北京航空航天大學
BEIHANG UNIVERSITY

Formation and critical dynamics of topological defects in Lifshitz holography

Zhi-Hong Li（李志红）

Cooperator： Chuan-Yin Xia Hua-Bi Zeng, Hai-Qing Zhang

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Outlines

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- Lifshitz geometry & Holographic Setup

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- Magnetic fluxoids and order parameter vortices
- Vortex number density and freeze-out time

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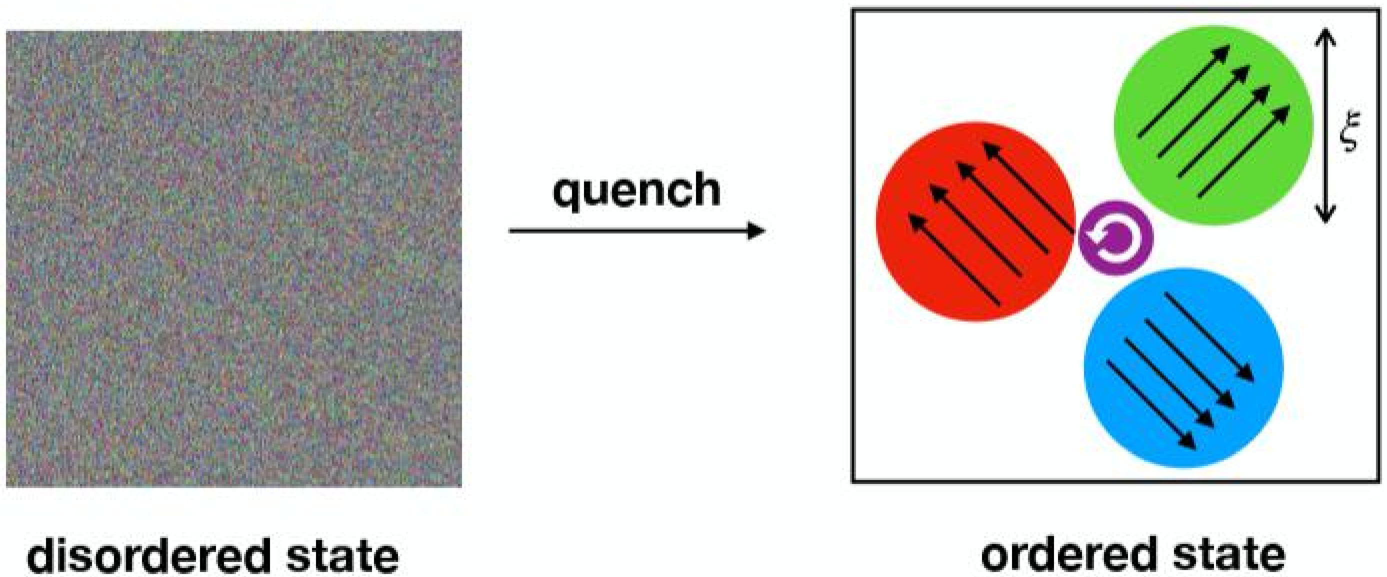
Kibble-Zurek mechanism

The Kibble-Zurek mechanism (KZM) is a paradigmatic theory to describe the critical dynamics with the spontaneous generation of topological defects when the system undergoes a continuous phase transition through a quench.

Sketchy picture to illustrate KZM:



Kibble-Zurek mechanism

A continuous second-order phase transition is characterized by the divergence of both the correlation length ξ and relaxation time τ near the critical point.

$$\xi(\epsilon) = \xi_0 |\epsilon|^{-\nu}$$

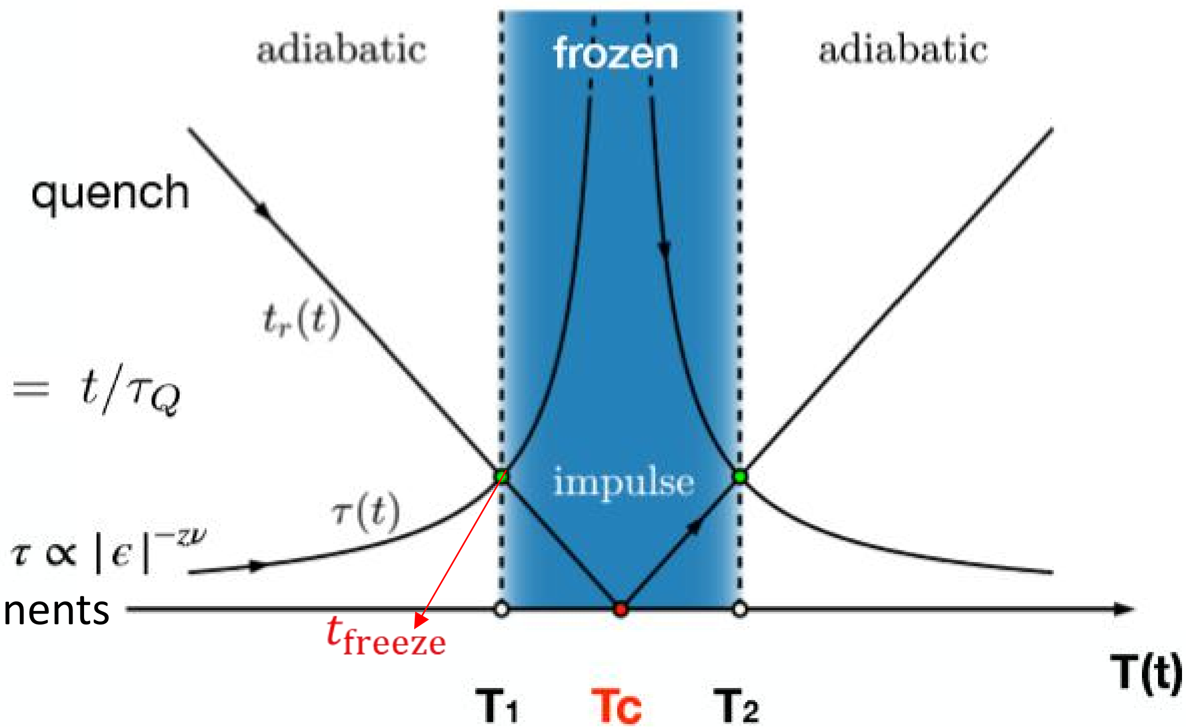
$$\tau(\epsilon) = \tau_0 |\epsilon|^{-\nu z_{\mathrm{d}}}$$

$$\epsilon = 1 - T/T_c \quad \epsilon(t) = t/\tau_Q$$

ξ_0, τ_0 : constant coefficients

ν,z_d: boundary critical exponents

ϵ:the dimensionless distance



Adolfo del Campo and Wojciech H. Zurek.arXiv:1310.1600v3

Kibble-Zurek mechanism

KZM predicts the relation between the number density of topological defects n and the quench rate (strength) τ_Q

$$n \propto \tau_Q^{\frac{-D\upsilon}{1+\upsilon z_d}}$$

$$t_{\text{freeze}} \propto \tau_Q^{\frac{\upsilon z_d}{1+\upsilon z_d}}$$

D is the spatial dimension

For 2-dim mean-field theory : $\nu = 1/2$ $z_d = 2$

$$t_{\text{freeze}} = \tau_Q^{1/2}, n \propto \tau_Q^{-1/2}$$

To find various scaling exponents in KZM has become a prime and important subject recently.

Lifshitz geometry

- Gauge/gravity duality (AdS/CFT correspondence) is a “first-principle” to study the strongly coupled field theories from weakly coupled gravitational theories in one higher dimensions.

- The Lifshitz geometry

$$ds^2_{d+2} = -\left(\frac{r}{L}\right)^{2z} dt^2 + \frac{L^2}{r^2} dr^2 + \left(\frac{r}{L}\right)^2 dx_i^2$$

z : dynamic critical exponent
(Lifshitz exponent)

The geometry is invariant under an anisotropic scaling $t \to \lambda^z t\,,\quad x^i \to \lambda x^i,\quad r \to r/\lambda,$

The Lifshitz geometry is conjectured to describe a quantum critical point on the boundary.

- Natsuume (2018)

Perturbation arXiv:1801.03154

Whatever the value of z a quantum critical point (Lifshitz exponent) has, the T $\neq$ 0 critical point is likely to have z_d = 2 at the mean-field level.

Motivations

Without any perturbation

We directly studied the holographic KZM in the background of a Lifshitz geometry with various Lifshitz exponents.

- To test the KZM.

- To identify the dynamical critical exponent z_d on the boundary field theory is irrespective of the Lifshitz scaling z in the bulk.

Holographic Setup

- **Action:**

$$S=\int d^4x\sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-|D\Psi|^2-m^2|\Psi|^2\right]$$

- **Background in the probe limit :**

AdS_4 black brane with Lifshitz scaling

$$ds^2=-\left(\frac{L}{u}\right)^{2z}f(u)dt^2-2\left(\frac{L}{u}\right)^{z+1}dtdu+\frac{L^2}{u^2}\left(dx^2+dy^2\right),$$

$$f(u)=1-(u/u_h)^{2+z}$$

L : the AdS radius   u : AdS radial coodinate

u=0 is AdS boundary u_h is the horizon

With ansatz:

$$\Psi=\Psi(t,u,x,y),A_t=A_t(t,u,x,y),A_x=A_x(t,u,x,y),$$

$$A_y=A_y(t,u,x,y)\text{ and }A_u=0.$$

Holographic Setup

In order to solve the systems, we need to provide suitable boundary conditions.

- At the horizon, we demand the regularity of the fields.

- Expansions of the fields near boundary

$$\Psi = \Psi_0 u^{\Delta_-} + \Psi_1 u^{\Delta_+}, \qquad \text{with} \quad \Delta_\pm = \frac{z+2\pm\sqrt{(z+2)^2+4m^2}}{2}$$

$$A_i = a_i + b_i u^z, (i = x, y)$$

$$A_t = a_t + b_t \log u, \qquad z = 2,$$
$$A_t = a_t + b_t u^{2-z}, \qquad z \neq 2.$$

Holographic Setup

Holographic Renormalization

- For z=1

The counter term: $C_1 = \int d^3x \sqrt{-\gamma}\,(n^\mu (D_\mu \Psi)^* \Psi + c.c.)$

The surface term: $C_{\text{surf.}} = \int d^3x \sqrt{-\gamma} n^\mu F_{\mu\nu} A^\nu$

γ : determinant of the reduced metric on the boundary
n^μ : the normal vector perpendicular to the boundary.

In order to get dynamical gauge fields on the boundary, we need to impose Neumann boundary conditions.

➢ Expanding the u-component of Maxwell equation implies the conservation equation on the boundary

$$\partial_t b_t + \partial_i J^i = 0 \qquad b_t = -\rho \qquad J^i = -b_i - (\partial_i a_t - \partial_t a_i)$$

Holographic Setup

Holographic Renormalization

- For z=3/2

The counter term: $$C_1 = \int d^3x \sqrt{-\gamma}\,(n^\mu (D_\mu \Psi)^* \Psi + c.c.)$$

The surface term: $$C_{\text{surf.}} = \int d^3x \sqrt{-\gamma} n^\mu F_{\mu\nu} A^\nu$$

γ : determinant of the reduced metric on the boundary
n^μ: the normal vector perpendicular to the boundary.

➢ Expanding the u-component of Maxwell equation implies the conservation equation on the boundary

$$\partial_t b_t + \partial_i J^i = 0 \qquad b_t = -\rho \qquad J^i = -3b_i/2 - (\partial_i a_t - \partial_t a_i)$$

Holographic Setup

Holographic Renormalization

- For z=2

The counter term:

$$C_2 = \int d^3x\sqrt{-\gamma}\Psi\Psi^*$$

$$C_3 = \int d^3x\sqrt{-\gamma}F_{ui}F^{ui}\log(\Lambda)$$

The surface term:

$$C_{\text{surf.}} = \int d^3x\sqrt{-\gamma}n^\mu F_{\mu\nu}A^\nu$$

Λ : an ultraviolet cut-off scale

➢ Expanding the u-component of Maxwell equation implies the conservation equation on the boundary

$$\partial_t b_t + \partial_i J^i = 0 \qquad b_t = -\rho \qquad J^i = -2b_i - (\partial_i a_t - \partial_t a_i).$$

Holographic Setup

Method

- We take advantage of the Chebyshev pseudo-spectral method with 21 grids in the radial direction u and use the fourier decomposition in the (x, y)-directions since the periodic boundary condition along (x, y) was imposed.
- We use the fourth order Runge-Kutta methed with time step.
- We thermalize the system by adding small random seeds in the normal state before quench. The reason is to make sure that the system before quench is in a symmetrical phase , which is the requirement of KZM.
- We quench the system by linearly decreasing the temperature through the critical point, then stop and keep the temperature at T=0.8Tc.
  t=0 is the instant to cross the critical temperature Tc.

Results

➢ Magnetic fluxoids and order parameter vortices

$$\Phi_c = \int dxdy\ B(x,y) \approx 6.2690 \approx 1.9955\pi$$

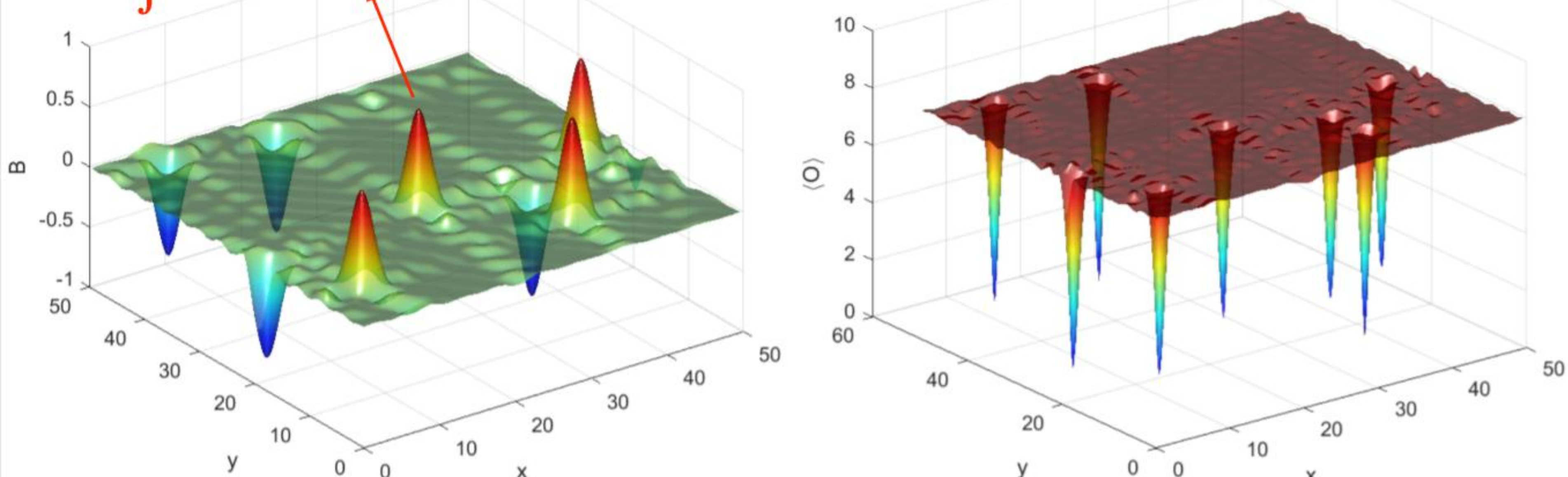


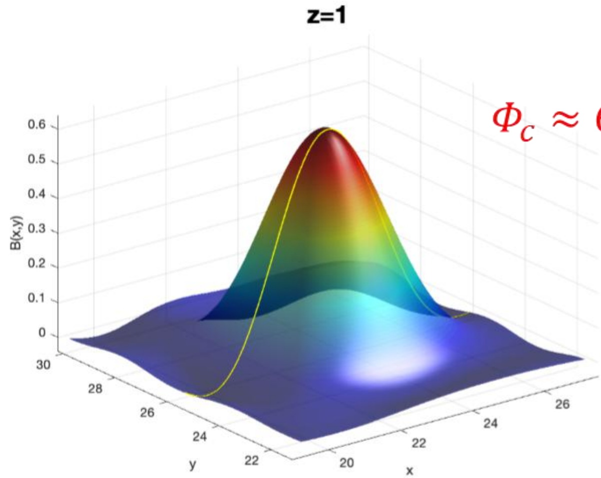
Figure 2: Configurations of magnetic fluxons (left panel) and superconducting vortices (right panel) at temperature T/Tc = 0.8 with the quench time τ_Q = 1800 in the Lifshitz exponent z = 2.

Requirements of minimal free energy and periodicity of the phase of order parameter imply the quantization of magnetic flux $\Phi_c = 2\pi N$, N is an integer.

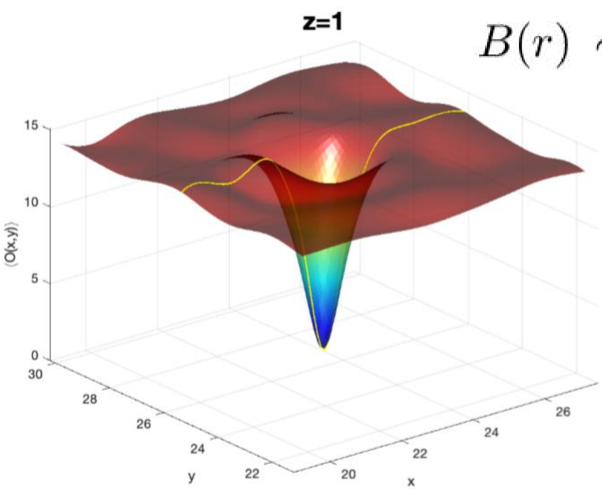
Results

$$O(r) \;\sim\; O(\infty)\tanh(r/(\sqrt{2}\xi))$$

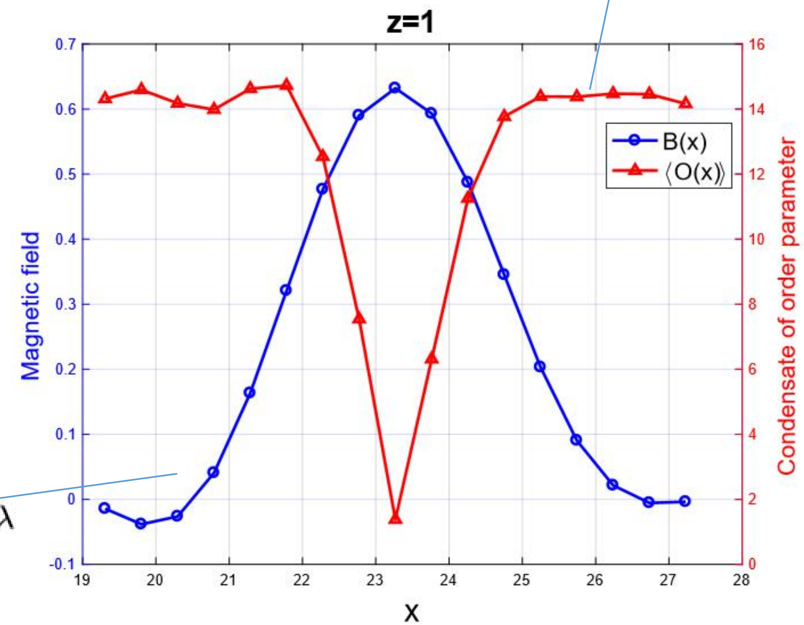
➢ Magnetic fluxoids and order parameter vortices



$\Phi_c \approx 6.11$



$$B(r) \;\sim\; B_0 e^{-r/\lambda}$$



From Abrikosov's criterion the Landau-Ginzburg parameter:

$$\kappa = \frac{\lambda}{\xi} = \frac{1.69}{0.88} \approx 1.92 > 1/\sqrt{2}$$

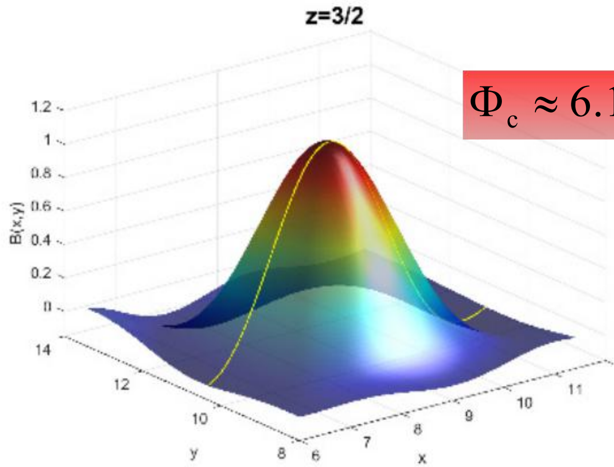
Type II superconductor

M. Tinkham, "Introduction to Superconductivity", 2nd Edition, McGraw-Hill Inc. press (1996).

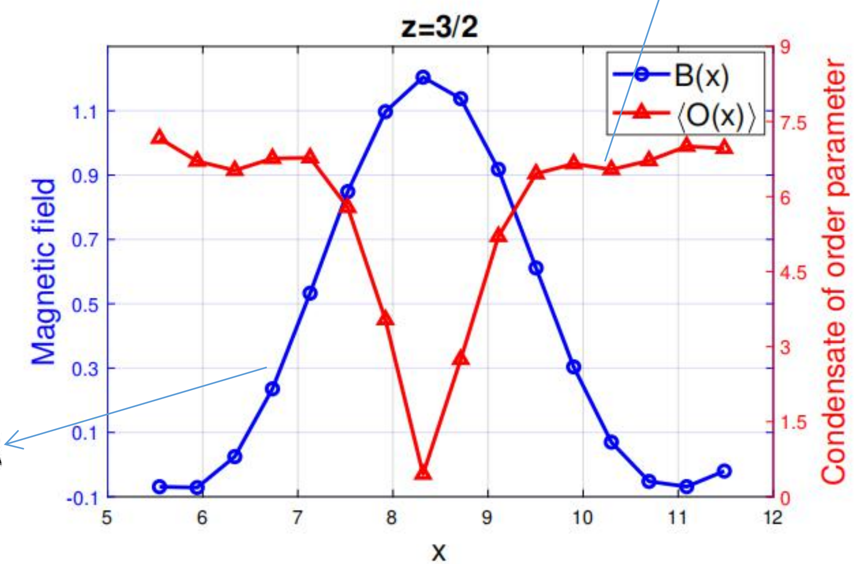
Results

➢ **Magnetic fluxoids and order parameter vortices**

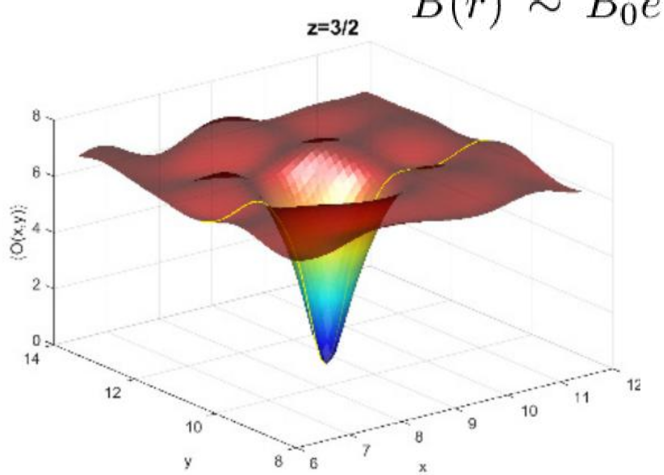
$$O(r) \sim O(\infty)\tanh(r/(\sqrt{2}\xi))$$



$\Phi_c \approx 6.14$



$$B(r) \sim B_0 e^{-r/\lambda}$$



From Abrikosov's criterion the Landau-Ginzburg parameter:

$$\kappa_{z=3/2} = \lambda/\xi = 1.10/0.71 = 1.55 > 1/\sqrt{2}$$

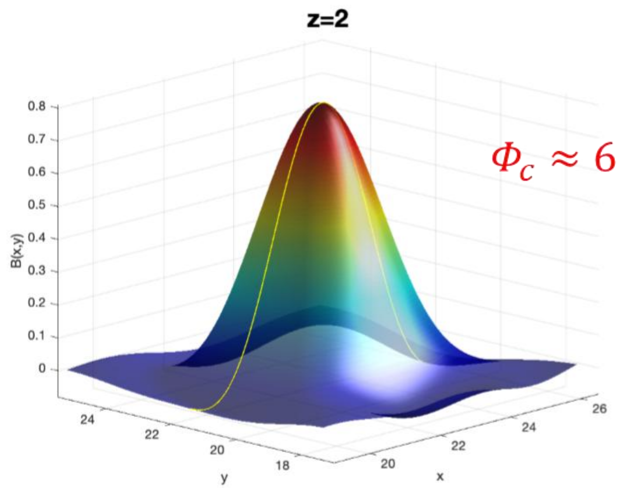
Type II superconductor

M. Tinkham, "Introduction to Superconductivity", 2nd Edition, McGraw-Hill Inc. press (1996).

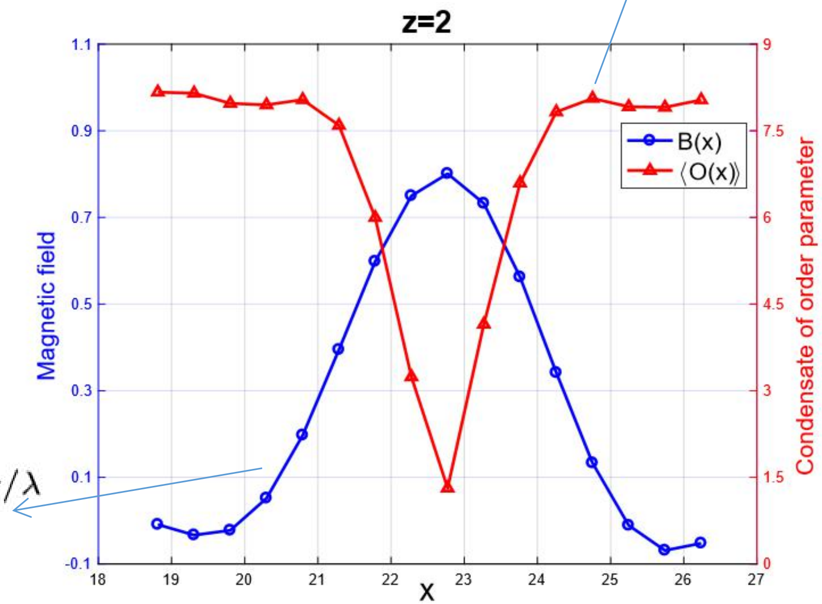
Results

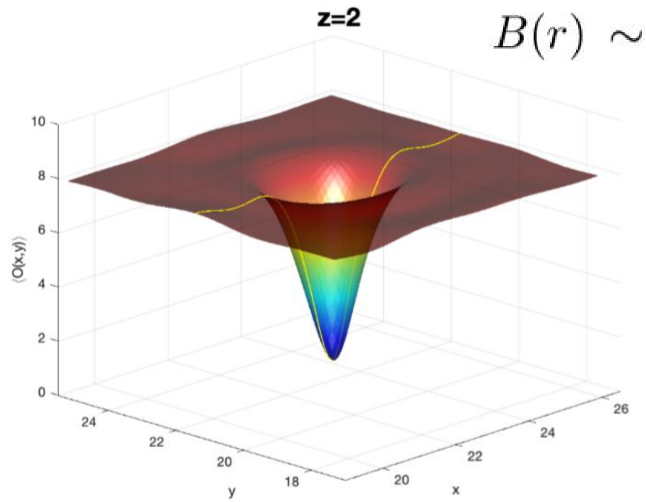
$$O(r) \sim O(\infty)\tanh(r/(\sqrt{2}\xi))$$

➢ Magnetic fluxoids and order parameter vortices



$\Phi_c \approx 6.18$





$$B(r) \sim B_0 e^{-r/\lambda}$$

From Abrikosov's criterion the Landau-Ginzburg parameter:

$$\kappa_{z=2} = \lambda/\xi = 1.35/0.75 = 1.8 > 1/\sqrt{2}.$$

Type II superconductor

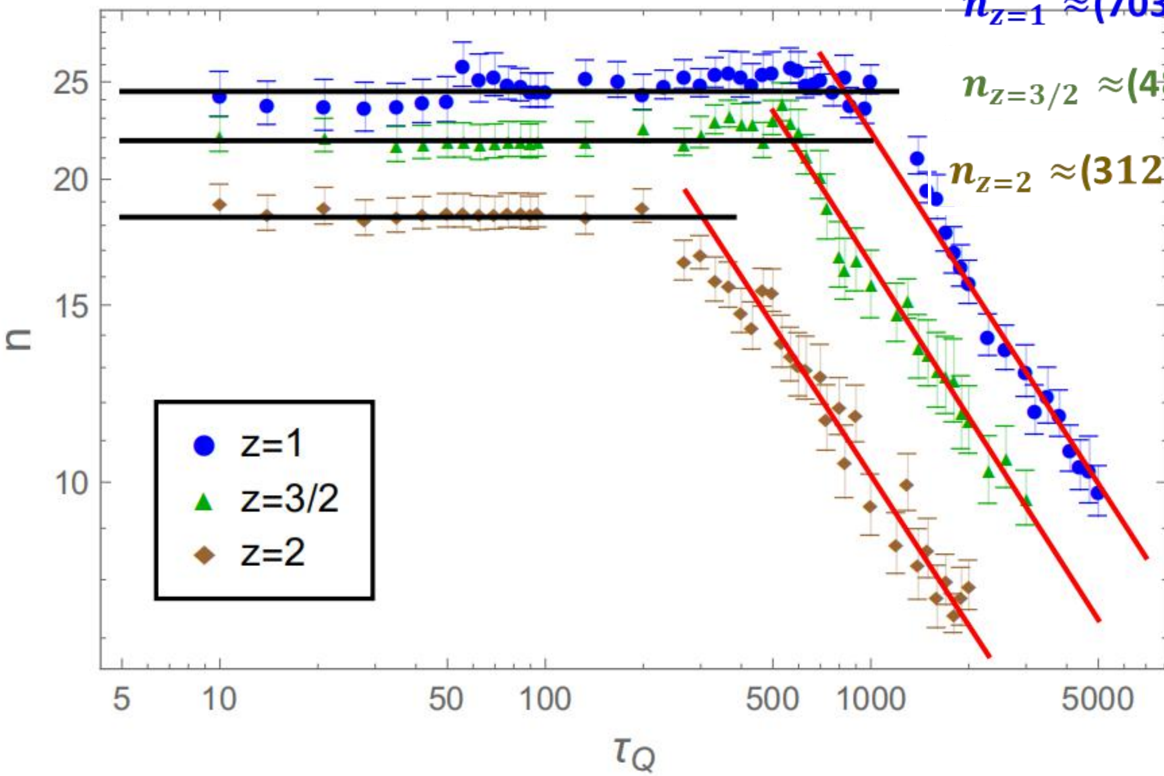
M. Tinkham, "Introduction to Superconductivity", 2nd Edition, McGraw-Hill Inc. press (1996).

Results

For slow quench, we can read taht

$$n \propto \tau_Q^{\frac{-2\upsilon}{1+\upsilon z_d}}$$

➢ Vortex number density



$$n_{z=1} \approx (703.9170 \pm 1.1333) \times \tau_Q^{(-0.4998 \pm 0.0162)}$$

$$n_{z=3/2} \approx (482.62 \pm 1.1431) \times \tau_Q^{(-0.4897 \pm 0.0183)}$$

$$n_{z=2} \approx (312.9211 \pm 1.2379) \times \tau_Q^{(-0.4961 \pm 0.0315)}$$

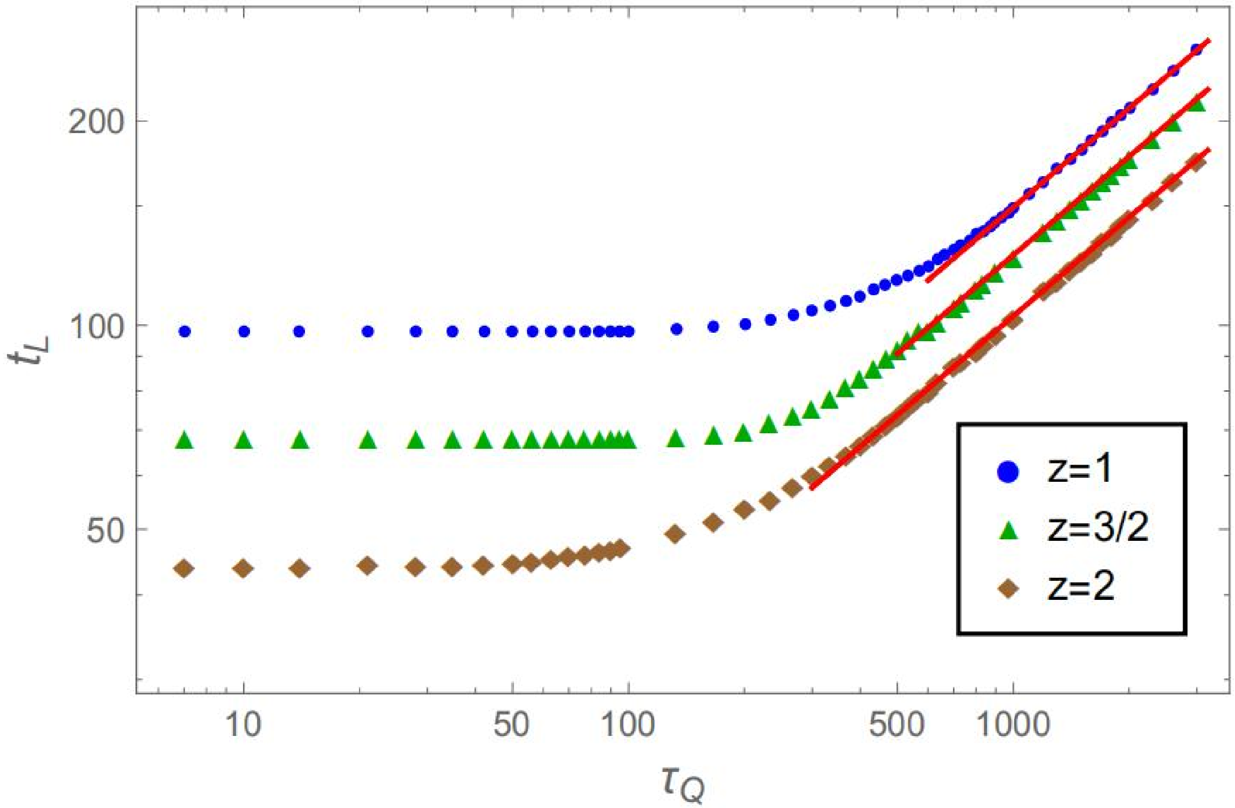
- For the slow quench (bigger τ_Q and red lines), both scaling relations satisfy the KZ scaling laws very well.
- For the fast quench (smaller τ_Q and blue lines), vortex number n is almost constant independent of τ_Q.

This is consistent with previous results in condensed matter or holography.

Results

➢ Freeze-out time

For slow quench, we can read taht



$$t_{\mathrm{L}} \propto \tau_Q^{\frac{\upsilon z_d}{1+\upsilon z_d}}$$

$$t_{L(z=1)} \approx 5.0540 \times \tau_Q{}^{0.4845}$$

$$t_{L(z=3/2)} \approx 4.5001 \times \tau_Q{}^{0.4834}$$

$$t_{L(z=2)} \approx 3.6064 \times \tau_Q{}^{0.4845}$$

- For slow quench the relation between t_L and t_Q satisfy the KZ scalings very well.
- For fast quench the lag time t_L keeps almost constant.

Results

➢ Vortex number density and freeze-out time

$$\left.\begin{aligned} n &\propto \tau_Q^{\frac{-2\upsilon}{1+\upsilon z_d}} \\ t_{\text{freeze}} &\propto \tau_Q^{\frac{\upsilon z_d}{1+\upsilon z_d}} \end{aligned}\right\} \Rightarrow$$

$z = 1$:	$\nu = 0.4893$	$z_d = 1.9579$
$z = 3/2$:	$\nu = 0.4740$	$z_d = 1.9743$
$z = 2$:	$\nu = 0.4812$	$z_d = 1.9532$

- The holographic results for z_d and ν are very close to the mean-field theory values with $z_d = 2$ and $\nu = 1/2$.
- We can see that the dynamic critical exponent z_d on the boundary is irrespective of the bulk Lifshitz exponent z.

Conclusions & Outlooks

◆ We studied the spontaneous formation of topological defects from KZM in Lifshitz holography at the finite temperature.

- The magnetic fluxes were found to be quantized and belonged to the type II superconductor.

- The KZ scaling relations : the vortex number density and the lag time to quench time matched KZM very well.

- These two scaling relations implied that the dynamical critical exponent on the boundary field theory was irrespective of the Lifshitz exponent in the bulk.

◆At zero temperature ????

Thanks

2020.08.04