Asymmetric Symmetry Breaking: Unequal Probabilities of Vacuum Selection

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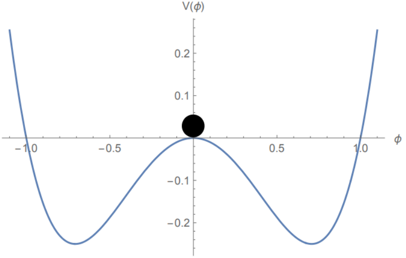
Outline

Spontaneous symmetry breaking (SSB) is an important concept in contemporary physics.
A typical picture of SSB is a field to roll down a potential with Mexican-hat profile.



$$V(\phi) = -a\phi^2 + b\phi^4, \quad (a, b > 0)$$

Figure 1: Schematic picture to illustrate the rolling down of the fields.

Asymmetric potential

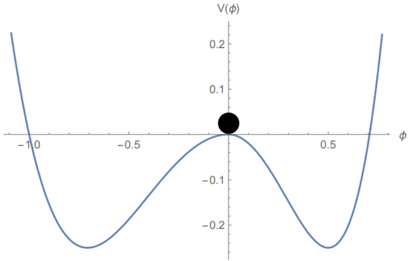


Figure 2: Schematic picture to illustrate the rolling down of the fields.

If the top of the potential is not smooth, such as 1st derivative is continuous, but 2nd derivative is discontinuous.

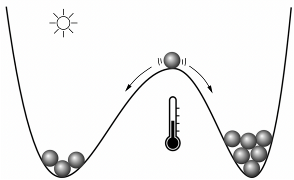


Figure 3: Schematic picture to illustrate the rolling down of the fields. The system is subject to thermal perturbations, and the top of the potential is only C^1 continuous. Fields tend to roll down to the steeper side.

Main result:

$$P_+ = \frac{\sqrt{V''_+/V''_-}}{1+\sqrt{V''_+/V''_-}}.$$

$$P_- = \frac{\sqrt{V''_-/V''_+}}{1+\sqrt{V''_-/V''_+}}.$$

$$\frac{P_+}{P_-} = \sqrt{\frac{V''_+}{V''_-}}.$$

We consider the time-dependent Ginzburg-Landau equation with an external random force $F(t)$,

$$\frac{d\phi}{dt} + \Gamma \frac{\partial V(\phi)}{\partial \phi} = F(t) \tag{1}$$

ϕ is a real scalar field; Γ is a coefficient related to the dissipations. $F(t)$ satisfies the Gaussian white noise relation

$$\langle F(t) \rangle = 0, \langle F(t)F(t') \rangle = Q\delta(t - t'). \tag{2}$$

Q is the fluctuation strength. Due to "fluctuation-dissipation theorem"

$$Q = 2k_B \Gamma T. \tag{3}$$

From stochastic process, the probability to find the field in the range $[\phi, \phi + d\phi]$ at time t is $P(\phi, t)$, which satisfies the Fokker-Planck equation,

$$\frac{\partial P}{\partial t} = \Gamma \frac{\partial}{\partial \phi}\left(\frac{\partial V}{\partial \phi} P\right) + \frac{Q}{2}\frac{\partial^2 P}{\partial \phi^2}. \tag{4}$$

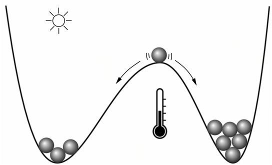


Figure 4: Schematic picture to illustrate the rolling down of the fields.

To derive our result:

1. Expand the potential near the top $\phi = 0$,

$$V(\phi) \approx V_0 + V_0' \phi + \frac{1}{2} V_0'' \phi^2 + \ldots . \tag{5}$$

2. Since the top is a maximum, we get $V_0' = 0$. Hence,

$$\partial V / \partial \phi \approx V_0'' \phi + \cdots . \tag{6}$$

3. Ignore higher terms of ϕ in the expansion of $\partial V / \partial \phi$, we get the reduced Fokker-Planck equation

$$\frac{\partial P}{\partial t} = \Gamma V_0'' \frac{\partial}{\partial \phi} (\phi P) + \frac{Q}{2} \frac{\partial^2 P}{\partial \phi^2} \tag{7}$$

4. Imagine that we do experiments many enough times. Therefore, the probability to roll down to the left/right reach a stable value $\partial P / \partial t = 0$

$$P_{\text{st.}}(\phi) = \sqrt{\frac{\Gamma |V_0''|}{\pi Q}} e^{-\frac{\Gamma V_0'' \phi^2}{Q}} . \tag{8}$$

5. ϕ fluctuates from $\phi = 0$ to a small value $\phi = \epsilon$, fields will permanently roll down to the left/right. Hence, ignore the small term in the exponential

$$P_{\mathrm{st.}}(\phi) = \sqrt{\frac{\Gamma |V_0''|}{\pi Q}}. \tag{9}$$

6. V_0'' are different in left/right sides. Define $V_\pm'' = \partial^2 V / \partial \phi^2$ at $\phi = 0_\pm$

$$\frac{P_+}{P_-} = \frac{\sqrt{\Gamma |V_+''|/(\pi Q)}}{\sqrt{\Gamma |V_-''|/(\pi Q)}} = \sqrt{\frac{V_+''}{V_-''}}. \tag{10}$$

7. Since $P_+ + P_- = 1$, we can get

$$P_+ = \frac{\sqrt{V_+''/V_-''}}{1 + \sqrt{V_+''/V_-''}}. \tag{11}$$

Or, equivalently, $P_- = \frac{\sqrt{V_-''/V_+''}}{1+\sqrt{V_-''/V_+''}}$.

Higgs-type potential with discontinuous 2nd derivative at the top,

$$V(\phi) = \theta(-\phi)(-\alpha_- \phi^2 + \phi^4) + \theta(\phi)(-\alpha_+ \phi^2 + \phi^4), \tag{12}$$

θ is the step function

$$\theta(x) = \begin{cases} 1, & x > 0 \\ \frac{1}{2}, & x = 0 \\ 0, & x < 0. \end{cases} \tag{13}$$

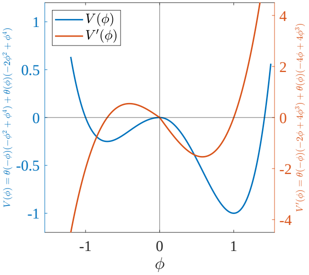


Figure 5: The Higgs-type potential $V(\phi)$ in Eq.(12) with $\alpha_- = 1$ and $\alpha_+ = 2$.

Evolution of $\phi(t)$

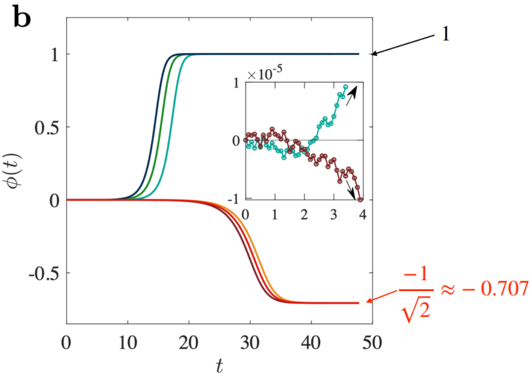


Figure 6: Time evolutions of the scalar fields indicating the rolling down from the top to the left and right minimum. The inset plot shows the very early fluctuations around the top due to the temperature fluctuations.

30000 times independent simulations

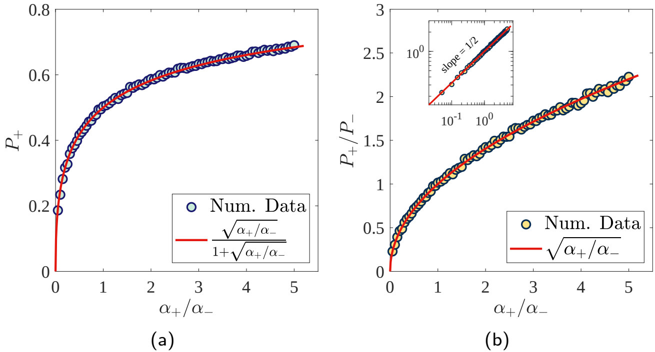


Figure 7: **a**. Relation between the probability P_+ to the ratio α_+/α_-. ; **b**. Relations between the ratio P_+/P_- to the ratio α_+/α_-.

$$\frac{\alpha_+}{\alpha_-} = \frac{V''_+}{V''_-}. \quad P_+ = \frac{\sqrt{V''_+/V''_-}}{1+\sqrt{V''_+/V''_-}}. \quad \frac{P_+}{P_-} = \sqrt{\frac{V''_+}{V''_-}}.$$

Possibility to the right side is P_+, to the left is $P_- = 1 - P_+$. Therefore, each time of the rolling down is a Bernoulli trial.

After n times of trials, k times to the right should has the binomial distributions

$$P(n,k) = \binom{n}{k} P_+^k (1 - P_+)^{n-k}. \tag{14}$$

Footnotes

The counting statistics of the rolling down to the right side with the Higgs-type potential

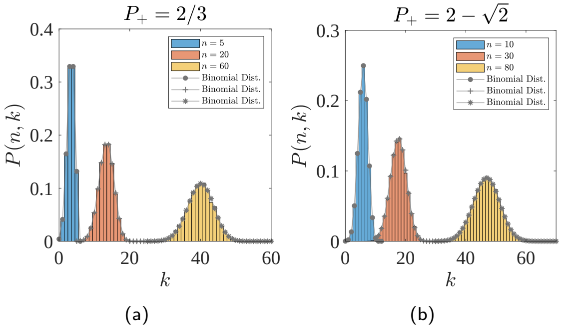


Figure 8: **a.** The histogram of $P(n,k)$ for three different trial numbers n, with the probability $P_+ = 2/3$; **b.** The histogram of $P(n,k)$ with the probability $P_+ = 2-\sqrt{2}$.

$\alpha_- = 1, \alpha_+ = 4, P_+ = \frac{\sqrt{4}}{1+\sqrt{4}} = 2/3.$ $\alpha_- = 1, \alpha_+ = 2, P_+ = \frac{\sqrt{2}}{1+\sqrt{2}} = 2-\sqrt{2}.$

Footnotes

Extremal cases: $P(n, k=0) = (1-P_+)^n$

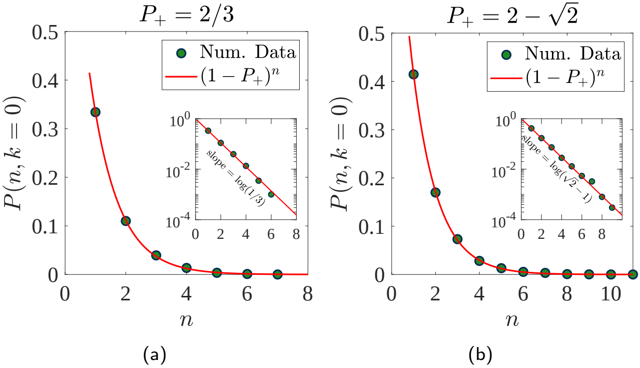
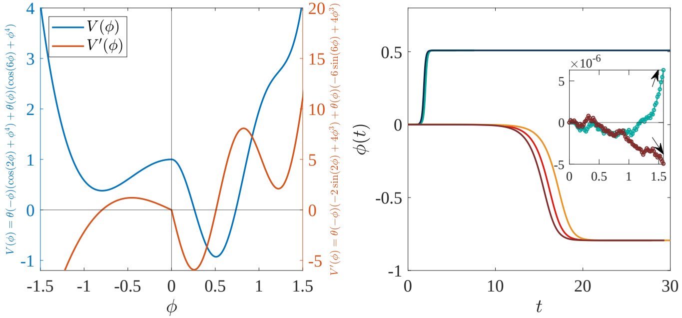


Figure 9: **a.** The vanishing probability of $P(n, k=0)$ in n trials with $P_+ = 2/3$. **b.** The vanishing probability of $P(n, k=0)$ in n trials with $P_+ = 2-\sqrt{2}$.

$\alpha_- = 1, \alpha_+ = 4, P_+ = \frac{\sqrt{4}}{1+\sqrt{4}} = 2/3$. $\alpha_- = 1, \alpha_+ = 2, P_+ = \frac{\sqrt{2}}{1+\sqrt{2}} = 2-\sqrt{2}$.

The cosine-type poetential

$$V(\phi) = \theta(-\phi)(\cos(\beta_-\phi) + \phi^4) + \theta(\phi)(\cos(\beta_+\phi) + \phi^4). \qquad (15)$$



(a) (b)

Figure 10: **a**. The cosine-type potential $V(\phi)$ (blue) in Eq.(15) with $\beta_- = 2$ and $\beta_+ = 6$, and its derivative $V'(\phi)$ (red); **b**. Time time evolution of the scalar fields.

30000 times independent simulations

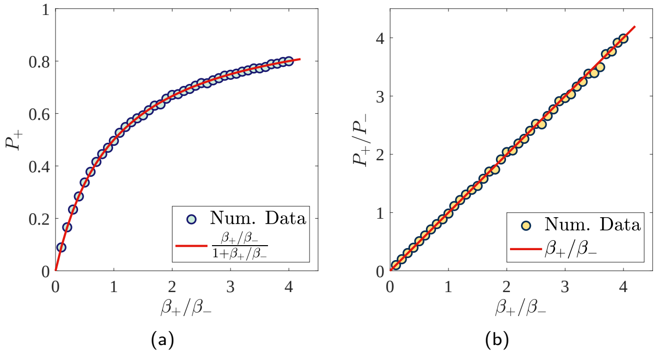


Figure 11: **a**. Relation between the probability P_+ to the ratio β_+/β_-.**b**. Relation between the ratio P_+/P_- to the ratio β_+/β_-.

The counting statistics of the rolling down to the right side with the cosine-type potential

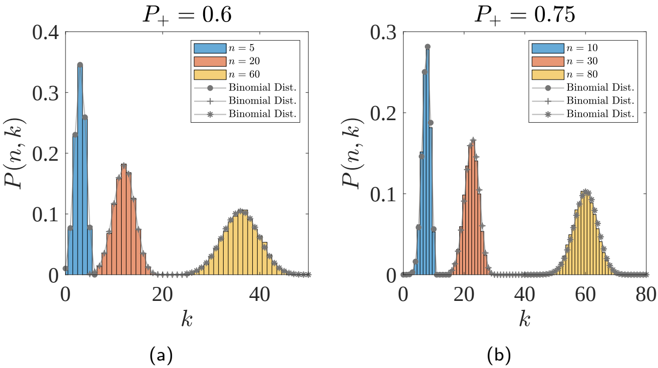


Figure 12: **a.** The histogram of $P(n,k)$ for three different trial numbers n, with the probability $P_+ = 3/5$; **b.** The histogram of $P(n,k)$ for three different trial numbers n, with the probability $P_+ = 3/4$.

$$\beta_- = 2, \beta_+ = 3, P_+ = \frac{3/2}{1+3/2} = 3/5. \quad \beta_- = 2, \beta_+ = 6, P_+ = \frac{6/2}{1+6/2} = 3/4.$$

Extremal cases: $P(n, k=0) = (1 - P_+)^n$

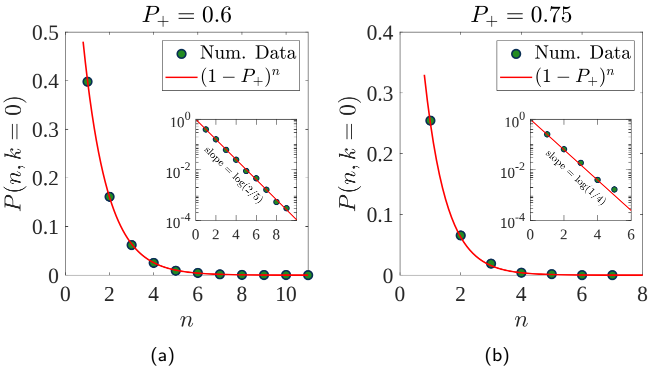


Figure 13: **a.** The vanishing probability of $P(n, k=0)$ in n trials with $P_+ = 3/5$. **b.** The vanishing probability of $P(n, k=0)$ in n trials with $P_+ = 3/4$.

$\beta_- = 2, \beta_+ = 3, P_+ = \frac{3/2}{1+3/2} = 3/5.$ $\quad \beta_- = 2, \beta_+ = 6, P_+ = \frac{6/2}{1+6/2} = 3/4.$

Our results are robust, we already checked that they are robust against:

- varying the reservoir temperature T;
- varying the coefficient Γ;
- adding ϕ^3 terms in the potential;
- changing the coefficients in front of ϕ^4;
- adding higher order terms in the potential, such as ϕ^5 and etc.

Further applications: any theory containing the Higgs potentials can be deformed in this case.

- cosmology;
- elementary particle physics;
- condensed matter physics;

Further problems:

- how to extend it to continuous symmetry, ϕ is complex;
- what about local gauge symmetry;
- the problem of Goldstone bosons;

Thank you very much!