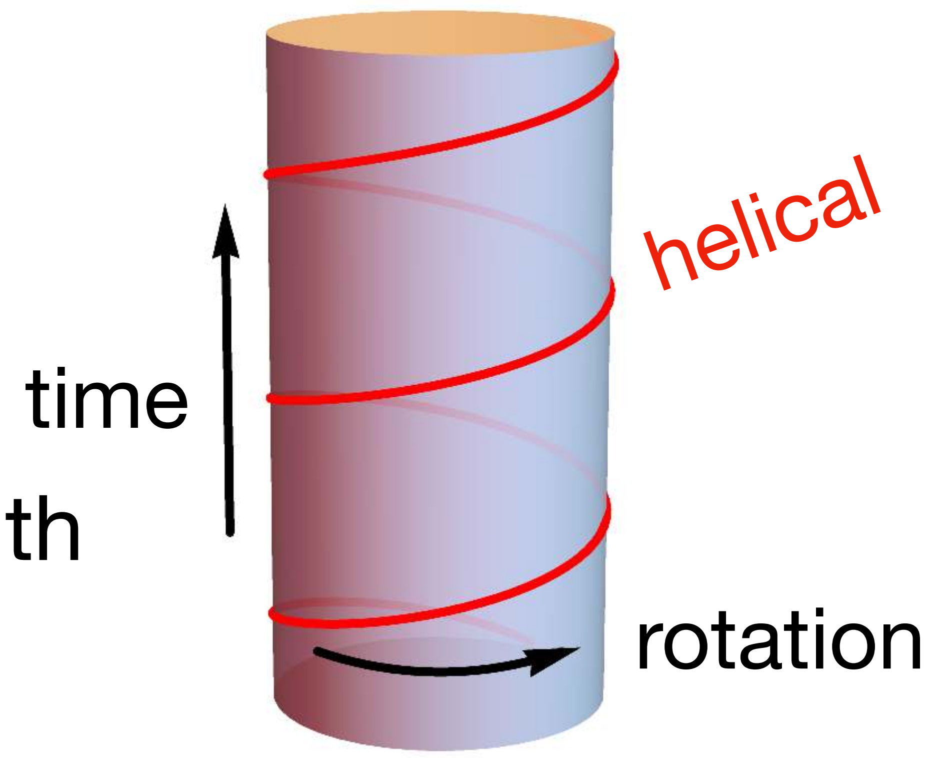
We search the onset frequency Ω, at which this equation is solved by nontrivial normal modes δα≠0.



Isometry breaking by the perturbation

This perturbation breaks the U(1) shift $\chi \to \chi + \mathrm{const.}$
(because $\sigma_1^2 - \sigma_2^2$ is not invariant by this shift).

$$\sigma_1 = -\sin\chi d\theta + \cos\chi \sin\theta d\phi$$
$$\sigma_2 = \cos\chi d\theta + \sin\chi \sin\theta d\phi$$
$$\sigma_3 = d\chi + \cos\theta d\phi$$



There will be a new BH solution with

$$R_\tau \times U(2) \to R_\tau \times SU(2)$$

We will see that R_τ is indeed a **helical isometry.**

Black resonators in AdS_5

Outline

Black resonators branch off from the onset of the superradiant instability.

Cohomogeneity-1 black resonator solutions can be constructed in 5D by imposing the SU(2) isometry.

Photonic black resonators can be also obtained in Einstein-Maxwell theory.

Black resonators (in AdS_4)

Black holes with a single Killing vector field: black resonators

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We numerically construct asymptotically anti-de Sitter (AdS) black holes in four dimensions that contain only a single Killing vector field. These solutions, which we coin *black resonators*, link

[Dias-Santos-Way 1505.04793]

Black holes with a helical isometry were first constructed in 4D and named **black resonators.**

These are **time-periodic black holes.**

5D cohomogeneity-1 black resonator

We consider a metric ansatz with the SU(2) isometry.

The Einstein eq. reduce to ODEs for (f,g,h,α,β).

[TI-Murata]

$$
\begin{aligned}
ds^2 = &-(1+r^2)f(r)d\tau^2 + \frac{dr^2}{(1+r^2)g(r)} \\
&+ \frac{r^2}{4}\left[\alpha(r)\sigma_1^2 + \frac{1}{\alpha(r)}\sigma_2^2 + \beta(r)(\sigma_3 + 2h(r)d\tau)^2\right]
\end{aligned}
$$

MPAdS: $\alpha(r)=1$ ⇒ black resonator: $\alpha(r)\neq 1$

<u>Boundary conditions</u>

$r \to r_h$: BH horizon $f=g=0$ with $\mathbf{h(r_h)=0}$

$r \to \infty$: asymp. AdS: $(f,g,\alpha,\beta) \to 1$ with $\mathbf{h(\infty)=\Omega}$

Time periodic metric

If we change the frame so that **$\bar{h}(\infty)=0$** (AdS boundary is static), the metric is **explicitly time periodic.**

$$dt = d\tau,\ \ d\bar{\chi} = d\chi + 2\Omega d\tau,\ \ \bar{h}(r) = h(r) - \Omega$$

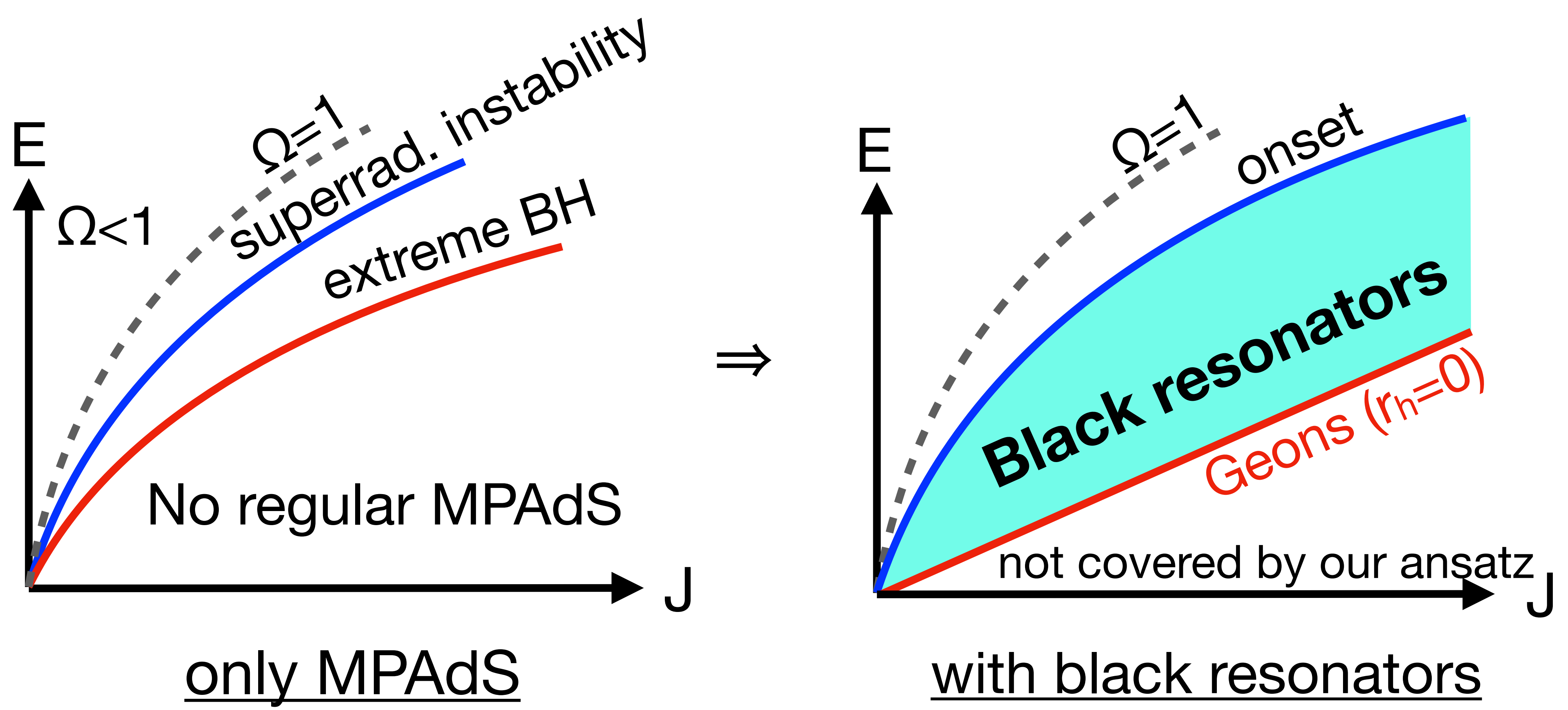
$$ds^2 = -(1+r^2)f(r)dt^2 + \frac{dr^2}{(1+r^2)g(r)} + \frac{r^2}{4}\Bigg[\frac{\alpha(r)^2+1}{2\alpha(r)}(\bar{\sigma}_1^2+\bar{\sigma}_2^2) + \frac{\alpha(r)^2-1}{2\alpha(r)}\left(\cos(4\Omega t)(\bar{\sigma}_1^2-\bar{\sigma}_2^2)+2\sin(4\Omega t)\bar{\sigma}_1\bar{\sigma}_2\right) + \beta(r)(\bar{\sigma}_3+2\bar{h}(r)dt)^2\Bigg]$$

So, this black hole is time-periodic: **black resonator**

There is a **helical Killing vector** $K = \partial_\tau = \partial_t + \Omega\partial_{\bar{\chi}/2}$

(E,J) phase diagram

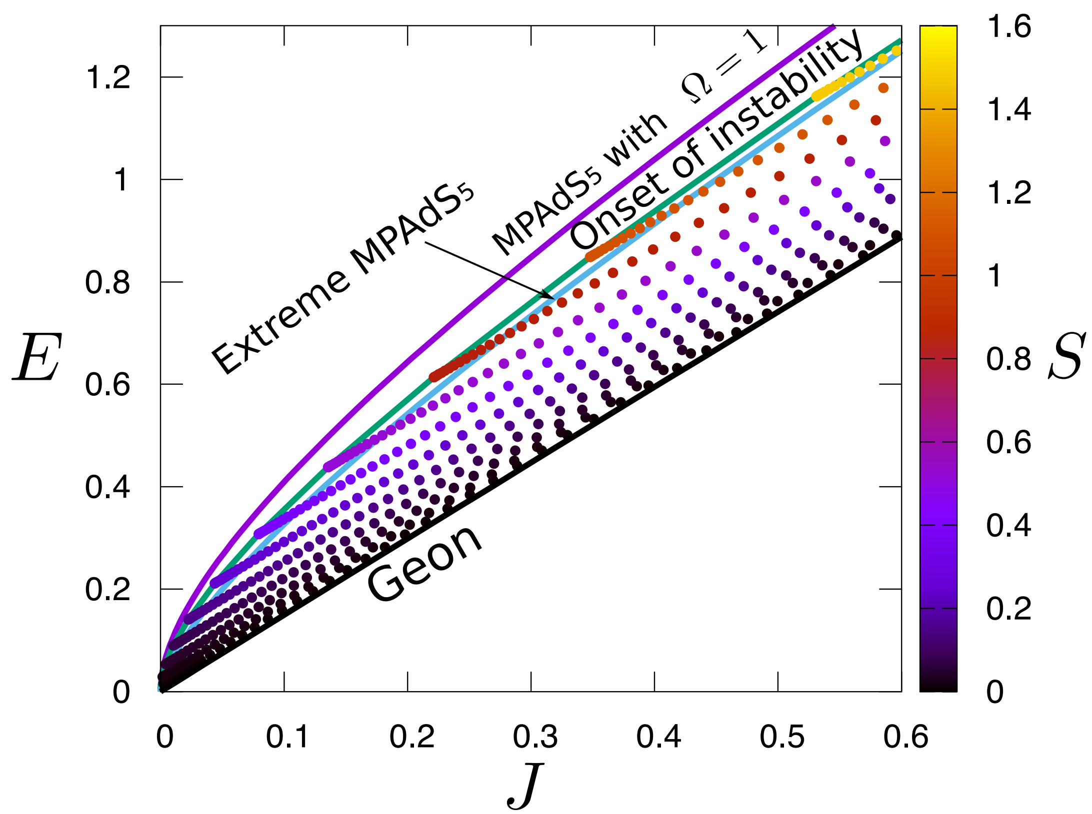
Black resonators cover the region where the MPAdS solution is super-extreme.

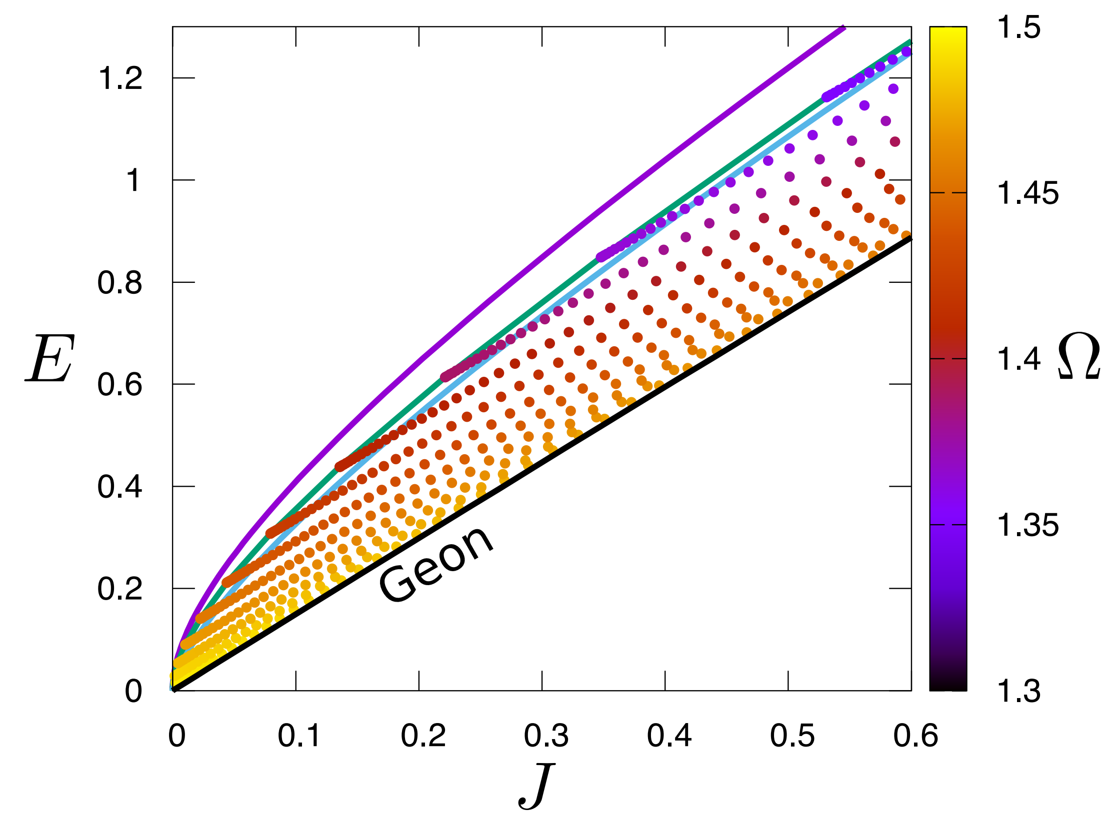


Remarks

- Entropy→0 for the regular horizonless geon (r_h→0).
  (BH's shape is time dependent but area is constant.)

- All black resonators we constructed have Ω>1.





Photonic black resonators

Black resonators with **time-periodic electromagnetic waves** can also be obtained in Einstein-Maxwell theory.

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

$$\begin{aligned} ds^2 = &-(1+r^2)f(r)d\tau^2 + \frac{dr^2}{(1+r^2)g(r)} \\ &+ \frac{r^2}{4}\left[\alpha(r)\sigma_1^2 + \alpha(r)^{-1}\sigma_2^2 + \beta(r)(\sigma_3 + 2h(r)d\tau)^2\right] \end{aligned}$$

Gauge field $\quad A = \gamma(r)\sigma_1 \;(= \gamma(r)\left[\cos(2\Omega t)\bar{\sigma}_1 + \sin(2\Omega t)\bar{\sigma}_2\right])$

[TI-Murata]

Superradiant instability of black resonators

Outline

The black resonators, having $\Omega > 1$, are subject to further **superradiant instability.**

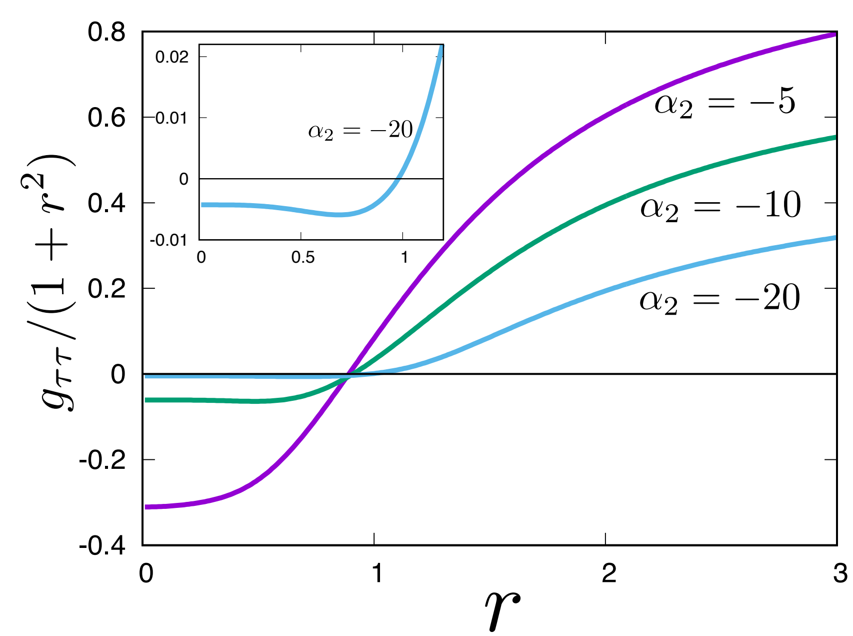
Studying **linear perturbations of black resonators** is viable for the cohomogeneity-1 metric.

Here I focus on **scalar field perturbation** (we also studied Maxwell and gravitational perturbations).

Instability of black resonators

The norm of the helical Killing vector satisfies **K²>0 (i.e. spacelike)** near the AdS boundary **if Ω>1.**

$$K^2 = g_{\tau\tau} \to -r^2(1-\Omega) \quad (r \to \infty)$$



And there is a **theorem** that says that such solutions should be **unstable.** [Green-Hollands-Ishibashi-Wald]

Scalar field perturbation

Consider a probe scalar field: $\Box\Phi = 0$

The scalar field can be expanded by the **Wigner-D matrices,** which are **spherical harmonics on S^3.**

$$D^j_{mk}(\theta,\phi,\chi) \qquad \begin{aligned} j &= 0,\ 1/2,\ 1,\ 3/2, \dots\ , \\ m &= -j,\ -j+1, \dots,\ j\ , \\ k &= -j,\ -j+1, \dots,\ j\ . \end{aligned}$$

$$\Phi(\tau,r,\theta,\phi,\chi) = e^{-i\omega\tau}\sum_{|k|\leq j}\phi_k(r)D_k(\theta,\phi,\chi)$$

We suppress (j,m) indices: $D_k \equiv D^j_{mk}$

Modes with different m decouple.

ODEs with mode coupling

In the MPAdS (α=1), c_k=0: the modes with different k decouple because of the U(1) isometry.

For black resonators, **different k modes couple** because they do not have the U(1) isometry.

$$L_k\phi_k + c_{k-1}\phi_{k-2} + c_{k+1}\phi_{k+2} = 0$$

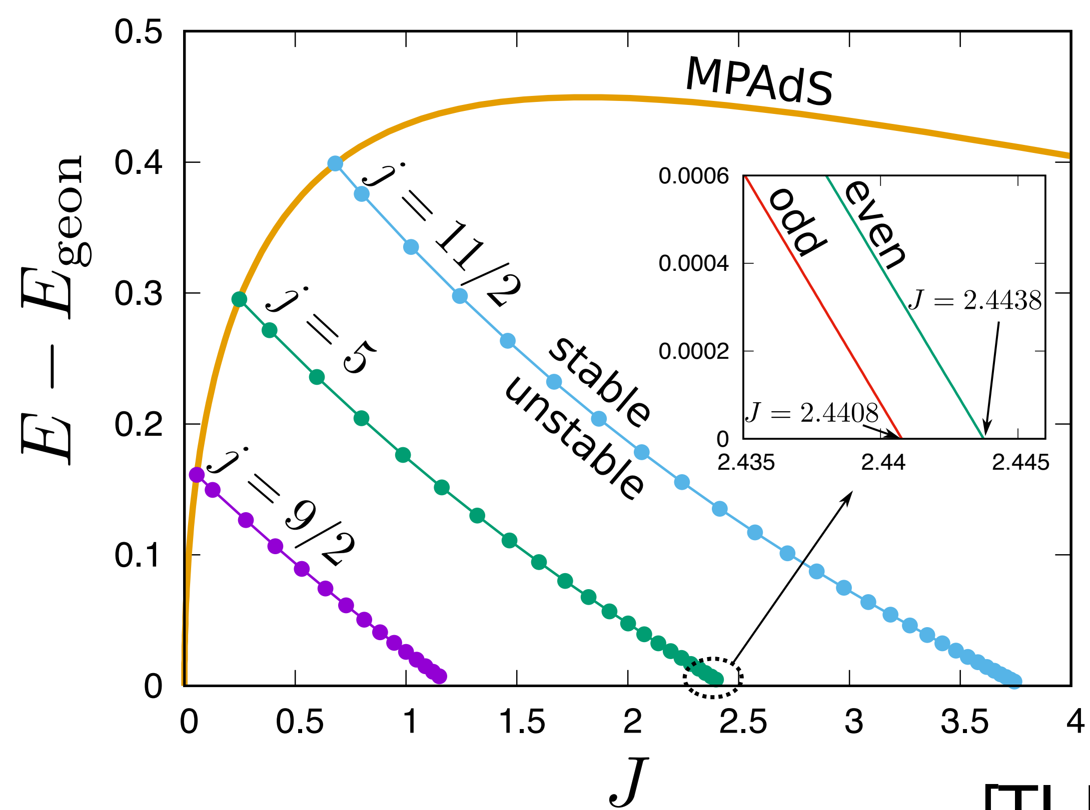
$$c_k = -\frac{\epsilon_k\epsilon_{k+1}}{r^2}\left(\alpha - \frac{1}{\alpha}\right)$$

$$L_k = (1+r^2)g\frac{d^2}{dr^2} + \left[\frac{1+r^2}{2}\left(\frac{f'}{f}+\frac{g'}{g}+\frac{\beta'}{\beta}\right) + \frac{3+5r^2}{r}\right]g\frac{d}{dr} \\ - \frac{\epsilon_k^2+\epsilon_{k+1}^2}{r^2}\left(\alpha+\frac{1}{\alpha}\right) - \frac{4k^2}{r^2\beta} + \frac{(\omega-2kh)^2}{(1+r^2)f}$$

Instability of black resonators

We solved the coupled equations for ϕₖ and identified their instability in the black resonator background.

More and more parameter regions are unstable for larger-j perturbations.



[TI-Murata-Santos-Way]

Summary

We studied the superradiant instability of **5D Myers-Perry black hole with equal angular momenta.**

We constructed **cohomogeneity-1 black resonators** in 5D asymptotically AdS space.

We studied **linear perturbations of black resonators** for the cohomogeneity-1 metric.