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Holographic Pseudo Entropy

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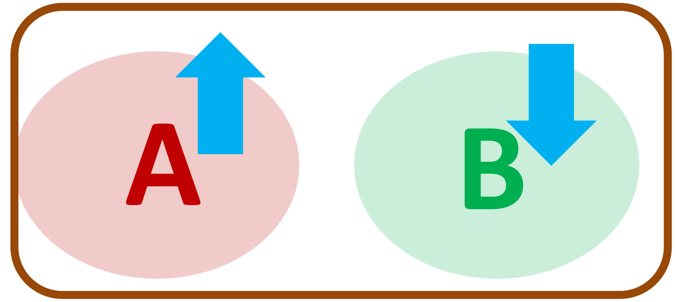
It from Qubit Simons Collaboration

① Introduction



Quantum Entanglement (QE)

Two parts (subsystems) A and B in a total system are quantum mechanically correlated.

e.g. Bell state : $|\Psi_{Bell}\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle_A \otimes |\downarrow\rangle_B + |\downarrow\rangle_A \otimes |\uparrow\rangle_B\right]$ ➡ **Minimal Unit of Entanglement**

Pure States: Non-zero QE ⇔ $|\Psi\rangle_{AB} \neq |\Psi_1\rangle_A \otimes |\Psi_2\rangle_B$.

Direct Product

The best (or only) measure of quantum entanglement for pure states is known to be **entanglement entropy (EE)**.

EE ＝ # of Bell Pairs between A and B

Entanglement entropy (EE) in "HEP"

Divide a quantum system into two subsystems A and B:

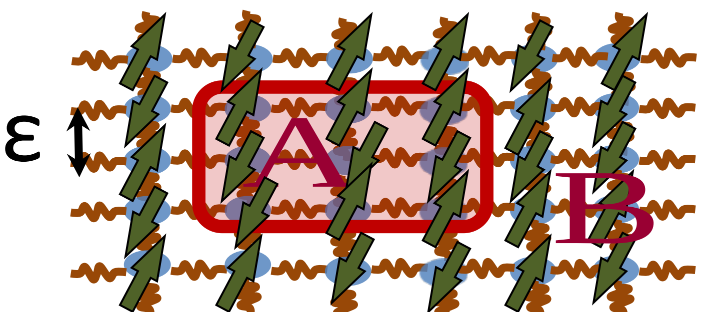
$$H_{tot} = H_A \otimes H_B \quad .$$

Define the reduced density matrix by $\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$.

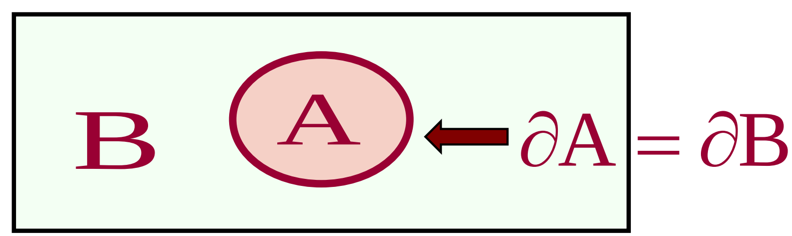
The entanglement entropy S_A is defined by

$$\boxed{S_A = -\mathrm{Tr}_A\, \rho_A \log \rho_A \,.}$$ (von-Neumann entropy)

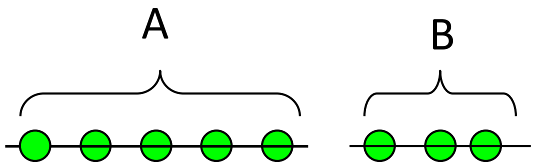
Quantum Many-body Systems | **Quantum Field Theories (QFTs)**



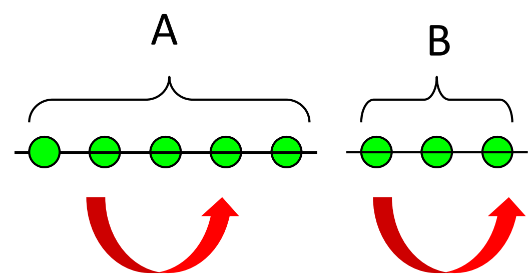
Continuum Limit ε→0



Entanglement Entropy (EE) in QI Text Book

Setup 

$\Rightarrow H_{tot} = H_A \otimes H_B$



LO (=Local Operations)

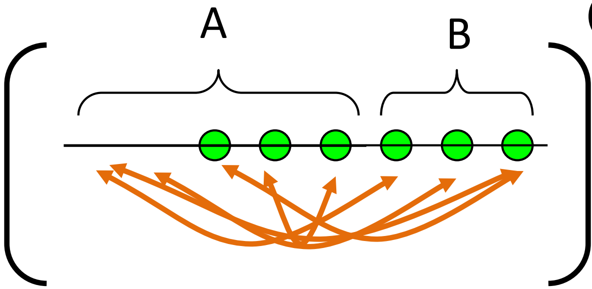
Projection measurements and unitary trfs. which act either A or B only.

CC (=Classical Communications between A and B)

⇒These operations are combined and called LOCC.

A basic example of LOCC: quantum teleportation

$$\left(|\Psi\rangle_{AB}\langle\Psi|\right)^{\otimes M}$$

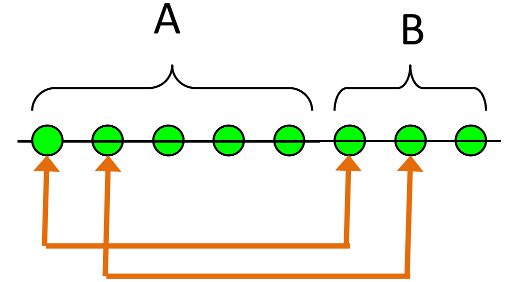


$\otimes M$

Entangled in a very complicated way

LOCC

Distillation



N Bell pairs

$$\left(|\Psi\rangle_{AB}\langle\Psi|\right)^{\otimes M} \Rightarrow \left(|\mathrm{Bell}\rangle\langle\mathrm{Bell}|\right)^{\otimes N}$$

Well-known fact in QI:

$$\boxed{S(\rho_A) = \lim_{M\to\infty} \frac{N}{M}}$$

$$\rho_A \equiv \mathrm{Tr}_B\left[|\Psi\rangle_{AB}\langle\Psi|\right]$$

[Bennett–Bernstein–Popescu–Schumacher 95, Nielsen 98]

Holographic Calculations of EE via AdS/CFT

Ver. 1, Ver. 2 and new Ver. 3

Ver. 1 Holographic EE for Static Spacetimes

[Ryu-TT 06; derived by Lewkowycz-Maldacena 13]

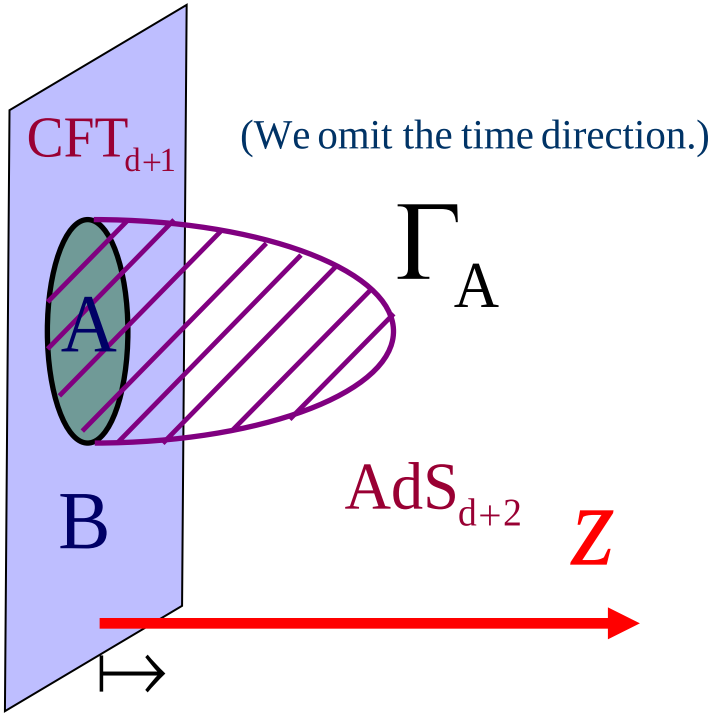
For static asymptotically AdS spacetimes:

$$\boxed{S_A = \underset{\substack{\partial\Gamma_A = \partial A \\ \Gamma_A \approx A}}{\mathrm{Min}} \left[\frac{\mathrm{Area}(\Gamma_A)}{4G_N} \right]}$$

Γ_A is the minimal area surface (codim.=2) on the time slice such that

$$\partial A = \partial \gamma_A \text{ and } A \sim \gamma_A \ .$$

homologous



$z > \varepsilon$ (UV cut off)

$$ds^2 = R^2 \cdot \frac{dz^2 - dt^2 + \sum_{i=1}^{d} dx_i^2}{z^2}$$

Ver. 2 Covariant Holographic Entanglement Entropy

[Hubeny-Rangamani-TT 07, derived by Dong-Lewkowycz-Rangamani-TT 16]

A generic Lorentzian asymptotic AdS spacetime is dual to a time dependent state $|\Psi(t)\rangle$ in the dual CFT.

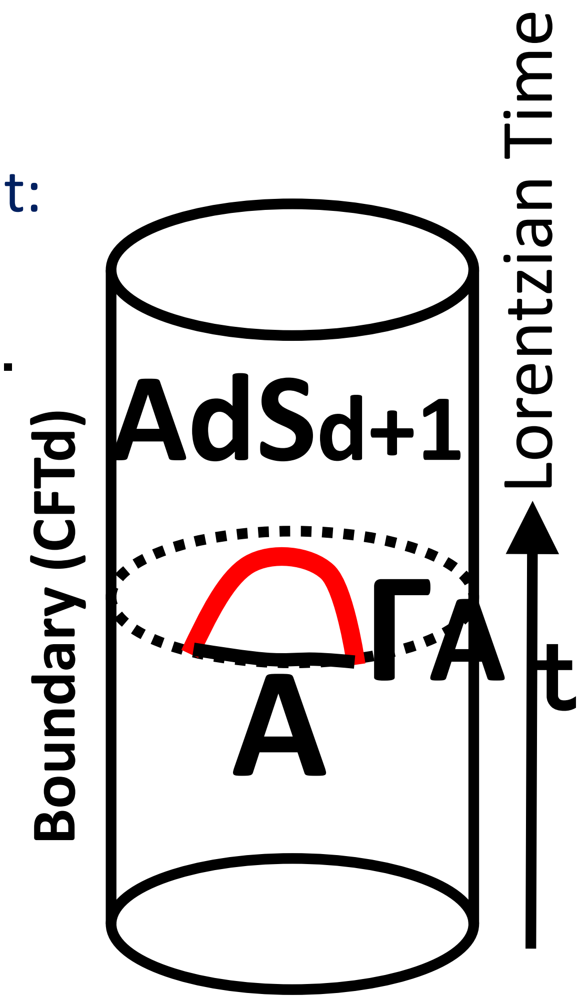
The entanglement entropy gets time-dependent:

$$\rho_A(t) = \mathrm{Tr}_B[|\Psi(t)\rangle\langle\Psi(t)|] \Rightarrow S_A(t).$$

This is computed by the holographic formula:

$$\boxed{S_A(t) = \mathrm{Min}_{\Gamma_A}\mathrm{Ext}_{\Gamma_A}\left[\frac{A(\Gamma_A)}{4G_N}\right]}$$

$$\partial A = \partial \gamma_A \text{ and } A \sim \gamma_A \ .$$

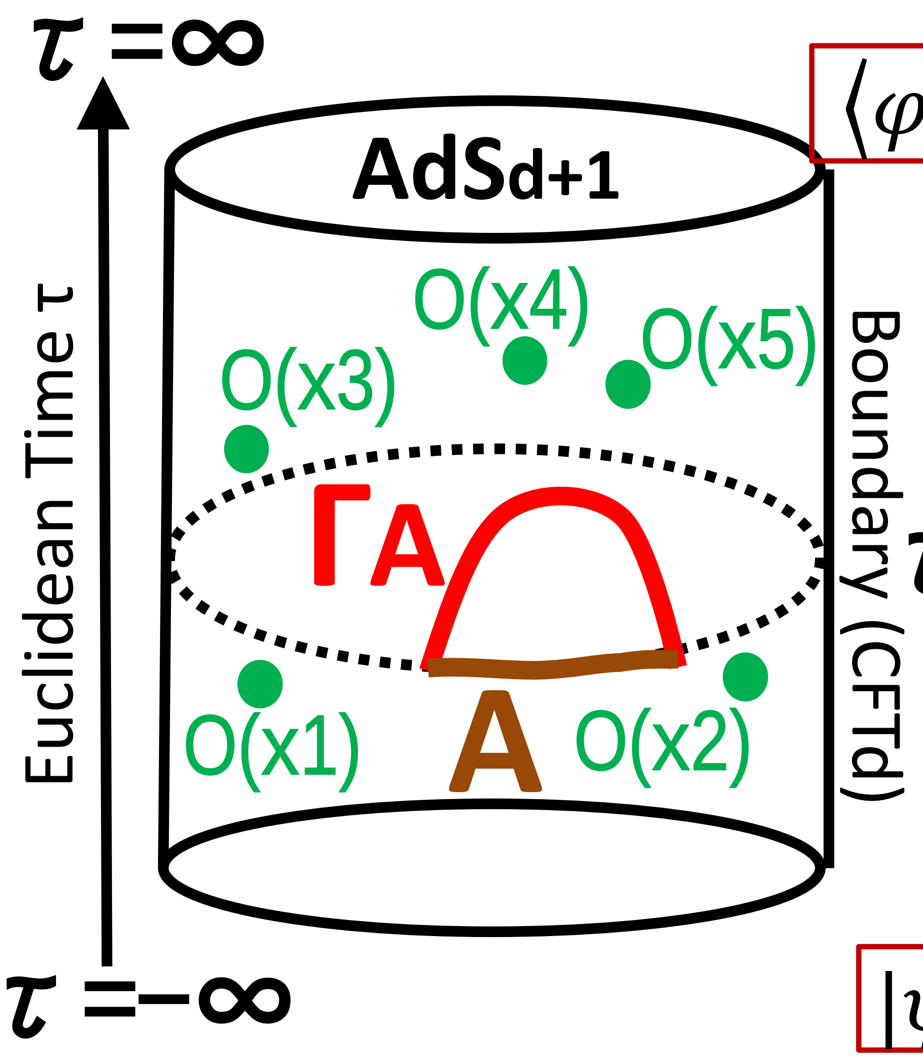


Question for Ver 3.

Minimal areas in ***Euclidean time dependent*** asymptotically AdS spaces

= What kind of QI quantity (Entropy ?) in CFT ?

Guiding Principle: Euclidean Path-integral on boundary = Bulk gravity partition function



$$\langle\varphi| = \langle 0|O(x_3)O(x_4)O(x_5)$$

Final State $\langle\varphi|$

$\tau = 0$

$|\Psi\rangle$ Initial State

$$|\psi\rangle = O(x_1)O(x_2)|0\rangle$$

New Feature: two different states are involved !

Our Answer: The minimal calculates the following quantity, which we call ***pseudo (entanglement) entropy***:

$$\boxed{S\left(\tau_A^{\psi|\varphi}\right) = -\mathrm{Tr}\left[\tau_A^{\psi|\varphi}\log\tau_A^{\psi|\varphi}\right].}$$

Here , the reduced transition matrix $\tau_A^{\psi|\varphi}$ is defined as

$$\tau_A^{\psi|\varphi} = \mathrm{Tr}_B\left[\tau^{\psi|\varphi}\right],$$

from the transition matrix:

$$\tau^{\psi|\varphi} = \frac{|\psi\rangle\langle\varphi|}{\langle\varphi|\psi\rangle}.$$

Note that in general this transition matrix is not Hermitian.

[cf. "conditional entropy of post-selected states" Salek-Schbert-Wiesner 2013]

Contents

② Basics of Pseudo (Renyi) Entropy

(2-1) Definition of Pseudo (Renyi) Entropy

Consider two quantum states $|\psi\rangle$ and $|\varphi\rangle$, and define the transition matrix: $\tau^{\psi|\varphi} = \dfrac{|\psi\rangle\langle\varphi|}{\langle\varphi|\psi\rangle}.$

We decompose the Hilbert space as $H_{tot} = H_A \otimes H_B$ and introduce the reduced transition matrix:

$$\tau_A^{\psi|\varphi} = \mathrm{Tr}_B\left[\tau^{\psi|\varphi}\right]$$

The pseudo n-th Renyi entropy is defined by

$$\boxed{S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = \frac{1}{1-n}\log\mathrm{Tr}\left[\left(\tau_A^{\psi|\varphi}\right)^n\right].}$$

The n=1 limit defines the (von-Neumann) pseudo entropy:

$$\boxed{S\left(\tau_A^{\psi|\varphi}\right)=S^{(n=1)}\left(\tau_A^{\psi|\varphi}\right)}$$

Note 1: Since $\tau_A^{\psi|\varphi}$ is not a quantum state i.e. Hermitian and positive semi-definite, the pseudo (Renyi) entropy is complex valued in general.

Note 2: When A=total system (B=empty), we obtain

$$\left(\tau^{\psi|\varphi}\right)^n = \tau^{\psi|\varphi} \quad \Rightarrow \quad \mathrm{Tr}\left[\left(\tau^{\psi|\varphi}\right)^n\right]=1.$$

Thus, $S^{(n)}\left(\tau^{\psi|\varphi}\right) = 0.$

(2-2) Basic Properties of Pseudo Entropy

[1] If either $|\psi\rangle$ or $|\varphi\rangle$ has no entanglement (i.e. direct product state) , then

$$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = 0.$$

→Connection to quantum entanglement !

[2] $$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = \left[S^{(n)}\left(\tau_A^{\varphi|\psi}\right)\right]^* .$$

[3] $$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = S^{(n)}\left(\tau_B^{\psi|\varphi}\right).$$

→"SA=SB"

[4] If $|\psi\rangle=|\varphi\rangle$, then $S^{(n)}\left(\tau_A^{\psi|\varphi}\right)$ = Renyi entropy.

Comment: In quantum theory, transition matrices arise when we consider *post-selection*.

$$\boxed{\frac{\langle\varphi|O_A|\psi\rangle}{\langle\varphi|\psi\rangle} = \mathrm{Tr}[O_A \tau_A^{\psi|\varphi}]}$$

Final state after post-selection

Initial State

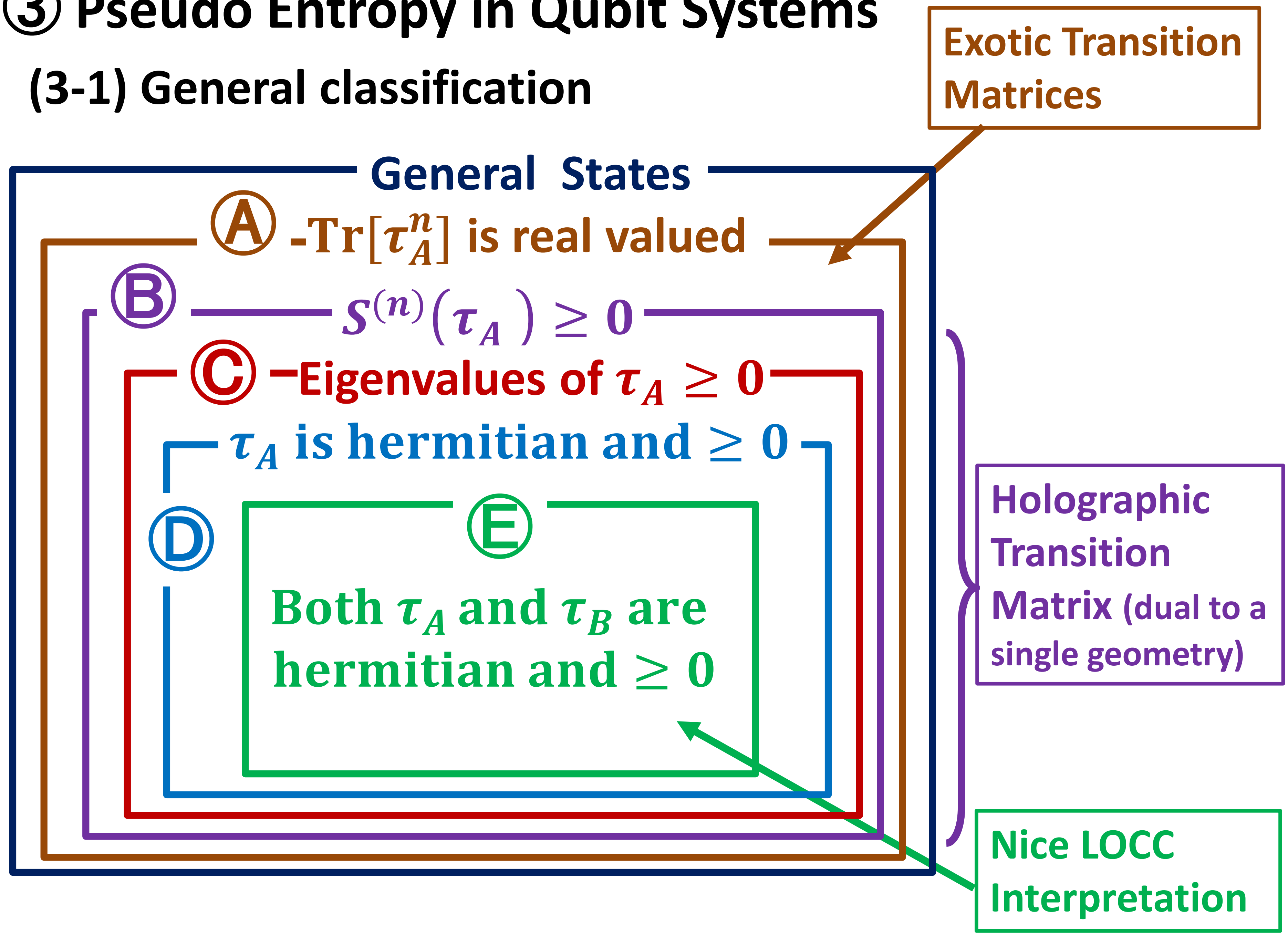
This quantity is called **weak value** and is complex valued in general. [Aharanov-Albert-Vaidman 1988,…]

Thus, "**hol. pseudo entropy =weak value of area operator**":

$$\boxed{S\left(\tau_A^{\psi|\varphi}\right) = \frac{\langle\varphi|\boldsymbol{A}|\psi\rangle}{\langle\varphi|\psi\rangle}},$$

③ Pseudo Entropy in Qubit Systems

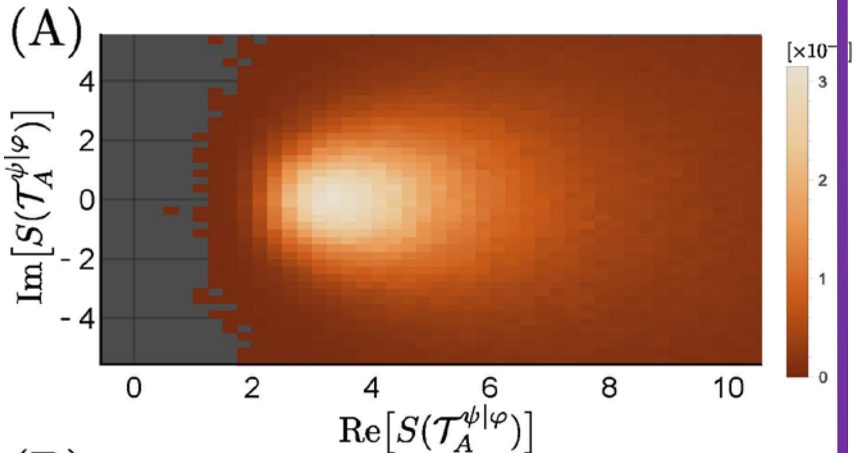
(3-1) General classification



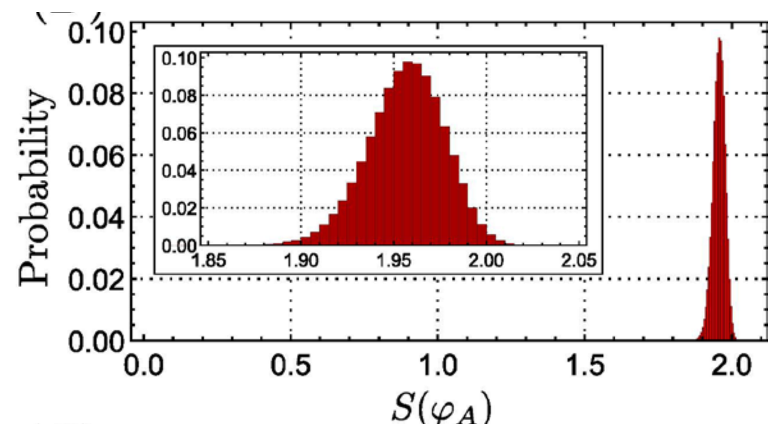
Values of Pseudo Entropy for Random States

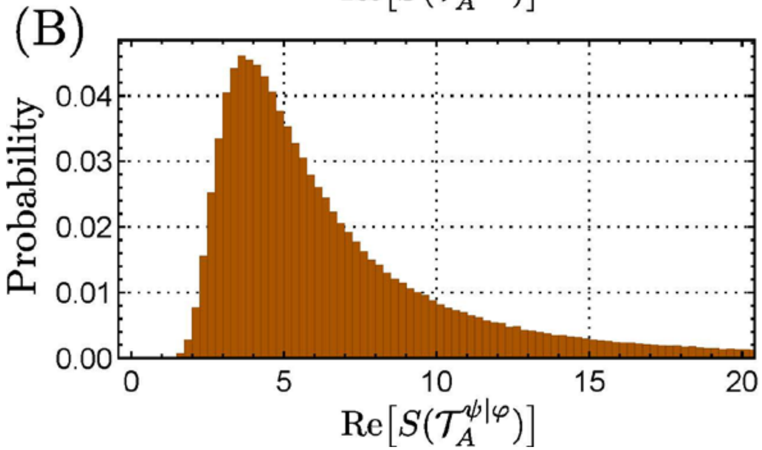
dim A=8,
dim B=32,
655360 samples

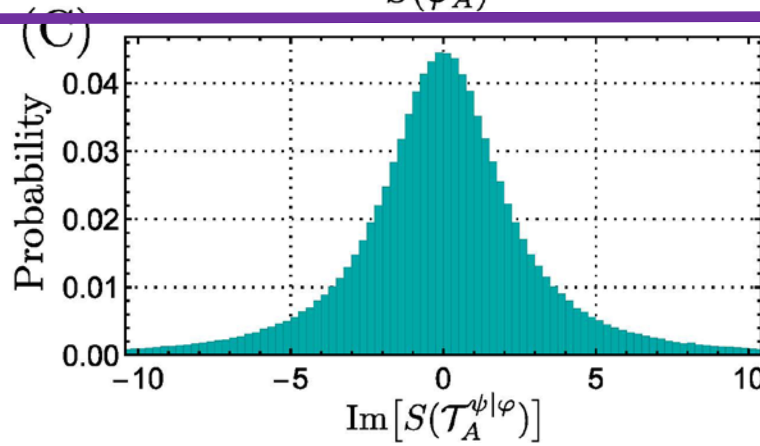
Pseudo Entropy



Standard Entanglement Entropy







Pseudo Entropy

Pseudo Entropy

(3-2) Pseudo Entropy as Entanglement Distillation

Let us focus on the class E i.e. $\tau_A^{\psi|\varphi}$ and $\tau_B^{\psi|\varphi}$ are Hermitian and semi-positive definite.

Remarkably, in this case we can show a quantum information theoretical interpretation of pseudo entropy:

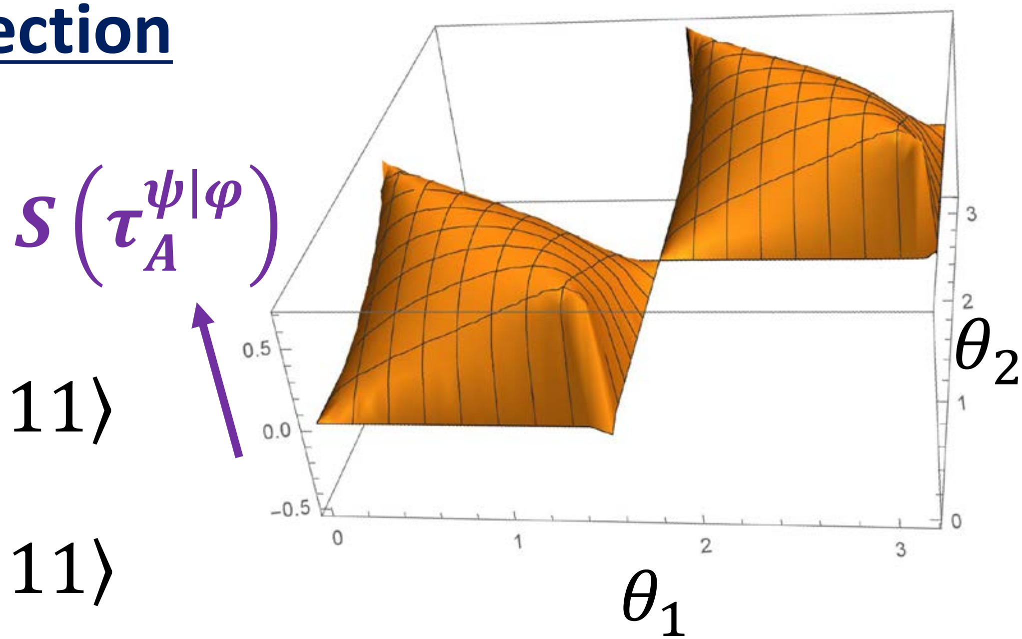
Claim **Pseudo Entropy** $S\left(\tau_A^{\psi|\varphi}\right)$

= # of Distillable Bell Pairs
as an intermediate states
of post-selection $|\psi\rangle \rightarrow |\varphi\rangle$.

More precisely, we take asymptotic limit M→∞.

Distillation from Post-selection

In class E, we can write



$$|\psi\rangle = \cos\theta_1|00\rangle + \sin\theta_1|11\rangle$$

$$|\varphi\rangle = \cos\theta_2|00\rangle + \sin\theta_2|11\rangle$$

$$\tau_A^{\psi|\varphi} = \frac{\cos\theta_1\cos\theta_2|0\rangle\langle 0| + \sin\theta_1\sin\theta_2|1\rangle\langle 1|}{\cos(\theta_1 - \theta_2)}$$

$$S\left(\tau_A^{\psi|\varphi}\right) =$$

$$-\frac{\cos\theta_1\cos\theta_2}{\cos(\theta_1-\theta_2)}\cdot\log\frac{\cos\theta_1\cos\theta_2}{\cos(\theta_1-\theta_2)} - \frac{\sin\theta_1\sin\theta_2}{\sin(\theta_1-\theta_2)}\cdot\log\frac{\sin\theta_1\sin\theta_2}{\sin(\theta_1-\theta_2)}$$

$$(|\psi\rangle)^{\otimes M} = (\cos\theta_1|00\rangle + \sin\theta_1|11\rangle)^{\otimes M}$$
$$= \sum_{k=0}^{M} (c_1)^{M-k} (s_1)^k \sum_{a=1}^{{}_M C_k} |P(k), a\rangle_A |P(k), a\rangle_B$$

$$c_1 \equiv \cos\theta_1,\ s_1 \equiv \sin\theta_1$$

$k = 0$: $\quad |P(0), 1\rangle = |00\cdots 0\rangle$

$k = 1$: $\quad |P(1), 1\rangle = |10\cdots 0\rangle,\ |P(1), 2\rangle = |01\cdots 0\rangle,\ \cdots$

Projection to maximally entangled states with **Log[MCk]** entropy:

MCk=M!/(M-k)!k!

$$\Pi_k = \sum_{a=1}^{{}_M C_k} |P(k), a\rangle_A \langle P(k), a|$$

probability: $p_k = \langle\varphi|\Pi_k|\psi\rangle / \langle\varphi|\psi\rangle = \dfrac{(c_1 c_2)^{M-k} (s_1 s_2)^k}{(c_1 c_2 + s_1 s_2)^M} \cdot {}_M C_k$

# of Distillable Bell pairs: $\mathbf{N} = \sum_{k=0}^{M} p_k \cdot \mathrm{Log}[{}_M C_k]$

$$\approx M \cdot S\left(\tau_A^{\psi|\varphi}\right)\ !$$

(3-3) Behaviors under Unitary Transformations

Monotonicity in 2 Qubit systems

We can prove the following monotonicity under unitary transformation:

Claim Consider two states related by local unitary trf.

$$|\psi\rangle = (U_A \otimes V_B)|\varphi\rangle.$$

If $\tau_A^{\psi|\varphi}$ has non-negative eigenvalues (i.e. class B=C), then

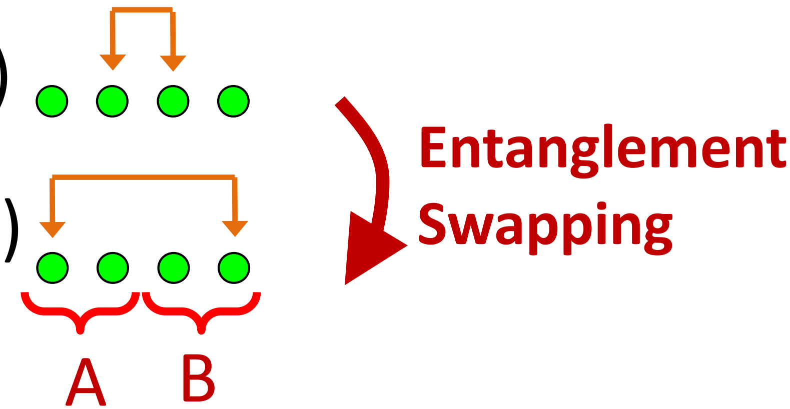
$$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) \geq S^{(n)}(\mathrm{Tr}_B[|\psi\rangle\langle\psi|]) = S^{(n)}(\mathrm{Tr}_B[|\varphi\rangle\langle\varphi|]).$$

Decreasing Pseudo Entropy in Multiple Qubits

Ex1

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |0110\rangle)$$

$$|\varphi\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1001\rangle)$$



$$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = 0$$

$$S^{(n)}(\mathrm{Tr}_B[|\psi\rangle\langle\psi|]) = S^{(n)}(\mathrm{Tr}_B[|\varphi\rangle\langle\varphi|]) = \log 2.$$

Ex2 Thermofield States in CFTs

$$|\psi_i\rangle = \frac{1}{\sqrt{Z(\beta_1)}}\sum_n e^{-\frac{\beta_i}{2}E_n}|n\rangle|n\rangle \qquad (i = 1{,}2)$$

$$S_{th}\left(\frac{\beta_1+\beta_2}{2}\right) \leq \frac{1}{2}[S_{th}(\beta_1) + S_{th}(\beta_2)]$$

$$\Rightarrow S\left(\tau_A^{\psi_1|\psi_2}\right) \leq \frac{1}{2}[S(\mathrm{Tr}_B[|\psi_1\rangle\langle\psi_1|]) + S(\mathrm{Tr}_B[|\psi_2\rangle\langle\psi_2|])].$$

④ Holographic Pseudo Entropy

(4-1) Holographic Pseudo Entropy and General Properties

Holographic Pseudo Entropy (HPE) Formula

$$\boxed{S\left(\tau_A^{\psi|\varphi}\right) = \mathrm{Min}_{\Gamma_A}\left[\frac{A(\Gamma_A)}{4G_N}\right] \geq 0}$$

Basic Properties of HPE

(i) If ρ_A *is pure*, $S\left(\tau_A^{\psi|\varphi}\right) = 0.$

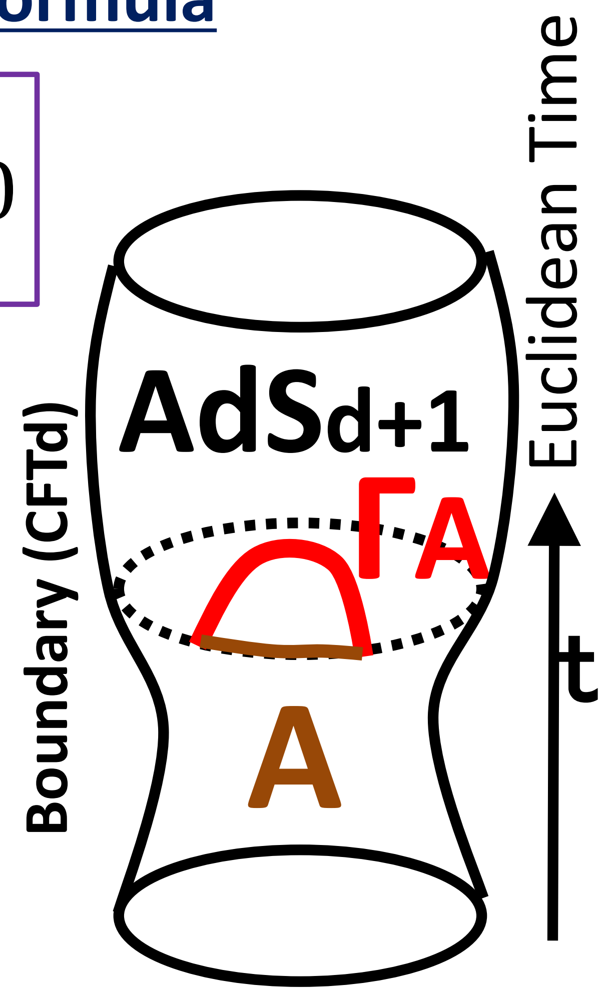
(ii) If ψ *or* φ *is not entangled*,

$S\left(\tau_A^{\psi|\varphi}\right) = 0.$

→This follows from AdS/BCFT [TT 2011]

(iii) $S\left(\tau_A^{\psi|\varphi}\right) = S\left(\tau_B^{\psi|\varphi}\right)$. **"SA=SB"**

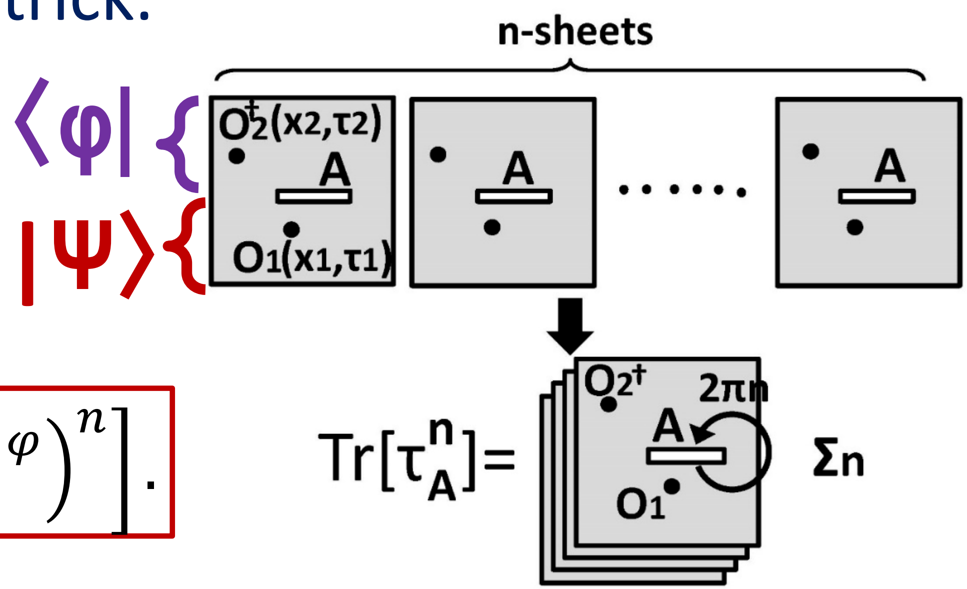
(iv) $S\left(\tau_A^{\psi|\varphi}\right) + S\left(\tau_B^{\psi|\varphi}\right) \geq S\left(\tau_{AB}^{\psi|\varphi}\right)$. **"Subadditivity"**



- However, it is not clear whether the strong subadditivity holds for the holographic pseudo entropy.

  (cf. Proof of SSA in Lorentzian AdS [Wall 2012])

- We can derive the holographic pseudo entropy formula as in [Lewkowycz-Maldacena 13] .
  This is because we can calculate the pseudo entropy via the standard replica trick.



$$\boxed{S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = \frac{1}{1-n}\log\mathrm{Tr}\left[\left(\tau_A^{\psi|\varphi}\right)^n\right].}$$

(4-2) Simple Example: Janus AdS3/CFT2

Consider a gravity dual of 2d CFT perturbed by the exactly marginal operator O(x) such that

$$\tau < 0: \quad \phi_- \int d^2x\, O(x), \qquad \tau > 0: \quad \phi_+ \int d^2x\, O(x).$$

$\Rightarrow |\psi\rangle$ $\qquad$ $\Rightarrow |\varphi\rangle$

AdS3 gravity coupled to a massless scalar

$$I = \frac{1}{16\pi G_N}\int d^3x\sqrt{g}(R - \partial_a\phi\partial^a\phi + 2),$$

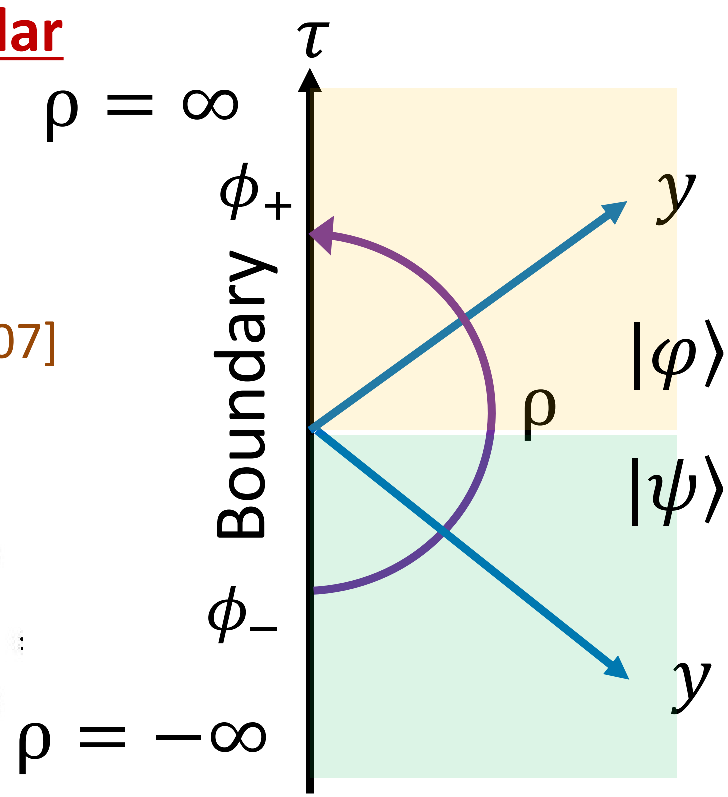
Janus Solution

[Bak-Gutperle-Hirano 2007]

$$ds^2 = d\rho^2 + f(\rho)\frac{dx^2 + dy^2}{y^2},$$

$$\phi = \phi_0 + \frac{1}{\sqrt{2}}\log\left[\frac{1+\sqrt{1-2\gamma^2}+\sqrt{2}\gamma\tanh\rho}{1+\sqrt{1-2\gamma^2}-\sqrt{2}\tanh\rho}\right],$$

$$f(\rho) = \frac{1}{2}\left(1+\sqrt{1-2\gamma^2}\cosh 2\rho\right).$$



Holographic Pseudo Entropy

We take A=[0,l]. The minimal surface is given by

$$\tau = 0, \qquad x^2 + y^2 = l^2.$$

Thus the holographic pseud entropy reads

$$S\left(\tau_A^{\psi_1|\psi_2}\right) = \frac{c}{3} \cdot \sqrt{\frac{1+\sqrt{1-2\gamma^2}}{2}} \cdot \log\frac{l}{\delta}.$$

$$\left(g_{xx} = \epsilon^{-2} \to \delta = \sqrt{\frac{1+\sqrt{1-2\gamma^2}}{2}} \cdot \epsilon \right)$$

When γ is small, we obtain

$$S\left(\tau_A^{\psi_1|\psi_2}\right) \approx \frac{c}{3} \cdot \left(1 - \frac{\gamma^2}{4}\right) \cdot \log\frac{l}{\epsilon} + \frac{c}{12}\gamma^2.$$

Agreeing with the CFT perturbation (universal in any 2d CFTs):

$$\Delta S_A^{(n)} = \frac{J^2}{1-n}\left[\int dx^2 \int dy^2 \langle O(x)O(y)\rangle_{\Sigma_n} - n\int dx^2 \int dy^2 \langle O(x)O(y)\rangle_{\Sigma_1}\right].$$

⑤ Mixed State Generalization

Can we define pseudo entropy when the total system AB is mixed ?

Assume $\tau_{AB}^{\psi|\varphi}$ is given.

One possibility is to extend the reflected entropy. [Dutta-Faulkner 2019, Kusuki-Tamaoka 2019,..]

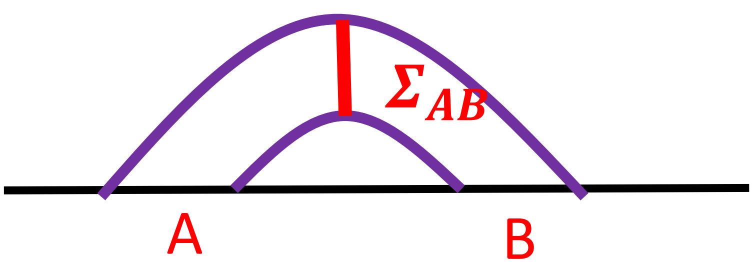
$$X_{AB} = \sum_{i,j} X_{ij} |i\rangle \langle j|$$

$$\Rightarrow |X\rangle_{AB\tilde{A}\tilde{B}} = \sum_{i.j} X_{ij} |i\rangle_{AB\tilde{A}\tilde{B}} |j^*\rangle_{AB\tilde{A}\tilde{B}}$$ **(canonical purification)**

Define pseudo reflected entropy: $S_R(\tau_{AB}^{\psi|\varphi}) = S\left(\mathrm{Tr}_{B\tilde{B}}\left[\frac{|(\tau_{AB}^{\psi|\varphi})^{1/2}\rangle\langle(\tau_{AB}^{\psi|\varphi})^{\dagger 1/2}|}{\langle(\tau_{AB}^{\psi|\varphi})^{\dagger 1/2}|(\tau_{AB}^{\psi|\varphi})^{1/2}\rangle}\right]\right).$

This coincides with the twice of entanglement wedge cross section:

$$\boldsymbol{S_R(\tau_{AB}^{\psi|\varphi}) = 2 \cdot \frac{A(\Sigma_{AB})}{4G_N}.}$$



[cf. EW=EoP: Umemoto-TT 2017
EW=Odd entropy: Tamaoka 2018
EW∝Negativity: Kudler-Flam-Ryu 2019]

⑥ Conclusions

Pseudo Entropy = Area of Minimal surface in Euclidean asymptotically AdS with time-dependence

$$S\left(\tau_A^{\psi|\varphi}\right) = \frac{\langle\varphi|A|\psi\rangle}{\langle\varphi|\psi\rangle}$$

- Pseudo Entropy is in general complex valued.
  ⇒ Why is holographic pseudo entropy non-negative ?
  [Euclidean path-integrals are positive valued !]

- Pseudo Entropy for `non-exotic states' measures the amount of quantum entanglement in the intermediate states.
  [How about general states ? What is the meaning of complex values?]

- Pseudo often tends to decrease when excited.
  ⇒ Any systematic understanding ?
  [Mollabashi-Shiba-Tamaoka-Wei-TT, work in progress]

Thank you very much !

Example: Pseudo Entropy for Locally Excited States

(a) Pseudo Renyi Entropy in Free CFT

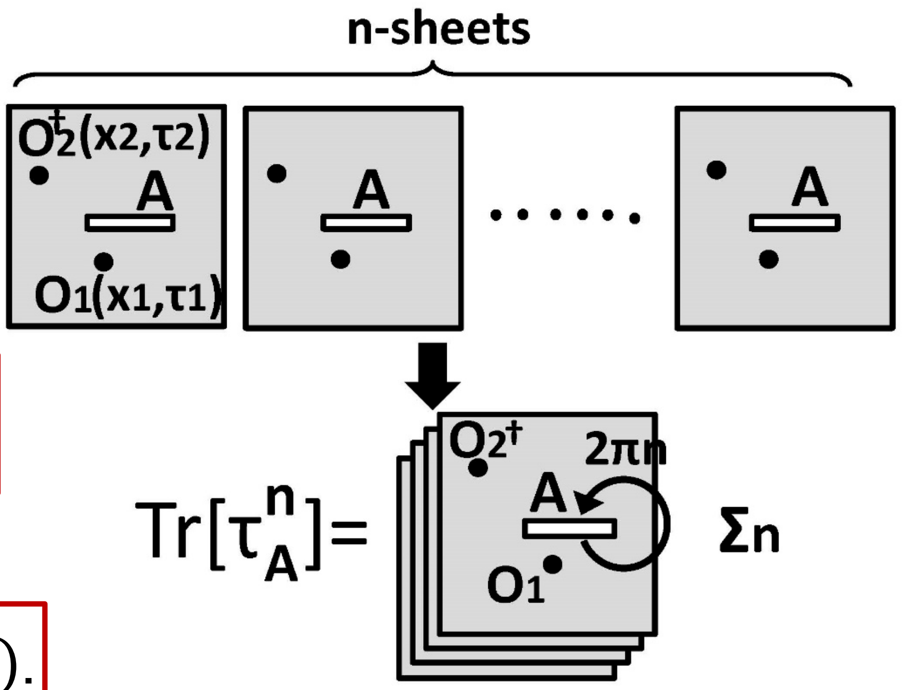
Consider the following 2 locally excited states:

$$|\psi\rangle = N_1 \cdot O(x_1, \tau_1)|0\rangle, \qquad |\varphi\rangle = N_2 \cdot O(x_2, \tau_2)|0\rangle$$

We would like to calculate their pseudo Renyi entropy.

We take the replica of complex plane $w = x + i\tau$.

$\Rightarrow \Sigma_n$



$$\boxed{S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = \frac{1}{1-n}\log\mathrm{Tr}\left[\left(\tau_A^{\psi|\varphi}\right)^n\right].}$$

We compute the difference:

$$\boxed{\Delta S_A^{(n)} = S^{(n)}\left(\tau_A^{\psi|\varphi}\right) - S^{(n)}(\mathrm{Tr}_B|0\rangle\langle 0|).}$$

For simplicity, we will focus on 2nd Renyi pseudo entropy in the 2d massless free scalar CFT.

[EE in the same setup: Nozaki-Numasawa-TT, He-Numasawa-Watanabe-TT 2014]

$$\Delta S_A^{(2)} = -\log \frac{\langle O(w_1)O^\dagger(w_2)O(w_3)O^\dagger(w_4)\rangle_{\boldsymbol{\Sigma 2}}}{\langle O(w_1)O^\dagger(w_2)\rangle_{\boldsymbol{\Sigma 1}}^2}$$

We choose $O(x,\tau) = e^{i\phi(x,\tau)/2} + e^{-i\phi(x,\tau)/2}$ (~Bell pair).

After we perform the conformal map

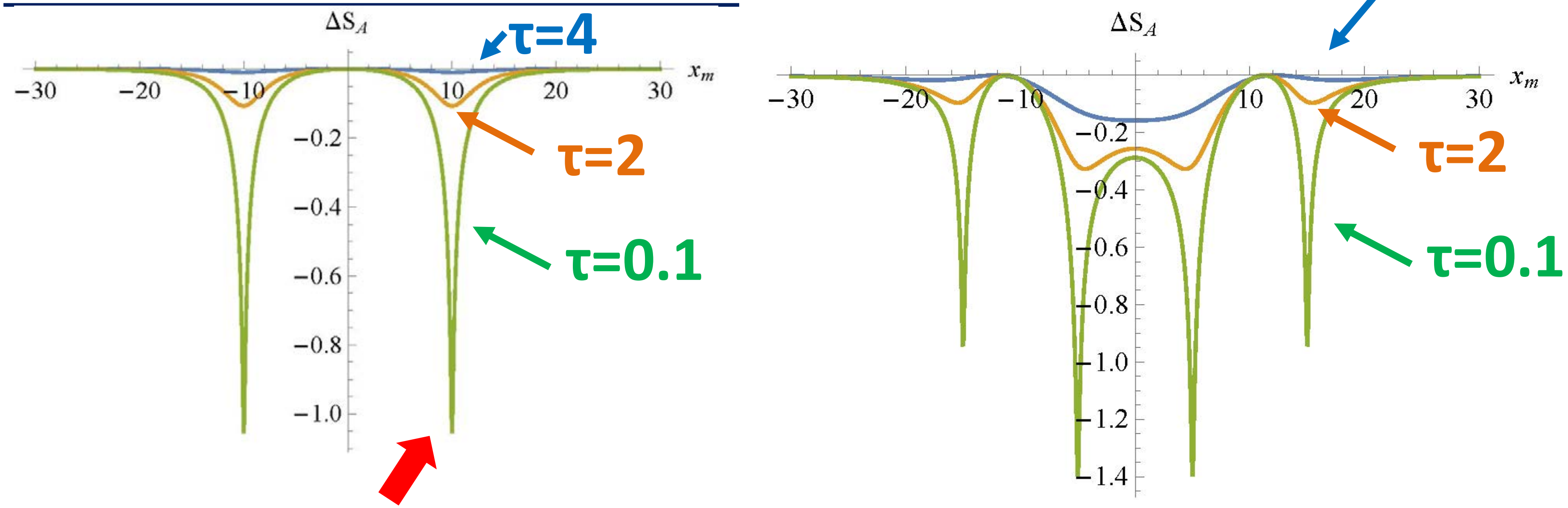
$$\frac{w-a}{w-b} = z^2$$

we obtain $\Delta S_A^{(2)} = \log\left(\frac{2}{1+|\eta|+|1-\eta|}\right) \leq 0.$

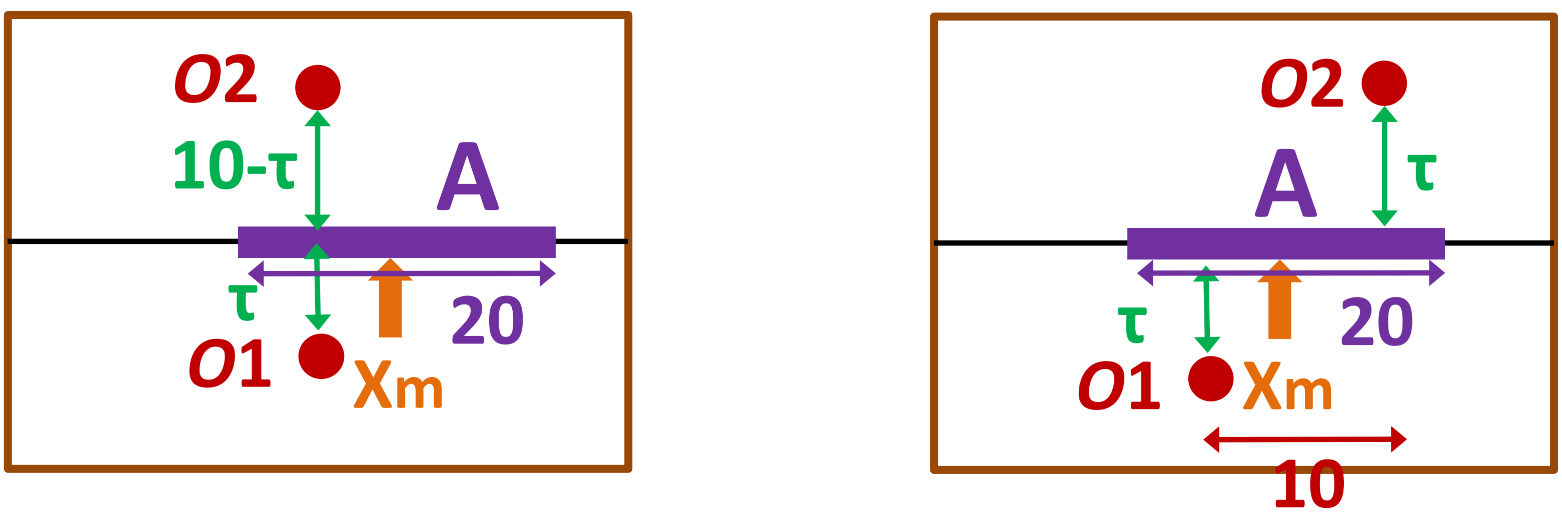
Cross ratio:

$$\eta = \frac{(z_1-z_2)(z_3-z_4)}{(z_1-z_3)(z_2-z_4)}.$$

Plots of Free CFT Results

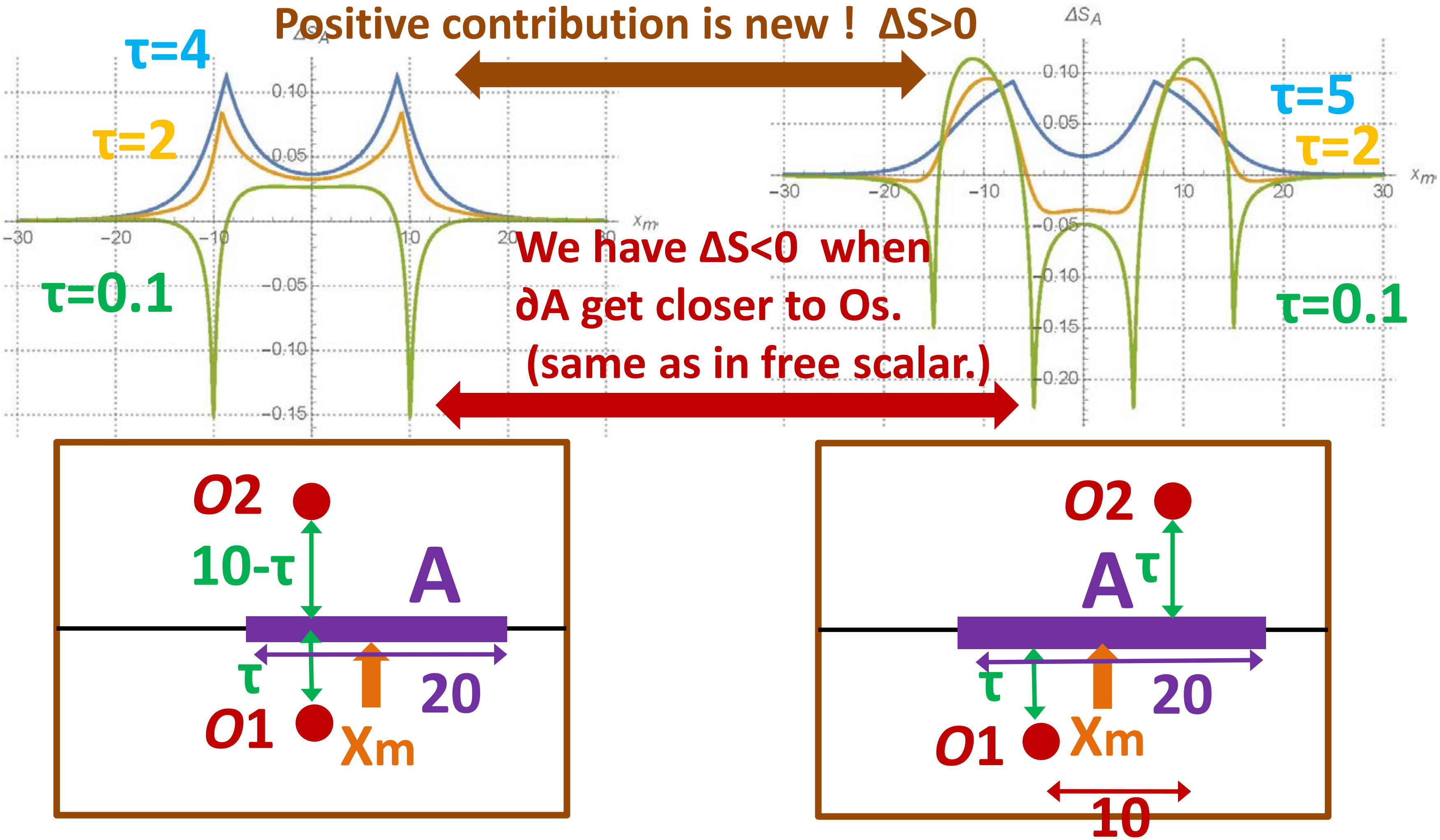


Pseudo Entropy is reduced when ∂A gets closer to the excitation.
⇒ This is due to the decreasing property under entanglement swap.



(b) Holographic CFT Calculations (ho/c=1/32)

We can calculate the pseudo entropy for the same setup in holographic CFTs:



An Example of Exotic Transition Matrix

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + e^{i\theta}|11\rangle)$$

$$|\varphi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\tau_A^{\psi|\varphi} = \frac{1}{1+e^{i\theta}}\left(|0\rangle\langle 0| + e^{i\theta}|1\rangle\langle 1|\right).$$

→ Complex conjugate pair of Eigenvalues

$$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = \frac{1}{1-n}\log\left[\frac{\cos\frac{n\theta}{2}}{2^{n-1}\cos^n\frac{\theta}{2}}\right]$$

→Only special values of θ can give positive values pseudo entropy.