From Rindler Fluid to Dark Fluid on the Holographic Cutoff Surface

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Mainly based on

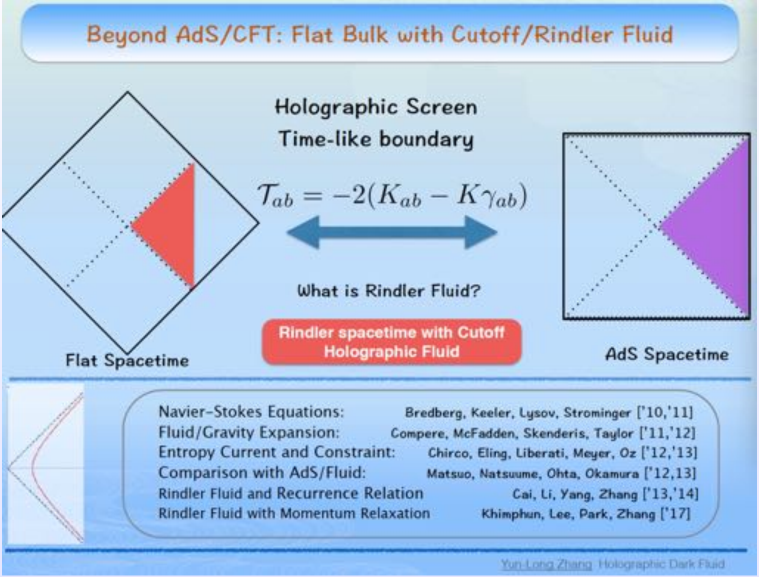
R. G. Cai, S. Khimphun, B. H. Lee, S. Sun, G. Tumurtushaa, Y. L. Zhang, Phys. Dark Univ 26, 100387 (2019)

S. Khimphun, B. H. Lee, C. Park and Y. L. Zhang, JHEP 01(2018)058

R.-G. Cai, L. Li, Q. Yang, and Y.-L. Zhang, JHEP 1304 (2013) 118,

4th International Conference on Holography, String Theory and Discrete Approach
(Aug 4, 2020@Hanoi Email: zhangyunlong001@gmail.com)

Major Motivations of Cutoff Holography



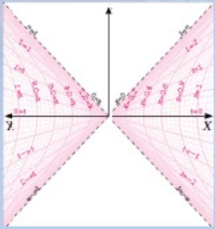
Holographic Screens in Flat Spacetime

— Rindler Screen & de-Sitter Screen

I. Holographic Rindler Fluid

— Accelerating Screen

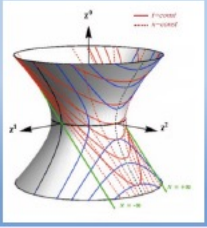
— Relation to AdS/CFT



II. Holographic dS Fluid

— de-Sitter&FRW Screen

— Relation to DGP brane?



(figures from web)

Hydrodynamics from Holography

- **QGP, Neutron star, Superfluid, High-energy astrophysics, ... (p+1)-dimensional relativistic viscous fluid**

$$
\begin{aligned}
T^{ab} =& T^{ab}_{(0)} + T^{ab}_{(1)} + T^{ab}_{(2)} \dots \quad \nabla_a T^{ab} = 0, \\
T^{ab}_{(0)} =& \mathbf{e} u^a u^b + \mathbf{p} h^{ab}, \quad h^{ab} \equiv \gamma^{ab} + u^a u^b, \\
T^{ab}_{(1)} =& -2\eta\,\sigma^{ab} - \zeta\,\theta\,h^{ab}, \qquad T^{ab}_{(2)} = \dots
\end{aligned} \tag{1}
$$

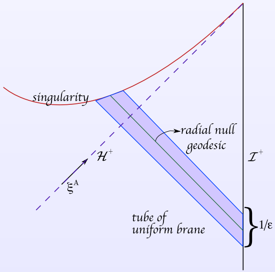
$$
\sigma^{ab} = h^{ac}\,h^{bd}\left(\nabla_{(c}\,u_{d)} - \frac{1}{p}\,\theta\,h_{cd}\right), \quad \theta \equiv \nabla_c\,u^c, \tag{2}
$$

- **Gravity/Fluid correspondence: Gravity $\Leftrightarrow$ a special fluid.**
- **Connection to the AdS/CFT and Cosmological Applications**
- **Is there a recursive relation between different orders?**

AdS/Conformal Fluid

- **Conformal fluid lives on the AdS boundary**
  S. Bhattacharyya, V. EHubeny, S. Minwalla and M. Rangamani, JHEP **0802**, 045 (2008)

$$ds^2 = -2\, u_a\, dx^a dr - r^2 f(b\, r)\, u_a u_b\, dx^a dx^b + r^2\, h_{ab}\, dx^a dx^b$$
$$+ g^{(1)}_{\mu\nu} dx^\mu dx^\nu + g^{(2)}_{\mu\nu} dx^\mu dx^\nu \ldots$$
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0, \qquad \Lambda = -\frac{p(p+1)}{2}, \qquad f(br) = \frac{1}{(br)^{p+1}}$$



M. Rangamani, Class. Quant. Grav. **26**, 224003 (2009)

- **The dual stress tensor**

$$T^{ab} = \lim_{r_c \to \infty} r_c^{p+1} \left(2K\gamma^{ab} - 2K^{ab} + T^{ab}_{ct} \right)$$
$$= (\pi T_H)^{p+1} \left(p u^a u^b + h^{ab} \right) + T^{ab}_{(1)} + T^{ab}_{(2)}$$
$$T^{ab}_{(1)} = -2\eta\sigma^{ab} + \zeta h^{ab},$$
$$\eta = \frac{s}{4\pi} = \left(\frac{4\pi T_H}{p+1} \right)^p, \quad \zeta = 0.$$

Black Holes/Membrane fluid

- **Membrane paradigm**
  Black Holes $\Longleftrightarrow$ lower dimensional fluid

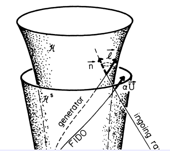
- **The horizon responds like a viscous fluid**

  S. W. Hawking, J. B. Hartle, T. Damour, 1970s

- **Universal relation $\eta/s = 1/4\pi$**

  P. Kovtun, D. T. Son and A. O. Starinets, JHEP 0310, 064 (2003)

(fig from web)



R. H. Price and K. S. Thorne,
Phys. Rev. D **33**, 915 (1986)

Rindler/Fluid

- **Ingoing Rindler metric**

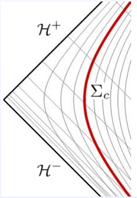
$$G_{\mu\nu} = 0, \quad ds^2 = -r\mathrm{d}\tau^2 + 2\mathrm{d}\tau\mathrm{d}r + \mathrm{d}x_i\mathrm{d}x^i$$

I. Bredberg, C. Keeler, V. Lysov and A. Strominger, JHEP 1207, 146 (2012)

- **Derivative expansion** $G_{\mu\nu} = O(\partial^3)$**,**

$$\mathrm{d}s^2 = -2\mathbf{p}u_a\mathrm{d}x^a\mathrm{d}r + g_{ab}\mathrm{d}x^a\mathrm{d}x^b + O(\partial^3)$$

$$g_{ab} = \left[-\mathbf{p}^2(r-r_c)u_a u_b + \gamma_{ab}\right] + g^{(1)}_{ab} + g^{(2)}_{ab}$$



G. Compere, P. McFadden, K. Skenderis, M. Taylor, JHEP **1203**, 076 (2012)

- **The dual stress tensor**

$$T_{ab} = 2(K\gamma_{ab} - K_{ab})$$
$$= 0 + \mathbf{p}P_{ab} + T^{(1)}_{ab} + T^{(2)}_{ab} + \ldots$$

Rindler Hydrodynamics

- **The Hamiltonian constraint equation**

$$2G_{\mu\nu}n^{\mu}n^{\nu}|_{\Sigma_c} = (K^2 - K_{ab}K^{ab}) = 0 \Longrightarrow T^2 - p\,T_{ab}T^{ab} = 0, \quad (3)$$

the equation of state relating the pressure and energy density.

- **The momentum constraint equation**

$$2G_{\mu b}n^{\mu}|_{\Sigma_c} = 2(\partial^a K_{ab} - \partial_b K) = 0 \Longrightarrow \partial^a T_{ab} = 0. \quad (4)$$

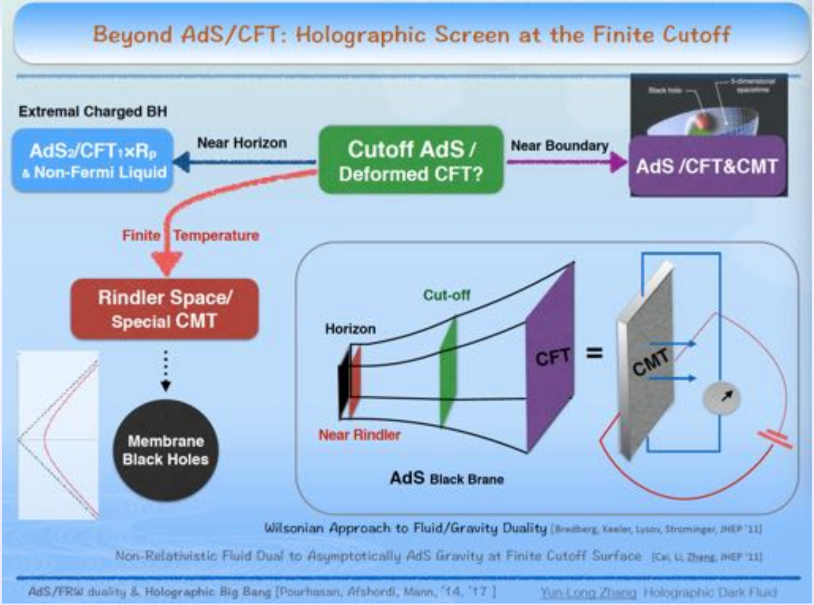
It reduce to the incompressible Navier-Stokes equations

$$\partial_i v^i = 0, \quad (5)$$

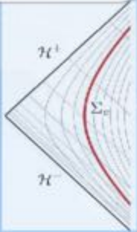
$$\partial_\tau v_i + v^j \partial_j v_i - r_c \partial^2 v_i + \partial_i P = 0, \quad (6)$$

in non-relativistic hydrodynamic limit
$v_i \sim \epsilon,\ P \sim \epsilon^2,\ \partial_i \sim \epsilon,\ \partial_\tau \sim \epsilon^2$**,**

Outline


Beyond AdS/CFT: Holographic Screen at the Finite Cutoff
Extremal Charged BH
AdS2/CFT1xRp
& Non-Fermi Liquid
Near Horizon
Cutoff AdS /
Deformed CFT?
Near Boundary
AdS /CFT&CMT
Finite Temperature
Rindler Space/
Special CMT
Membrane
Black Holes
Cut-off
Horizon
Near Rindler
CFT
=
CMT
AdS Black Brane
Wilsonian Approach to Fluid/Gravity Duality [Bredberg, Keeler, Lysov, Strominger, JHEP '11]
Non-Relativistic Fluid Dual to Asymptotically AdS Gravity at Finite Cutoff Surface [Cai, Li, Zhang, JHEP '11]
AdS/FRW duality & Holographic Big Bang [Pourhasan, Afshordi, Mann, '14, '17]
Yun-Long Zhang Holographic Dark Fluid

Rindler Holography: Flat Bulk with Cutoff <=> Rindler Fluid



Bulk Metric: Rindler $ds^2 = -r d\tau^2 + \frac{1}{r} dr^2 + dx_i dx^i$

Induced Metric: Flat $ds^2 = -r_c d\tau^2 + dx_i dx^i$

Dual Stress Energy Tensor $\boxed{T_{ab} = -2(K_{ab} - K\gamma_{ab})}$

Constraint Equations -> Self Consistent Relativistic Hydrodynamics

$$2G_{\mu b} n^\mu|_{r_c} = 2\partial^a (K_{ab} - \gamma_{ab} K) = 0 \;\Rightarrow\; \partial^a T_{ab} = 0$$

$$2G_{\mu\nu} n^\mu n^\nu|_{r_c} = (K^2 - K_{ab}K^{ab}) = 0 \;\Rightarrow\; T^2 - pT_{ab}T^{ab} = 0$$

Non-Relativistic limit -> Incompressible Navier-Stokes Equations

$$\boxed{\partial_i v^i = 0 \qquad \partial_i P + \partial_\tau v_i + v^j \partial_j v_i - \eta \partial^2 v_i = 0}$$

Ref: (JHEP 07 (2012) 146) by Bredberg, Keeler, Lysov, Strominger   Yun-Long Zhang Holographic Dark Fluid

Cutoff AdS Fluid: Universal Hydrodynamic Viscosities

AdS Metric $$ds^2_{p+2} = -r^2 f(r) d\tau^2 + \frac{1}{r^2 f(r)} dr^2 + r^2 dx_i dx^i$$

Induced Metric $$ds^2_{p+1} = -r_c^2 f(r_c) d\tau^2 + r_c^2 dx_i dx^i$$

Dual Tensor $$\boxed{\mathcal{T}_{ab} = -2(K_{ab} - K\gamma_{ab} + C\gamma_{ab})}$$

Constraint equations $$2G_{\mu b} n^\mu|_{r_c} = 2\partial^a(K_{ab} - \gamma_{ab}K) = 0 \;\Rightarrow\; \partial^a T_{ab} = 0$$

Cutoff AdS Fluid: Universal holographic conductivities in first order

Holographic Cutoff AdS Fluid in Non-relativistic limit
[Cai, Li, Zhang, JHEP 1107(2011)027]
$$\partial_r \sim \epsilon^0, \quad \partial_i \sim v_i \sim \partial_i\phi \sim \epsilon^1, \quad \partial_\tau \sim P \sim \epsilon^2$$

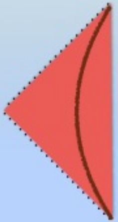
Holographic Forced Fluid Dynamics on finite Cutoff
[Cai, Li, Nie, Zhang, NPB 864 (2012) 260]
$$\partial_i v^i = 0, \quad \partial_\tau v_i + v^j\partial_j v_i + \partial_i P - \nu\partial^2 v_i = f_i^\phi + f_i^q$$

Incompressible Navier-Stokes from Chern-Simons Modified Gravity
Cai, Li, Qi, Zhang, PRD 86 (2012) 086008]
$$\partial_\tau v_i + v^j\partial_j v_i + \partial_i P - \nu\partial^2 v_i - (\tilde{\nu}\epsilon_{ij}\partial^2 v_j + \tilde{\zeta}\epsilon^{jk}\partial_i\partial_j v_k) = f_i$$

Holographic Charged Fluid with Anomalous Current at Finite Cutoff
[Bai, Hu, Lee, Zhang, JHEP 1211 (2012) 054]
$$\xi_B = c\left(\mu - \frac{1}{2}\frac{n\mu^2}{\rho+p}\right), \quad \xi_V = c\left(\mu^2 - \frac{2}{3}\frac{n\mu^3}{\rho+p}\right)$$

Yun-Long Zhang Holographic Dark Fluid

Rindler Fluid with Weak Momentum Relaxation



$$S_0 = \frac{1}{16\pi G_{p+2}}\int \mathrm{d}^{p+2}x\sqrt{-g}\left[R - \frac{1}{2}\sum_{I=1}^{p}(\partial\phi_I)^2\right] - \frac{1}{8\pi G_{p+2}}\int \mathrm{d}^{p+1}x\sqrt{-\gamma}K.$$

$$\mathrm{d}s^2_{p+2} = -2\kappa_0(r-r_0)\mathrm{d}t^2 + 2\mathrm{d}t\mathrm{d}r + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j - \frac{p}{4}(r-r_0)(r-r_c)k^2\mathrm{d}t^2 - \frac{(r-r_c)}{2\kappa_0}k^2\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j + O(k^4),$$

$$\phi_I = kx_I, \quad x_I = x_i = x_1, x_2, ..., x_p.$$

Ward Identity $\quad \partial_t\langle T^t{}_i\rangle + \partial_i\mathbb{p}_k = -\bar{\tau}_0^{-1}\langle T^t{}_i\rangle - (\ell_0)k^2\partial_t v_i + \cdots. \quad \bar{\tau}_0^{-1} = \frac{k^2 s_k}{4\pi(\mathbb{e}_k + \mathbb{p}_k)}.$

Thermal Conductivity $\quad \bar{\kappa}_\omega = \frac{1}{1 - \mathbb{i}\omega\tau_0}\frac{4\pi s_k T_k}{k^2}, \quad \tau_0^{-1} = \frac{k^2}{4\pi T_k}\left[1 - \frac{(\ell_0)T_0}{s_0}\frac{k^2}{T_0^2}\right] + O(k^6)$

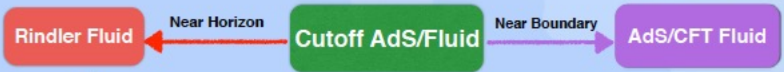
Correction to Relaxation Rate $\quad \ell_0 = -\frac{2}{\mathbb{p}} - \delta\ell_0 = -\frac{1}{\mathbb{p}}, \quad \boxed{\xi_0 \equiv \frac{\ell_0 T_0}{s_0} = -1}$

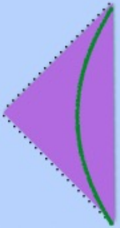
"Momentum relaxation from the fluid/gravity correspondence" Blake [15']

"hydrodynamic of transports with momentum relaxation" Hartnoll, Kovtun, Muller, Sachdev[07']

S. Khimphun, B. H. Lee, C. Park and Y. L. Zhang, JHEP01(2018)058

Cutoff AdS Fluid with Momentum Relaxation





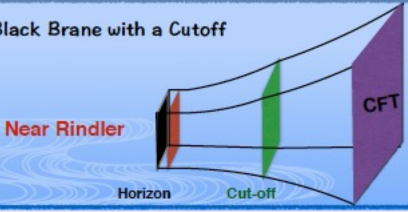
Ward Identity

$$\partial_t\langle \mathcal{T}^t{}_i\rangle + \partial_i\tilde{\mathcal{P}} = -\tilde{\tau}_c^{-1}\langle \mathcal{T}^t{}_i\rangle - \ell_c k^2\partial_t v_i + \cdots, \quad \tilde{\tau}_c^{-1} = \frac{k^2\tilde{s}_c}{4\pi(\tilde{\mathcal{E}}+\tilde{\mathcal{P}})}.$$

Thermal Conductivity & Relaxation Rate

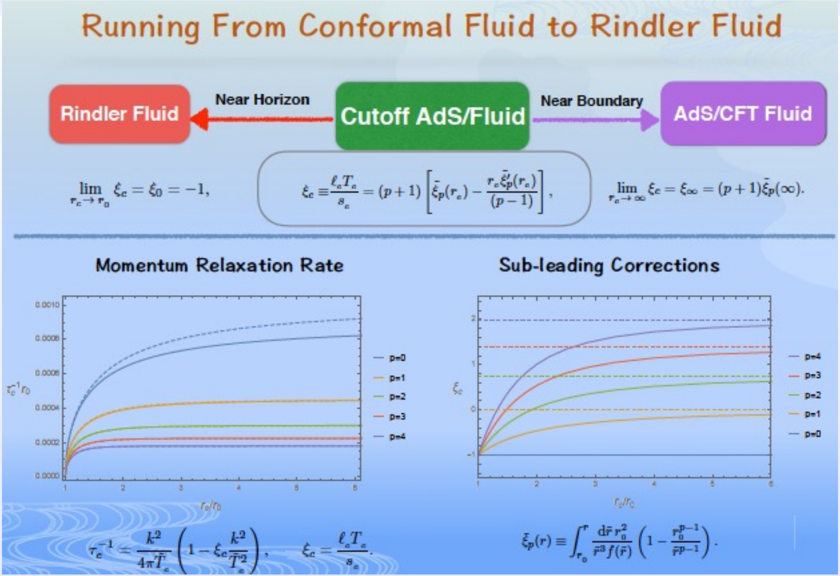
$$\tilde{\kappa}_\omega = \frac{1}{1-i\omega\tau_c}\frac{4\pi\tilde{s}_c\tilde{T}_c}{k^2}, \qquad \tau_c^{-1} = \frac{k^2}{4\pi\tilde{T}_c}\left[1-\frac{\ell_c T_c}{s_c}\frac{k^2}{T_c^2}\right] + O(k^6).$$

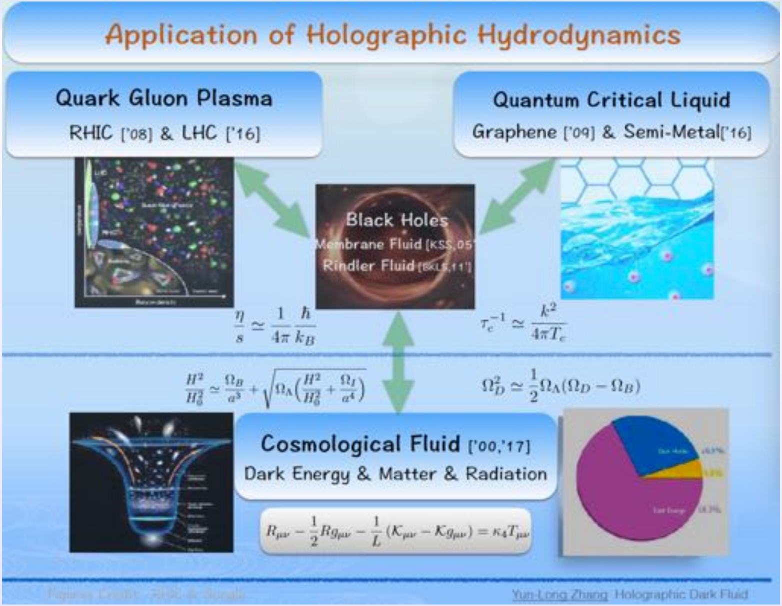
AdS Black Brane with a Cutoff



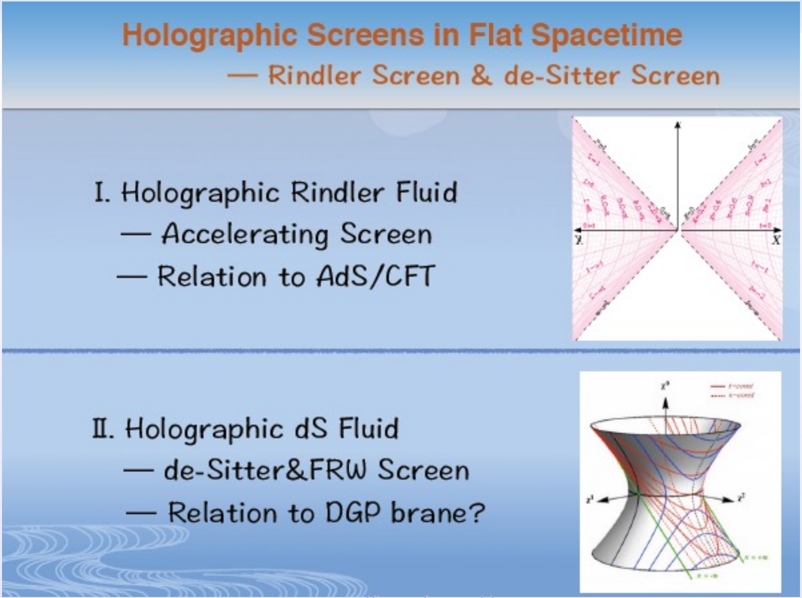
Sub-Leading Correction

$$\xi_c \equiv \frac{\ell_c T_c}{s_c} = (p+1)\left[\tilde{\xi}_p(r_c) - \frac{r_c\tilde{\xi}_p'(r_c)}{(p-1)}\right],$$


Running From Conformal Fluid to Rindler Fluid
Rindler Fluid
Near Horizon
Cutoff AdS/Fluid
Near Boundary
AdS/CFT Fluid
$\lim_{r_c \to r_0} \xi_c = \xi_0 = -1,$
$\xi_c \equiv \frac{\ell_c T_c}{s_c} = (p+1)\left[\bar{\xi}_p(r_c) - \frac{r_c \bar{\xi}_p'(r_c)}{(p-1)}\right],$
$\lim_{r_c \to \infty} \xi_c = \xi_\infty = (p+1)\bar{\xi}_p(\infty).$
Momentum Relaxation Rate
$\tau_c^{-1} r_0$
r_c/r_0
p=0
p=1
p=2
p=3
p=4
Sub-leading Corrections
ξ_c
r_c/r_0
p=4
p=3
p=2
p=1
p=0
$\tau_c^{-1} = \frac{k^2}{4\pi T_c}\left(1 - \xi_c \frac{k^2}{T_c^2}\right), \quad \xi_c = \frac{\ell_c T_c}{s_c}.$
$\bar{\xi}_p(r) \equiv \int_{r_0}^{r} \frac{d\bar{r}\, r_0^2}{\bar{r}^3 f(\bar{r})}\left(1 - \frac{r_0^{p-1}}{\bar{r}^{p-1}}\right).$


Application of Holographic Hydrodynamics
Quark Gluon Plasma
RHIC ['08] & LHC ['16]
Quantum Critical Liquid
Graphene ['09] & Semi-Metal['16]
Black Holes
Membrane Fluid [KSS,05']
Rindler Fluid [BKLS,11']
$\frac{\eta}{s} \simeq \frac{1}{4\pi}\frac{\hbar}{k_B}$
$\tau_c^{-1} \simeq \frac{k^2}{4\pi T_c}$
$\frac{H^2}{H_0^2} \simeq \frac{\Omega_B}{a^3} + \sqrt{\Omega_\Lambda\left(\frac{H^2}{H_0^2} + \frac{\Omega_I}{a^4}\right)}$
$\Omega_D^2 \simeq \frac{1}{2}\Omega_\Lambda(\Omega_D - \Omega_B)$
Cosmological Fluid ['00,'17]
Dark Energy & Matter & Radiation
$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{1}{L}(K_{\mu\nu} - Kg_{\mu\nu}) = \kappa_4 T_{\mu\nu}$
Yun-Long Zhang Holographic Dark Fluid

Outline

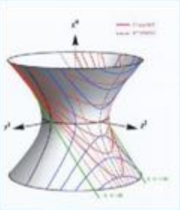


(figures from web)

Cutoff Flat/dS Surface: Minkowski Bulk <=> de-Siter Universe

1) Holographic Stress Tensor — Dark Sectors

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa_4 T_{\mu\nu} + \kappa_4 \langle\mathcal{T}\rangle_{\mu\nu}, \qquad \langle\mathcal{T}\rangle_{\mu\nu} \equiv \frac{1}{\kappa_4 L}(\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu})$$



Modified Einstein equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{1}{L}(\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu}) = \kappa_4 T_{\mu\nu}$$

$$\langle\mathcal{T}\rangle^{\Lambda}_{\mu\nu} = -\frac{\Lambda}{\kappa_4}g_{\mu\nu}, \qquad \Lambda = \frac{3}{L^2} \qquad L = \frac{\kappa_5}{\kappa_4}$$

Hamiltonian Constraint

$$0 = 2\mathcal{G}^{(5)}_{AB}\mathcal{N}^A\mathcal{N}^B \equiv \mathcal{K}^2 - \mathcal{K}_{\mu\nu}\mathcal{K}^{\mu\nu} - R.$$

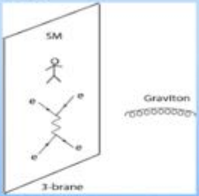
$$\frac{\langle\mathcal{T}\rangle^2}{3} - \langle\mathcal{T}\rangle_{\mu\nu}\langle\mathcal{T}\rangle^{\mu\nu} = -\frac{\kappa_4}{(\kappa_5)^2}\frac{1}{2}(T + \langle\mathcal{T}\rangle).$$

2) Brane World Models — Embedding into higher dimensions —

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \mathcal{T}^{\mathcal{M}}_{\mu\nu} + T^{B}_{\mu\nu},$$

$$\mathcal{T}^{\mathcal{M}}_{\mu\nu} \equiv (\mathcal{K}g_{\mu\sigma} - \mathcal{K}_{\mu\sigma})\mathcal{K}^{\sigma}{}_{\nu} + \mathcal{M}_{\mu\nu} - \frac{1}{2}(\mathcal{K}^2 - \mathcal{K}_{\rho\sigma}\mathcal{K}^{\rho\sigma})\,g_{\mu\nu},$$

$$\mathcal{M}_{\mu\nu} \equiv g_{\mu}{}^{M}g_{\nu}{}^{N}R^{(d+1)}_{MN} - g_{\mu}{}^{M}\mathcal{N}^{P}g_{\nu}{}^{N}\mathcal{N}^{Q}R^{(d+1)}_{MPNQ}.$$



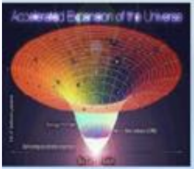
[Maeda, Mukohyama, Sasaki, Shiromizu, ..., ['99, '10]

Ref: 1106.2476 [Living Rev. '10] Modified Gravity and Cosmology

Ref: [arXiv: JHEP 1810 (2018) 009] by Cai, Sun, Zhang

Yun-Long Zhang Holographic Dark Fluid

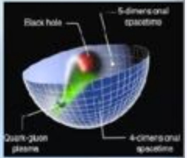
II. Emergent Dark Universe: Cutoff Flat Bulk <=> FRW Universe



The Total Action

$$\mathcal{S}_{tot} = \int_{\mathcal{H}} \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2\kappa_4}R + \mathcal{L}_m\right) + \mathcal{S}_5\,,$$

$$\mathcal{S}_5 \equiv \int_{\mathcal{M}} \mathrm{d}^5x\sqrt{-\tilde{g}}\left(\frac{1}{2\kappa_5}\mathcal{R}\right) + \int_{\mathcal{H}} \mathrm{d}^4x\sqrt{-g}\,\frac{1}{\kappa_5}\mathcal{K},$$



Einstein Field Equations
$$\frac{1}{\kappa_4}G_{\mu\nu} = T^m_{\mu\nu} + \langle\mathcal{T}\rangle^d_{\mu\nu},$$

Holographic Dark Fluid?

Stress Energy Tensors
$$T^m_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta(S_m)}{\delta g^{\mu\nu}}, \qquad \langle\mathcal{T}\rangle^d_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\mathcal{S}_5)}{\delta g^{\mu\nu}} = \frac{1}{\kappa_5}(\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu})$$

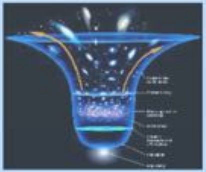
In the Bulk => Modified GR
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{\kappa_4}{\kappa_5}(\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu}) = \kappa_4 T^m_{\mu\nu}$$

Hamilton Constraint Equation
& Emergent de-Sitter Universe
$$\tilde{\Omega}_D^2 = \frac{\tilde{\Omega}_\Lambda}{2(1+3\tilde{w}_D)}\left[\tilde{\Omega}_D(1-3\tilde{w}_D) - \tilde{\Omega}_B\right].$$

Cutoff Flat Bulk <=> Dark Fluid in FRW Universe?

Induced Metric $\mathrm{d}s_4^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -c^2\mathrm{d}t^2 + a(t)^2\left[\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2\right].$

Bulk Metric



$$\mathrm{d}s_5^2 = \tilde{g}_{AB}\mathrm{d}x^A\mathrm{d}x^B = \mathrm{d}y^2 + \tilde{g}_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu.$$

$$= \mathrm{d}y^2 - \mathbf{n}(y,t)^2\, c^2\mathrm{d}t^2 + \mathbf{a}(y,t)^2\left[\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2\right]$$

$$\mathbf{a}(y,t)^2 = a(t)^2 + y^2\frac{\dot{a}(t)^2}{c^2} \pm 2y\sqrt{a(t)^2\frac{\dot{a}(t)^2}{c^2} + I},$$

$$\mathbf{n}(y,t) = \frac{\partial_t \mathbf{a}(y,t)}{\dot{a}(t)}.$$

Holographic Stress energy Tensor

$$\langle T\rangle^d_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\mathcal{S}_5)}{\delta g^{\mu\nu}} = \frac{1}{\kappa_5}\left(\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu}\right).$$

$$(\kappa_5 c^2)\rho_d = 3\sqrt{\frac{H(t)^2}{c^2} + \frac{I}{a(t)^4}},$$

$$(\kappa_5)p_d = -\frac{\frac{\dot{H}(t)+3H(t)^2}{c^2} + \frac{I}{a(t)^4}}{\sqrt{\frac{H(t)^2}{c^2} + \frac{I}{a(t)^4}}}.$$

Hamiltonian Constraint from Bulk: Add-on Matters on the Cutoff

$$\frac{\langle\mathcal{T}\rangle^2}{3}-\langle\mathcal{T}\rangle_{\mu\nu}\langle\mathcal{T}\rangle^{\mu\nu}=-\frac{\tilde{\rho}_\Lambda c^2}{3}\left[T+\langle\mathcal{T}\rangle\right].$$

$$0=2\mathcal{G}^{(5)}_{AB}\mathcal{N}^A\mathcal{N}^B\equiv\mathcal{K}^2-\mathcal{K}_{\mu\nu}\mathcal{K}^{\mu\nu}-R.$$

$$T^{\mu\nu}=-\rho_B u^\mu u^\nu+(\rho_R u^\mu u^\nu+p_R h^{\mu\nu})$$

$$\langle\mathcal{T}\rangle^{\mu\nu}=-\rho_\Lambda g^{\mu\nu}+(\rho_D u^\mu u^\nu+p_D h^{\mu\nu})$$

$$\rho_D^2=\frac{\rho_\Lambda}{2(1+3\tilde{w}_D)}\left[\rho_D(1-3\tilde{w}_D)-\rho_B\right],\qquad \tilde{w}_D\equiv\frac{p_D}{\rho_D c^2}.$$

$$\tilde{\Omega}_\Lambda\equiv\rho_\Lambda/\rho_c,\quad \tilde{\Omega}_D\equiv\rho_D/\rho_c,\quad \tilde{\Omega}_B\equiv\rho_B/\rho_c,$$

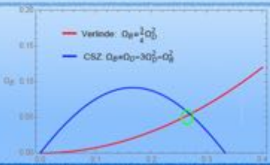
$$\frac{H(t)^2}{H_0^2}=\Omega_\Lambda+\frac{\Omega_D}{a(t)^3}+\frac{\Omega_B}{a(t)^3}\equiv\tilde{\Omega}_\Lambda+\tilde{\Omega}_D+\tilde{\Omega}$$

$$\boxed{\tilde{\Omega}_D^2=\frac{\tilde{\Omega}_\Lambda}{2(1+3\tilde{w}_D)}\left[\tilde{\Omega}_D(1-3\tilde{w}_D)-\tilde{\Omega}_B\right].}$$

$$\frac{H(t)^2}{H_0^2}\simeq\frac{\Omega_B}{a(t)^3}+\Omega_\Lambda^{1/2}\left[\frac{H(t)^2}{H_0^2}+\frac{\Omega_I}{a(t)^4}\right]^{1/2}$$

Late Time Evolution $\Omega_\Lambda\simeq 0.685,\quad \Omega_D\simeq 0.265,\quad \Omega_B\simeq 0.050,$

$$w_D\simeq 0\qquad \boxed{\begin{aligned}\delta_{CSZ}&\equiv\Omega_D^2-\frac{1}{2}\Omega_\Lambda(\Omega_D-\Omega_B)\simeq-0.003,\\ \delta_V&\equiv\Omega_D^2-\frac{4}{3}\Omega_B\simeq 0.004.\end{aligned}}$$

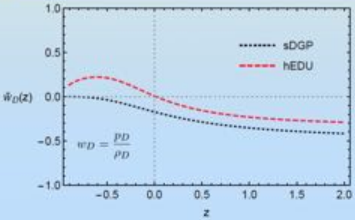


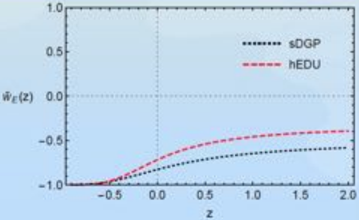
Hamilton Constraint Equation & Emergent de-Sitter Universe

Ref: [arXiv: JHEP 1810 (2018) 009] by Cai, Sun, Zhang

Yun-Long Zhang Holographic Dark Fluid

Holographic Emergent Dark Universe(hEDU): Late-Time Evolution





Friedmann Equation

$$\frac{H(z)^2}{H_0^2} = \frac{\Omega_\Lambda}{2} + \Omega_m(1+z)^3 + \frac{\Omega_\Lambda}{2}\sqrt{1 + \frac{4}{\Omega_\Lambda}\left[\Omega_m(1+z)^3 + \Omega_I(1+z)^4\right]}$$

Bulk Hamiltonian Constraint on the FRW Hypersurface

$$\boxed{\tilde{\Omega}_D^2 = \frac{\tilde{\Omega}_\Lambda}{2(1+3\tilde{w}_D)}\left[\tilde{\Omega}_D(1-3\tilde{w}_D) - \tilde{\Omega}_B\right].}$$

$$\tilde{\Omega}_\Lambda = \Omega_\Lambda, \qquad \tilde{\Omega}_D = \Omega_{\mathcal{H}}(t) - \Omega_\Lambda, \qquad \tilde{\Omega}_B \equiv \frac{\Omega_M}{a(t)^3} = \frac{H(t)^2}{H_0^2} - \Omega_{\mathcal{H}}(t),$$

$$\tilde{w}_D = -1 - \frac{1}{3H(t)}\frac{\dot{\Omega}_{\mathcal{H}}(t)}{\Omega_{\mathcal{H}}(t) - \Omega_\Lambda}, \qquad \Omega_{\mathcal{H}}(t) \equiv \frac{\rho_{\mathcal{H}}}{\rho_c} = \Omega_\Lambda^{1/2}\left[\frac{H(t)^2}{H_0^2} + \frac{\Omega_I}{a(t)^4}\right]^{1/2}.$$

Ref: [arXiv: JHEP 1810 (2018) 009] Cai, Sun, Zhang, Emergent Dark Matter in Late Time Universe on Holographic Screen

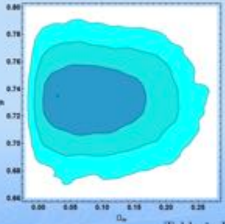
Holographic Emergent Dark Universe (hEDU): SNIa Data & H_0

Friedmann Equations

$$\text{LCDM}: \frac{H(z)^2}{H_0^2} = \Omega_\Lambda + \Omega_m(1+z)^3$$

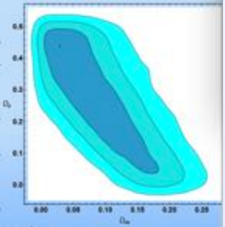
$$\text{sDGP}: \frac{H(z)^2}{H_0^2} = \frac{\Omega_\Lambda}{2} + \Omega_m(1+z)^3 + \frac{\Omega_\Lambda}{2}\sqrt{1+\frac{4\Omega_m}{\Omega_\Lambda}(1+z)^3}$$

$$\text{hEDU}: \frac{H(z)^2}{H_0^2} = \frac{\Omega_\Lambda}{2} + \Omega_m(1+z)^3 + \frac{\Omega_\Lambda}{2}\sqrt{1+\frac{4\Omega_m}{\Omega_\Lambda}(1+z)^3+\frac{4\Omega_I}{\Omega_\Lambda}(1+z)^4}$$



Parameters	LCDM	hEDU
h	0.7330 ± 0.0180	0.7349 ± 0.0179
Ω_m	0.2969 ± 0.0352	0.0299 ± 0.0515
Ω_I	—	0.4382 ± 0.1317
α	0.1403 ± 0.0068	0.1409 ± 0.0068
β	3.1081 ± 0.0892	3.1144 ± 0.0896
$\chi^2_{\rm min}$	695.063	694.321
ΔAIC	0	1.258
ΔBIC	0	5.866

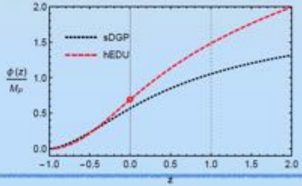
Table 1: Fitting values and uncertainties of the cosmological parameters.



Single Scalar Field: Reconstruct the Effective Potential

Total Action $\mathcal{S}_{tot} = \int_{\mathcal{H}} \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2\kappa_4}R + \mathcal{L}_m + \frac{1}{2\kappa_5}\mathcal{K}\right) + \int_{\mathcal{M}} \mathrm{d}^5x\sqrt{-\bar{g}}\left(\frac{1}{2\kappa_5}\mathcal{R}\right)$

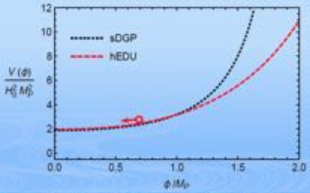
Effective Action $\mathcal{S}_{tot} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2\kappa_4}R + \mathcal{L}_m - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right].$



$$\boxed{\begin{aligned} V[\phi(t)] &= \frac{1}{2}\left[\rho_d(t) - p_d(t)\right], \\ \dot{\phi}(t) &= -\sqrt{\rho_d(t) + p_d(t)}. \end{aligned}}$$

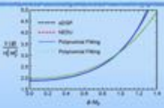
$$\rho_d(t) = \rho_c\sqrt{\Omega_\Lambda}\sqrt{\frac{H(t)^2}{H_0^2} + \frac{\Omega_I}{a(t)^4}}, \qquad p_d(t) = -\frac{\dot{\rho}_d}{3H(t)} - \rho_d,$$

Friedmann Equation $\frac{H(t)^2}{H_0^2} \simeq \frac{\Omega_B}{a(t)^3} + \Omega_\Lambda^{1/2}\left[\frac{H(t)^2}{H_0^2} + \frac{\Omega_I}{a(t)^4}\right]^{1/2}$



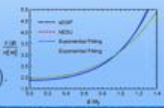
Polynomial fittings

$$\frac{V(\phi)}{H_0^2 M_p^2} = \frac{\Lambda_0}{H_0^2} + \frac{h_2}{2}\frac{\phi^2}{M_p^2} + \frac{h_3}{3!}\frac{\phi^3}{M_p^3} + \cdots$$



Exponential fittings

$$\frac{V(\phi)}{M_p^2} = \frac{\Lambda_0}{\lambda_+ + \lambda_-}\left(\lambda_+ e^{-\lambda_-\frac{\phi}{M_p}} + \lambda_- e^{\lambda_+\frac{\phi}{M_p}}\right).$$



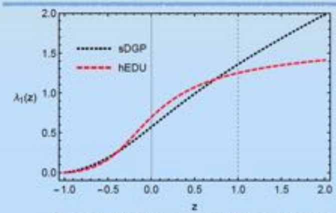
Ref: [arXiv: 1812.11105] Emergent Dark Universe and the Swampland Criteria — Yun-Long Zhang, Holographic Dark Fluid

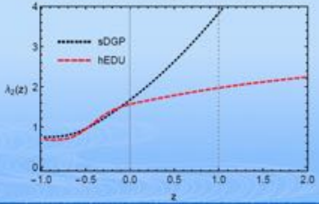
Effective Potential Swampland: Checking the Parameters

Swampland Criteria

Criterion 1 : $\frac{|\Delta\phi|}{M_P} \le d_0,$ $\qquad d_0, c_1, c_2 \sim \mathcal{O}(1)$

Criterion 2 : $M_P|\nabla V| \ge c_1 V$ or $M_P^2 \min[\nabla_i\nabla_j V] \le -c_2 V,$





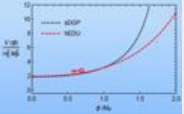
$$\lambda_1 \equiv M_P\frac{V'}{V}, \qquad V' \equiv \frac{dV(\phi)}{d\phi} = \frac{\dot V(t)}{\dot\phi(t)},$$

$$\lambda_2 \equiv M_P^2\frac{V''}{V}, \qquad V'' \equiv \frac{d}{d\phi}\frac{dV(\phi)}{d\phi} = \frac{1}{\dot\phi(t)}\frac{d}{dt}\left[\frac{\dot V(t)}{\dot\phi(t)}\right].$$

$$\frac{H(z)^2}{H_0^2} = \frac{\Omega_\Lambda}{2} + \Omega_m(1+z)^3 + \frac{\Omega_\Lambda}{2}\sqrt{1+\frac{4}{\Omega_\Lambda}\left[\Omega_m(1+z)^3+\Omega_I(1+z)^4\right]}$$

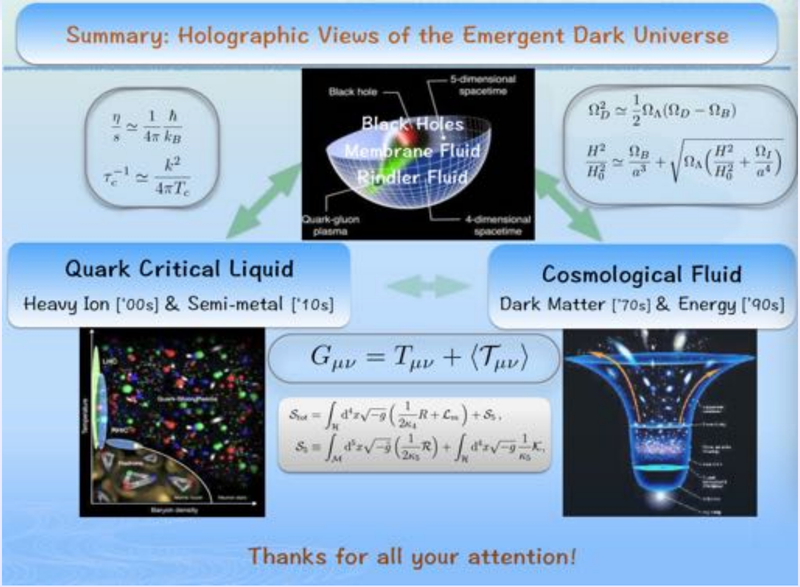
$$1 = \Omega_m + \sqrt{\Omega_\Lambda(1+\Omega_I)}$$

Models	Ω_m	Ω_I	Ω_Λ
sDGP	0.21	0	0.62
hEDU	0.03	0.44	0.65



$$M_P^2|\nabla_i\nabla_j V| \ge c_3 V, \qquad c_3 \sim \mathcal{O}(1).$$

Ref: [arXiv: 1812.11105] Emergent Dark Universe and the Swampland Criteria　Yun-Long Zhang　Holographic Dark Fluid


Summary: Holographic Views of the Emergent Dark Universe
$\frac{\eta}{s} \simeq \frac{1}{4\pi}\frac{\hbar}{k_B}$
$\tau_c^{-1} \simeq \frac{k^2}{4\pi T_c}$
Black hole
5-dimensional spacetime
Black Holes
Membrane Fluid
Rindler Fluid
Quark-gluon plasma
4-dimensional spacetime
$\Omega_D^2 \simeq \frac{1}{2}\Omega_\Lambda(\Omega_D - \Omega_B)$
$\frac{H^2}{H_0^2} \simeq \frac{\Omega_B}{a^3} + \sqrt{\Omega_\Lambda\left(\frac{H^2}{H_0^2} + \frac{\Omega_I}{a^4}\right)}$
Quark Critical Liquid
Heavy Ion ['00s] & Semi-metal ['10s]
Cosmological Fluid
Dark Matter ['70s] & Energy ['90s]
$G_{\mu\nu} = T_{\mu\nu} + \langle \mathcal{T}_{\mu\nu} \rangle$
$\mathcal{S}_{tot} = \int_N d^4x\sqrt{-g}\left(\frac{1}{2\kappa_4}R + \mathcal{L}_m\right) + \mathcal{S}_5,$
$\mathcal{S}_5 = \int_M d^5x\sqrt{-g}\left(\frac{1}{2\kappa_5}\mathcal{R}\right) + \int_N d^4x\sqrt{-g}\frac{1}{\kappa_5}\mathcal{K},$
Thanks for all your attention!

Summaries

1. Gravity/Fluid Duality on Cutoff
2. Connection with AdS/CFT
3. Holographic Dark Fluid in the Universe
4. Appendix: More about Rindler Fluid

Petrov type I condition

Obtain a recurrence relation $\mathbb{P}_{ab} \equiv n^r h_a^c n^r h_b^d C_{rcrd}$

$$\boxed{T_{ab}^{(0)} = \mathcal{E} u_a u_b + \mathcal{P} h_{ab} \overset{\mathbb{P}_{ab}=0}{\Longrightarrow} T_{ab}^{(1)} = -2\eta\sigma_{ab} + \dots \overset{\mathbb{P}_{ab}=0}{\Longrightarrow} T_{ab}^{(2)} = \dots}$$

Rindler-Fluid	→	AdS Cutoff-Fluid	→	AdS-CFT Fluid
Up to 2nd order How about Higher orders? Cai *et al.* 1401.7792		Up to to 0th order Modified Condition? Y. Ling *et al.* 1306.5633		Up to to 0th order AdS/Rindler correspondence?

JHEP 1304, 118 (2013) , Phys. Rev. D 90, no. 4, 041901 (2014)

Appendix: More about Rindler Fluid

- **Rindler Fluid up to second order**

$$
\begin{aligned}
T^{(0)}_{ab} &= 0 + \mathbf{p} h_{ab}, \\
T^{(1)}_{ab} &= (\zeta' D \ln \mathbf{p}) u_a u_b - 2\eta \mathcal{K}_{ab}, \\
T^{(2)}_{ab} &= \mathbf{p}^{-1} \Big\{ \Big[d_1 \mathcal{K}_{ab}\mathcal{K}^{ab} + d_2 \Omega_{ab}\Omega^{ab} + d_3 (D \ln \mathbf{p})^2 \\
&\quad + d_4 DD \ln \mathbf{p} + d_5 (D_\perp \ln \mathbf{p})^2 \Big] u_a u_b + \big(c_1 \mathcal{K}_{ac}\mathcal{K}^c{}_b \\
&\quad + c_2 \mathcal{K}_{c(a}\Omega^c{}_{b)} + c_3 \Omega_{ac}\Omega^c{}_b + c_4 h^c_a h^d_b \partial_c \partial_d \ln \mathbf{p} \\
&\quad + c_5 \mathcal{K}_{ab} D \ln \mathbf{p} + c_6 D^\perp_a \ln \mathbf{p}\, D^\perp_b \ln \mathbf{p} \big) \Big\}.
\end{aligned} \tag{7}
$$

- **Coefficients from holographic calculation**

$$
\begin{aligned}
\zeta' &= 0, \qquad \eta = 1, \\
d_1 &= -2, \quad d_2 = d_3 = d_4 = d_5 = 0, \\
c_1 &= -2, \quad c_2 = c_3 = c_4 = c_5 = -c_6 = -4\,.
\end{aligned} \tag{8}
$$

More about Rindler Fluid

- **The metric components**

$$
\begin{aligned}
g^{(0)}_{ab} =& -\mathbf{p}^2(r-r_c)u_a u_b + \gamma_{ab},\\
g^{(1)}_{ab} =& 2\mathbf{p}(r-r_c)\left(D\ln\mathbf{p}\,u_a u_b + 2a_{(a}u_{b)}\right),\\
g^{(2)}_{ab} =& 2(r-r_c)\Big[(\mathcal{K}_{cd}\mathcal{K}^{cd})u_a u_b - 2u_{(a}h^c_{b)}\partial_d\mathcal{K}^d{}_c\\
& - \mathcal{K}_a{}^c\mathcal{K}_{cb} + 2\mathcal{K}_{c(a}\Omega^c{}_{b)} - 2h^c_a h^d_b D\mathcal{K}_{cd}\Big]\\
& + \mathbf{p}^2(r-r_c)^2\Big\{\big(\frac{1}{2}\mathcal{K}_{cd}\mathcal{K}^{cd} + a_c a^c\big)u_a u_b\\
& + 2u_{(a}h^c_{b)}\big[\partial_d\mathcal{K}^d{}_c - (\mathcal{K}_{cd}+\Omega_{cd})a^d\big] - \Omega_{ac}\Omega^c{}_b\Big\}\\
& + \mathbf{p}^4(r-r_c)^3\big(\frac{1}{2}\Omega_{cd}\Omega^{cd}\big)u_a u_b. \qquad (9)
\end{aligned}
$$

where $\mathcal{K}_{ab} = h^c_a h^d_b\partial_{(c}u_{d)}$, $\Omega_{ab} = h^c_a h^d_b\partial_{[c}u_{d]}$, $D^\perp_a = h^c_a\partial_c$.

G. Compere, P. McFadden, K. Skenderis and M. Taylor, JHEP **1203**, 076 (2012)

2. Petrov type I condition

- **The spacetime is at least Petrov type I , if there exists a frame**

$$\ell^2 = k^2 = 0, \quad k_\mu \ell^\mu = 1, \quad {m^i}_\mu {m_j}^\mu = \delta^i_j, \tag{10}$$

- **Such that**

$$\mathbb{P}_{ij} = 0, \quad \mathbb{P}_{ij} \equiv C_{(\ell)i(\ell)j} \equiv \ell^\mu m_i^\nu \ell^\alpha m_j^\beta C_{\mu\nu\alpha\beta}. \tag{11}$$

V. Lysov and A. Strominger, “From Petrov-Einstein to Navier-Stokes,” arXiv: 1104.5502 .

T.-Z. Huang, Y. Ling, W.-J. Pan, Y. Tian, and X.-N. Wu, JHEP 1110 (2011) 079

- **For the Rindler spacetime, the Newman-Penrose-like vector fields can be choosed as**

$$m_i = \partial_i, \;\; \sqrt{2}\ell = \partial_0 - n, \;\; \sqrt{2}k = -\partial_0 - n, \tag{12}$$

Projections on the hypersurface

- **Consider the following projections**

$$\begin{aligned}
\gamma^{\alpha}_{a}\gamma^{\beta}_{b}\gamma^{\gamma}_{c}\gamma^{\delta}_{d}C_{\alpha\beta\gamma\delta} &= K_{ad}K_{bc} - K_{ac}K_{bd}\,,\\
\gamma^{\alpha}_{a}\gamma^{\beta}_{b}\gamma^{\gamma}_{c}n^{\delta}C_{\alpha\beta\gamma\delta} &= \partial_a K_{bc} - \partial_b K_{ac}\,,\\
\gamma^{\alpha}_{a}n^{\beta}\gamma^{\gamma}_{c}n^{\delta}C_{\alpha\beta\gamma\delta} &= KK_{ac} - K_{ab}K^{b}{}_{c}\,,
\end{aligned} \tag{13}$$

and the Brown-York stress tensor $T_{ab} = 2(K\gamma_{ab} - K_{ab})$**.**

- **The Petrov type I condition turns out to be** $\mathbb{P}_{ij} = 0$**, where**

$$\begin{aligned}
2\mathbb{P}_{ij} \equiv\; & T^{\tau}{}_{\tau}T_{ij} + 2r_c T^{\tau}{}_{i}T^{\tau}{}_{j} - 4r_c^{-1/2}\partial_\tau T_{ij} - T_{ik}T^{k}{}_{j} - 4r_c^{1/2}\partial_{(i}T^{\tau}{}_{j)}\\
& + p^{-2}\left[T\left(T - pT^{\tau}{}_{\tau}\right) + 4pr_c^{-1/2}\partial_\tau T\right]\delta_{ij}.
\end{aligned} \tag{14}$$

Petrov type I condition

- **A stress tensor T_{ab} on a $(p+1)$-dimensional timelike hypersurface has $(p+1)(p+2)/2$ arbitrary components,**
- **Petrov type I condition $\mathbb{P}_{ij} = 0$ gives $(p-1)(p+2)/2$ constraints.**
- **This leaves $\frac{(p+1)(p+2)}{2} - \frac{(p-1)(p+2)}{2} = (p+2)$ independent components.**
- **Exactly the number of the physical quantities of fluid: Local energy density e, pressure p, velocity v^i.**

Non-relativistic hydrodynamic expansion parameter ϵ

- **The stress tensor can be expanded as**

$$
\begin{aligned}
T^{\tau}{}_{i} &= T^{\tau}{}_{i}^{(1)} + T^{\tau}{}_{i}^{(3)} + O(\epsilon^5), \\
T^{\tau}{}_{\tau} &= T^{\tau}{}_{\tau}^{(0)} + T^{\tau}{}_{\tau}^{(2)} + T^{\tau}{}_{\tau}^{(4)} + O(\epsilon^6), \\
T_{ij} &= T_{ij}^{(0)} + T_{ij}^{(2)} + T_{ij}^{(4)} + O(\epsilon^6), \\
T &= T^{(0)} + T^{(2)} + T^{(4)} + O(\epsilon^6).
\end{aligned} \tag{15}
$$

- **Impose the Hamiltonian constraint and Petrov type I condition**

$$
\begin{aligned}
\mathbb{H}^{(0)} &= 0 \Rightarrow T^{\tau}{}_{\tau}^{(2)} = -T^{\tau}{}_{i}^{(1)} T^{\tau}{}_{j}^{(1)} \delta^{ij}, \\
\mathbb{P}_{ij}^{(0)} &= 0 \Rightarrow T_{ij}^{(2)} = p^{-1} T^{(2)} \delta_{ij} + r_c^{3/2}\, T^{\tau}{}_{i}^{(1)} T^{\tau}{}_{j}^{(1)} - 2\, r_c\, \partial_{(i} T^{\tau}{}_{j)}^{(1)},
\end{aligned}
$$

Recover the dual fluid

- **Identify** $T^{\tau(1)}_{i} = {r_c}^{-3/2} v_i$, $T^{(2)} = {r_c}^{-3/2} pP$, **we can recover the stress tensor**

$$T^{\tau}{}_{i} = + {r_c}^{-3/2} v_i + O(\epsilon^3), \tag{16}$$

$$T^{\tau}{}_{\tau} = - {r_c}^{-3/2} v^2 + O(\epsilon^3), \tag{17}$$

$$T_{ij} = + {r_c}^{-1/2} \delta_{ij} + {r_c}^{-3/2} \left[P\delta_{ij} + v_i v_j - 2 r_c \sigma_{ij}\right] + O(\epsilon^3), \tag{18}$$

$$T \;\; = T^{\tau}{}_{\tau} + T^{i}{}_{i} = p\, {r_c}^{-1/2} + p\, {r_c}^{-3/2} P + O(\epsilon^3), \tag{19}$$

- $\partial_a T^{ab} = 0 \rightarrow$ **incompressiable Navier-Stokes equations**

$$\partial_i v^i = O(\epsilon^4), \quad \partial_\tau v_i + v^j \partial_j v_i + \partial_i P - r_c \partial^2 v_i = O(\epsilon^5), \tag{20}$$

The next non-trivial order

- **Hamiltonian constraint** $\mathbb{H}^{(4)} = 0$ **and Petrov type I condition** $\mathbb{P}_{ij}^{(4)} = 0$ **lead to**

$$\begin{aligned} T^{\tau}{}_{\tau}^{(4)} =& - r_c^{3/2} T^{\tau}{}_{i}^{(1)} T^{\tau}{}_{j}^{(3)} \delta^{ij} \\ & + \frac{1}{2}\left[r_c^{1/2} T_{ij}^{(2)} T_{(2)}^{ij} + r_c^{1/2} (T^{\tau}{}_{\tau}^{(2)})^2 - p^{-1} r_c^{1/2} (T^{(2)})^2 \right], \quad (21) \\ T_{ij}^{(4)} =& \, 2\, r_c^{3/2} T^{\tau}{}_{(i}^{(1)} T^{\tau}{}_{j)}^{(3)} - 2 r_c\, \partial_{(i} T^{\tau}{}_{j)}^{(3)} + \frac{1}{2} r_c^{1/2} T^{\tau}{}_{\tau}^{(2)} T_{ij}^{(2)} \\ & - \frac{1}{2} T_{ik}^{(2)} T_{jl}^{(2)} \delta^{kl} - \partial_\tau T_{ij}^{(2)} \\ & + \frac{1}{2} p^{-1} \left[p^{-1} r_c^{1/2} (T^{(2)})^2 - r_c^{1/2} T^{(2)} T^{\tau}{}_{\tau}^{(2)} + 4 \partial_\tau T^{(2)} + 2\, T^{(4)} \right] \delta_{ij}. \end{aligned}$$

But it fail to recover the holographic stress tensor.

Check the Petrov type I condition

- **Inserting the stress tensor into $\mathbb{P}_{ij}$ and expanding in powers of parameter ϵ, one has**

$$\mathbb{P}_{ij} = \mathbb{P}_{ij}^{(0)} + \mathbb{P}_{ij}^{(2)} + \mathbb{P}_{ij}^{(4)} + O(\epsilon^6). \tag{22}$$

- **One can see that $\mathbb{P}_{ij}^{(0)}$ and $\mathbb{P}_{ij}^{(2)}$ vanish identically, but**

$$\mathbb{P}_{ij}^{(4)} = r_c^{-3}\left[-6r_c v^k v_{(i}\omega_{j)k} - 2r_c^2 v_{(i}\partial^2 v_{j)} + 4r_c^2 v^k \partial_{(i}\omega_{j)k} + r_c^3\partial^2\sigma_{ij}\right]. \tag{23}$$

Unless additional constraint, eg the irrotational condition $\omega_{ij} \sim O(\epsilon^4)$ R. -G. Cai, L. Li, Q. Yang and Y. -L. Zhang, JHEP **1304**, 118 (2013)

See more recent discussions by C. Keeler, T. Manton and N. Monga, "From Navier-Stokes to Maxwell via Einstein," [arXiv:2005.04242 [hep-th]].

Slightly change the frame

- **In general, these basses can be written as**

$$\begin{aligned}\sqrt{2}\ell =&\gamma(\partial_0+\beta^i\partial_i)-n^\mu\partial_\mu,\\ \sqrt{2}k =&-\gamma(\partial_0+\beta^i\partial_i)-n^\mu\partial_\mu,\\ m_i =&\partial_i+\gamma\beta_i\partial_0+(\gamma-1)\beta^{-2}\beta_i\beta^j\partial_j,\end{aligned} \tag{24}$$

- **Only one term $\mathbb{P}^{(4)}_{ij}=\partial^2\sigma_{ij}=\partial^2\partial_{(i}v_{j)}$ is left,**

3. Relativistic Rindler fluid in derivative expansion

- **Choose a covariant frame** $\sqrt{2}\ell = -n + u, \ \sqrt{2}k = -n - u,$

$$g_{\mu\nu} = \ell_\mu k_\nu + k_\mu \ell_\nu + \delta_{ij} m^i{}_\mu m^j{}_\nu. \tag{25}$$

- **The Petrov type I condition becomes** $\mathbb{P}_{ij} = m_i{}^a m_j{}^b \mathbb{P}_{ab} = 0$**,**

$$\begin{aligned} 2\mathbb{P}_{ab} =& h_a^c h_b^d \big[(T_{mc} T_{nd} - T_{mn} T_{cd})\, u^m u^n - T_{cm} T^m{}_d \big] \\ &+ 4 h_a^c h_b^d \big[- u^m \partial_m T_{cd} + u^m \partial_{(c} T_{d)m} \big] \\ &+ p^{-2} \big[T(T + p\, T_{mn} u^m u^n) + 4p\, u^c \partial_c T \big] h_{ab}. \end{aligned} \tag{26}$$

From Petrov type I condition to Relativistic Fluid

- **Define the fluid velocity u^a such that $h_a^b T_{bc} u^c = 0$,**
- **Choose the isotropy gauge of the pressure p,**
- **The stress tensor can be decomposed as**

$$T_{ab} = \mathbf{e} u_a u_b + \Pi_{ab}, \tag{27}$$

with

$$\mathbf{e} \equiv T_{ab} u^a u^b, \quad \Pi_{ab} \equiv h_a^c h_b^d T_{cd}. \tag{28}$$

- **Then the Hamiltonian constraint becomes $\mathbb{H} = 0$, where**

$$\mathbb{H} \equiv 2p\mathbf{e}\mathbf{p} + (p-1)\mathbf{e}^2 + p\Pi_{ab}\Pi^{ab} - p^2\mathbf{p}^2. \tag{29}$$

- **The covariant Petrov type I condition is $\mathbb{P}_{ab} = 0$, where**

$$\begin{aligned} 2\mathbb{P}_{ab} \equiv & -\mathbf{e}\Pi_{ab} - \Pi_{ac}\Pi^c{}_b - 4u^c\partial_c\Pi_{ab} - 4\Pi_{(a}{}^c D^{\perp}_{b)}u_c \\ & - 4\mathbf{e}\mathcal{K}_{ab} + \left[\mathbf{p}\left(\mathbf{e}+\mathbf{p}\right) + 4\mathbf{p}\, u^c\partial_c \ln \mathbf{p}\right] h_{ab}. \end{aligned} \tag{30}$$

R. G. Cai, Q. Yang and Y. L. Zhang, Phys. Rev. D **90** (R)041901 (2014)

Recover the stress tensor of dual fluid

- **Expanding the stress tensor**

$$\begin{aligned} \mathbf{e} &= 0 + \mathbf{e}^{(1)} + \mathbf{e}^{(2)} + O(\partial^3), \\ \Pi_{ab} &= \mathbf{p} h_{ab} + \Pi^{(1)}_{ab} + \Pi^{(2)}_{ab} + O(\partial^3). \end{aligned} \tag{31}$$

- **The first order**

$$\begin{aligned} \mathbb{H}^{(1)} &= 0 \Rightarrow \mathbf{e}^{(1)} = 0, \\ \mathbb{P}^{(1)}_{ab} &= 0 \Rightarrow \Pi^{(1)}_{ab} = -2\mathcal{K}_{ab}. \end{aligned} \tag{32}$$

- **The second order**

$$\begin{aligned} \mathbb{H}^{(2)} &= 0 \Rightarrow \mathbf{e}^{(2)} = -2\mathbf{p}^{-1}\mathcal{K}_{ab}\mathcal{K}^{ab}, \\ \mathbb{P}^{(2)}_{ab} &= 0 \Rightarrow \Pi^{(2)}_{ab} = \mathbf{p}^{-1}\left[2\mathcal{K}_{ac}\mathcal{K}^{c}{}_{b} - 4\mathcal{K}_{c(a}\Omega^{c}{}_{b)} + 4h^c_a h^d_b D\mathcal{K}_{cd}\right]. \end{aligned} \tag{33}$$

- **Recover the stress tensor of dual fluid**

$$
\begin{aligned}
T^{(0)}_{ab} =& 0 + \mathbf{p} h_{ab}, \\
T^{(1)}_{ab} =& -2\mathcal{K}_{ab}, \\
T^{(2)}_{ab} =& -2\mathbf{p}^{-1}\mathcal{K}_{ab}\mathcal{K}^{ab} \\
& - \big(2\mathcal{K}_{ac}\mathcal{K}^{c}{}_{b} + 4\mathcal{K}_{c(a}\Omega^{c}{}_{b)} + 4\Omega_{ac}\Omega^{c}{}_{b} + 4h^{c}_{a}h^{d}_{b}\partial_c\partial_d \ln \mathbf{p} \\
& + 4\mathcal{K}_{ab}\, D\ln \mathbf{p} - 4D^{\perp}_{a}\ln \mathbf{p}\, D^{\perp}_{b}\ln \mathbf{p}\big)\,.
\end{aligned} \tag{34}
$$

- **With exactly the same holographic coefficients**

$$
\begin{aligned}
\zeta' &= 0, & \eta &= 1, \\
d_1 &= -2, & d_2 &= d_3 = d_4 = d_5 = 0, \\
c_1 &= -2, & c_2 &= c_3 = c_4 = c_5 = -c_6 = -4\,.
\end{aligned} \tag{35}
$$

4. Consider the Gauss-Bonnet correction

- **The action of vacuum Einstein gravity with Gauss-Bonnet correction $\mathcal{L}_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\lambda}R^{\mu\nu\sigma\lambda}$,**

$$S = \int d^{p+2}x\sqrt{-g}(R + \alpha\mathcal{L}_{GB}), \tag{36}$$

The field equation

$$G_{\mu\nu} + 2\alpha H_{\mu\nu} = 0, \tag{37}$$

with the tensor

$$H_{\mu\nu} = RR_{\mu\nu} - 2R_{\mu\lambda}R^{\lambda}_{\ \nu} - 2R^{\sigma\lambda}R_{\mu\sigma\nu\lambda} + R_{\mu}^{\ \sigma\lambda\rho}R_{\nu\sigma\lambda\rho} - \frac{1}{4}g_{\mu\nu}\mathcal{L}_{GB}.$$

- **The dual stress tensor,**

$$\delta T_{ab} = \delta \tilde{T}_{ab} + \delta \bar{T}_{ab} = -4\alpha \mathbf{p} \left(\mathcal{K}_{ac}\mathcal{K}^{c}{}_{b} + 3p^{-1}\Omega_{ac}\Omega^{c}{}_{b} \right) . \tag{38}$$

- **We can recover the first and second order transport coefficients from fluid/gravity calculation**

$$\begin{aligned} &\zeta' = 0, \quad \eta = 1, \quad d_1 = -2, \quad d_2 = d_3 = d_4 = d_5 = 0, \\ &c_1 = -\, 2(1 + 2\alpha \mathbf{p}^2), \quad c_3 = -4(1 + 3\alpha \mathbf{p}^2), \\ &c_2 = c_4 = c_5 = -c_6 = -4 \, . \end{aligned} \tag{39}$$

C. Eling, A. Meyer and Y. Oz, JHEP **1208**, 088 (2012)

R. G. Cai, Q. Yang and Y. L. Zhang, JHEP **1412**, 147 (2014)

Conclusions on the Petrov type condition

- A recursive relation of the fluid dual to Rindler spacetime.
- 4 dimensional as special case? Petrov type I .
- Equivalent choice of the regularity condition on horizon?
- Generalize to other holographic system?

Thanks for all your attention!